MAGNETIC AMPLIFICATION AND THE SEARCH FOR MAGNETO-ROTATIONAL-INSTABILITY IN BAR-MODE UNSTABLE NEUTRON STARS

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NUMERICAL GENERAL RELATIVITY (WITH MATTER)

- There is a wide zoology of phenomena in the Sky where General-Relativistic effect play a very importa role.
- Solve the Einstein Equations without approximations in order to:
  - investigate the physics of gravitational collapse
  - investigate structure and stability of the most relativistic compact objects
  - model the most catastrophic events in the Universe (GRB, magnetars,...)
  - model real sources of gravitational waves : core colapse in supernova and binary mergers (NS-NS, NS-BH, BH-BH)

GENERAL RELATIVITY IS IMPORTANT WHEN NEUTRON STARS (OR BLACK HOLES) ARE PRESENT

- Besides, NS are quite peculiar since all the four fundamental forces play a crucial role in determining their structure and dynamics and therefore they arouse interest in all fields of modern physics.

- Two of the most intriguing puzzles that one day could be solved with NS are:
  —The behaviour of matter at supra nuclear densities
  —The study of gravitational waves (GW) signals

GW are ripples of space-time traveling at the speed of light. To this day, they remain the most elusive prediction of the Einstein's theory of General Relativity.

NEUTRON STARS

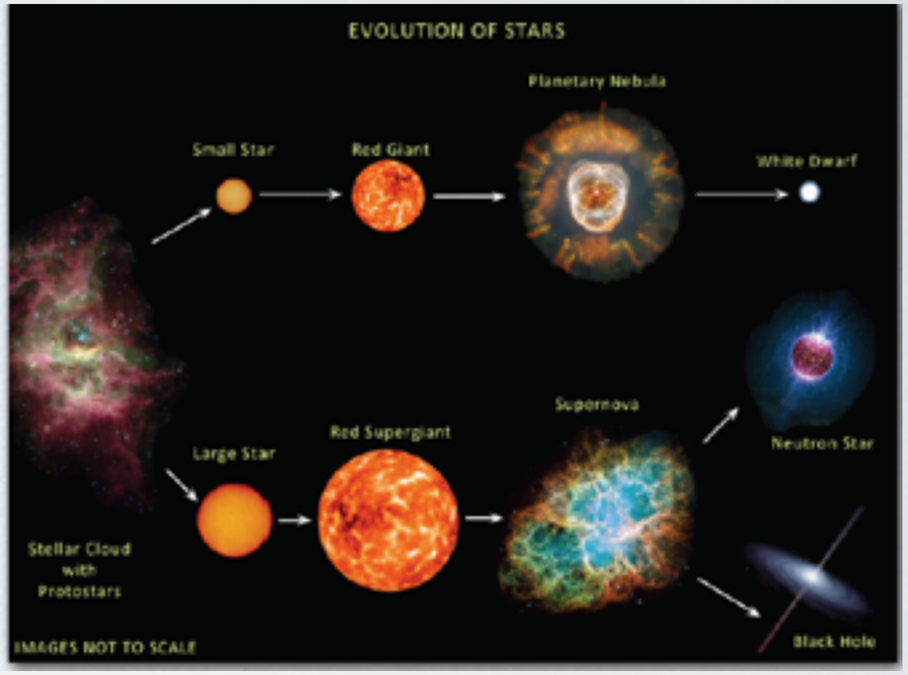
- Neutron stars are extremely compact star with a typical mass $M \approx 1.4 M_{\odot}$ and a typical radius of $R \approx 10 km$, resulting in a compactness $M/R \approx 100$ times higher than the Sun.

- These properties make them the most compact objects endowed with a structure in our Universe, such that they can not be properly described without resorting to General Relativity.

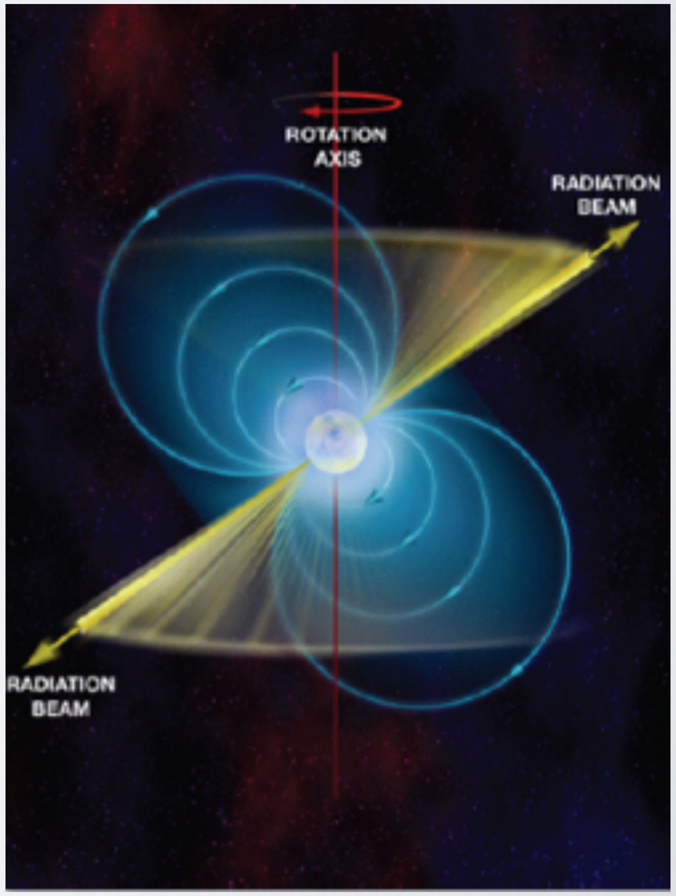
- They exhibit very high densities (nuclear density) $\rho_0 \approx 2.8 \times 10^{14} g/cm^3$ very fast rotation (around 716 Hz) and a strong magnetic field (magnetic fields range between $\mathbf{10^{12}}$ G to $\mathbf{10^{16}}$ G in magnetars

NEUTRON STAR

- NS are formed from the collapse of large stars at the end of their lives, as remnant of Supernova explosion



- A pulsar is highly magnetized rotating NS that emits a beam of EM radiation.



- NS have surface magnetic fields strength of the order of **10^{12}** Gauss.
- Magnetars have ultra-strong magnetic fields up to **10^{16}** Gauss.
- 21 magnetars are known, with 5 more candidates awaiting for confirmation.

PARMA GROUP EFFORTS

- Stiffness effects on the dynamics of the bar-mode instability in full General Relativity.
- Dynamical bar-mode instability in rotating and magnetized relativistic stars.
- New Project: Magnetic Evolution during Binary Neutron Star Merger
- The TEONGRAV has related project on BNS-merger (here in Trento) and on Magnetic Evolutions in Magnetars (Florence)

THE ROLE OF THE INFN
SUMA PROJECT

- The INFN theoretical community is active in several scientific areas that require significant computational support. These areas stretch over a wide spectrum, requiring in some cases fairly limited computing resources ……….. SUMA plans to support all these physics goals, and at the same time **aims to explore all suitable ways in which the technological developments made at INFN can be put to good use for the present and future needs of computational physics**.

- Numerical Relativity ….. aim to SIMULATE (not compute) simplified models of matter at high-density … that may play a significant role on the physics of NS and/or of the generation of Gravitational Waves.

