Minimal defect production
vs
Kibble-Zurek Mechanism

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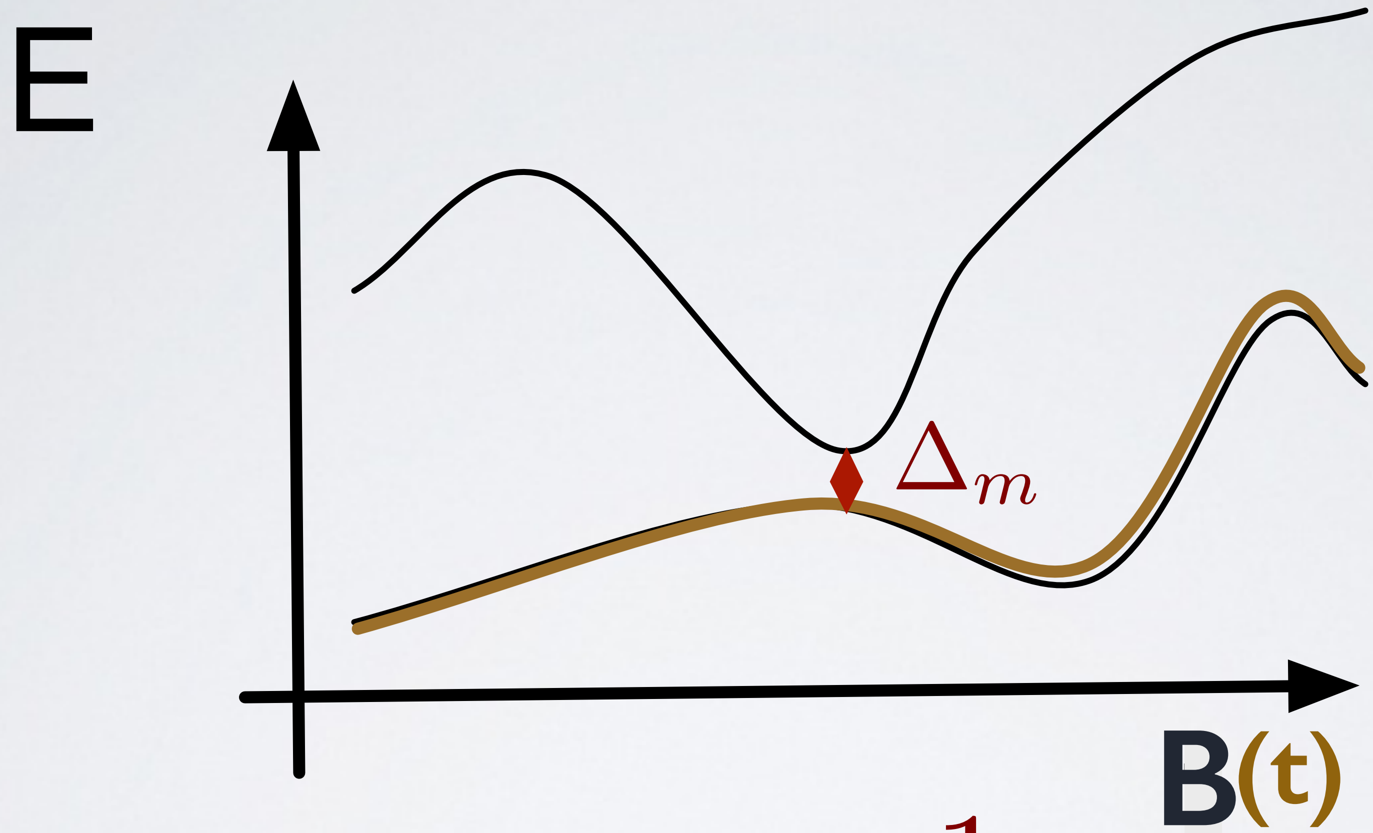
ADIABATIC DYNAMICS



$$T \gg \frac{1}{\Delta_m}$$

ADIABATIC DYNAMICS

There are cases in which the minimum gap closes in the thermodynamic limit:
"*failure*" *of the adiabatic regime*

- Adiabatic dynamics across critical points:
  Kibble-Zurek mechanism for defect formation

