

# Testing New Physics with the lepton $g - 2$

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DaMeSyFla: CP Violation 50 years after discovery  
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- **The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:**
  - ▶ Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?
  - ▶ Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?
- **Related important questions are:**
  - ▶ Which is the role of **flavor physics** in the **LHC** era?
  - ▶ Do we expect to understand the (SM and NP) **flavor puzzles** through the synergy and interplay of **flavor physics** and the **LHC**?

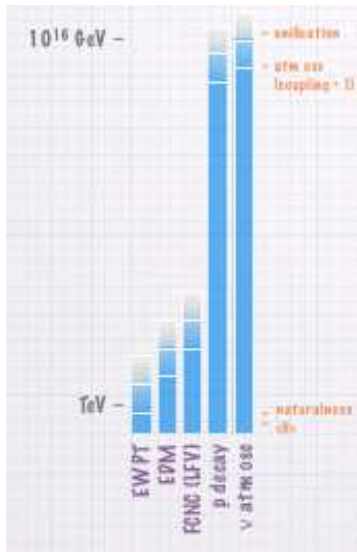
# The NP “scale”

- **Gravity**  $\implies \Lambda_{\text{Planck}} \sim 10^{18-19}$  GeV
- **Neutrino masses**  $\implies \Lambda_{\text{see-saw}} \lesssim 10^{15}$  GeV
- **BAU**: evidence of CPV beyond SM
  - ▶ Electroweak Baryogenesis  $\implies \Lambda_{\text{NP}} \lesssim \text{TeV}$
  - ▶ Leptogenesis  $\implies \Lambda_{\text{see-saw}} \lesssim 10^{15}$  GeV
- **Hierarchy problem**:  $\implies \Lambda_{\text{NP}} \lesssim \text{TeV}$
- **Dark Matter**  $\implies \Lambda_{\text{NP}} \lesssim \text{TeV}$

## SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d \geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{\text{NP}}^{d-4}} \mathcal{O}_{ij}^{(d)}$$

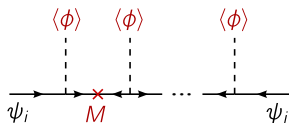
- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_{\nu}^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi \phi$ ,
- $\mathcal{L}_{\text{eff}}^{d=6}$  generates FCNC operators



$$\text{BR}(l_i \rightarrow l_j \gamma) \sim \frac{1}{\Lambda_{\text{NP}}^4}$$

- Can the SM and NP flavour problems have a common explanation?
- **Froggat-Nielsen '79: Hierarchies from SSB of a Flavour Symmetry**

$$\epsilon = \frac{\langle \phi \rangle}{M} \ll 1 \Rightarrow Y_{ij} \propto \epsilon^{(a_i+b_j)}$$



- **Flavor protection from flavor models:** [Lalak, Pokorski & Ross '10]

Operator	$U(1)$	$U(1)^2$	$SU(3)$	MFV
$(\bar{Q}_L X_{LL}^Q Q_L)_{12}$	$\lambda$	$\lambda^5$	$\lambda^3$	$\lambda^5$
$(\bar{D}_R X_{RR}^D D_R)_{12}$	$\lambda$	$\lambda^{11}$	$\lambda^3$	$(y_d y_s) \times \lambda^5$
$(\bar{Q}_L X_{LR}^D D_R)_{12}$	$\lambda^4$	$\lambda^9$	$\lambda^3$	$y_s \times \lambda^5$

- Is this flavor protection enough?
- Is it possible to disentangle among different flavour models by means of their predicted pattern of deviation w.r.t. the SM predictions in flavour physics?

## • Why CP violation? Motivation:

- ▶ **Baryogenesis** requires extra sources of CPV
- ▶ The QCD  $\bar{\theta}$ -term  $\mathcal{L}_{CP} = \bar{\theta} \frac{\alpha_s}{8\pi} G\tilde{G}$  is a CPV source beyond the CKM
- ▶ Most UV completion of the SM, e.g. the MSSM, have many CPV sources
- ▶ However, TeV scale NP with  $\mathcal{O}(1)$  CPV phases generally leads to EDMs many orders of magnitude above the current limits  $\Rightarrow$  the New Physics CP problem.

## • How to solve the New Physics CP problem?

- ▶ **Decoupling** some NP particles in the loop generating the EDMs (e.g. hierarchical sfermions, split SUSY, 2HDM limit...)
- ▶ Generating **CPV phases radiatively**  $\phi_{CP}^f \sim \alpha_w/4\pi \sim 10^{-3}$
- ▶ Generating **CPV phases** via **small flavour mixing angles**  $\phi_{CP}^f \sim \delta_{ij}\delta_{ij}$  with  $f = e, u, d$ : maybe the suppression of FCNC processes and EDMs have a common origin?