- No experiment available …. indeed … we need computer experiment to understand which physical ingredients are import to our understanding of the fundamental physics at very-high density

AN EXAMPLE OF SYSTEMS THAT ARE STUDIED

An evolution that can be studied on a small system

G2_I12vs12_D4R25_41_5_km

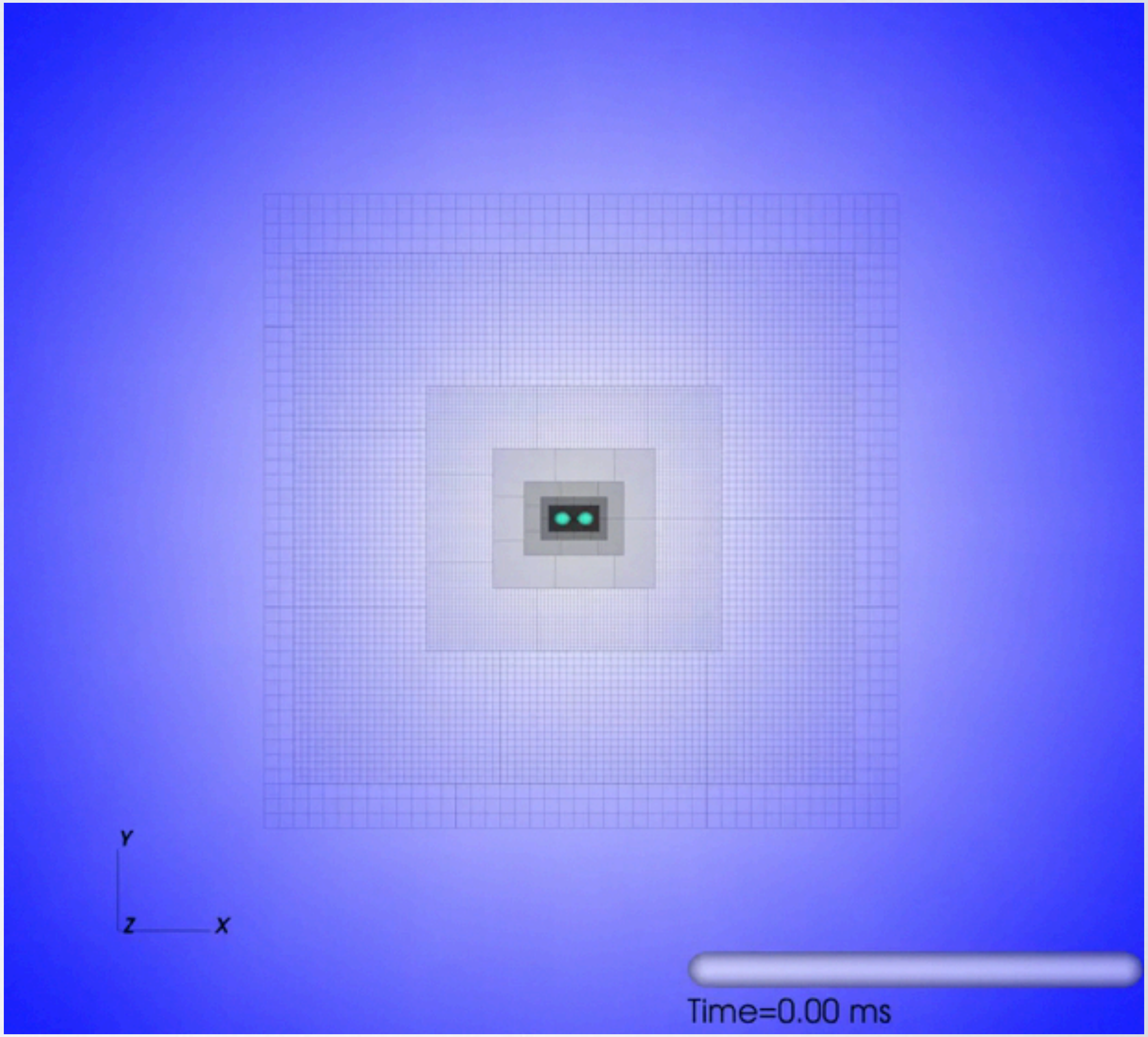
M_ADM=3.251

separation 45Km

K = 123.6

Gamma = 2

dx=0.28125 (415 m)



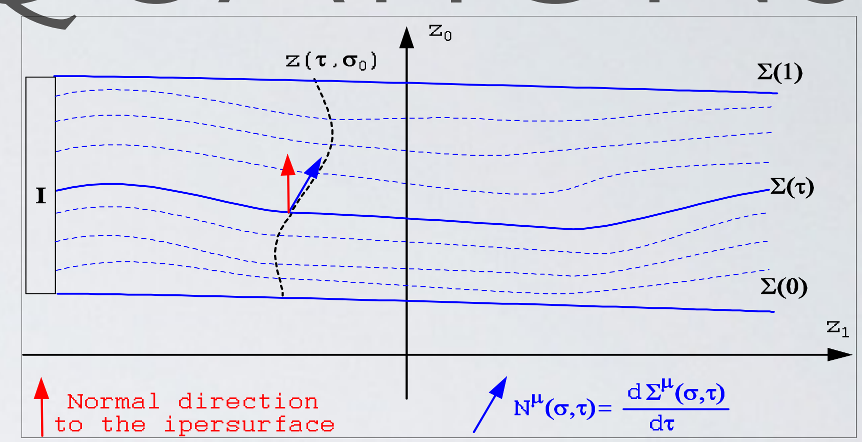
COMPUTATIONAL NEED ARE AT THE FOREFRONT OF THE ACTUAL TECHNOLOGY

- Need to solve in (coordinate time) a very complex set of Partial Differential Equations
- Very high number of physical fields that should be evolved (10 for the metric + 7 for perfect fluid matter + 3 for magnetic fields)
- Need of auxiliary fields variable (order of 20 or more) to properly formulate the initial values problem, i.e. to have the equations set in Hyperbolic-Form.
- Grid size to be at least of the order 200x200x200 to have enough (spatial) resolution
- Need to grid refinement setting (grid-nesting) in order to cover the space around the source to properly set the boundary condition and to extract gravitational wave signal. (between 4 to 9 levels of mesh-refinement structure)

THE EINSTEIN EQUATIONS

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

$$ds^2 = -\alpha^2 dt^2 + g_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$



- BSSN version of the Einstein's equations that introduce additional conformal variables:

$$R_{ij} = -\tfrac{1}{2}\tilde{g}^{lm}\tilde{g}_{ij,lm} - \tilde{g}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \tilde{\Gamma}^k\tilde{\Gamma}_{(ij)k} + \tilde{g}^{lm}(2\tilde{\Gamma}^k_{l(i}\tilde{\Gamma}_{j)km} + \tilde{\Gamma}^k_{im}\tilde{\Gamma}_{klj})$$

$$R^{TF}_{ij} = R_{ij} - \tfrac{1}{3} g_{ij} R$$

$$\partial_t\varphi = -\tfrac{1}{6}\alpha K + \beta^i\partial_i\varphi + \tfrac{1}{6}\partial_i\beta^i$$

$$\partial_t K = -g^{ij}\nabla_i\nabla_j\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \tfrac{1}{3}K) + \beta^i\partial_i K$$

$$\partial_t\tilde{g}_{ij} = -2\alpha K_{ij} + \tilde{g}_{jk}\partial_i\beta^k + \tilde{g}_{ik}\partial_j\beta^k - \tfrac{2}{3}\tilde{g}_{ij}\partial_k\beta^k$$

$$\begin{aligned}\partial_t\tilde{\Gamma}^i = {} & -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha(\Gamma^i_{jk}\tilde{A}^{jk} - \tfrac{2}{3}\tilde{g}^{ij}\partial_j K + 6\tilde{A}^{ij}\partial_j\varphi) + \\ & + \beta^k\partial_k\tilde{\Gamma}^i - \tilde{\Gamma}^k\partial_k\beta^i + \tfrac{2}{3}\tilde{\Gamma}^i\partial_k\beta^k + \tfrac{1}{3}\tilde{g}^{ij}\partial_j\partial_k\beta^k + \tilde{g}^{jk}\partial_j\partial_k\beta^i\end{aligned}$$

$$\begin{aligned}\partial_t\tilde{A}_{ij} = {} & e^{-4\varphi}(-(\nabla_i\nabla_j\alpha)^{TF} + \alpha R^{TF}_{ij}) + \alpha\left(\tilde{A}_{ij}K - 2\tilde{A}_{ik}\tilde{A}^k{}_j\right) - \partial_i\partial_j\alpha + \\ & + \beta^k\partial_k\tilde{A}_{ij} + \left(\tilde{A}_{ik}\partial_j + \tilde{A}_{jk}\partial_i\right)\beta^\kappa - \tfrac{2}{3}\tilde{A}_{ij}\partial_k\beta^k\end{aligned}$$

[4] M. Shibata, T. Nakamura: "**Evolution of three dimensional gravitational ..**", **Phys. Rev. D52(1995)5429**
[5] T.W. Baumgarte, S.L. Shapiro: "**On the numerical integration of Einstein..**", **Phys. Rev. D59(1999)024007**

- Matter and MAGNETIC evolution using shock capturing methods in the General-Relativistic Ideal-Magneto-Hydrodynamics approximation (infinite conductivity) using GRHydro code that implements a Constrain Transporte methods to preserve zero divergence condition.

