EGAN - 3rd Data-Analysis School

LNL, October 1 – 3, 2014

MEASURING LINEAR POLARIZATION OF GAMMA RAYS WITH AGATA

(Firenze – Padova – Legnaro Collaboration)

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Two fundamental questions

Polarization Measurements:

WHY? and HOW?

Polarization measurements: Why?

Apart from some more fundamental questions (which would be outside the purpose of these lectures),
Polarization Measurements (even of moderate precision)
can be useful, e.g,

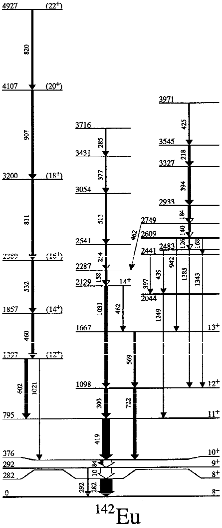
- **To assign the parity (Electric or Magnetic) to a transition of known multipole order**
- **To remove the ambiguity in the multipole mixing ratio given by angular distributions**

A couple of examples will better clarify these points.

A (rather old) example (1990)

Problem:
Parity of the high-spin cascade feeding the 8^- isomer of ^{142}Eu
Measurements with ESSA30 at Daresbury and MIPAD at LNL.
Linear polarization measurements with a segmented Ge detector at LNL for the 282 kev and 192 keV transitions

These results have been confirmed by electron conversion coefficients



A.M. Bizzeti-Sona et al: Z. für Physik A, 337 (1990) 235.

A second example (2001)

From E.Farnea, Ph.D. Thesis

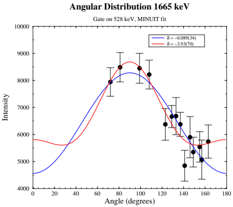
Problem: Multipolarity of the $5^- \rightarrow 4^+$ transition in ^{64}Ge

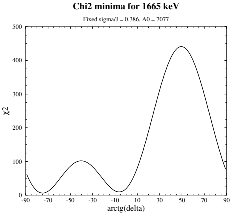
Experiment: $^{40}Ca(^{32}S,2\alpha)^{64}Ge$ reaction

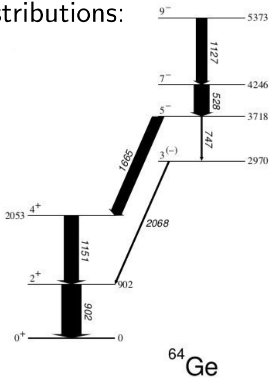
EUROBALL III + ISIS at IReS Strasbourg

Multipole mixing ratio δ from angular distributions:

$\mathrm{arctg}(\delta) = -5.1^\circ$ or $\mathrm{arctg}(\delta) = -75.7^\circ$







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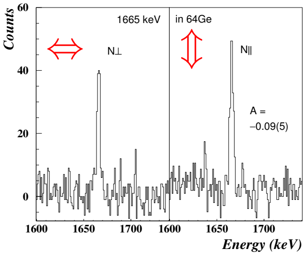
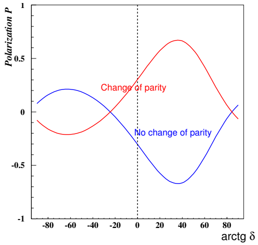
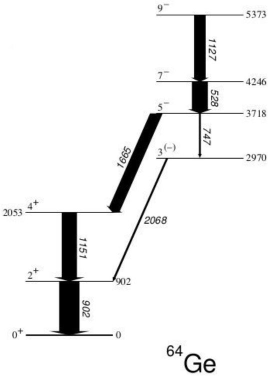
Multipole mixing ratio δ from angular distributio[n]
and linear polarization (from Clover Detectors)
arctg(δ) $= -75.7°$, $\delta = -3.93$

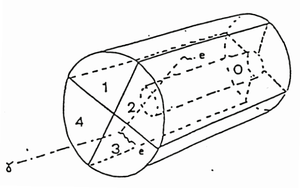




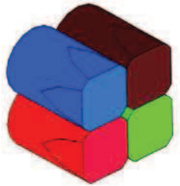


Polarization measurements: How?

- Azimuthal distribution of Compton scattering
- Polarized photon are preferentially scattered
  in the plane perpendicular to the the Electric Field
- Examples of simple experimental set-up:



Segmented Ge detector



EUROBALL Clover

One plane of sector separation along the polarization plane

Asymmetry $[N(\updownarrow)/N(\leftrightarrow)]$ proportional to Polarization

Another (presumably better) solution:
Tracking of the Compton scattering with AGATA
explores the complete angular distribution

Layout of the rest of this talk

- Theoretical preliminaries:
  - Polarization and Stokes parameters
  - Compton scattering and polarization
- Measurements with AGATA
  - AGATA as a segmented detector
  - Agata as a tracking detector
- MonteCarlo simulations
  - Introducing Polarization in MC results
  - comparison with CoulEx results
- Perspectives for future improvements

Polarization and the Stokes parameters

Vector potential (plane electromagnetic wave)

$$\vec{\mathcal{A}}(\vec{r},t) = \vec{A}(\vec{r})\exp(-i\omega t),\ \text{with}\ \vec{E} = -(1/c)\partial\vec{A}/\partial t,\ \ \vec{B} = \text{rot}\vec{A}.$$

For a plane wave propagating in the direction $\vec{e}_z$

$\vec{A}(\vec{r}) \propto (\alpha_x\vec{e}_x + \alpha_y\vec{e}_y)\exp(ikz)$ with $\alpha_x\alpha_x^* + \alpha_y\alpha_y^* = 1$

The three Stokes parameters are defined as

$$\begin{aligned} P_1 &= \alpha_x\alpha_x^* - \alpha_y\alpha_y^* \\ P_2 &= \alpha_x\alpha_y^* + \alpha_y\alpha_x^* \\ P_3 &= i(\alpha_x\alpha_y^* - \alpha_y\alpha_x^*) \end{aligned}$$

For a pure state $P_1^2 + P_2^2 + P_3^2 = 1$.

- $P_3 = \pm 1$ for pure circular polarization;
- $|P_1|^2 + |P_2|^2 = 1$ for pure linear polarization.
  - $P_1 = +1\ (\text{or}\ -1) \Rightarrow \tilde{\mathrm{A}}$ along e_x(or e_y)
  - $P_2 = \pm 1 \quad \Rightarrow \vec{A}$ along $(e_x \pm e_y)/\sqrt{2}$

Stokes parameters as matrix elements of the density matrix

For a statistical mixture of different polariation states k with probability $p(k)$ and Stokes parameters $P_1(k), \; P_2(k), \; P_3(k)$
$P_i = \sum_k p(k) P_i(k)$ and $P_1^2 + P_2^2 + P_3^2 \leq 1.$
In the helicity representation is expressed in terms of the Stokes parameters:

$$\rho = \left| \begin{array}{cc} (1+P_3)/2 & -(P_1 - iP_2)/2 \\ -(P_1 + iP_2)/2 & (1-P_3)/2 \end{array} \right|$$

Stokes parameter do not transform as the component of a vector!
E.g., for a rotation of an angle ϕ around the e_z axis,

$$\begin{aligned} P_1' &= P_1 \cos(2\phi) - P_2 \sin(2\phi) \\ P_2' &= P_1 \sin(2\phi) + P_2 \cos(2\phi) \\ P_3' &= P_3 \end{aligned}$$

Angular distribution of Compton scattering for linearly polarised radiation

For a complete linear polarization of the photons (moving in the direction of the axis z with electric field $\mathbf{E}$ along the x axis)

$$\mathrm{d}\sigma(\theta, \Theta_E) = \frac{r_0^2}{4}\left(\frac{\nu'}{\nu_0}\right)^2\left(\frac{\nu_0}{\nu'} + \frac{\nu'}{\nu_0} - 2 + 4\cos^2\Theta_E\right)\mathrm{d}\Omega$$

with $\quad \nu_0 \;/\; \nu' = 1 + \alpha(1 - \cos\theta), \qquad \alpha = h\nu_0/m_0c^2$
and θ is the scattering angle.
The dependence on the azimuthal angle φ (between the scattering plane and the plane xz) is contained in the angle Θ_E between the electric field of the primary photon and of the scattered photon.