- **High-energy frontier**: A unique effort to determine the NP scale
- **High-intensity frontier** (flavor physics): A collective effort to determine the flavor structure of NP

Where to look for **New Physics** at the low energy?

- Processes very **suppressed** or even **forbidden** in the SM
  - ▶ FCNC processes ( $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow eee$ ,  $\mu \rightarrow e$  in N,  $\tau \rightarrow \mu\gamma$ ,  $B_{s,d}^0 \rightarrow \mu^+\mu^- \dots$ )
  - ▶ CPV effects in the electron/neutron EDMs,  $d_{e,n} \dots$
  - ▶ FCNC & CPV in  $B_{s,d}$  &  $D$  decay/mixing amplitudes
- Processes predicted with **high precision** in the SM
  - ▶ EWPO as  $(g-2)_{\mu,e}$ :  $a_{\mu}^{exp} - a_{\mu}^{SM} \approx (3 \pm 1) \times 10^{-9}$ , a discrepancy at  $3\sigma$
  - ▶ LU in  $R_M^{e/\mu} = \Gamma(M \rightarrow e\nu)/\Gamma(M \rightarrow \mu\nu)$  with  $M = \pi, K$

# Experimental status

process	current exp.	future exp.
$K^0$ mixing	$\epsilon_K = (2.228 \pm 0.011) \times 10^{-3}$	—
$D^0$ mixing	$A_\Gamma = (-0.02 \pm 0.16)\%$	$\pm 0.007\%$ LHCb $\pm 0.06\%$ Belle II
$B_d$ mixing	$\sin 2\beta = 0.68 \pm 0.02$	$\pm 0.008$ LHCb $\pm 0.012$ Belle II
$B_s$ mixing	$\phi_s = 0.01 \pm 0.07$	$\pm 0.008$ LHCb
$d_{\text{Hg}}$	$< 3.1 \times 10^{-29}$ ecm	—
$d_{\text{Ra}}$	—	$\lesssim 10^{-29}$ ecm
$d_n$	$< 2.9 \times 10^{-26}$ ecm	$\lesssim 10^{-28}$ ecm
$d_p$	—	$\lesssim 10^{-29}$ ecm
$d_e$	$< 1.05 \times 10^{-27}$ ecm YbF	$\lesssim 10^{-30}$ ecm YbF, Fr
$\mu \rightarrow e\gamma$	$< 5.4 \times 10^{-13}$ MEG	$\lesssim 6 \times 10^{-14}$ MEG upgrade
$\mu \rightarrow 3e$	$< 1.0 \times 10^{-12}$ SINDRUM I	$\lesssim 10^{-16}$ Mu3e
$\mu \rightarrow e$ in Au	$< 7.0 \times 10^{-13}$ SINDRUM II	—
$\mu \rightarrow e$ in Al	—	$\lesssim 6 \times 10^{-17}$ Mu2e

**Table:** Summary of current and selected future expected experimental limits on CP violation in meson mixing, EDMs and lepton flavor violating processes.

LFV process	Experiment	Future limits	Year (expected)
$\text{BR}(\mu \rightarrow e\gamma)$	MEG	$\mathcal{O}(10^{-13})$	$\sim 2013$
	Project X	$\mathcal{O}(10^{-15})$	$> 2021$
$\text{BR}(\mu \rightarrow eee)$	Mu3e	$\mathcal{O}(10^{-15})$	$\sim 2017$
	Mu3e	$\mathcal{O}(10^{-16})$	$> 2017$
	MUSIC	$\mathcal{O}(10^{-16})$	$\sim 2017$
	Project X	$\mathcal{O}(10^{-17})$	$> 2021$
$\text{CR}(\mu \rightarrow e)$	COMET	$\mathcal{O}(10^{-17})$	$\sim 2017$
	Mu2e	$\mathcal{O}(10^{-17})$	$\sim 2020$
	PRISM/PRIME	$\mathcal{O}(10^{-18})$	$\sim 2020$
	Project X	$\mathcal{O}(10^{-19})$	$> 2021$
$\text{BR}(\tau \rightarrow \mu\gamma)$	Belle II	$\mathcal{O}(10^{-8})$	$> 2020$
$\text{BR}(\tau \rightarrow \mu\mu\mu)$	Belle II	$\mathcal{O}(10^{-10})$	$> 2020$
$\text{BR}(\tau \rightarrow e\gamma)$	Belle II	$\mathcal{O}(10^{-9})$	$> 2020$
$\text{BR}(\tau \rightarrow \mu\mu\mu)$	Belle II	$\mathcal{O}(10^{-10})$	$> 2020$