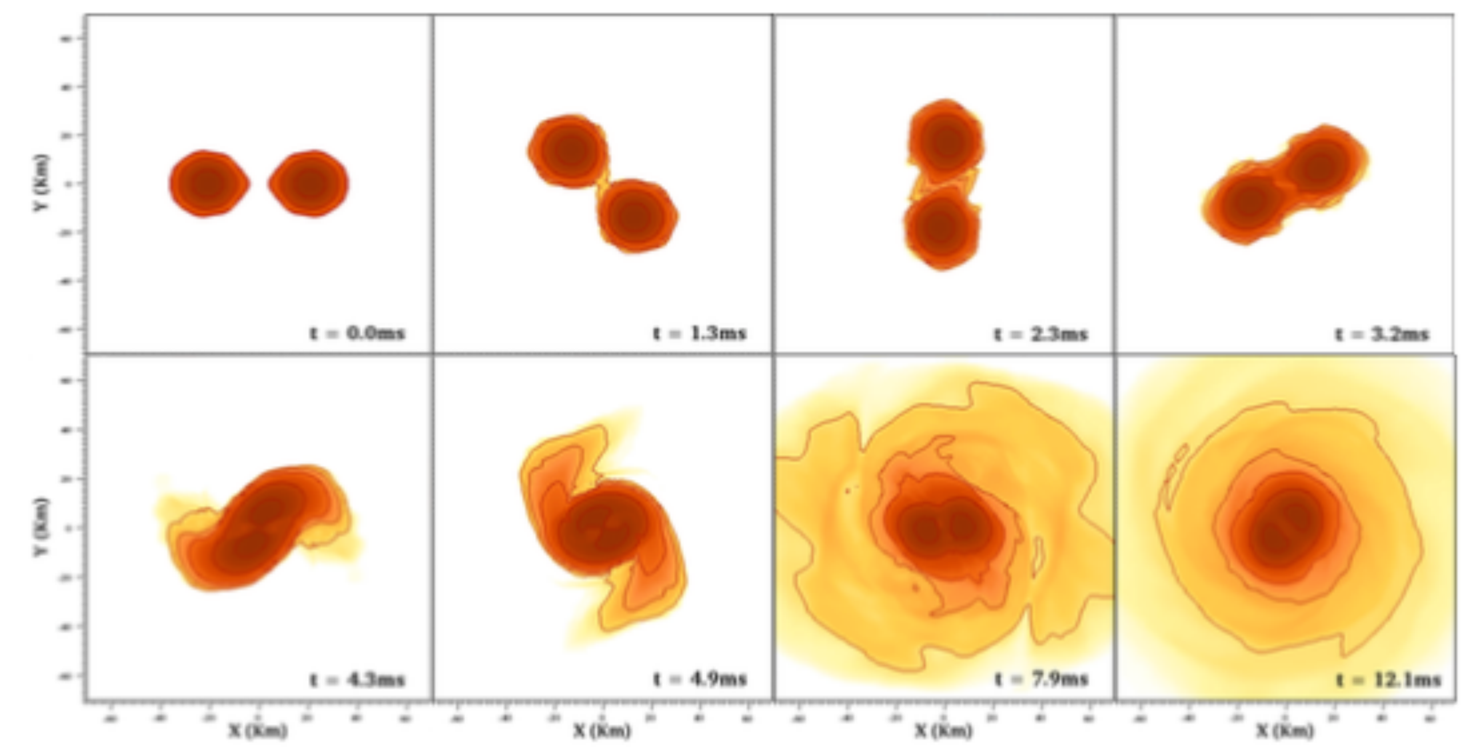
NS MAY BE UNSTABLE

- NS in nature are rotating and subject to non-axisymmetric rotational instabilities. Now the question are….
  which types of instabilities will develop?
  does a fully developed instability persist for long and, if not, what induces its decay?
  does an unstable NS radiate GW and how much?
  what is the threshold of instabilities?(dependence on EOS, …)

- Previous work in literature usually focus on polytropic models with gamma=2. The expected value for real NS is more likely around gamma=2.5-3 in the interior.

- Our aim is to obtain properties that resemble a more realistic case and yet maintaining computational simplicity.

DYNAMICAL BAR-MODE INSTABILITY

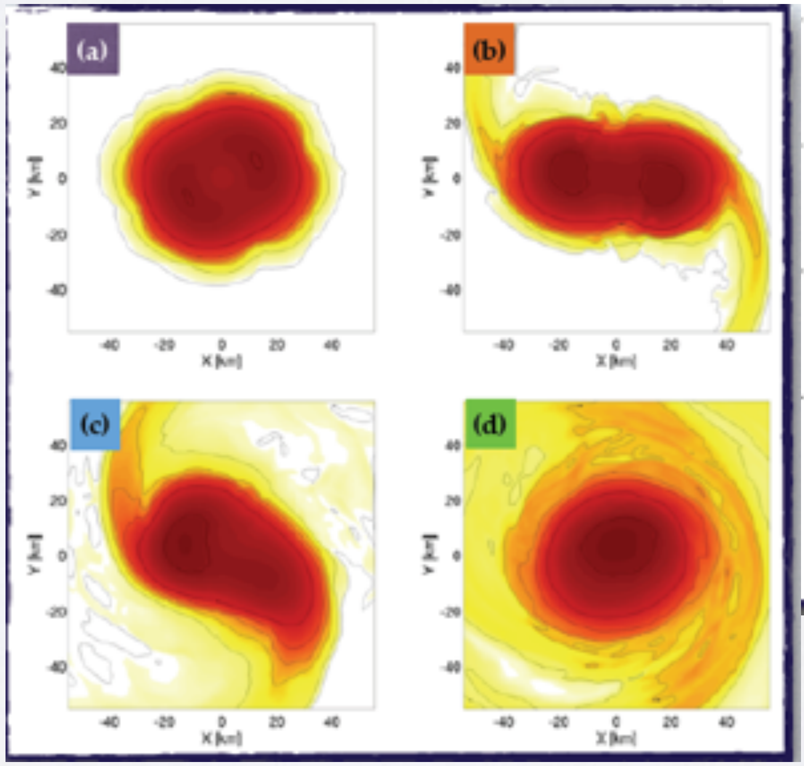


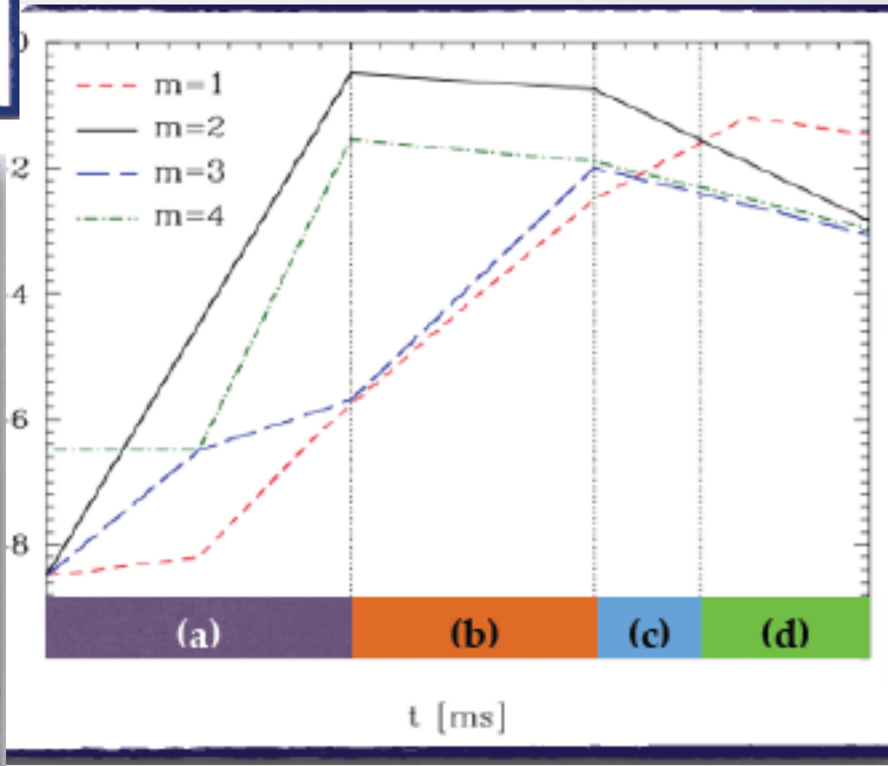
- Dynamics of a Binary Neutron Merger.. … just after the formation of an HyperMassive Neutron Star there is a BAR-deformed stage