- Adiabatic quantum computation

- Quantum state preparation

Here we want to stay always in the ground state

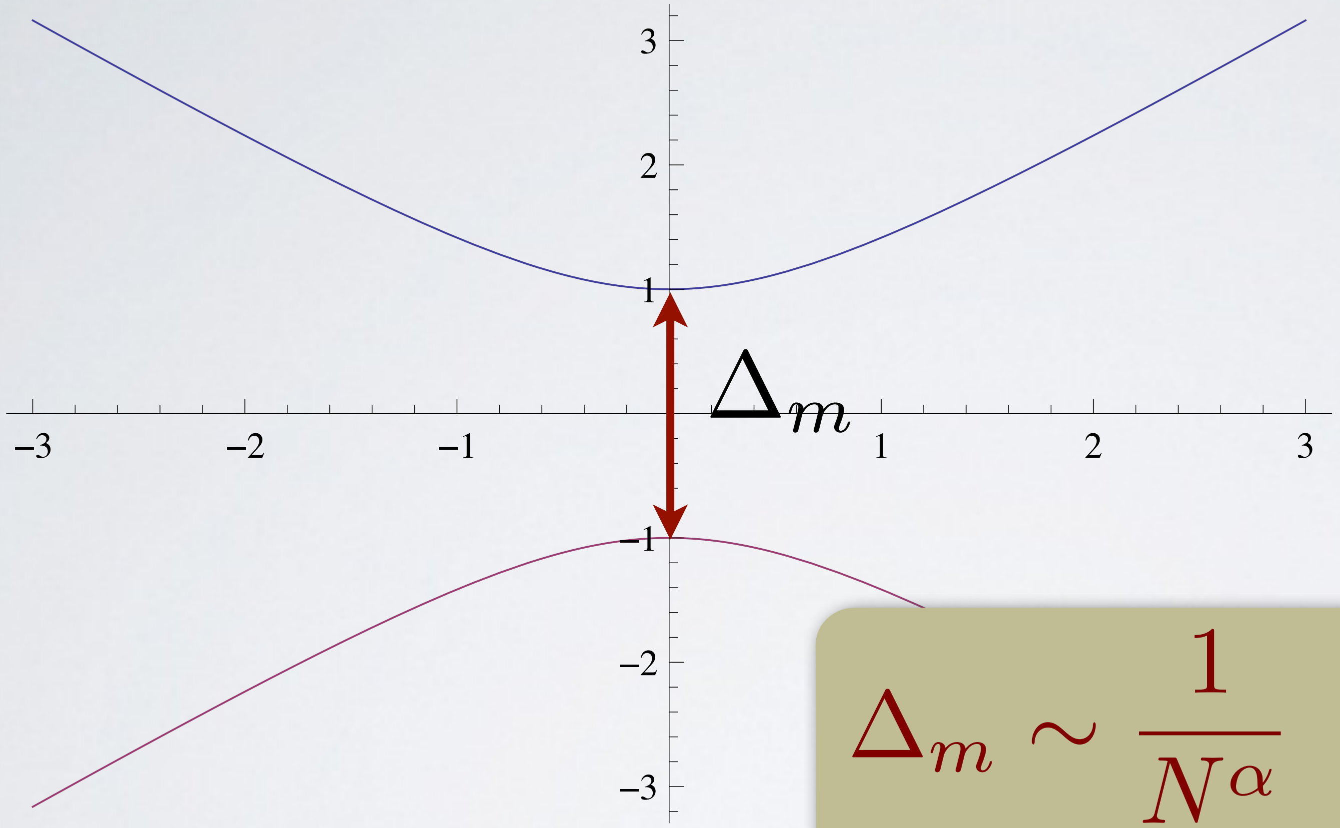
ADIABATIC QUANTUM COMPUTATION

$\mathcal{H}_i$ The ground state is known

$\mathcal{H}_f$ The ground state is the solution to our problem

$$\boxed{\mathcal{H}(t) = \frac{T-t}{T}\mathcal{H}_i + \frac{t}{T}\mathcal{H}_f}$$

ADIABATIC QUANTUM COMPUTATION



$$\Delta_m \sim \frac{1}{N^\alpha}$$ Easy problem

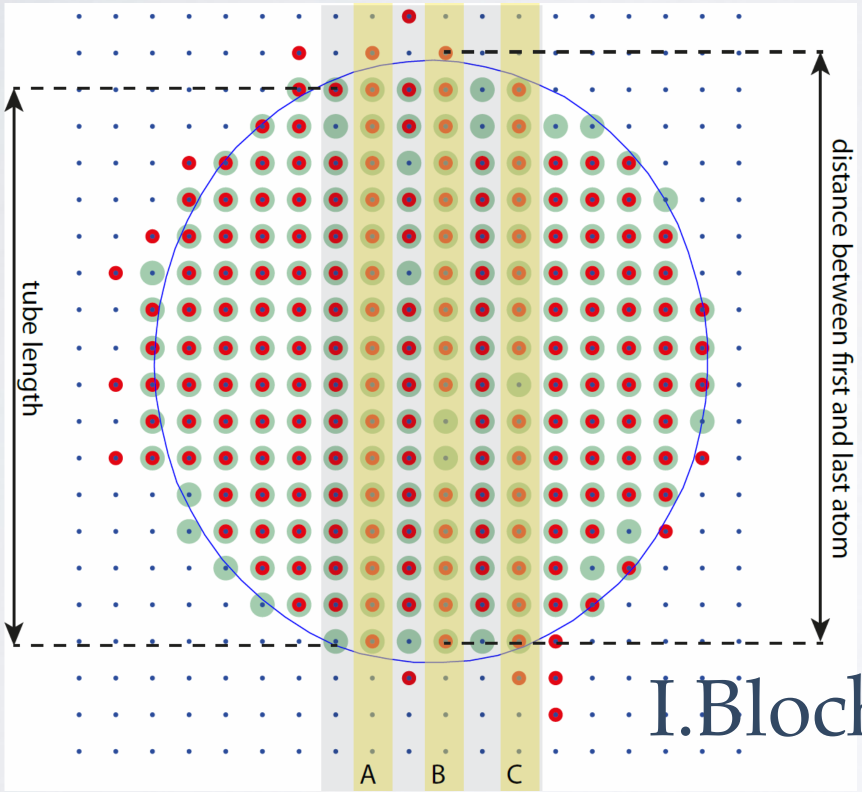
$$\Delta_m \sim e^{-N^\alpha}$$ Difficult problem