**Table:** Future sensitivities of next-generation experiments.



- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e \frac{m_\ell}{2} (\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A_{\ell\ell'}^* \ell_R) F^{\mu\nu} \quad \ell, \ell' = e, \mu, \tau,$$

$$A_{\ell\ell'} = \frac{1}{(4\pi \Lambda_{\text{NP}})^2} \left[ \left( g_{\ell k}^L g_{\ell' k}^{L*} + g_{\ell k}^R g_{\ell' k}^{R*} \right) f_1(x_k) + \frac{v}{m_\ell} \left( g_{\ell k}^L g_{\ell' k}^{R*} \right) f_2(x_k) \right],$$

- ▶  $\Delta a_\ell$  and leptonic EDMs are given by

$$\Delta a_\ell = 2m_\ell^2 \text{Re}(A_{\ell\ell}), \quad \frac{d_\ell}{e} = m_\ell \text{Im}(A_{\ell\ell}).$$

- ▶ The branching ratios of  $\ell \rightarrow \ell' \gamma$  are given by

$$\frac{\text{BR}(\ell \rightarrow \ell' \gamma)}{\text{BR}(\ell \rightarrow \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3 \alpha}{G_F^2} \left( |A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2 \right).$$

- **“Naive scaling”:**

$$\Delta a_{\ell_i} / \Delta a_{\ell_j} = m_{\ell_i}^2 / m_{\ell_j}^2, \quad d_{\ell_i} / d_{\ell_j} = m_{\ell_i} / m_{\ell_j}.$$

(for instance, if the new particles have an underlying SU(3) flavor symmetry in their mass spectrum and in their couplings to leptons, which is the case for gauge interactions).

- $(g-2)_\ell$  assuming “Naive scaling”  $\Delta a_{\ell_i}/\Delta a_{\ell_j} = m_{\ell_i}^2/m_{\ell_j}^2$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}, \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}.$$

- EDMs assuming “Naive scaling”  $d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}$

$$d_e \simeq \left( \frac{\Delta a_e}{7 \times 10^{-14}} \right) 10^{-24} \tan \phi_e \text{ e cm},$$

$$d_\mu \simeq \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \tan \phi_\mu \text{ e cm},$$

$$d_\tau \simeq \left( \frac{\Delta a_\tau}{8 \times 10^{-7}} \right) 4 \times 10^{-21} \tan \phi_\tau \text{ e cm},$$

- $\text{BR}(\ell_i \rightarrow \ell_j \gamma)$  vs.  $(g-2)_\mu$

$$\text{BR}(\mu \rightarrow e \gamma) \approx 3 \times 10^{-13} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{e\mu}}{10^{-5}} \right)^2,$$

$$\text{BR}(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-8} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left( \frac{\theta_{\ell\tau}}{10^{-2}} \right)^2.$$

[Giudice, P.P., & Passera, '12]

- **Challenge:** Large effects for  $g-2$  keeping under control  $\mu \rightarrow e\gamma$  and  $d_e$
- **“Disoriented A-terms”** [Giudice, Isidori & P.P., '12]:

$$(\delta_{LR}^{ij})_f \sim \frac{A_f \theta_{ij}^f m_{f_j}}{m_{\tilde{f}}} \quad f = u, d, \ell,$$

- ▶ Flavor and CP violation is restricted to the trilinear scalar terms.
- ▶ Flavor bounds of the down-sector are naturally satisfied thanks to the smallness of down-type quark/lepton masses.
- ▶ This ansatz arises in scenarios with partial compositeness where we a natural prediction is  $\theta_{ij}^\ell \sim \sqrt{m_i/m_j}$  [Rattazzi et al., '12].
- $\mu \rightarrow e\gamma$  and  $d_e$  are generated only by  $U(1)$  interactions

$$A_L^{\mu e} \sim \frac{\alpha}{\cos^2 \theta_W} \delta_{LR}^{\mu e}, \quad \frac{d_e}{e} \sim \frac{\alpha}{\cos^2 \theta_W} \text{Im} \delta_{LR}^{ee}.$$