$$P_m \equiv \int \mathrm{d}^3x\, \rho\, \mathrm{e}^{im\phi}.$$

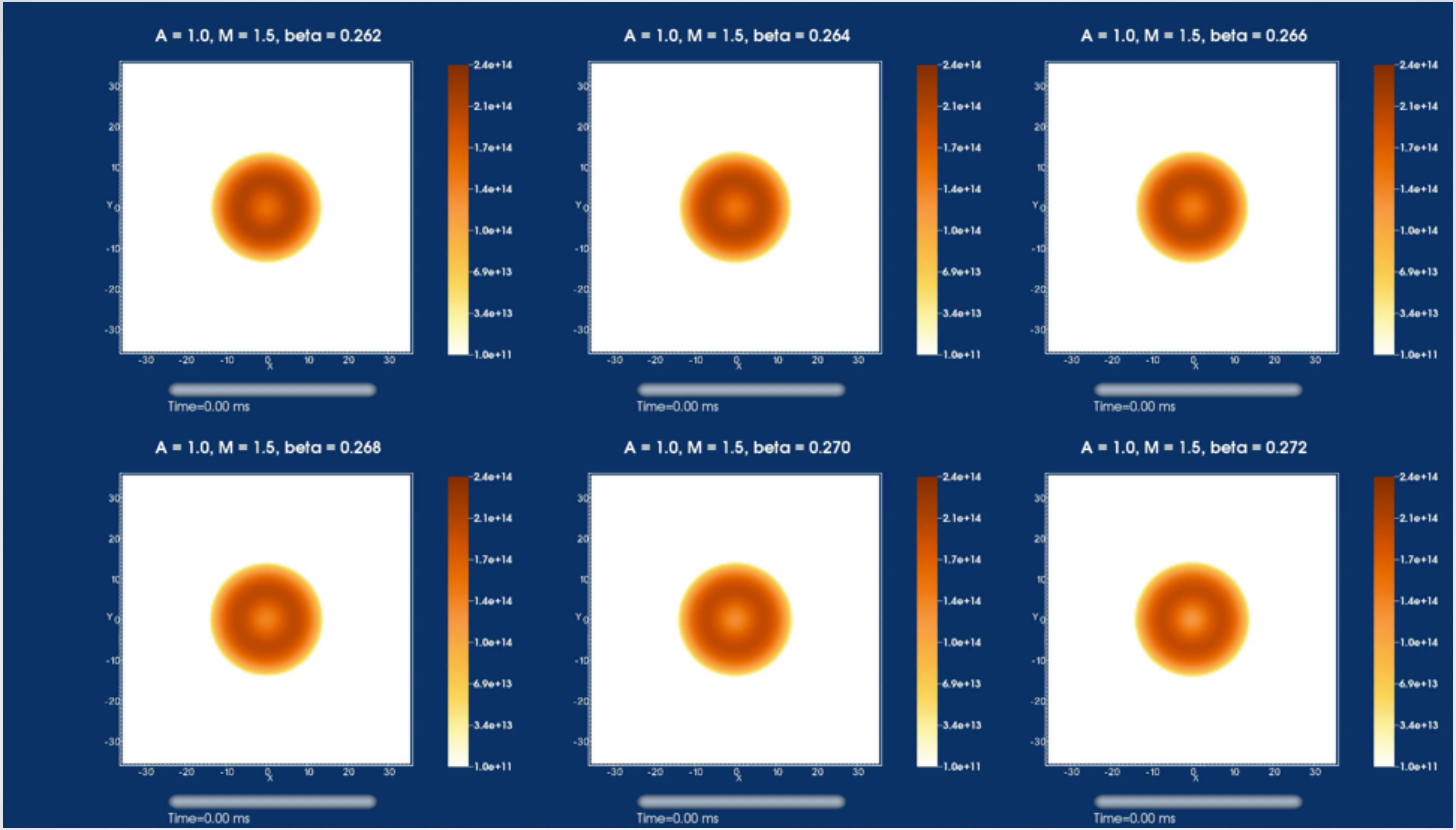
- BAR-MODE unstable stars show a stage that have a similar stage