QUANTUM STATE PREPARATION

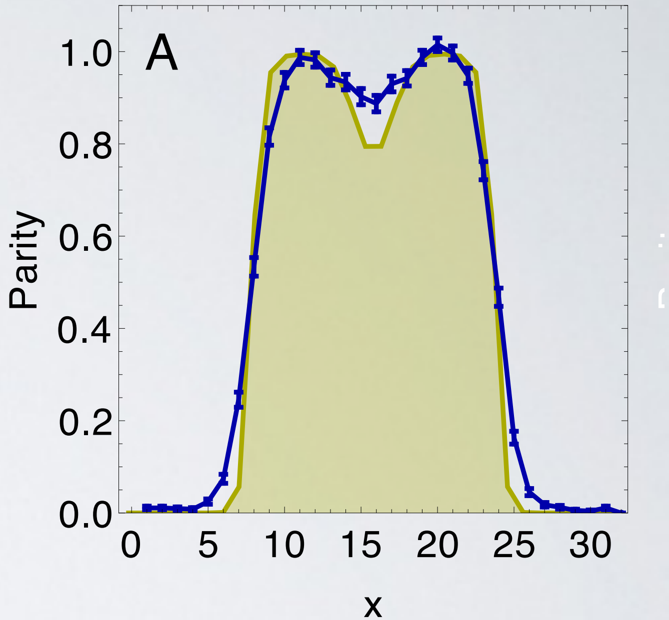
Courtesy of Simone Montangero

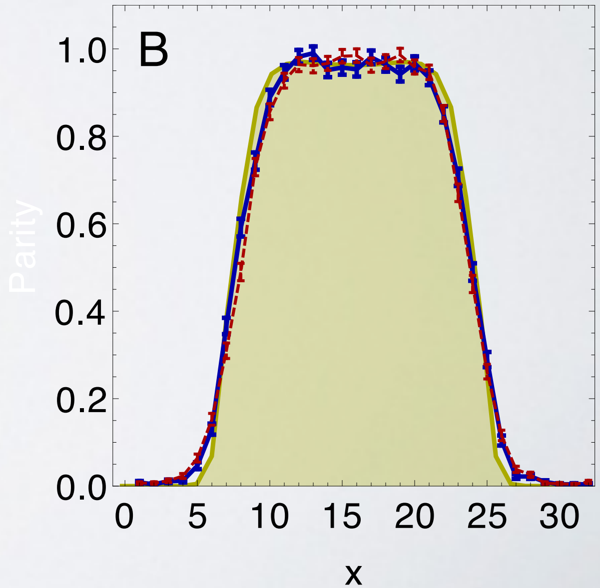
$$\mathrm{H}=\sum_j[-J(b_j^\dagger b_{j+1}+\mathrm{h.c.})+\Omega(j-\frac{N}{2})^2 n_j+\frac{U}{2}(n_j^2-n_j)]$$

Preparation of a Mott state



I.Bloch's group





T. Pichler, et. al. in preparation

OUTLINE(*)

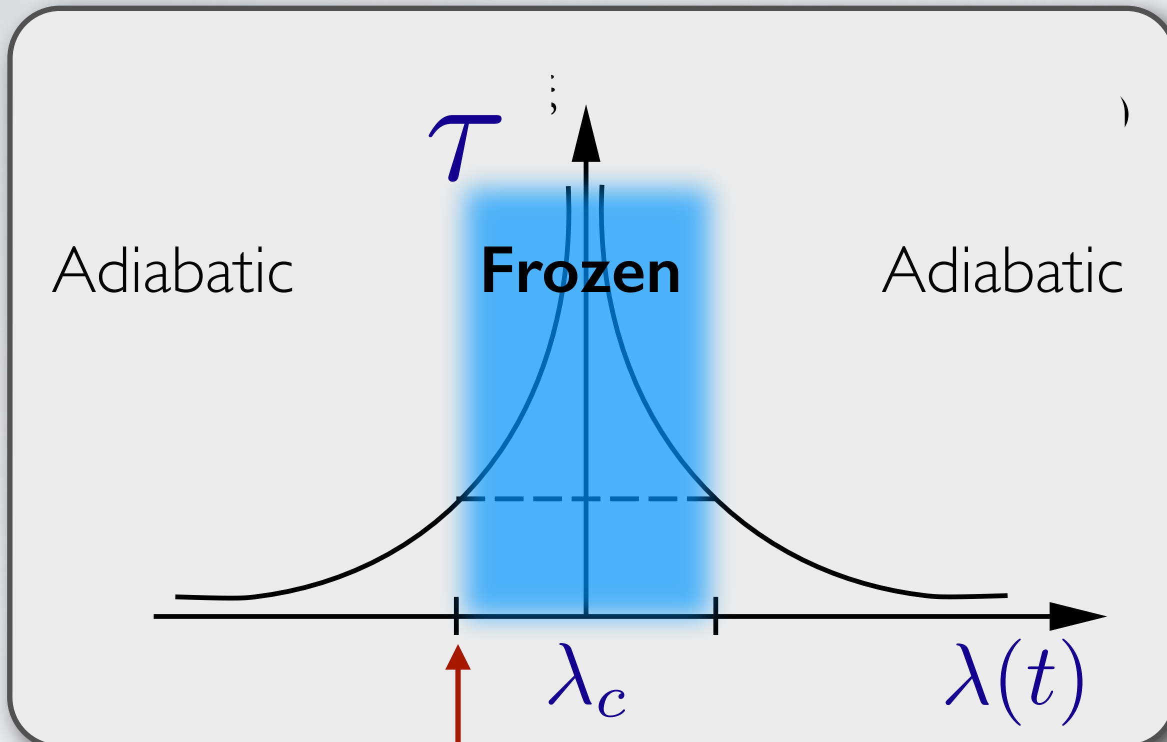
- Kibble-Zurek mechanism
- Optimal control in many-body systems
- Transitionless quantum driving

(*)-From now on only gaps closing as power laws

IN COLLABORATION WITH

• Tommaso Calarco, Tommaso Caneva, Simone Montangero	Ulm University
• Giuseppe Santoro	SISSA (Trieste)
• Mauro Paternostro, Gabriele De Chiara, Steve Campbell	Queen’s University
• Massimo Palma	Uni Palermo

KIBBLE-ZUREK MECHANISM



$$\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{H}_I$$

$$\lambda_c = \text{critical point}$$

$$\lambda - \lambda_c = vt$$

The adiabatic approximation breaks down when $\frac{\dot{\lambda}}{\lambda} \sim \tau$

$\hat{\xi}$ controls the density of defects which will be left after crossing the critical point

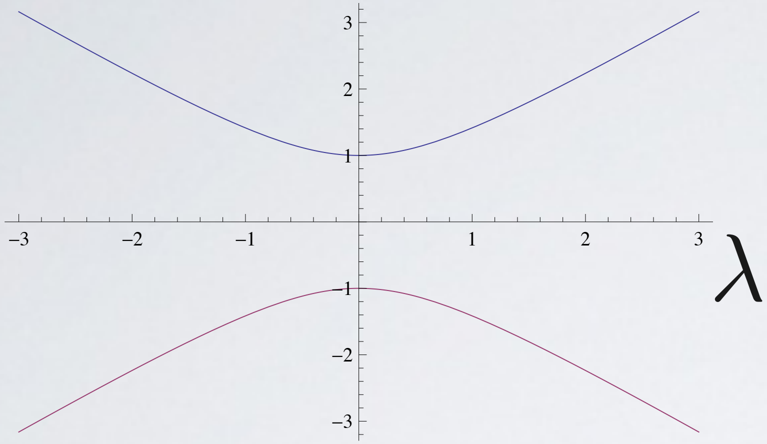
KIBBLE-ZUREK MECHANISM

$$\rho_{def} \sim \hat{\xi}^{-d} \sim v^{\frac{d\nu}{z\nu+1}}$$

$$\mathcal{E}_{res} \sim \mathrm{J}\rho_{def}$$

–W. Zurek, U. Dorner and P. Zoller (2005)
– A. Polkovnikov (2005)