Azimuthal distribution of Compton scattering

If the polarization of the scattering photon is not measured, $\cos^2\Theta_E$ can be replaced by its average value (over polarization states of the scattered photon)

$$\overline{\cos^2\Theta_E} = \left(1 - \sin^2\theta\cos^2\varphi\right)/2$$

to obtain

$$\mathrm{d}\sigma(\theta,\varphi) = \frac{r_0^2}{4}\left(\frac{\nu'}{\nu_0}\right)^2\left[\frac{\nu_0}{\nu'} + \frac{\nu'}{\nu_0} + \sin^2\theta(1-\cos 2\varphi)\right]\mathrm{d}\Omega$$

For unpolarized radiation, taking the average over φ

$$\overline{\mathrm{d}\sigma(\theta)} = \frac{r_0^2}{4}\left(\frac{\nu'}{\nu_0}\right)^2\left(\frac{\nu_0}{\nu'} + \frac{\nu'}{\nu_0} + \sin^2\theta\right)\mathrm{d}\Omega$$

Compton scattering for a partially polarized radiation

For a radiation characterized by the Stokes parameters $P_1,\ P_2,\ P_3$ the differential cross section for Compton scattering at angles θ, φ, summed over polarization states of the outgoing radiation, is

$$\begin{aligned}
\mathrm{d}\sigma(\theta,\varphi) &= \frac{r_0^2}{4}\left(\frac{\nu'}{\nu_0}\right)^2\left[\frac{\nu_0}{\nu'}+\frac{\nu'}{\nu_0}-\sin^2\theta\,(1-P_1\cos 2\varphi-P_2\sin 2\varphi)\right]\mathrm{d}\Omega \\
&= \frac{r_0^2}{4}\left(\frac{\nu'}{\nu_0}\right)^2\left[\frac{\nu_0}{\nu'}+\frac{\nu'}{\nu_0}-\sin^2\theta\,\{1-P\cos 2(\varphi-\varphi_0)\}\right]\mathrm{d}\Omega
\end{aligned}$$

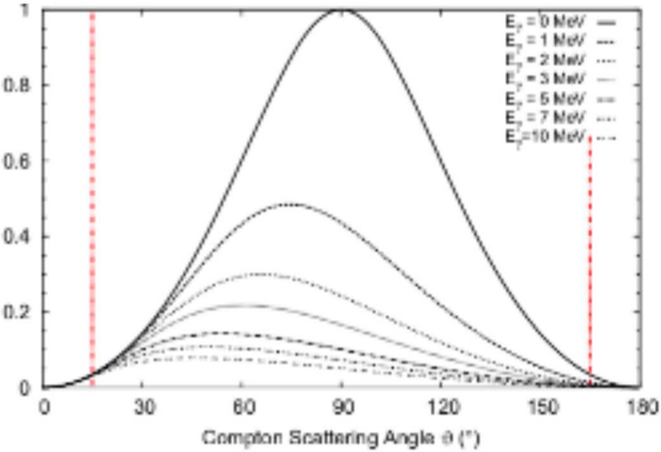
with $\quad P=\sqrt{P_1^2+P_2^2} \qquad$ and $\qquad \varphi_0=\frac{1}{2}\mathrm{arctg}(P_2/P_1)$

In the following,we always choose a reference frame in which $P_2=0,\ P_1=P$.

Analysing power *versus* scattering angle

The analysing power at the scattering angle θ can be defined as

$$A(\theta) = \frac{\mathrm{d}\sigma(\theta, \pi/2) - \mathrm{d}\sigma(\theta, 0)}{\mathrm{d}\sigma(\theta, \pi/2) + \mathrm{d}\sigma(\theta, 0)} = \frac{\sin^2\theta}{\frac{\nu_0}{\nu'} + \frac{\nu'}{\nu_0} - \sin^2\theta}$$



(from Alikhani et al, NIM A 675 (2012) 144)

Layout

- Theoretical preliminaries
- Measurements with AGATA
  - AGATA as a segmented detector
    - The Darmstadt test experiment
  - Agata as a tracking detector
- MonteCarlo simulations
  - Introducing Polarization in MC results
  - comparison with CoulEx results
- Perspectives for future improvements

Polarization measurements with AGATA

Exploiting the 6×6 segmentation of a single crystal?

Comparing $N(\leftrightarrow)$ with $N(\updownarrow)$:

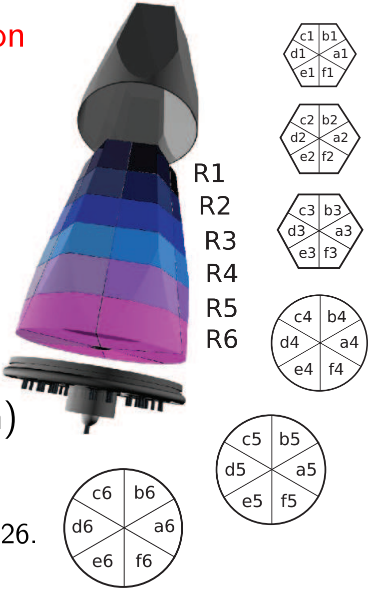
Coincidences $\leftrightarrow$: bc, ef, ad

Coincidences $\updownarrow$: ce, bf

are not equivalent!

Reference measurements with unpolarized radiation (and / or MonteCarlo simulation) are necessary!

(from Akkoyun et al., NIM A 668 (2012) 26.



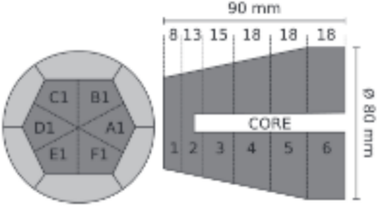
Compton Polarimeter with a 36–fold segmented HPGe detector of the AGATA–type

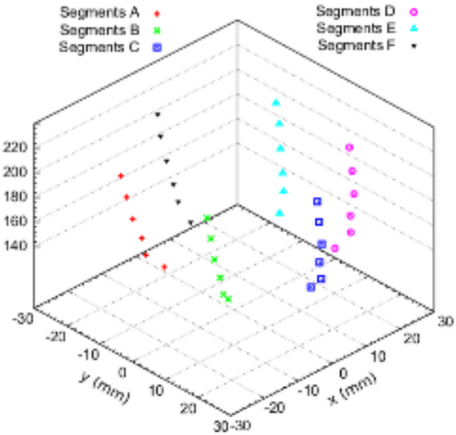
Alikhani et al., NIM A 675 (2012) 144

- **Experiment:** Linear polarization of one of the cascading γ rays (1173 keV, 1332 keV) from a ^{60}Co source in coincidence with the other one.
- **Set-up:** One AGATA-type segmented Ge and two supplementary detectors (coaxial Ge) to measure γ rays emitted in coincidence at 90^0 to one another.
- **Selection criteria:** Only events with full energy spent entirely in two interactions.
  Threshold for energy release $\approx$ 30 keV.
- **Reference data:** Unpolarised radiation not in coincidence with supplementary detectors.

Compton Polarimeter with a 36–fold segmented HPGe detector of the AGATA–type

Alikhani et al., NIM A 675 (2012) 144





For each volume elements all interactions inside are attributed to its center-of-mass.

Results

Angles θ and φ referred to the center of mass of the volume element.

Selection on θ from 15° to 165°

$|\cos\theta| < 0.97$

Values of φ binned in 36° intervals, symmetrized with respect to 90°

Fraction of events in the bin φ_i:

$F(i) = \sum_{k\in\varphi_i} N_k \ / \ \sum_k N_k$

Asymmetry for the bin φ_i

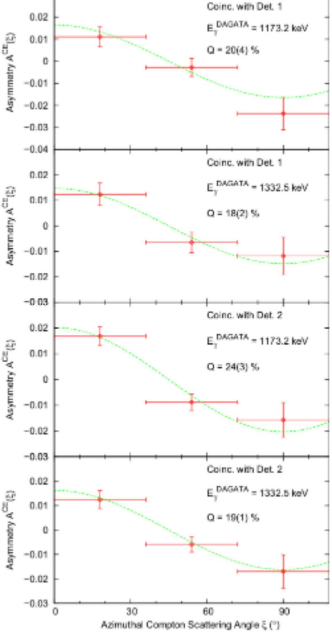
$A(i) = F_{coinc} \ / \ F_{unpol}$

Polarization efficiency Q defined by

$A(E_\gamma) = \frac{1}{2}P(E_\gamma)Q(E_\gamma)$:

$Q(1173\ \mathrm{keV}) = (22.8 \pm 2.6)10^{-2}$

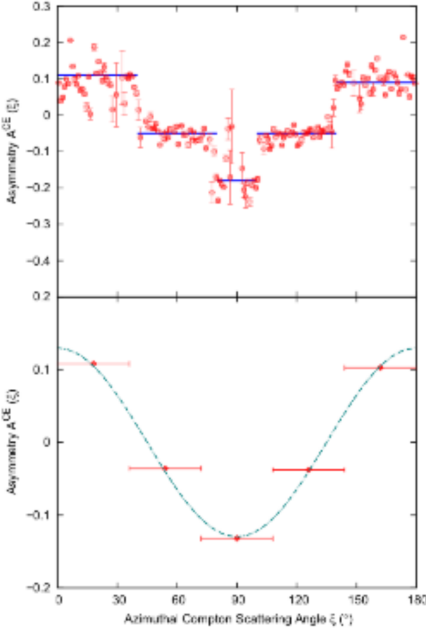
$Q(1332\ \mathrm{keV}) = (19.2 \pm 0.9)10^{-2}$



MonteCarlo simulation

Simulated asymmetry in different bins of φ for totally polarized radiation.