- $(g-2)_\mu$  is generated by  $SU(2)$  interactions and is  $\tan \beta$  enhanced therefore the relative enhancement w.r.t.  $\mu \rightarrow e\gamma$  and  $d_e$  is  $\tan \beta / \tan^2 \theta_W \approx 100 \times (\tan \beta / 30)$

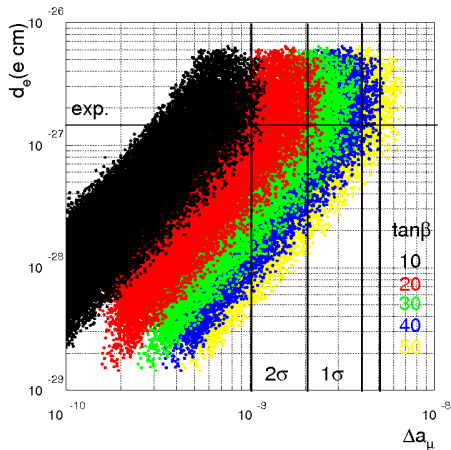
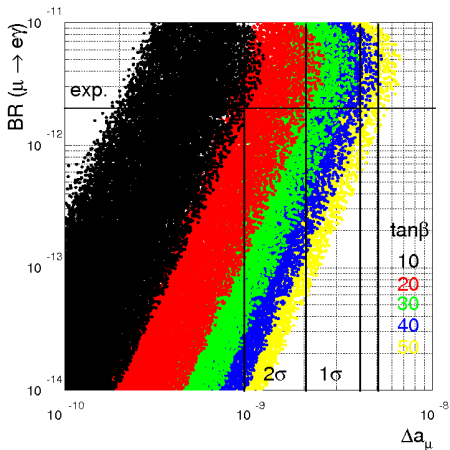
$$\Delta a_\ell \sim \frac{\alpha}{\sin^2 \theta_W} \tan \beta$$

- **Numerical example:**  $\tilde{m} = |A_e| = 1$  TeV,  $\sin \phi_{A_e} = 1$ ,  $M_2 = \mu = 2M_1 = 0.2$  TeV, and  $\tan \beta = 30$  [Giudice, P.P., & Passera, '12]

$$\begin{aligned}\text{BR}(\mu \rightarrow e\gamma) &\approx 6 \times 10^{-13} \left| \frac{A_\ell}{\text{TeV}} \frac{\theta_{12}^\ell}{\sqrt{m_e/m_\mu}} \right|^2 \left( \frac{\text{TeV}}{m_{\tilde{\ell}}} \right)^4, \\ d_e &\approx 4 \times 10^{-28} \text{Im} \left( \frac{A_\ell \theta_{11}^\ell}{\text{TeV}} \right) \left( \frac{\text{TeV}}{m_{\tilde{\ell}}} \right)^2 e \text{ cm}, \\ \Delta a_\mu &\approx 1 \times 10^{-9} \left( \frac{\text{TeV}}{m_{\tilde{\ell}}} \right)^2 \left( \frac{\tan \beta}{30} \right).\end{aligned}$$

- ▶ Disoriented A-terms can account for  $(g-2)_\mu$ , satisfy the bounds on  $\mu \rightarrow e\gamma$  and  $d_e$ , while giving predictions for  $\mu \rightarrow e\gamma$  and  $d_e$  within experimental reach.
- ▶ The electron  $(g-2)$  follows “naive scaling”.

# A concrete SUSY scenario: “Disoriented A-terms”



Predictions for  $\mu \rightarrow e\gamma$ ,  $\Delta a_\mu$  and  $d_e$  in the disoriented A-term scenario with  $\theta_{ij}^\ell = \sqrt{m_i/m_j}$ . Left:  $\mu \rightarrow e\gamma$  vs.  $\Delta a_\mu$ . Right:  $d_e$  vs.  $\Delta a_\mu$  [Giudice, P.P., & Passera, '12]

- **Longstanding muon  $g - 2$  anomaly**

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = 2.90(90) \times 10^{-9}, \quad \mathbf{3.5\sigma \text{ discrepancy}}$$

- **NP effects are expected to be of order  $a_\ell^{\text{NP}} \sim a_\ell^{\text{EW}}$**

$$a_\mu^{\text{EW}} = \frac{m_\mu^2}{(4\pi v)^2} \left( 1 - \frac{4}{3} \sin^2 \theta_W + \frac{8}{3} \sin^4 \theta_W \right) \approx 2 \times 10^{-9}.$$

- **Main question: how could we check if the  $a_\mu$  discrepancy is due to NP?**
- **Answer: testing new-physics effects in  $a_e$**  [Giudice, P.P. & Passera, '12]
- **“Naive scaling”:**  $\Delta a_{\ell_i} / \Delta a_{\ell_j} = m_{\ell_i}^2 / m_{\ell_j}^2$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}.$$

- ▶  $a_e$  has never played a role in testing beyond SM effects. From  $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ , we extract  $\alpha$  which is the most precise value of  $\alpha$  available today!
- ▶ The situation has now changed thanks to progresses both on the th. and exp. sides.