KIBBLE-ZUREK MECHANISM



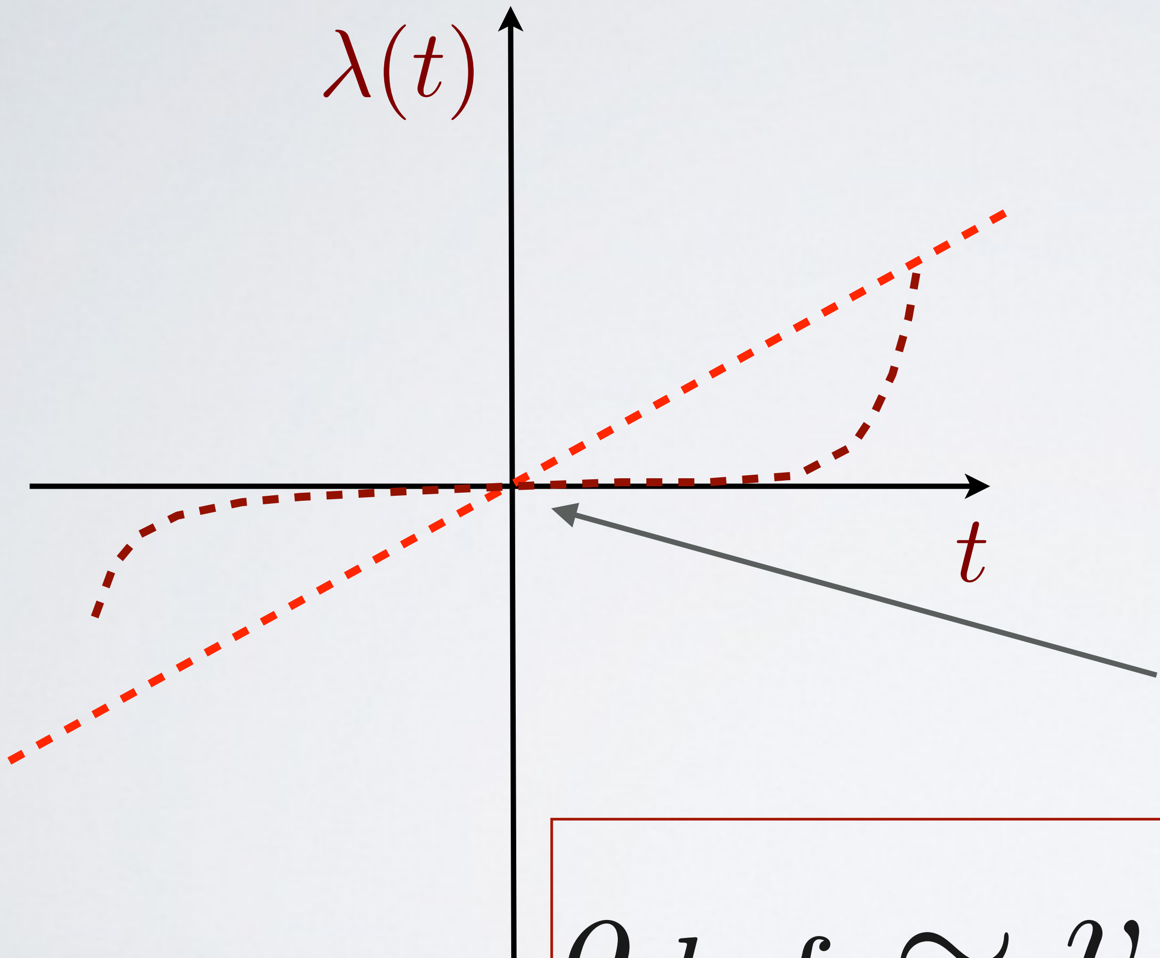
Landau-Zener problem

$$p_{LZ} \sim e^{-\frac{\pi \Delta}{2\hbar v}}$$

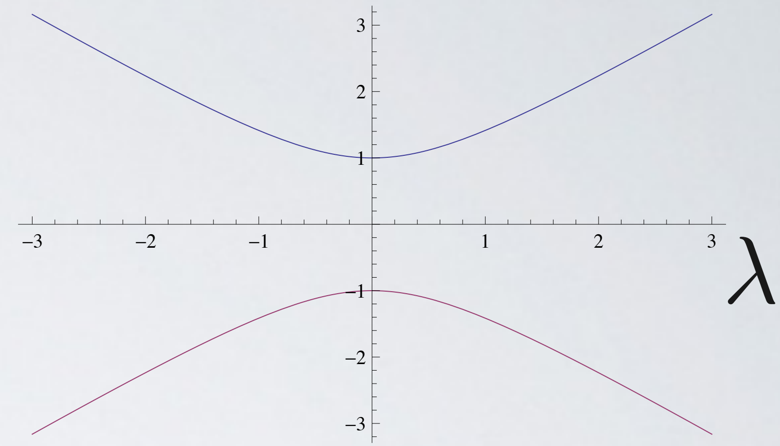
Estimate the the minimum length of the system "defect-free".