Upper panel: 1° bins
Lower panel: 36° bins



Layout

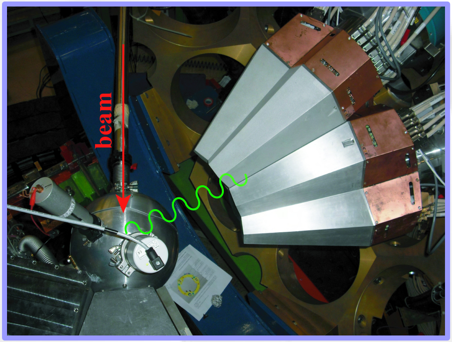
- Theoretical preliminaries
- Measurements with AGATA
  - AGATA as a segmented detector
  - Agata as a tracking detector
    - The LNL CoulEx experiment
      - Analysis procedure
      - Problems from instrumental effects
      - Examples of results
- MonteCarlo simulations
  - Introducing Polarization in MC results
  - comparison with CoulEx results
- Perspectives for future improvements

Measuring linear polarization with AGATA: PSA for identification of hit positions

If the position of every hit (and associated energy release) as well as the time order of hits, are known, one can determine the polar angle θ and the azimuthal angle φ for the first Compton scattering and deduce the linear polarization of the incoming γ's from the azimuthal distribution.

The hit positions can be derived from the measured Pulse Shapes in the different elements of the segmented crystal by means of the PSA procedure:

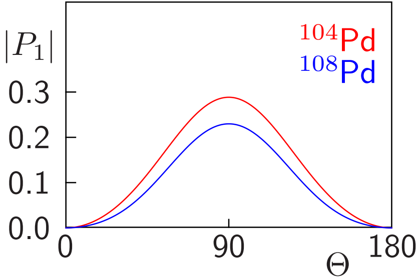
The test experiment at LNL



- Two AGATA triple clusters, mounted in the AGATA demonstrator at LNL.
- Partially polarized γ rays from CoulEx of ^{104}Pd (555.8 keV) and ^{108}Pd (443.9 keV).
- Unpolarized 661 keV γ rays from a ^{137}Cs source.

Linear polarization for CoulEx γ rays

- Reaction: 32 MeV ^{12}C beam onto 1 mg/cm^2 thick ^{104,108}Pd targets.
- Almost all Pd recoils stop in the target
  - $\Rightarrow$ Aligned (axial) symmetry for γ emission.
  - $\Rightarrow$ $P_2 \equiv 0$ for Reference axis perpendicular to the beam and to the γ-emission direction.
- Linear polarization of CoulEx γ rays emitted at angle Θ to the beam direction evaluated by means of the GOSIA code.



The three steps of the data analysis procedure

For each event, the digitized shapes of signals from the 36 elementary volumes of each crystal are first stored in a sequence of disk files.

1. These data are analysed with PSA to derive energies and positions (– *Dino Bazzacco*).

2. The output of PSA is sorted to reconstruct the hit sequence (– *Caterina Michelagnoli*).

3. Sorted data are analysed for the effects of polarization (– *Firenze*).

The PSA procedure

At the moment, only one hit per volume element is assumed. Only events with a single identified γ ray are used at later steps of the procedure.

The following information is recorded (in list mode):
For each γ:
- Number of hits, total energy.
- For every hit:
  - Counter Nr, element Nr; Released energy;
  - Space coordinates (on a 2mm lattice).

An option for reconstruction of data to be attributed to a not-working channel is also provided.

Data sorting with mgt code

- As the 2mm lattice of hit positions produced by PSA would result in unphysical spikes in the angular distributions, hit coordinates are randomly spread over a cube of 2mm a side around the original value.

- The most probable time sequence of hits is reconstructed via a χ^2-like procedure.

- A first selection of events (e.g. discarding those with one single hit) can be performed at this phase.

A (relatively small) fraction of errors is expected. Their origin will be discussed later.

Data analysis

For each event we determine:

- The γ emission angle Θ_γ and polarization $P(\Theta_\gamma)$
- The flight path r_{12}
- The polar scattering angle, as derived

  # from the coordinates of the 1st and 2nd hit

  $$cos\theta_G = \vec{r}_{12} \cdot \vec{r}_1 / (r_{12} r_1)$$

  # from the energy E_1 released at the first hit

  $$\cos\theta_E = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma - E_1}$$

- The azimuthal angle φ

Events have been classified according to the counter containing the first hit.

Further analysis for polarization

- Construction of the azimuthal angular distribution $f(\varphi; \mathrm{CoulEx})$ for the first Compton scattering of CoulEx γ rays (separately for each counter).
- Construction of the corresponding reference distribution $f(\varphi; \mathrm{ref})$ from ^{137}Cs data.
- Evaluation of the distribution of ratios $R(\varphi) = f(\varphi; \mathrm{CoulEx}) \ / \ f(\varphi; \mathrm{ref})$.
- Fit of $R(\varphi)$ with N $(1 + A \cos 2\phi)$ to obtain the asymmetry coefficient A.
- The ratio of A to the average polarization $\bar{P}$ gives the Analysing power.

Refinement of the analysis and instrumental effects

It would be easy to derive from the scattering angle θ of each event the theoretical analysing power and compare its average value with experimental results. The direct comparison would be disappointing, due to several instrumental effects which (although irrelevant for normal spectroscopy measurements) significantly reduce the measured asymmetry. Namely

- Uncertainties in the hit position
- Tracking errors
- Unresolved hits

We will briefly discuss their effects.

Effects of Errors on the coordinates

We assume $\Delta_x^2 = \Delta_y^2 = \Delta_z^2 \approx b/E_e$

If the tracking of the event is correct one can deduce (at the first order) the statistical uncertainties on the scattering angles:

$$\Delta_\varphi^2 = \frac{\Delta_x^2(E_1) + \Delta_x^2(E_2)}{(r_{12} \sin\theta)^2}$$

$$\Delta_{\cos\theta_G}^2 = \frac{\left[\Delta_x^2(E_1) + \Delta_x^2(E_2)\right] \sin^2\theta}{r_{12}^2}$$

The error on φ determines a decrease of the coefficient of $\cos 2\phi$. For a Gaussian distribution of the errors with variance $\Delta^2\pi$, the reduction coefficient is (again,at the first order)

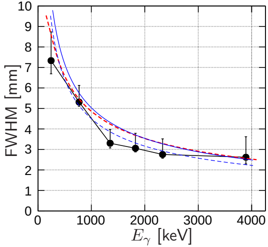
$$F_\Delta = e^{-2\Delta_\varphi^2}$$

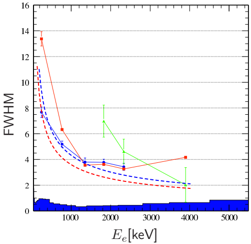
BUT: Is the first order sufficient?

and how to determine the value of b?

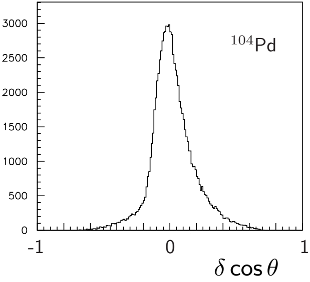
Experimental investigations of errors on hit position

(from S. Akkoyun et al., NIM A 668 (2012) 26; P.A. SöderstrÖ al., NIM A 638 (2011) 96.)





A first approximation has been obtained by a fit of these results...