- NICE PLAYGROUND to study magnetic DYNAMICS in NSs





DYNAMICAL BAR MODE INSTABILITY



A = 1.0, M = 1.5, beta = 0.262

A = 1.0, M = 1.5, beta = 0.264

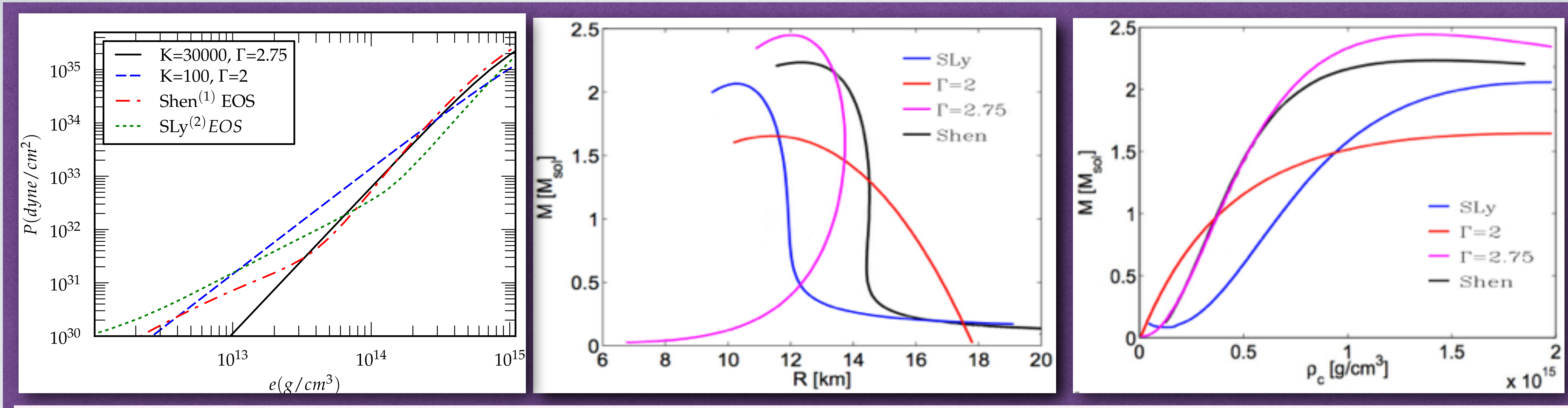
A = 1.0, M = 1.5, beta = 0.266

A = 1.0, M = 1.5, beta = 0.268

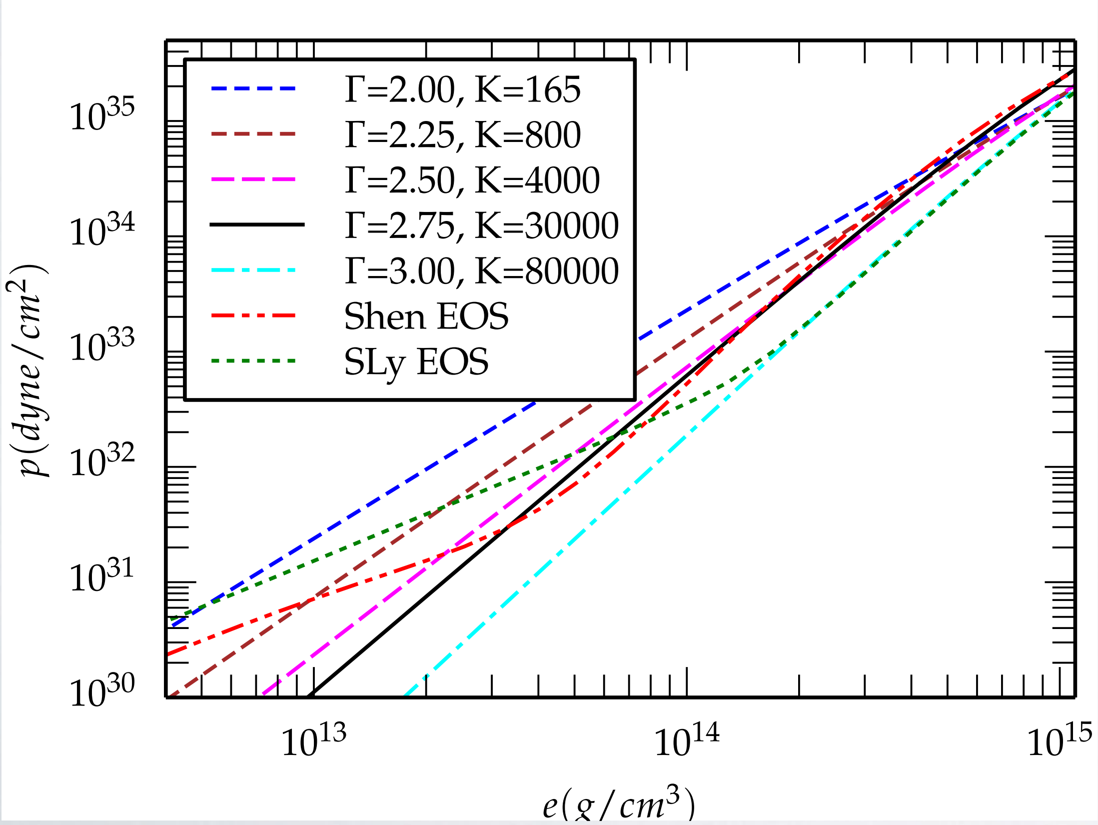
A = 1.0, M = 1.5, beta = 0.270

A = 1.0, M = 1.5, beta = 0.272

EFFECT OF THE EOS



[1] R. De Pietri, A. Feo, L. Franci and F. Loeffler *"Neutron star instabilities in full general relativity using a Γ=2.75 ideal fluid"* Phys. Rev. **D 90**, 024034 arXiv:1403.8066.



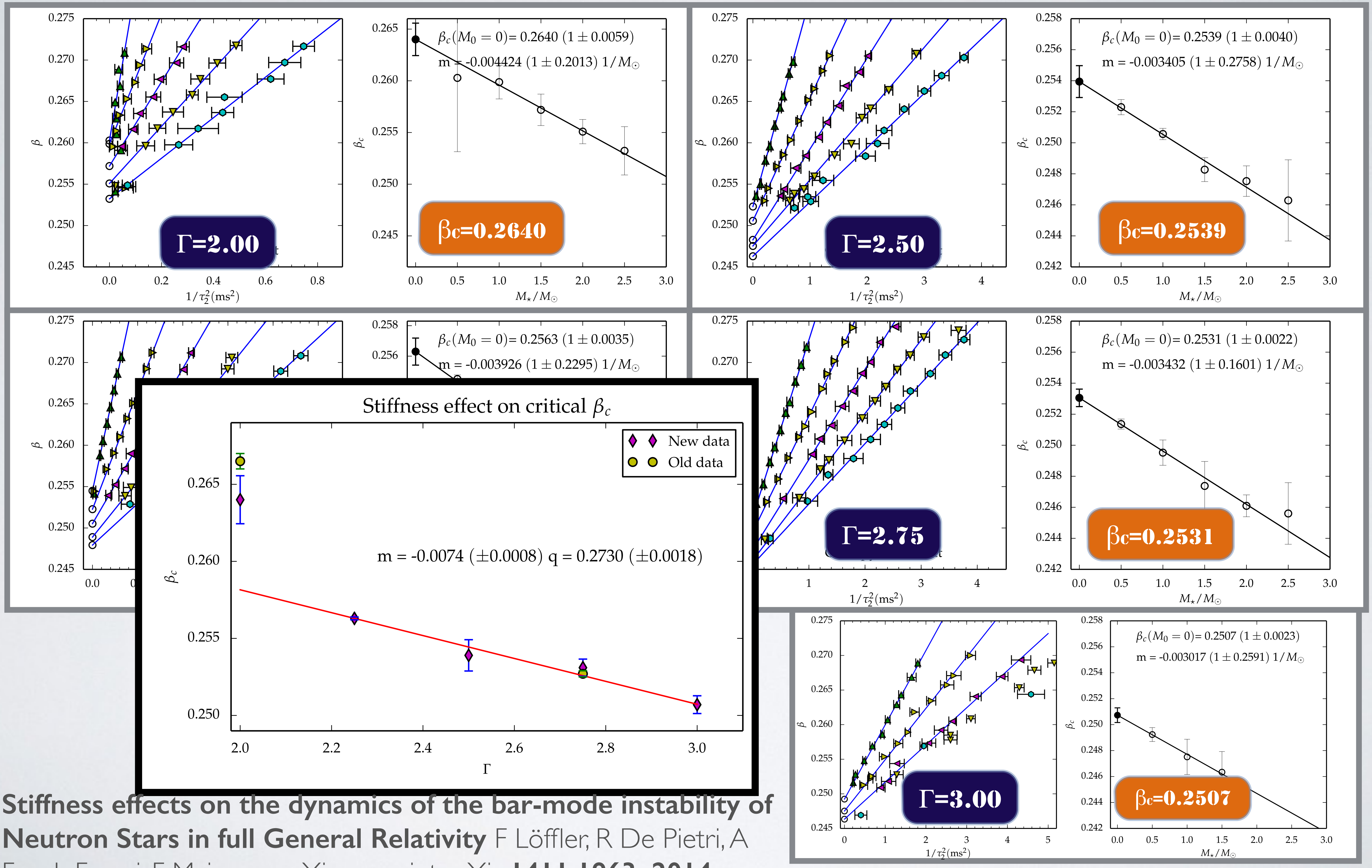
SLy: unified Sly EOS models high-density and cold (i.e. zero temperature) matter via a Skyrme effective potential for the nucleon-nucleon interactions

Shen: relativistic mean-field (RMF) framework

polytropic EoS $p = K\rho^{\Gamma}$

Γ = 2 -> 2.25, 2.50, 2.75, 3.00

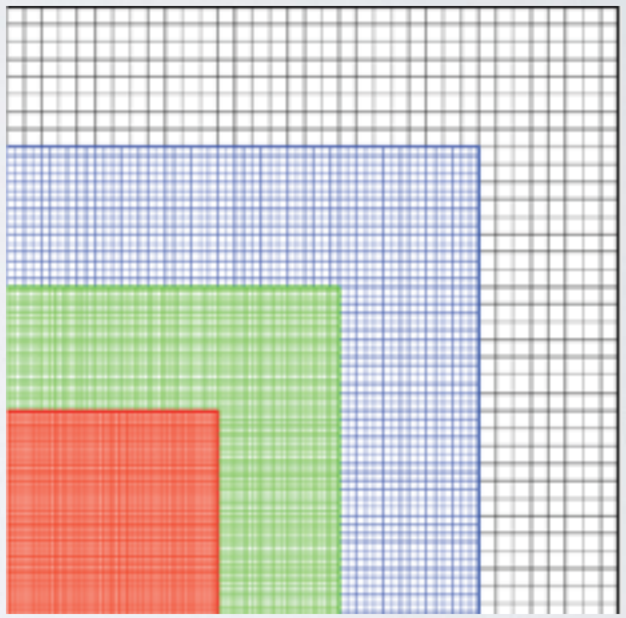
STUDYING STIFFNESS EFFECTS



Stiffness effects on the dynamics of the bar-mode instability of Neutron Stars in full General Relativity F Löffler, R De Pietri, A Feo, L Franci, F Maione - arXiv preprint arXiv:**1411.1963, 2014**