From here one can estimate the density of defects (in agreement with previous scaling)

KIBBLE-ZUREK MECHANISM



$$\sim t^r$$



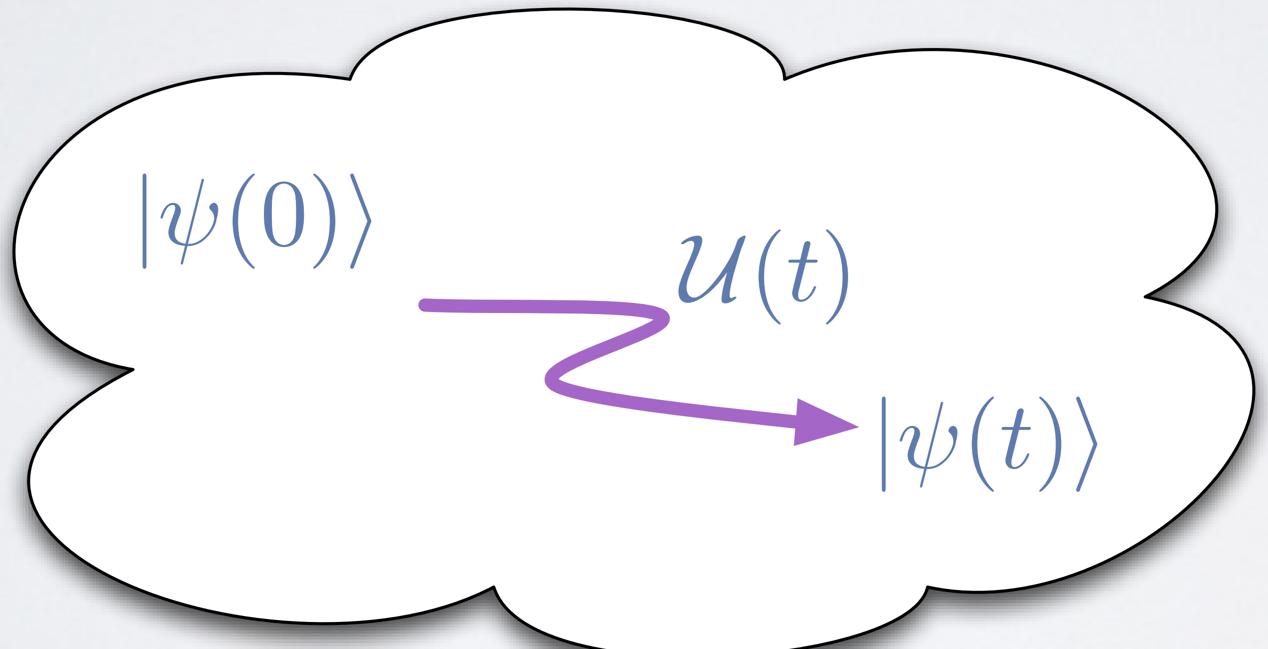
$$\boxed{\rho_{def} \sim v^{\frac{rd\nu}{rz\nu+1}}}$$

–D. Sen, K. Sengupta, and S. Mondal (2008)

Is it possible to minimize the defect production on crossing a phase transition?

Optimal control applied to defect formation across a QPT

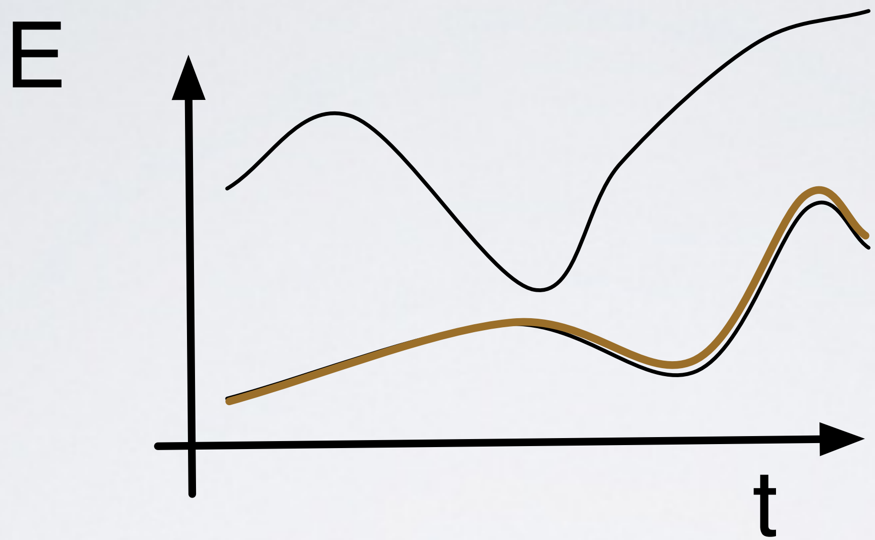
$$\mathcal{H} = \mathcal{H}(\{d_j(t)\}, t)$$



– T. Caneva, T. Calarco, R. F., G.E. Santoro, and S. Montangero (2011)

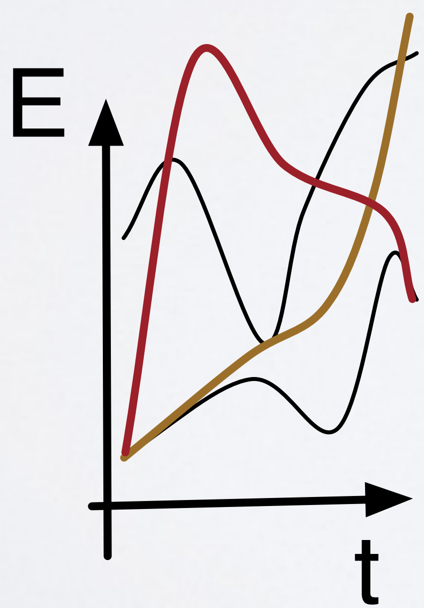
OPTIMAL DYNAMICS: A CARTOON

Slow



Adiabatic strategy

Fast



Optimal control

THE MODELS

1D Ising model

$$\mathcal{H} = \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda(t) \sum_i \sigma_i^z$$

Lipkin-Meshkov-Glick (LMG) model

$$\mathcal{H} = \frac{1}{N} \sum_{i<j} \sigma_i^x \sigma_j^x + \lambda(t) \sum_i \sigma_i^z$$

THE MODELS

Grover search model

$$\mathcal{H} = [1 - \lambda(t)](1 - \mid s\rangle\langle s \mid) + \lambda(t) \mid \bar{\psi}\rangle\langle\bar{\psi} \mid$$