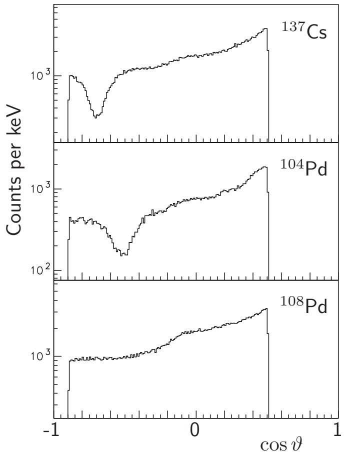


... but a fine tuning is obtained by comparing the estimate of $\Delta^2(\cos\theta_G)$ determined by the error Δ_x with the variance of the experimental distribution of $\delta\cos\theta = \cos\theta_G - \cos\theta_E$ (the error on $\cos\theta_E$ is negligible).

Tracking errors

Tracking errors can result as a consequence of the finite precision in the determination of hit positions.

Most of them (but not all!) will be discarded by the strict selection criteria. E.g., for $E_\gamma > m_e c^2$, the tracking of events consisting of only 2 hits is affected by an unresolvable ambiguity for a couple of angles θ_1 and $\theta_2 \approx \pi - \theta_1$ such that $E'_\gamma(E_\gamma, \theta_1) = E_\gamma - E'_\gamma(E_\gamma, \theta_2)$. In the distribution of $\cos\theta$ for ^{137}Cs and ^{104}Pd, a deep minimum at backward angles is apparent. Missing events in this region have been wrongly attributed to the corresponding forward angle.



Unresolved hits

In the current PSA procedure, only one hit per volume element is assumed.
If the 'first-interaction point' consists of two unresolved hits:

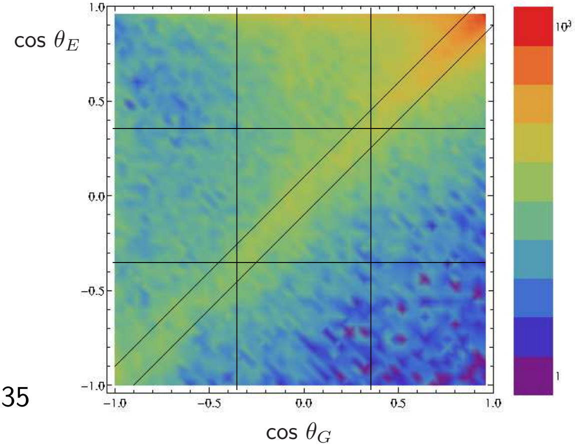
- The energy release and the scattering angle do not follow the Compton kinematics.
- The azimuthal angle φ keeps (almost) no memory of the initial polarization.

For a realistic evaluation of all these instrumental effects, a MonteCarlo simulation is necessary. This will be the subject of the second part of the lecture.

Selection criteria

- Total energy released in one triple cluster.
- Flight path of the scattered photon: $r_{12} > 15\text{mm}$
- Cuts on the scattering angle:
- We require:

$$|\cos\theta_G| < 0.35$$
$$|\cos\theta_E| < 0.35$$
$$|\cos\theta_G - \cos\theta_E| < 0.1$$



Correlation plot of $\cos\theta_E$ *vs.* $\cos\theta_G$

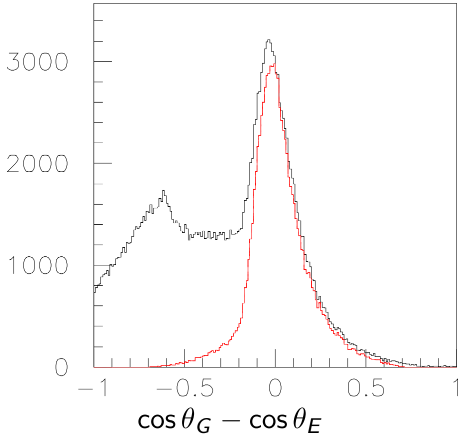
Solid lines: $\cos\theta = \pm 0,35$

Selection on the scattering angle θ

Selection criteria:

$|\cos\theta_G| < 0.35 \qquad |\cos\theta_E| < 0.35$

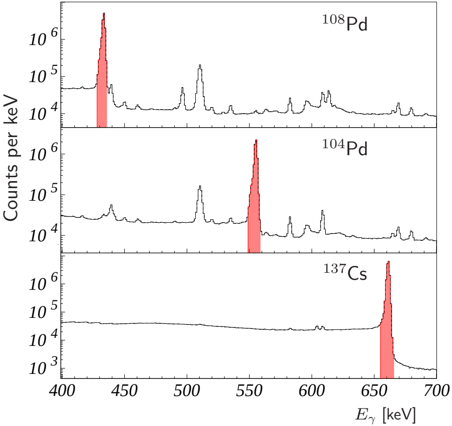
$|cos\theta_G - \cos\theta_E| < 0.1$



Distribution of the difference $\cos\theta_G - \cos\theta_E$
Black line: no cut on $\cos\theta_E$
Red line: $|\cos\theta_E| < 0.35$
(Counter C5)

Selection on total energy release

Selection thresholds: $|E_{tot} - E_\gamma| < \Delta E \approx 4$ keV



Fraction of underlying background in the full-energy gate:

^{108}Pd: $\approx 3\%$

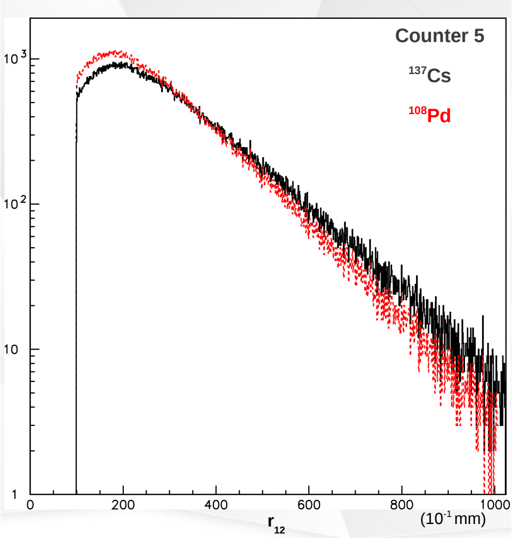
^{104}Pd: $\approx 3\%$

^{137}Cs: $\approx 1\%$

In case of a large underlying background it can be necessary to subtract, from the distributions corresponding to the full energy peak, those corresponding to an equivalent region of background.

Distributions of free path r_{12} for ^{108}Pd and ^{137}Cs

Selection threshold: $r_{12} > 15$mm.



The distributions of distances r_{12} for ^{137}Cs and ^{108}Pd are different due to the different energy of scattered photons at equal angle θ.

Moreover, also the angular distributions in θ are different.

Correction of the reference data

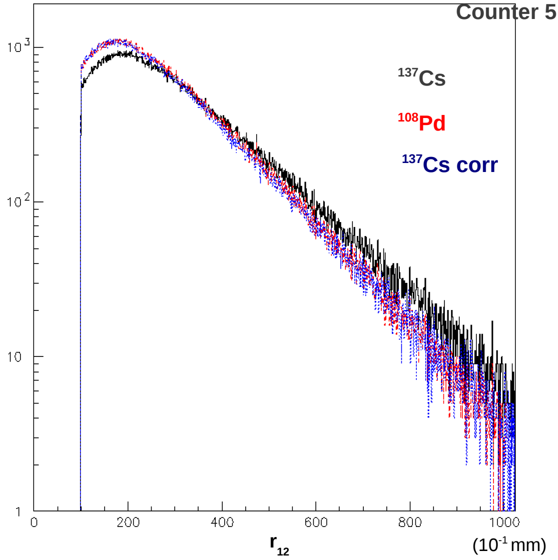
To remedy (at least partially) for these differences, corrections to reference angular distributions in φ (different for ^{104}Pd and ^{108}Pd) must be introduced. Namely, to each ^{137}Cs event is attributed a weight

$$w(\theta) = \frac{\mu(E'_{Pd}) \quad \exp[-\mu(E'_{Pd})r_{12}] \quad \mathrm{d}\sigma(E_{Pd}, \theta)/\mathrm{d}\Omega}{\mu(E'_{Cs}) \quad \exp[-\mu(E'_{Cs})r_{12}] \quad \mathrm{d}\sigma(E_{Cs}, \theta)/\mathrm{d}\Omega}$$

For each bin φ_k in the reference φ distribution, the resulting value and standard deviation are

$$N(\varphi_k) \pm \delta N(\varphi_k) = \sum_{i \in \varphi_k} w(\theta_i) \pm \sqrt{\sum_{i \in \varphi_k} w^2(\theta_i)}$$