- **Standard Model vs. measurement**

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -10.6 (8.1) \times 10^{-13},$$

- ▶ Beautiful test of QED at four-loop level!
- ▶  $\delta \Delta a_e = 8.1 \times 10^{-13}$  is dominated by  $\delta a_e^{\text{SM}}$  through  $\delta \alpha$  ( $^{87}\text{Rb}$ ).

- **Future improvements in the determination of  $\Delta a_e$**

$$\underbrace{(0.6)_{\text{QED4}}, (0.4)_{\text{QED5}}, (0.2)_{\text{HAD}}, (7.6)_{\delta\alpha}, (2.8)_{\delta a_e^{\text{EXP}}}}_{(0.7)_{\text{TH}}} \quad (1)$$

- ▶ The first error,  $0.6 \times 10^{-13}$ , stems from numerical uncertainties in the four-loop QED. It can be reduced to  $0.1 \times 10^{-13}$  with a large scale numerical recalculation. [Kinoshita]
  - ▶ The second error, from five-loop QED term may soon drop to  $0.1 \times 10^{-13}$ .
  - ▶ Experimental uncertainties  $2.8 \times 10^{-13}$  ( $\delta a_e^{\text{EXP}}$ ) and  $7.6 \times 10^{-13}$  ( $\delta \alpha$ ) dominate. We expect a reduction of the former error to a part in  $10^{-13}$  (or better). [Gabrielse] Work is also in progress for a significant reduction of the latter error. [Nez]
- **$\Delta a_e$  at the  $10^{-13}$  (or below) is not too far! This will bring  $a_e$  to play a pivotal role in probing new physics in the leptonic sector.**

- SUSY contributions to  $a_\ell$  comes from loops with exchange of chargino/sneutrino or neutralino/charged slepton.
- **Violations of “naive scaling”** can arise through sources of non-universalities in the slepton mass matrices in two possible ways
  - ▶ **Lepton flavor conserving (LFC) case.** The charged slepton mass matrix violates the global non-abelian **flavor symmetry**, but preserves  $U(1)^3$ . This case is characterized by non-degenerate sleptons ( $m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$ ) but vanishing mixing angles because of an exact alignment.
    - Interesting interplay with collider physics: violations of “naive scaling” imply a different number of events in the  $2\ell + \cancel{E}_T$  final states ( $\ell = e, \mu$ ) arising from the decay of slepton pairs produced via Drell-Yan [Calibbi, P.P., in progress].
  - ▶ **Lepton flavor violating (LFV) case.** The slepton mass matrix fully breaks **flavor symmetry** up to  $U(1)$  lepton number, generating mixing angles that allow for flavor transitions. Lepton flavor violating processes, such as  $\mu \rightarrow e\gamma$ , provide stringent constraints on this case. However, because of flavor transitions,  $a_e$  and  $a_\mu$  can receive new large contributions proportional to  $m_\tau$  (from a chiral flip in the internal line of the loop diagram), giving a new source of non-naive scaling.



- In the LFC case, we assume  $m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$  but flavor alignment between lepton and slepton mass matrices to avoid LFV. This is reminiscent of the alignment mechanism [Nir & Seiberg, '93], proposed to solve the supersymmetric flavor problem in the quark sector (which might arise naturally in the context of abelian flavor models).

$$\Delta a_{\ell}^{\text{LFC}} \approx 3 \times 10^{-9} \left( \frac{m_{\ell}}{m_{\mu}} \right)^2 \left( \frac{100 \text{ GeV}}{m_{\tilde{\ell}}} \right)^2 \left( \frac{\tan \beta}{3} \right).$$