Landau-Zener problem

$$\mathcal{H} = \lambda(t)\sigma^x + \omega\sigma^z$$

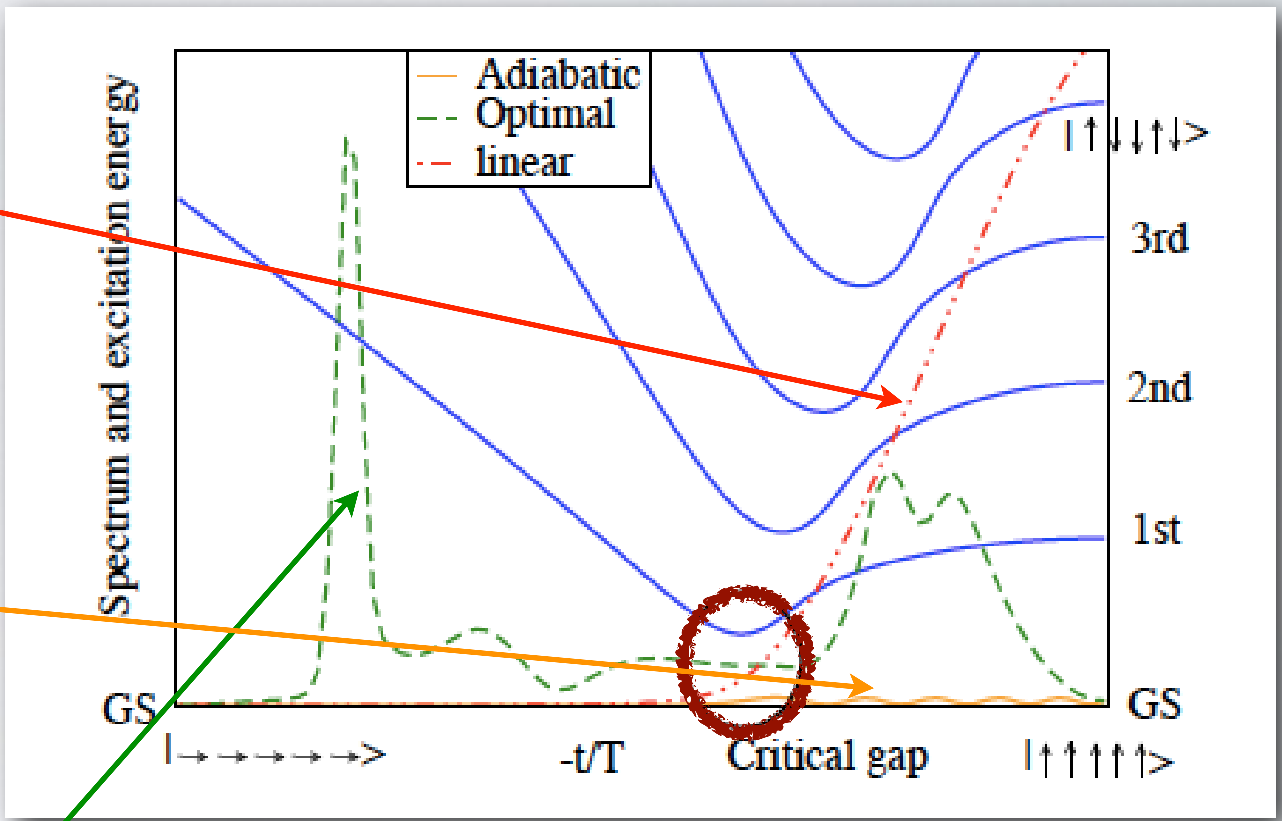
A "MEASURE FOR DEFECT FORMATION"

The infidelity

$$I(T) = 1 - |\langle \psi_G | \psi(T) \rangle|^2$$

Linear quench for a "short" time T

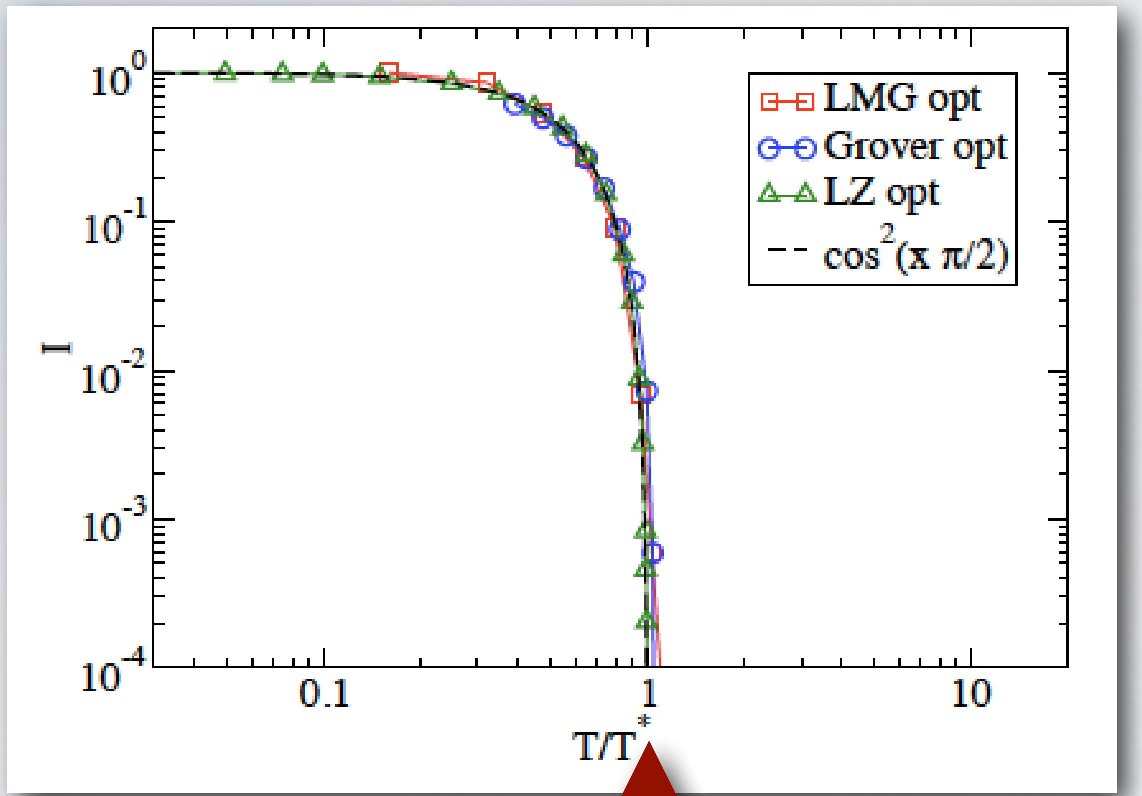
Linear quench in the adiabatic limit $T \sim T_{ad}$