Distributions of free path r_{12} after correction



I realize now that a further correction could be introduced for the different distributions in depth of the first Compton interaction:

$$w'(r_{01}) = \frac{e^{-\mu(E_{Pd})\ r_{01}}}{e^{-\mu(E_{Cs})\ r_{01}}}.$$

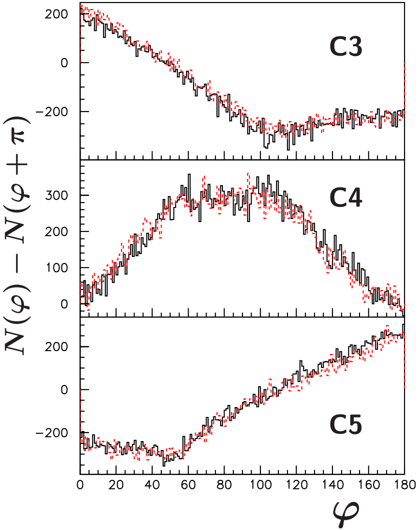
But its effect would be probably small.

r¡2¿

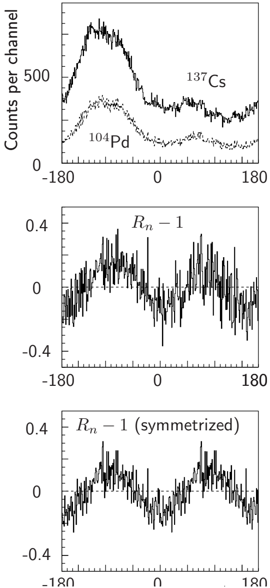
Consistency of results for ^{108}Pd and ^{137}Cs

The effects of polarization cancel (almost exactly) in the difference $N(\varphi) - N(\varphi + \pi)$

If the analysis is correct, the differences deduced from angular distributions of ^{108}Pd and ^{137}Cs (normalised to equal area) should overlap exactly.



Normalised ratios



Azimuthal distributions $N_{CE}(\varphi)$ (for CoulEx) and $N_{ref}(\varphi)$ (from Cs source) evaluated for the crystal containing the first interaction (in this example, C4).

Normalized ratios $R(\varphi)$ are deduced from the symmetrized distributions $N^s(\varphi) = [N(\varphi) + N(\varphi + \pi)]/2$:

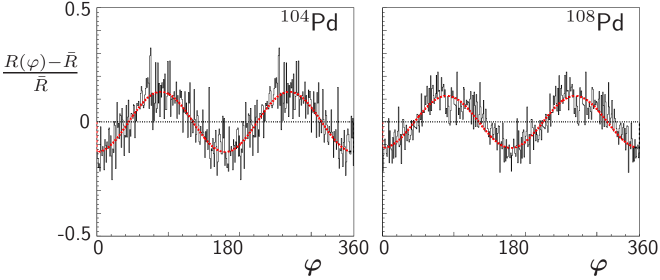
$$R(\varphi) = \frac{N^s_{CE}(\varphi)/\bar{N}^s_{CE}}{N^s_{ref}(\varphi)/\bar{N}^s_{ref}}$$

Asymmetries (C4)

Apart from second order corrections*,

Normalised ratios can be fitted with

$R(\varphi) = 1 + A \ \cos 2\varphi$

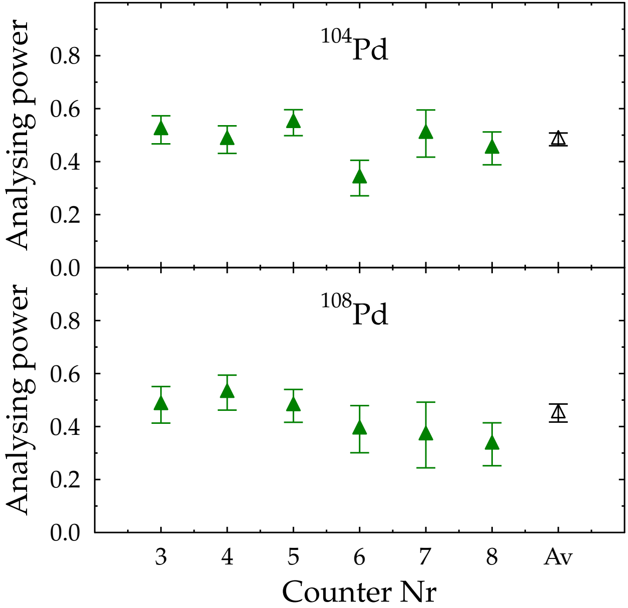


We define the Analysing Power $\mathcal{A} \equiv Q/2$

from the relation $A = \mathcal{A} \ \overline{P(\Theta)}$

* *More exactly* $E(a/b) \approx [E(a)/E(b))/\{1 + [D^2(b)/E^2(b)]\}$

Estimated Analysing Power



Layout

- Theoretical preliminaries
- Measurements with AGATA
  - AGATA as a segmented detector
  - Agata as a tracking detector
- MonteCarlo simulations
  - Introducing Polarization in MC results
  - comparison with CoulEx results
- Perspectives for future improvements

AGATA simulation in GEANT4

Realistic simulations of AGATA counters, triple clusters and various combinations of them have been developed in the frame of GEANT4.

I want to acknowledge here, once more, the fundamental contribution given by Enrico Farnea.

MonteCarlo and Polarization

Early attempt to account for Polarization in the first Compton scattering: $\approx$ 1990 polarization in GEANT 3 (Firenze).

The current version of AGATA MC includes the option for taking into account the polarization of primary γ rays.
However we have preferred to simulate events with a non polarized γ and introduce corrections for polarization later (as we did to derive reference distributions from ^{137}Cs data, correcting for different mean free path.)

How to introduce polarization in MonteCarlo results simulated without polarization

Suppose a simulated event k contains n hits at positions $\vec{X}_i$ corresponding to a sequence of $n-1$ Compton scatterings at angles $\theta_i(k), \varphi_i(k)$.

For a primary γ with linear polarization P, the probability of this event would be $W_P(k)$, while it is $W_0(k)$ for $P=0$ as assumed by the MonteCarlo.

By definition, the MonteCarlo procedure attributes equal weight to all the simulated events.

Instead, a weight $w(k) = W_P(k)/W_0(k)$ will be attributed to each event k, in order to deduce simulated results for polarization P.

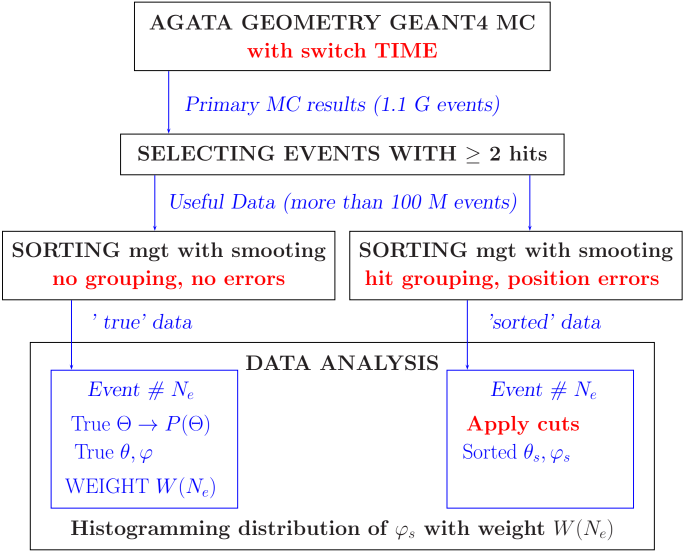
How to introduce polarization in MonteCarlo results simulated without polarization (2)

As a consequence, in every bin B_j of a simulated distribution, the simulated content will be

$$N_j = \sum_{k \in B_j} w(k) \pm \sqrt{\sum_{k \in B_j} w^2(k)}$$

This is a hybrid procedure, half-way between pure MonteCarlo and integration of the probability density over the available space of parameters. But, as we know, MonteCarlo itself can be considered as a form of numerical integration.