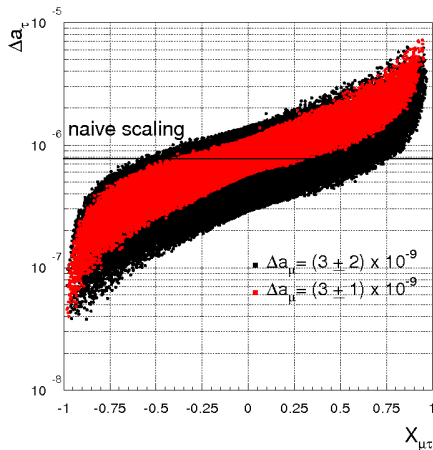
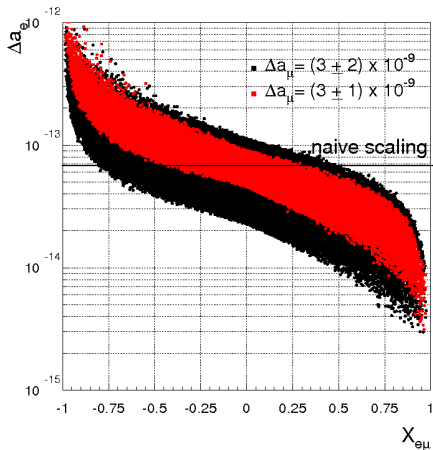
- Assuming that sleptons are the heaviest particles running in the loop

$$\Delta a_e \approx \Delta a_{\mu} \frac{m_e^2}{m_{\mu}^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \left( \frac{\Delta a_{\mu}}{3 \times 10^{-9}} \right) 10^{-13},$$

$$\Delta a_{\tau} \approx \Delta a_{\mu} \frac{m_{\tau}^2}{m_{\mu}^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \left( \frac{\Delta a_{\mu}}{3 \times 10^{-9}} \right) 10^{-6}.$$

- For values of  $\Delta a_{\mu}$  explaining the muon  $g-2$ , non-degenerate sleptons at the level  $m_{\tilde{\mu}} \approx 3 m_{\tilde{e}}$  lead to  $\Delta a_e \approx 10^{-12}$ , which is at the limit of present experimental sensitivity.

# Lepton flavor conserving case



Left:  $\Delta a_e$  as a function of  $X_{e\mu} = (m_\theta^2 - m_\mu^2)/(m_\theta^2 + m_\mu^2)$ . Right:  $\Delta a_\tau$  as a function of  $X_{\mu\tau} = (m_\mu^2 - m_\tau^2)/(m_\mu^2 + m_\tau^2)$ . Black points satisfy the condition  $1 \leq \Delta a_\mu \times 10^9 \leq 5$ , while red points correspond to  $2 \leq \Delta a_\mu \times 10^9 \leq 4$ .

- In SUSY, “naive scaling” violations for  $(g - 2)_\ell$  can arise through sources of non-universalities in the slepton masses.
- In turn, these non-universalities will induce violations of lepton flavor universality in  $P \rightarrow \ell\nu$ ,  $\tau \rightarrow P\nu$  (where  $P = \pi, K$ ),  $\ell_i \rightarrow \ell_j\bar{\nu}\nu$ ,  $Z \rightarrow \ell\ell$  and  $W \rightarrow \ell\nu$  through loop effects.
- LFU has been tested at the 0.1% level so far.
- It is interesting to study the **correlation** between such **LFU** and departures from “**naive scaling**” for  $\Delta a_\ell$ .
- Taking for example the process  $P \rightarrow \ell\nu$ , we can define the quantity

$$\frac{(R_P^{e/\mu})_{\text{EXP}}}{(R_P^{e/\mu})_{\text{SM}}} = 1 + \Delta r_P^{e/\mu} .$$

- ▶  $R_P^{e/\mu} = \Gamma(P \rightarrow e\nu)/\Gamma(P \rightarrow \mu\nu)$
- ▶  $\Delta r_P^{e/\mu} \neq 0$  signals the presence of new physics violating LFU.

- In SUSY, in the absence of LFV sources,  $\Delta r_P^{e/\mu}$  is induced at the loop level through sparticle exchange. The parametrical structure of  $\Delta r_P^{e/\mu}$  is