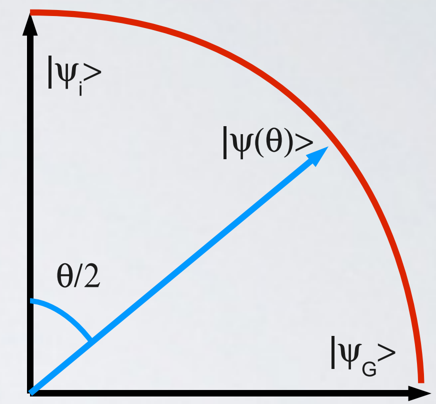


Optimal pulse at time $T \ll T_{ad}$

MINIMAL DEFECT FORMATION

– T. Caneva, T. Calarco, R. F., G.E. Santoro, and S. Montangero (2009)





An optimized evolution then can be interpreted as a uniform motion along a geodesic with speed π/T^*

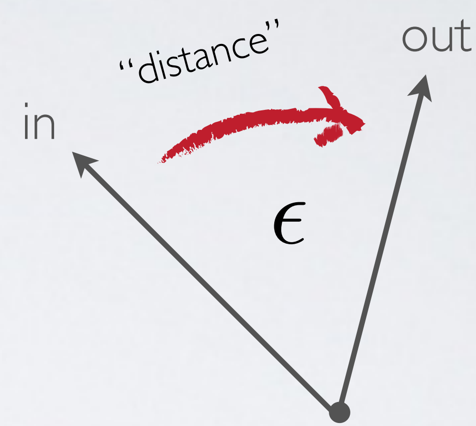
There is a minimum time, associated to the quantum speed limit, below which the optimization is ineffective

– T. Caneva, M. Murphy,T. Calarco, R. F., S. Montangero, V. Giovannetti, and G.E. Santoro (2009)

QUANTUM SPEED LIMIT

Time-independent case

Determine the Minimum time required for a quantum state to evolve to a different one placed at a certain distance from it.



$$E = \langle\Psi|H|\Psi\rangle \qquad \Delta E = \sqrt{\langle\Psi|(H-E)^2|\Psi\rangle}$$

Initial energy

Initial state

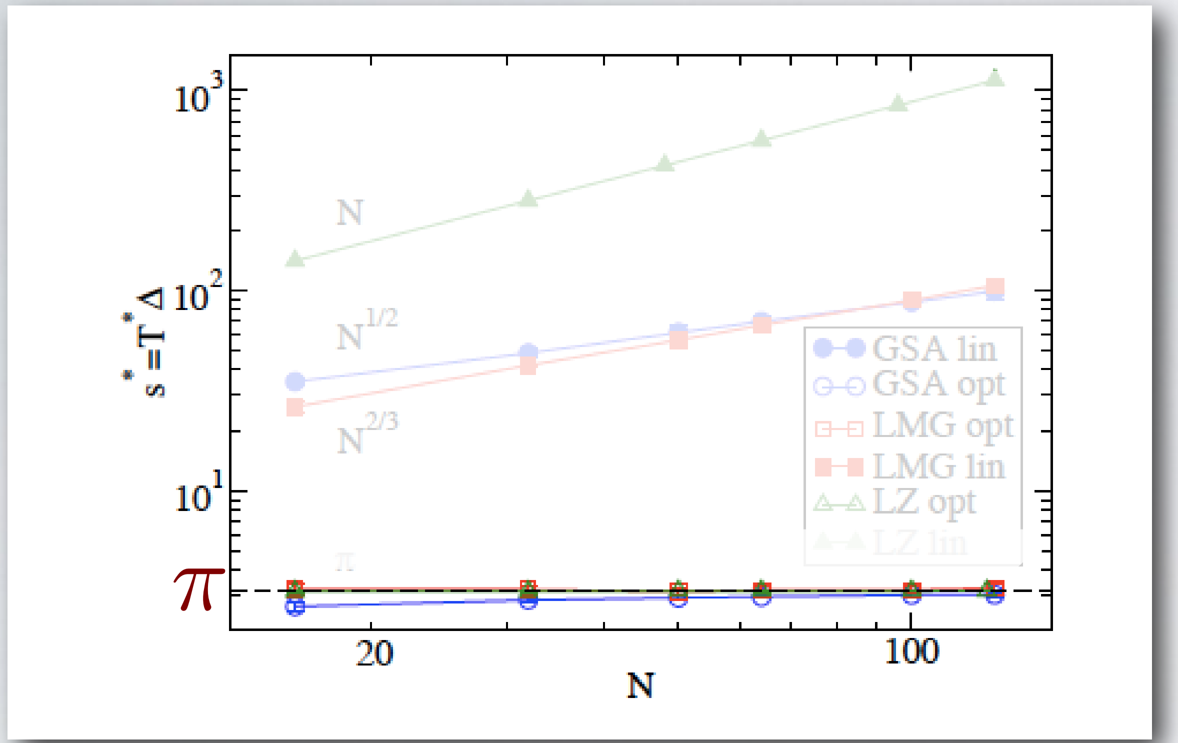
Energy variance

$$\boxed{T^* = \max\left(\alpha(\epsilon)\frac{\pi}{2E}, \beta(\epsilon)\frac{\pi}{2\Delta E}\right)}$$

- T. K. Bhattacharyya (1983)
- P. Pfeifer (1993)
- N. Margolus and L.B. Levitin (1998)
- V Giovannetti, S Lloyd, and L Maccone (2003)
- A. Carlini et al (2006)

MINIMAL DEFECT FORMATION

– T. Caneva, T. Calarco, R. F., G.E. Santoro, and S. Montangero (2009)