Block Diagram of the procedure



Analysis

Three azimuthal distributions $N(\varphi)$ are obtained:
$N_0(\varphi)$ for $P \equiv 0$ (no polarization)
$N_1(\varphi)$ for $P \equiv 1$ (full polarization)
$N_P(\varphi)$ for P as predicted for CoulEx.
Data are analysed separately according to the crystal containing the first interaction.
Ratios $R(\varphi)$ shows the expected dependence on φ:
$R_1(\varphi) = N_1(\varphi)/N_0(\varphi) \propto 1 + \mathcal{A} \cos 2\varphi$
$R_P(\varphi) = N_P(\varphi)/N_0(\varphi) \propto 1 + \mathrm{A} \cos 2\varphi$
where $\mathcal{A}$ is the Analysing power and A the asymmetry (different for each counter) to be compared with the experimental value.

BUT ....

is our complicated procedure really necessary?

One could use the results of 'Sorting with errors' and adjust the error parameters to reproduce ^{104}Pd. Yes, but ^{108}Pd will not be reproduced. In particular, for any choice of parameters, the predicted analysing power will be larger for ^{108}Pd than for ^{104}Pd, at variance with results of our procedure and experimental results.

This work is still in progress, and all data must be considered as preliminary results.

A word of caution ....

In the present analysis, effects of the polarization of the primary γ are considered only in the first Compton scattering.

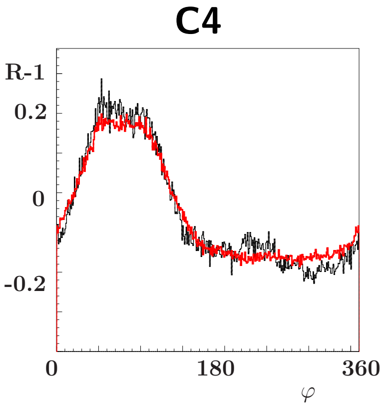
Consequences on the further scatterings are ignored.

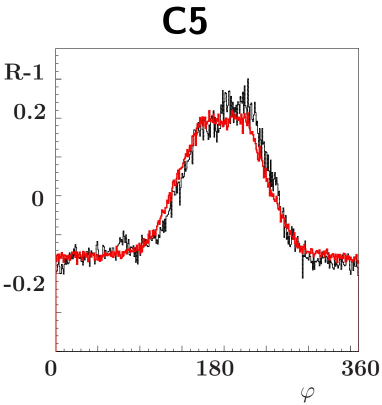
This is strictly valid for one Compton scattering, followed by total absorption of the scattered photon.

In fact, polarization of photons emerging from each Compton scattering will influence the azimuthal distribution of the next one and therefore the escape probability.

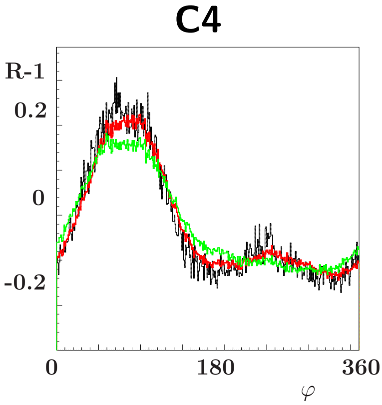
In principle, an exact calculation of W for more hits is possible. We will come again to this point later.

Comparison of Experimental Results for ^{137}Cs with MC Simulations

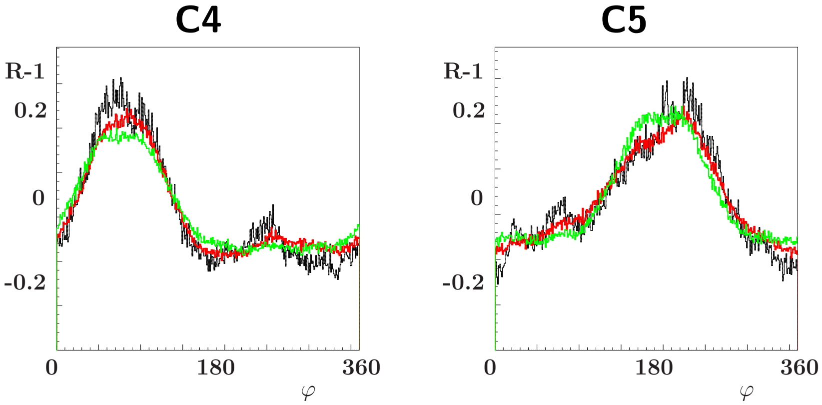




Comparison of Experimental Results for ^{104}Pd with MC Simulations (with polarization) and without polarization



Comparison of Experimental Results for ^{108}Pd with MC Simulations (with polarization) and without polarization



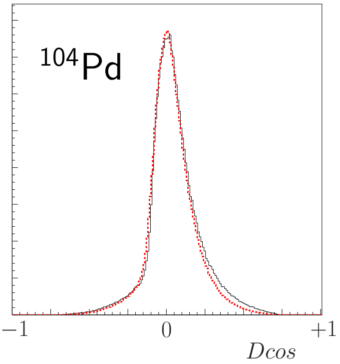
Experimental Results and MC Simulations

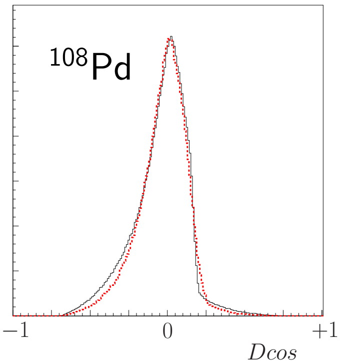
Distributions of $Dcos = \cos\theta_E - \cos\theta_G$ (C4)

Selection criteria:

$r_{12} > 15mm;$ $E_{tot} = E_\gamma \pm 4keV;$

$|\cos_{en}| < 0.35$ $|\cos_{ge}| < 0.35.$





Experimental Results and MC Simulations for Asymmetry ratios

Same selection criteria plus $|Dcos| < 0.1$

Experimental results: $R(\varphi) \Rightarrow$Pd / Cs

Simulated results:

simulation with expect P

divided by simulation with $P = 0$

Two possible Methods:

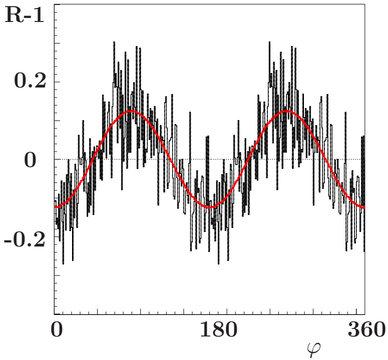
simulations with and without P

$\#1$ from the same set of MC data

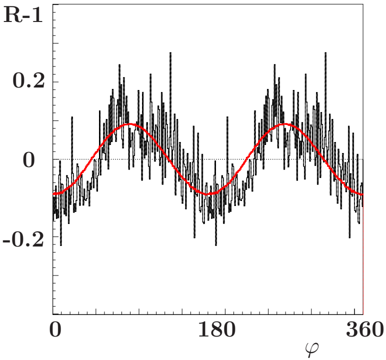
$\#2$ from independent sets

Comparison of Experimental Results with Simulation Method #1 (counter C4)

^{104}Pd

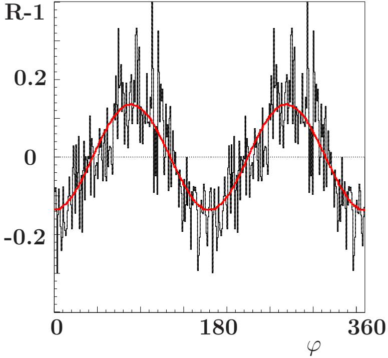


^{108}Pd

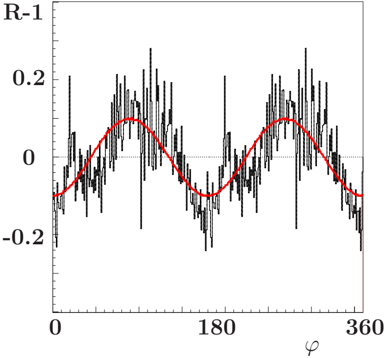


Comparison of Experimental Ratios with Simulation Method #1 (counter C5)