THE CODE: EINSTEIN TOOLKIT

- **Cactus**
  framework for parallel high performance computing (Grid computing, parallel I/O)
- **Einstein Toolkit** open set of over 100 Cactus thorns for computational relativity along with associated tools for simulation management and visualization
- Mesh refinement with **Carpet**
- Magnetic+Matter Evolution with **GRHydro:**
  CT evolution of Magnetic Field
  HLLE Riemann Solver
  ppm Reconstruction methods
  BSSN gravitational evolutions
- Initial data computed using
  RNS solver by Stergioulas
  + a B poloidal perturbation



- 3D cartesian grids with 4 refinement levels
- resolution: **0.25 $M_\odot$** ~ 0.360 km (typical)
- grid size:
  30 $M_\odot$ ~ 44 km (240x240x120)
  42 $M_\odot$ ~ 62 km
  84 $M_\odot$ ~ 124 km
  168 $M_\odot$ ~ 248 km

MAGNETIC FIELDS ARE AN IMPORTANT INGREDIENT ON THE PHYSICS OF NEUTRON STARS

- Newly born Neutron Stars are:
  - highly differentially rotating
  - magnetized (magnetic fields are potentially as strong as **10^{16}** Gauss) but the normal expected amplitude is **10^{12}** Gauss.
  - possible sources for Gravitational Waves

- Matter instabilities may enhance Gravitational Waves emission and the Hyper Massive Neutron formed after a binary merger are highly deformed!

- What happen to magnetic fields during unstable or highly deformed phase of neutron stars ? Are present instability that can magnify the amplitude of the magnetic field ?

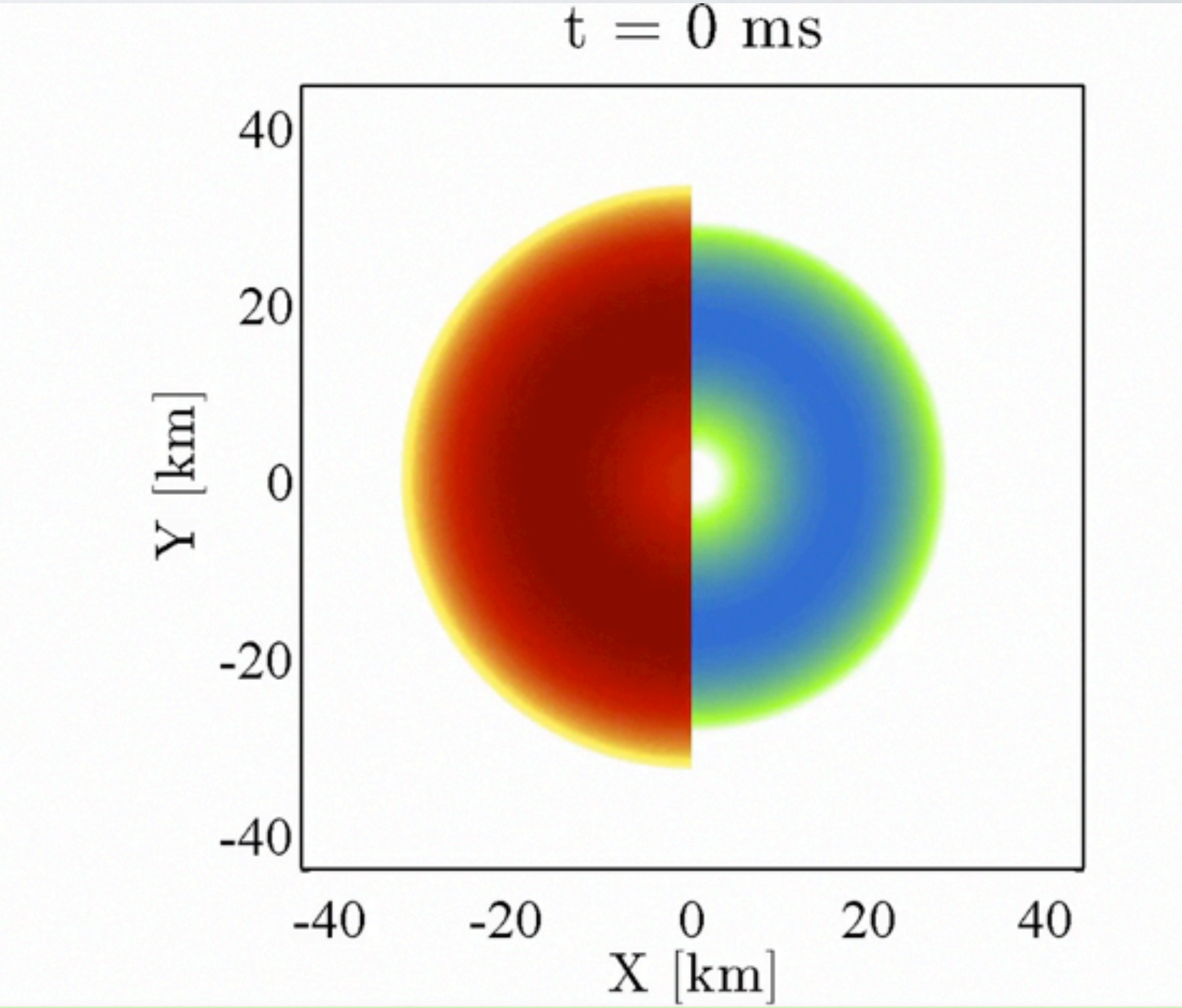
THE DYNAMICS OF AN UNSTABLE MODELS

- Dynamics of the evolution of a model with a seed magnetic field of **10^{14}** Gauss.

- Left: matter density

- Right: modulus of the magnetic fields [10^{12}-$10^{16,5}$ Gauss] in the xy plane a z=1.5