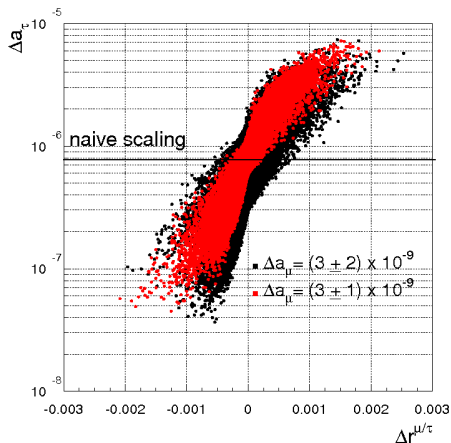
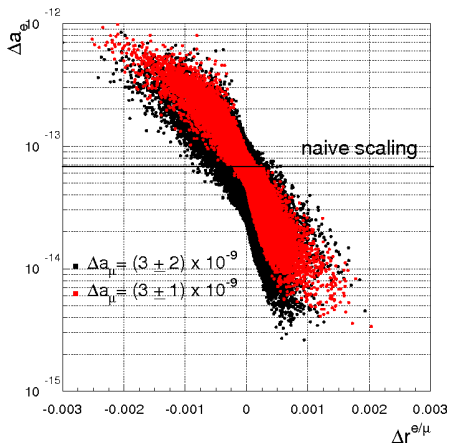
$$\Delta r_P^{e/\mu} \sim \frac{\alpha}{4\pi} \left( \frac{m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2 + m_{\tilde{\mu}}^2} \right) \frac{v^2}{\min(m_{\tilde{e},\tilde{\mu}}^2)},$$

- The term  $v^2/\min(m_{\tilde{e},\tilde{\mu}}^2)$  stems from SU(2) breaking effects which arise from 1) left-right soft breaking terms, 2) mixing terms in the chargino/neutralino mass matrices, or 3) D-terms.
- “**naive scaling**” violations for  $\Delta a_\ell$

$$\Delta a_e \approx \Delta a_\mu \frac{m_{\tilde{e}}^2}{m_\mu^2} \frac{m_{\tilde{\mu}}^2}{m_e^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-13},$$

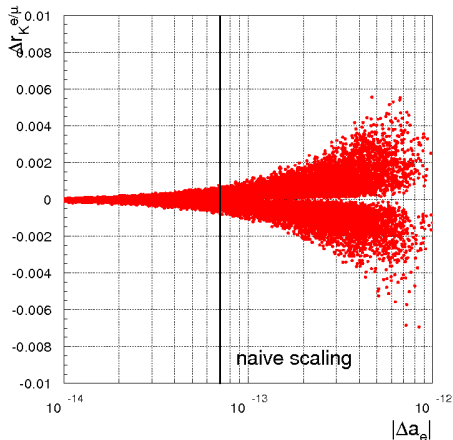
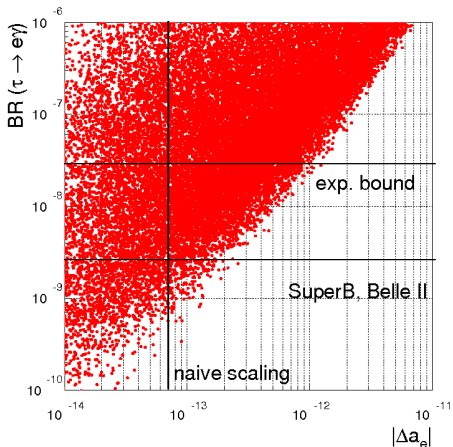
- For values of  $\Delta a_\mu \sim \text{few} \times 10^{-9}$  (explaining the muon  $g-2$  anomaly), non-degenerate sleptons at the level  $m_{\tilde{\mu}} \approx 3 m_{\tilde{e}}$  lead to  $\Delta a_e \approx 10^{-12}$ , ( $\Delta a_e \approx 10^{-13}$  in “naive scaling”) and  $\Delta r_P^{e/\mu} \approx 10^{-3}$ .

# Lepton flavor conserving case



Left:  $\Delta r_P^{e/\mu}$  vs.  $\Delta a_e$ , where  $\Delta r_P^{e/\mu}$  measures violations of lepton universality in  $\Gamma(P \rightarrow e\nu)/\Gamma(P \rightarrow \mu\nu)$  with  $P = K, \pi$ . Right:  $\Delta r_P^{\mu/\tau}$  vs.  $\Delta a_\tau$  where  $\Delta r_P^{\mu/\tau}$  measures violations of lepton universality in  $\Gamma(P \rightarrow \mu\nu)/\Gamma(\tau \rightarrow P\nu)$ .

# Lepton flavor violating case



Left:  $BR(\tau \rightarrow e\gamma)$  vs.  $|\Delta a_e|$ . Right:  $\Delta r_K^{e/\mu}$  vs.  $|\Delta a_e|$ . The vertical line corresponds to the prediction for  $\Delta a_e$  assuming NS, setting  $\Delta a_\mu$  equal to its central value  $\Delta a_\mu = 3 \times 10^{-9}$ .