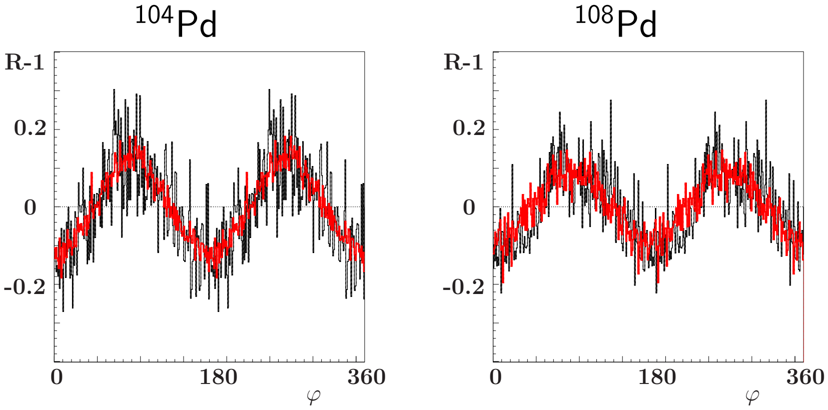
^{104}Pd



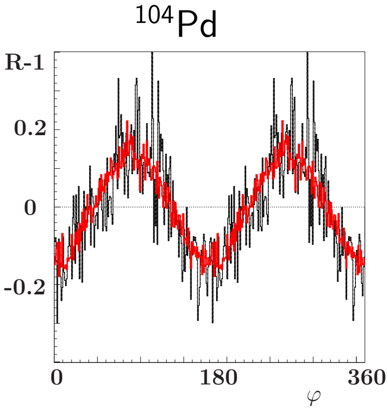
^{108}Pd

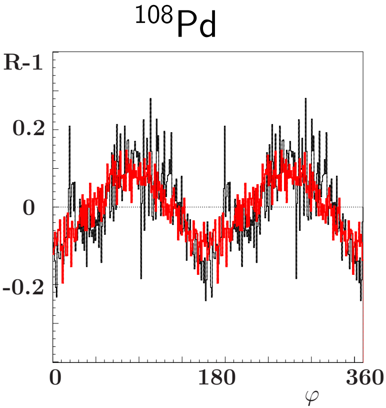


Comparison of Experimental Results with Simulation Method #2 (counter C4)

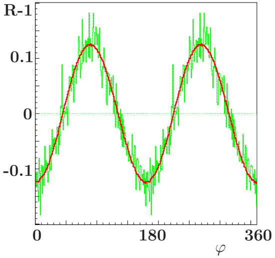


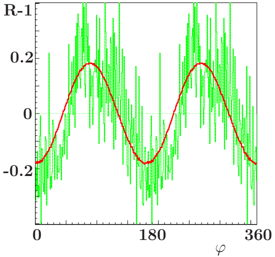
Comparison of Experimental Results with Simulation Method #2 (counter C5)





Comparison of MC Results for Ratios (counter C4) with Method #1 and Method #2





Which one is preferable?
It depends on the particular purpose:

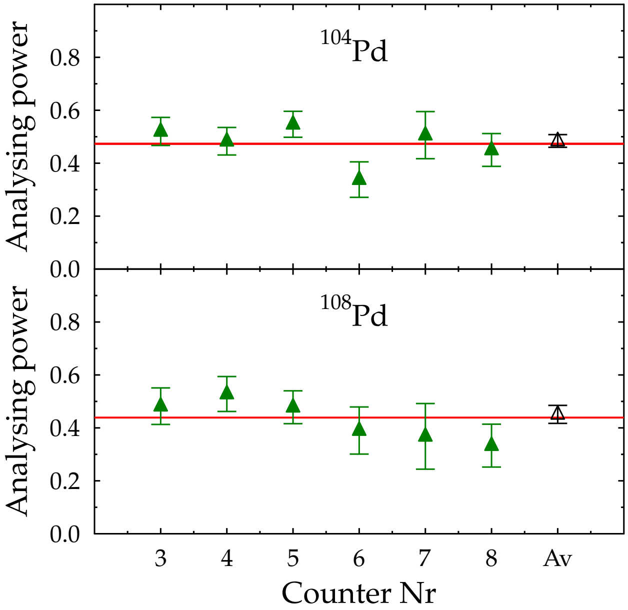
- Method #1:

More accurate,almost no fluctuations
Best used for Conclusions

- Method #2:

Realistic prediction of statistical errors
Best used for Proposals

Estimated Analysing Power (horizontal lines)



Layout

Theoretical preliminaries:

Measurements with AGATA

- AGATA as a segmented detector
- Agata as a tracking detector
  - The LNL CoulEx experiment

MonteCarlo simulations

Perspectives for future improvements

Possible improvements

Although present results show already a reasonable agreement between the analysis of experimental results and MonteCarlo simulation, we think some further improvement is possible on both.

- In the data analysis, e.g. in deriving from ^{137}Cs data the 'reference distributions' at the rather different energy of ^{108}Pd.
- In the MonteCarlo simulation, evaluating correctly the combined probability for the entire sequence of Compton scatterings.

Some comments on the MonteCarlo simulation for a sequence of Compton scatterings

Usually, MonteCarlo likes to work with a sequence of independent events, associated to given probabilities (cross sections). This is not possible in our case, as it is necessary to save memory of the polarization of intermediate photons (as it is correctly performed in GEANT4).

To this purpose, it is not sufficient to know the probabilities (cross section) for every step: we need the transition amplitudes. We shall see how they can be evaluated,

Pure polarization states

Pure polarization eigenstate:

for the photon: $|\mu>$; for the electron: $\nu = \pm 1/2 >$

Compton scattering amplitude from a pure state $|\mu\nu>$ to a pure state $|\mu'\nu'>$: $f(E, \theta, \varphi; \mu\nu, \mu'\nu')$

The amplitude for a process of two consecutive Compton scatterings, at angles θ_1, φ_1 and θ_2, φ_2, is the product of the two amplitudes, summed over the polarization states of the intermediate photon.

Cross section from a pure state $|\mu_1\nu_1\nu_2>$ to a pure state $|\mu_2'\nu_1'\nu_2'>$

$$\mathrm{d}\sigma \quad \propto \quad \left| \sum_{\mu_1'\mu_2} \delta_{\mu_1',\mu_2} f(E_1, \theta_1, \varphi_1; \mu_1\nu_1, \mu_1'\nu_1') f(E_2.\theta_2, \varphi_2; \mu_2\nu_2\mu_2'\nu_2') \right|^2$$

Mixed state !

We do not know the polarization of the two electrons.

$\Rightarrow$ Sum over the final electron polarizations
and average over the initial ones.

If also the final polarization of the photon is not measured, the average cross section for the entire process takes the form

$$\mathrm{d}\bar{\sigma} \propto \frac{1}{4} \sum_{\nu_1 \nu_1' \nu_2 \nu_2'} \sum_{\mu_2'} \sum_{\mu_1', \mu_1''} f^*(E_1, \theta_1, \varphi_1, \mu_1 \nu_1 \mu_1' \nu_1') f^*(E_2, \theta_2, \varphi_2, \mu_1' \nu_2 \mu_2' \nu_2') \\ f(E_1, \theta_1, \varphi_1, \mu_1 \nu_1 \mu_1'' \nu_1') f(E_2, \theta_2, \varphi_2, \mu_1'' \nu_2 \mu_2' \nu_2')$$

This expression cannot be factorised. Polarization of the intermediate photon must be taken into account.

Polarization transfer

The 2×2 density matrices $\rho_0,\ \rho_1$ and ρ_2 describe the polarization of the initial, intermediate and final photon.
We define the polarization-transfer matrix

$$T(\theta,\varphi;\mu_0,\mu_1,\mu_0',\mu_1') = \frac{1}{2}\sum_{\nu_1\nu_1'} f^*(\theta,\varphi,\mu_0\nu_1\mu_0'\nu_1')f(\theta,\varphi,\mu_1\nu_1\mu_1'\nu_1')$$

Then

$$\begin{aligned}
\rho_1 &= T(E_1,\theta_1,\varphi_1)\rho_0\tilde{T}(E_2,\theta_1,\varphi_1)\\
\rho_2 &= T(E_2,\theta_2,\varphi_2)\rho_1\tilde{T}(E_2,\theta_2,\varphi_2)\\
&= T(E_2,\theta_2,\varphi_2)T(E_1.\theta_1,\varphi_1)\rho_0\tilde{T}(E_1\theta_1,\varphi_1)\tilde{T}(E_2,\theta_2,\varphi_2)
\end{aligned}$$

Average cross section (associated to the expectation value of the operator $\mathcal{O}_2$ in the polarization space).

$$\mathrm{d}\bar{\sigma} \propto \mathrm{Tr}(\rho_2\mathcal{O}_2)$$

If the polarization of the final state is not observed, the operator $\mathcal{O}_2$ is the unit operator and $\mathrm{d}\bar{\sigma} \propto \mathrm{Tr}\ \rho_2$.