- grid = [207x407x407], i.e more the 300 points inside the stars



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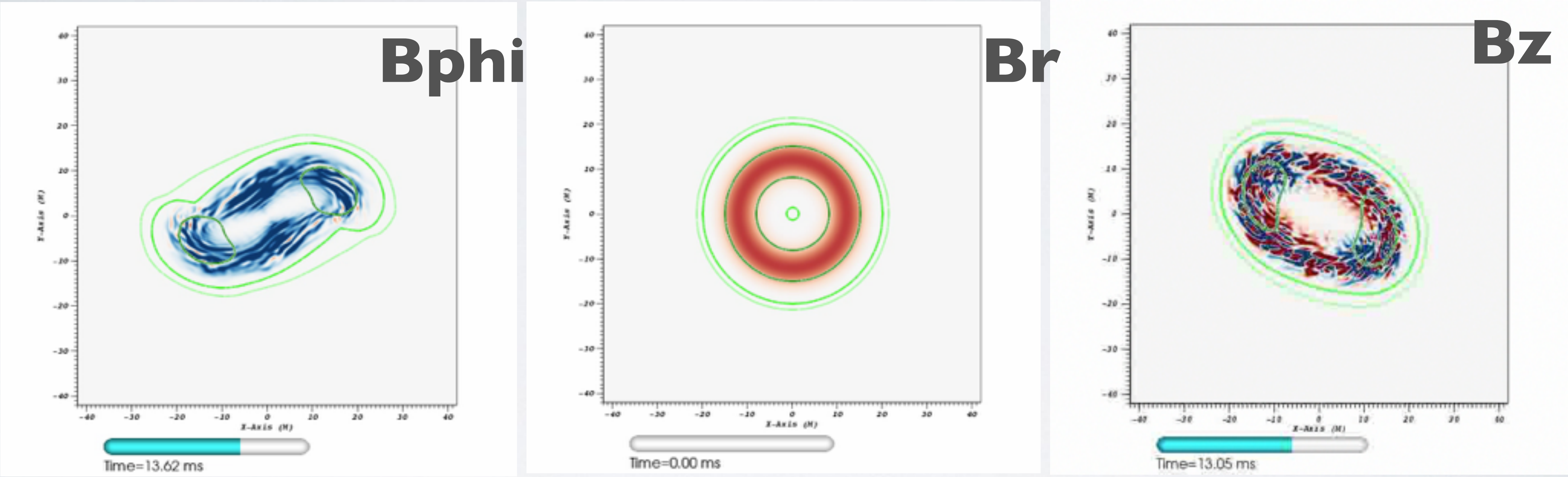
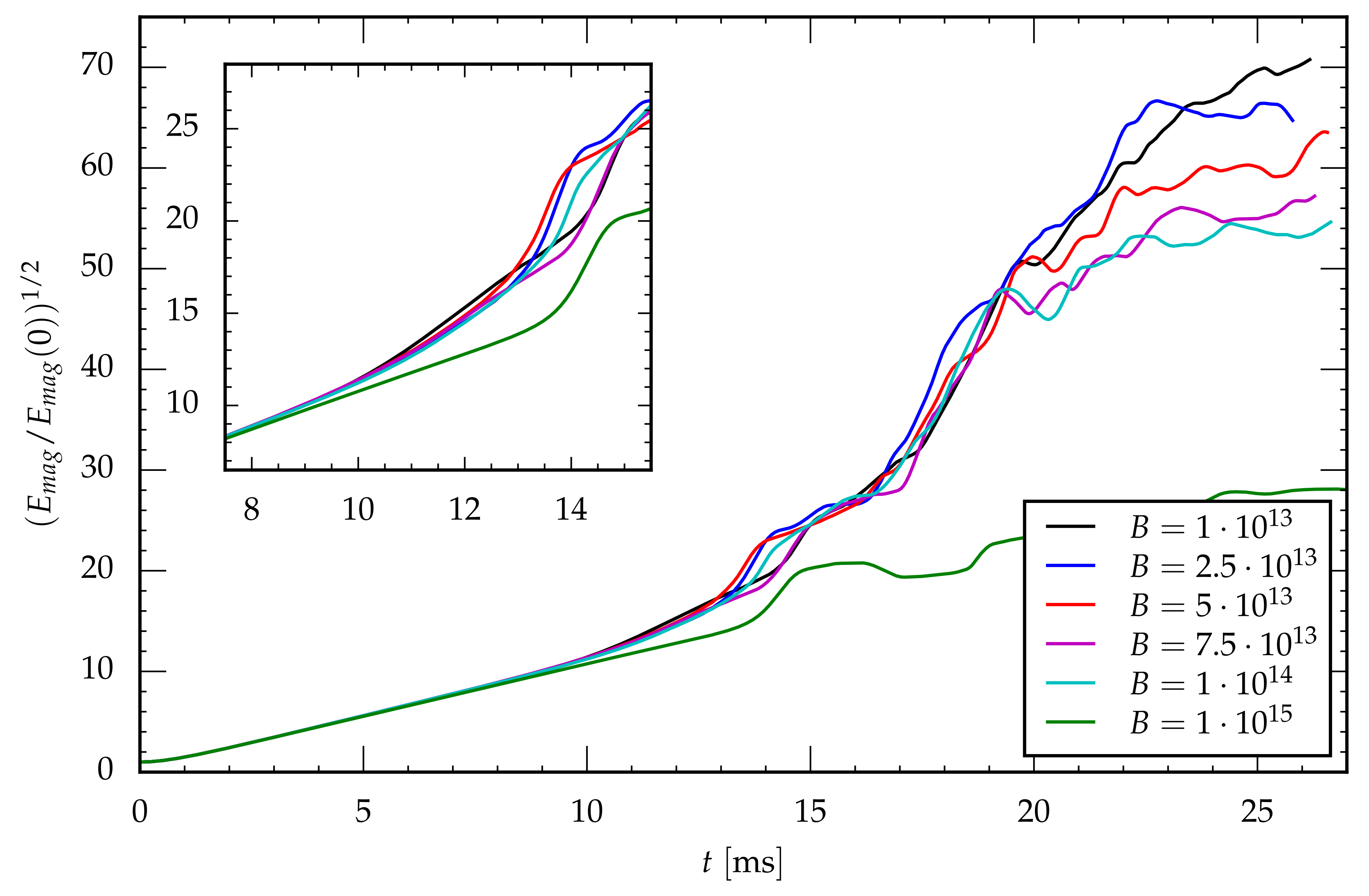
DYNAMICS OF THE MAGNETIC FIELD