Comparison of optimized and non-optimized evolutions

T^*

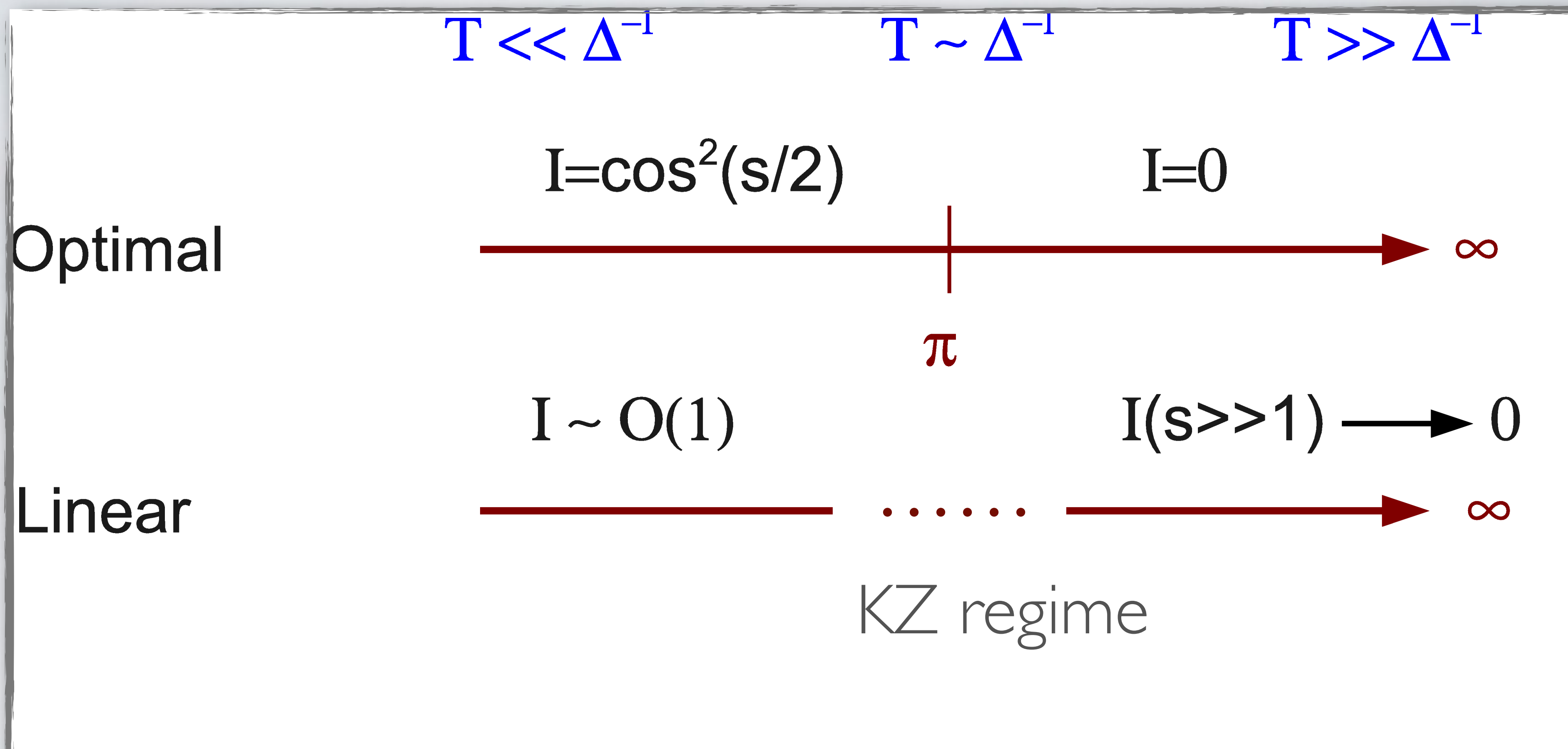
minimum time to achieve infidelity $I \sim 10^{-3}$

- Linear: scaling with N

- Optimal: motion along the geodesic at constant speed

MINIMAL DEFECT FORMATION

– T. Caneva, T. Calarco, R. F., G.E. Santoro, and S. Montangero (2009)



COMPLEXITY OF CONTROLLING A MANY-BODY CRITICAL SYSTEM?

TRANSITIONLESS QUANTUM DRIVING

Demirplak and Rice (2003)
Berry (2009)

Requirement $|\psi_0[\Gamma(t)]\rangle$

ground state of H

$$\tilde{H} = H + \delta H$$

$$\delta H = i \sum_n [|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|]$$

Experimental implementation M. Bason et al (2011) (Pisa group)

TRANSITIONLESS QUANTUM DRIVING THROUGH A CRITICAL POINT

A. del Campo, M.M. Rams, and W.H. Zurek (2012)
S. Campbell, G. De Chiara, M. Paternostro, G.M. Palma, and R.F. (2014)

$$\delta H = i \sum_n [|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle |n\rangle\langle n|]$$

One–dimensional Ising model A. del Campo, M.M. Rams, and W.H. Zurek (2012)

$$\delta \mathcal{H} = \sum_i \sum_{m<\xi} g(m) \sigma_i^\alpha \sigma_{i+1}^\beta \cdots \sigma_{i+m}^\delta$$

Complexity increases at the critical point

TRANSITIONLESS QUANTUM DRIVING THROUGH A CRITICAL POINT

S. Campbell, G. De Chiara, M. Paternostro, G.M. Palma, and R.F. (2014)

Lipkin-Meshkov-Glick (LMG) model

$$\mathcal{H} = \frac{1}{N} \sum_{i<j} \sigma_i^x \sigma_j^x + \lambda(t) \sum_i \sigma_i^z$$

Despite the correlation length being always infinite the closing of the gap at the critical point makes the driving Hamiltonian of increasing complexity also in this case.

TRANSITIONLESS QUANTUM DRIVING THROUGH A CRITICAL POINT

S. Campbell, G. De Chiara, M. Paternostro, G.M. Palma, and R.F. (2014)

Holstein-Primakov transformation → Mapping onto free bosons

$$\delta\mathcal{H} \sim \frac{1}{N\,|\,\lambda - 1\,|}\sum_{i<j}\left[\sigma_i^x\sigma_j^y + \sigma_i^y\sigma_j^x\right]$$

Divergence at the critical point

“Comparison”

- Optimal control may require “complex”pulses (robustness towards pulse deformation)
- Superadiabatic dynamics requires multi-spin interactions

SUMMARY

- Minimal defect formation by optimal quantum control
- Quantum speed limit related to the minimum gap
- Simple description in terms of two-level dynamics

Thank you for the attention!