The Compton cascade

Until now, we have considered the case of two Compton scatterings, but the procedure can be easily extended to an arbitrary number of interactions in the Compton scattering chain.

This treatment of polarization can be inserted in the MonteCarlo procedure by taking memory of the polarization parameters (Stokes parameter) at each step of the Compton cascade, Thanks for your attention

Practical Session

INDEX

1 - 2 # Data files (sorted data)
from Legnaro experiment

3 - 4 # Algorithms for deriving the Compton
scattering angles

5 # Suggested exercises.

6 - 7 # Polarization of gammas from aligned states
(fusion-evaporation reactions)

1.- Data files:

dati-104pd.root exp. ^{104}Pd
dati-137cs.root exp. ^{137}Cs
mc-104pd-1.root MC ^{104}Pd, part1
mc-104pd-2.root MC ^{104}Pd, part2

sector-c.txt center of volume elements of the 6 counters in general coordinates.

Meaning of 'weight' (W)

dat-104pd.txt $W \equiv 1$
dat-137cs.txt W to construct reference for ^{104}Pd
mc-104pd-*.txt W to construct distributions for polarized γ from Coulex or put $W = 1$ for no polarization

2.- Record structure of data files:

cosen, cosge, diffcos, phi,
en1, en2, etotd, r12, costetagamma,
nc1, nsec1, nc2, nsec2, ind3, nhits, weight

diff-cos= cos-en – cos-ge

ind3=1: energy entirely released in 1 crystal
=2: in 1 triple cluster
=3: in more clusters

3.- Algorithms for the Compton scattering angles

Here, the primary reference frame is defined as having the z axis pointing to a symmetry axis of AGATA demonstrator and the x axis perpendicular to it and to the beam direction. We define:

Beam direction: $\hat{b} = \hat{e}_y \cos\alpha + \hat{e}_z \sin\alpha$

Coordinates of the γ source: $x_s = y_s = z_s = 0$

Coordinates of the first Compton interaction:

$\vec{r}_1 = x_1 \hat{e}_x + y_1 \hat{e}_y + z_1 \hat{e}_z$

Coordinates of the second interaction: $\vec{r}_2 = x_2 \hat{e}_x + y_2 \hat{e}_y + z_2 \hat{e}_z$

Normal to the scattering plane: $\hat{n}_1$

Angle of γemission with respect to the beam axis: Θ_γ

We can uase the relations:

$$r_1 \cos\Theta_\gamma = \hat{b} \cdot \vec{r}_1 = y_1 \cos\alpha + z_1 \sin\alpha$$

$$r_1 \sin\Theta_\gamma \hat{n}_1 = \hat{b} \times \vec{r}_1 = (z_1 \cos\alpha - y_1 \sin\alpha)\ \hat{e}_x + x_1 \sin\alpha\ \hat{e}_y - x_1 \cos\alpha\ \hat{e}_z$$

$$r_1 \sin\Theta_\gamma \cos\phi_1 = \hat{b} \times \vec{r}_1 \cdot \hat{e}_x = z_1 \cos\alpha - y_1 \sin\alpha$$

$$\hat{e}_x \times (\hat{b} \times \vec{r}_1) = x_1 \cos\alpha\ \hat{e}_y + x_1 \sin\alpha\ \hat{e}_z = x_1\ \hat{b}$$

$$r_1 \sin\Theta_\gamma \sin\phi_\gamma = [(\hat{b} \times \vec{r}_1) \times \hat{e}x] \cdot \hat{b} = -x_1$$

4.- Compton scattering

For the first Compton scattering:
Direction of the scattered γ: $\hat{r}_{12} = \vec{r}_{12}/|r_{12}|$, with $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$
Polar angle θ between $\vec{r}_{12}$ and $\vec{r}_1$.
Azimuthal angle φ between the planes $\vec{r}_{12}$, $\vec{r}_1$ and $\vec{b}$, $\vec{r}_1$ (or between the normals to these planes, $\hat{n}_{12}$ and $\hat{n}_1$, both perpendicular to $\vec{r}_1$).
We obtain them from the relations

$$r_1 r_{12} \cos\theta = \vec{r}_{12} \cdot \vec{r}_1 = x_1 x_{12} + y_1 y_{12} + z_1 z_{12}$$

$$r_1 r_{12} \sin\theta = |\vec{r}_1 \times \vec{r}_{12}|$$

$$\vec{r}_1 \times \vec{r}_{12} = (y_1 z_{12} - z_1 y_{12})\hat{e}_x + (z_1 x_{12} - x_1 z_{12})\hat{e}_y + (x_1 y_{12} - y_1 x_{12})\hat{e}_z$$

$$\cos\varphi = \hat{n}_1 \cdot \hat{n}_{12}$$

$$\sin\varphi = |\hat{n}_1 \times \hat{n}_{12} \cdot \vec{r}_{12}|/r_{12}$$

5.-Suggested exercises

Ratio of the azimuthal distributions

1.- Read the first data file in ROOT ntuple format

2.- Select events with proper cuts (variation around suggested values are welcome).

Suggeste values: r12 > 15

$|\cos - \mathrm{en}| < 0.35$; $\cos - \mathrm{ge}| < 0.35$

$|$diff-cos$| < 0.1$

$|\mathrm{Etotd} - Etrue| < 4$

with .8; $Etrue(^{137}\mathrm{Cs})=661$

ind3 $= 2$ (values 1 and 3 could be tried!)

nhits: no limits (a limit to nhits=2 could be interesting)

3.- Construct the histograms of φ with proper cuts and weights.

Suggested step 1° other values welcome.

4.- Optionally: Symmetrize: $N^s(\varphi) = N(\varphi) + N(\varphi + 180)$

5.- Repeat points 1 to 4 for the second file. 6.- Construct the ratio of the relevant spectra (from 0° to 180°)

e.g. Pd / Cs(weighted) or MC(weighted) / MC(W=1)

7.- Fit with the function $A + B\cos 2\varphi$

6.- Polarization of gammas from aligned states

From Ferguson* Eq. 3.66:

$$P(\Theta_\gamma) = \frac{A_+}{A_-}$$

where

$$A_\pm = \sum_{kLL'} \rho_{k0}(aa)(-)^{b-a}\bar{Z}_1(LaL'a, bk)\ \delta^r \left[\ P_k(\cos\Theta_\gamma) \pm (-)^{\pi'} K_k(LL') P_k^2(\cos\Theta_\gamma)\ \right]$$

$$K_k(LL') = -\sqrt{\frac{(k-2)!}{(k+2)!}}\ \frac{(L1, L'1|k2)}{(L1, L'-1|k0)}$$

Where a (b) is the spin of the parent (daughter) state, L the multipole order of the transition, and the exponent r of the multipole mixing coefficient δ is 0, 1 or 2 according to the number of indexes L corresponding to the higher multipole.

.* D.J. Ferguson, Angular correlation methods in gamma-ray spectroscopy (Amsterdam 1965). Eq. 3.66 contains an obvious printing error, see eq. 3.63. .

7.- The coefficients

$$\bar{Z}_1(LbL'b';ck) = (-)^{k-L+L'-1}\hat{L}\hat{L}'\hat{b}\hat{b}'(L1,L'-1|k0)W(LbL'b';ck)$$

where $W(LbL'b';ck) = (-)^{L+L'+b+b'}W_{6J}(Lbc;b'L'k)$ is a Racah coefficient, and $\hat{L} \equiv \sqrt{2L+1}$. The coefficients $\bar{Z}_1$ are tabulated by Ferguson. $P_k(\cos\Theta_\gamma)$ is a Legendre polynomial, $P_k^2(\cos\Theta_\gamma)$ is an associated Legendre polynomial.

The statistical tensors are expressed as a function of the density matrix as

$$\rho_{k\kappa}(a,a) = \sum_{\alpha\alpha'}(-)^{a-\alpha'}(a\ \alpha, a-\alpha'|k\kappa)\left\langle a\ \alpha|\rho|a\ \alpha'\right\rangle$$

For an aligned (axially symmetric) system $\kappa \equiv 0$ and

$$\rho_{k0}(a,a) = \sum_{\alpha}(-)^{a-\alpha}(a\ \alpha, a-\alpha|k0)\left\langle a\ \alpha|\rho|a\ \alpha\right\rangle$$

Maximum alignment: $\langle a\ 0|\rho|a\ 0\rangle = 1$ for a even,
$\langle a\ \pm 1/2|\rho|a\ \pm 1/2\rangle = 1/2$ for a odd.