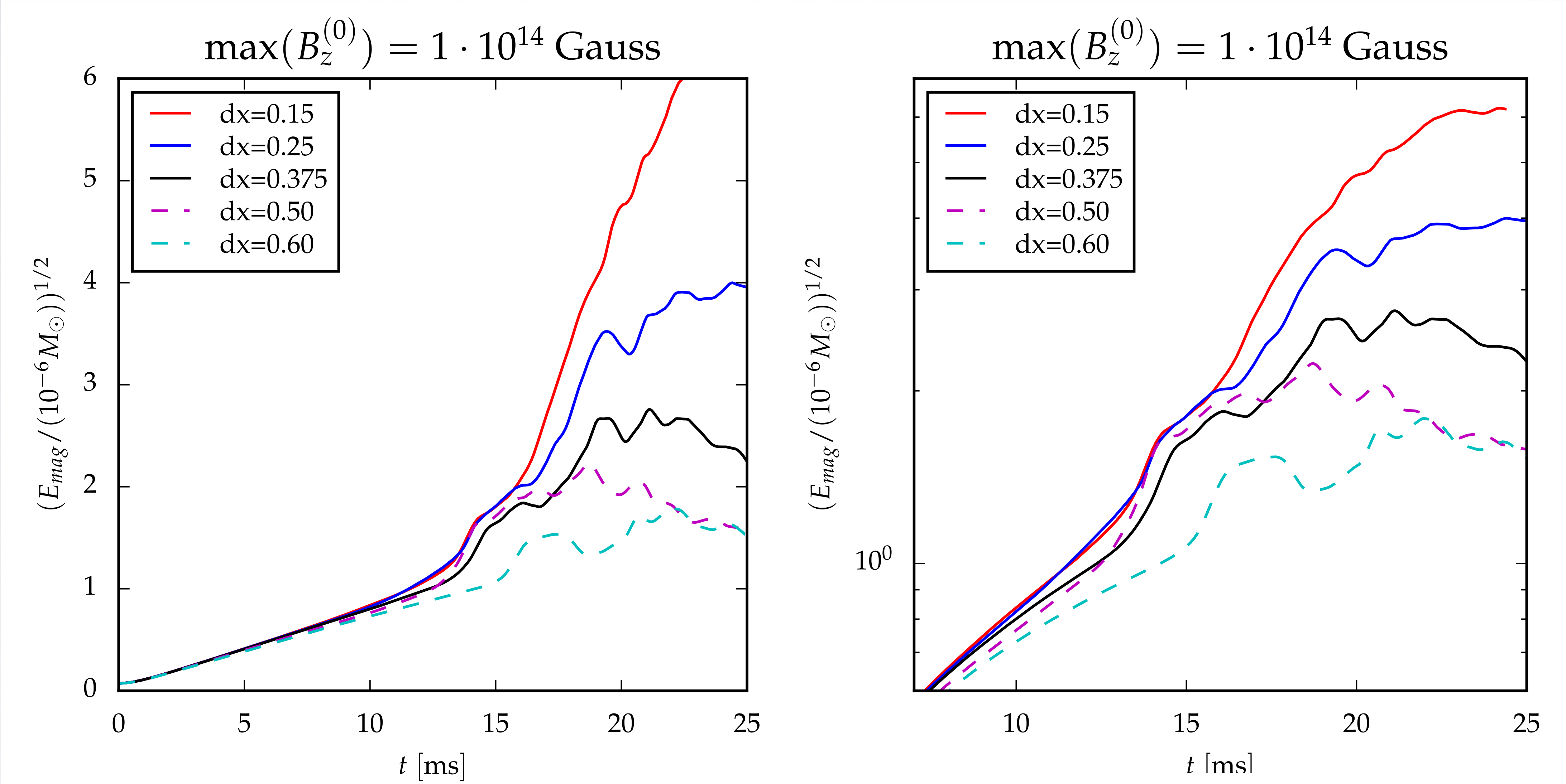
xy-plaze at z=6km

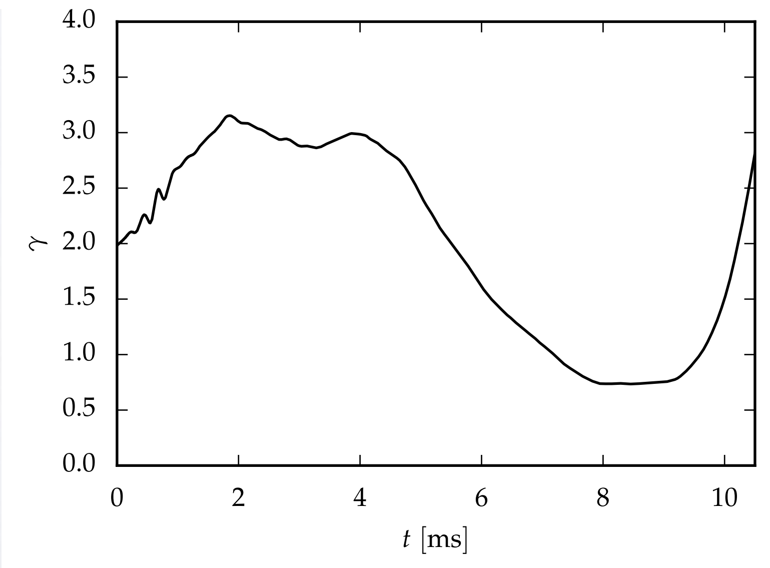
DIFFERENT VALUE OF B



the wavelength **λ_{MRI}** of the fast-growing modes is proportional to the magnetic field strength
the growth time **τMRI** is only related to rotation (independent from B field!)

DIFFERENT RESOLUTIONS

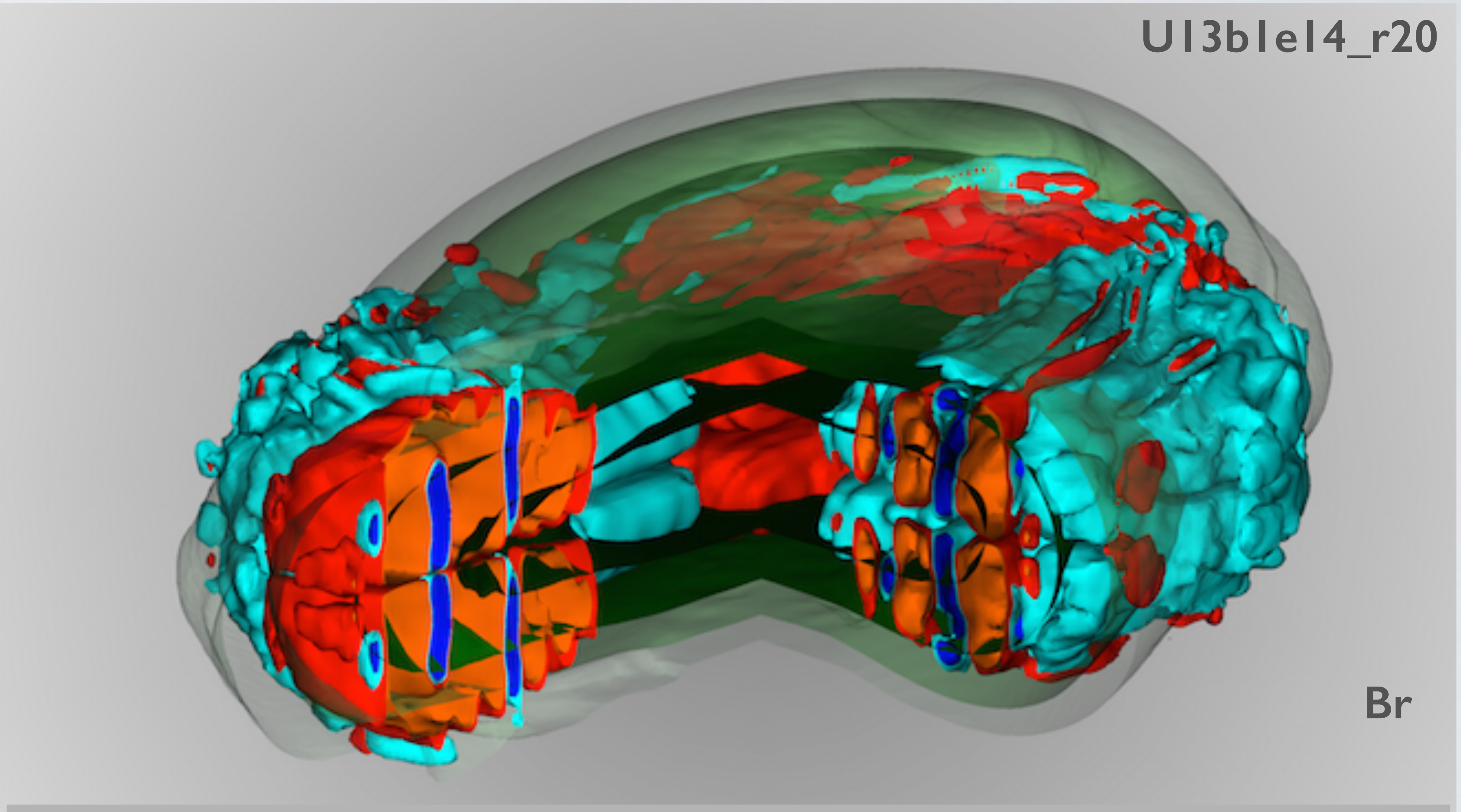




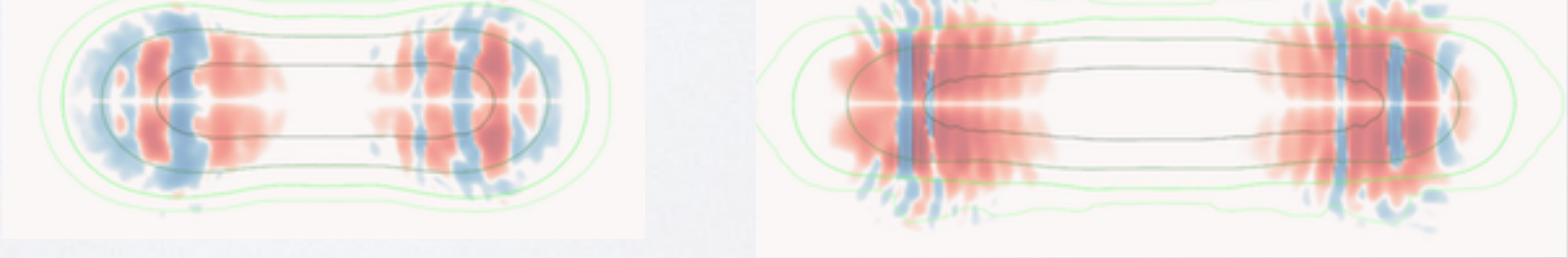
- Effect of the used spatial resolution on the simulated dynamics.

STRATIFICATION OF THE MAGNETIC FIELD

- In the dynamics of the model with a seed magnetic field of **10^{14}** Gauss.
- Image obtained using a spatial resolution of dx=0.20 (~295m)



Snapshot of the amplitude of the radial component of magnetic fields at time **t=13.5 ms** of the evolution of the stellar model U13-1.0e14. Please note, in the two sections on the xz-plane and yz-plane, the typical "wave" structure expected in the presence of MRIs.



MAGNETOROTATIONAL INSTABILITY

The magneto-rotational instability or MRI is an instability that :

- represents an important mechanism to amplify magnetic fields
- arises when the angular velocity of a conducting fluid in a magnetic field decreases as the distance from the rotation center increases
- shows rapidly growing and spatially periodic structure (channels flows)
- is very important in astrophysics (important part of the dynamics in accretion disks)

The MRI can be observed

- Local "Shearing boxes"
- Cylindrical disks (semi-global)
- Axisymmetric global simulations
- Full 3D global simulations (challenging due to computational limitations!) 3D global simulations (challenging due to computational limitations!

CONCLUSIONS

- SUMA project has bring to the group the possibility to do fore-front numerical simulations of ….. general relativistic compact object dynamics (NS,BH) in our universe.