- Yukawa interactions between a light scalar (pseudoscalar)  $\phi$  ( $A$ ) with leptons  $\ell$

$$\mathcal{L} = \left( \frac{gm_\ell}{2M_W} \right) C_\phi^\ell \bar{\ell}\ell\phi + i \left( \frac{gm_\ell}{2M_W} \right) C_A^\ell \bar{\ell}\gamma_5\ell A$$

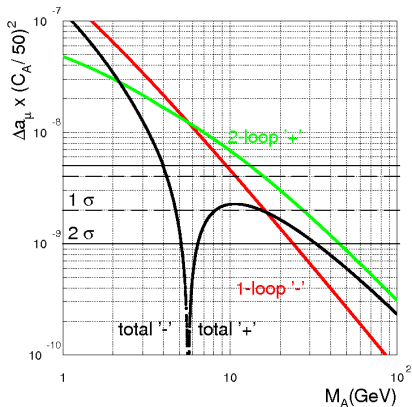
- ▶  $A$  could be a pseudo-Goldstone boson of an extended Higgs sector and  $\phi$  a light gauge singlet coupled through a dimension-five interaction to the Yukawa terms.
- ▶ Very light  $\phi$  and  $A$  are constrained by low-energy data (meson decays) as well as reactor experiments (most of these bounds disappear for  $M_A > 10$  GeV).
- One-loop effects to  $(g-2)_\ell$

$$(\Delta a_\ell^{\phi A})_{1\text{loop}} = \frac{g^2 m_\ell^4}{32\pi^2 M_W^2} \left( |C_\phi^\ell|^2 \frac{I_\phi^\ell}{M_\phi^2} - |C_A^\ell|^2 \frac{I_A^\ell}{M_A^2} \right),$$

- ▶ One-loop pseudoscalar (scalar) effect is unambiguously negative (positive).
- ▶ “Naive scaling”  $\Delta a_\ell \propto m_\ell^2$  for  $m_\ell \gg M_{\phi,A}$ ,  $\Delta a_\ell \propto m_\ell^4$  for  $m_\ell \ll M_{\phi,A}$ .
- Two-loop effects to  $(g-2)_\ell$

$$(\Delta a_\ell^{\phi A})_{2\text{loop}} = -\frac{\alpha^2}{8\pi^2 \sin^2 \theta_W} \frac{m_\ell^2}{M_W^2} m_\tau^2 \left( \text{Re} \left( C_\phi^\ell C_\phi^{\tau*} \right) \frac{L_\phi^\tau}{M_\phi^2} - \text{Re} \left( C_A^\ell C_A^{\tau*} \right) \frac{L_A^\tau}{M_A^2} \right),$$

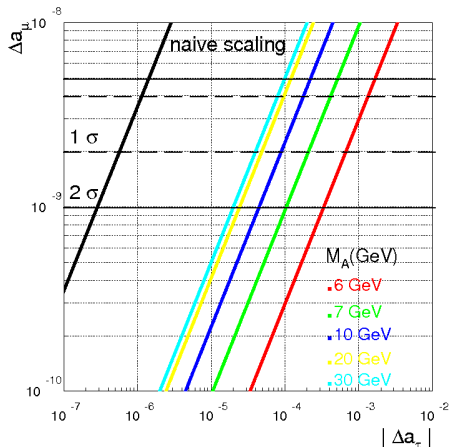
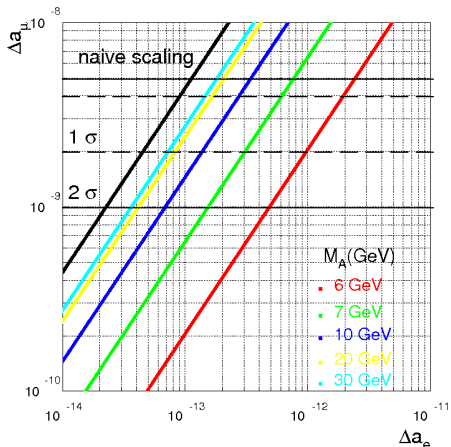
- ▶ For  $m_\ell \ll M_{\phi,A}$  we have the enhancement  $m_\tau^2/m_{e,\mu}^2$  w.r.t. one-loop effects.
- ▶ Two loop effects can be positive or negative depending on the sign of  $\text{Re}(C^\ell C^{\tau*})$ .



- For  $m_\ell \ll M_A$ , where the  $\Delta a_\mu$  anomaly can be explained, we have
  - ▶  $\Delta a_e$  is always dominated by two-loop effects
  - ▶  $\Delta a_\mu$  receives comparable one- and two-loop contributions
  - ▶  $\Delta a_\tau$  is always dominated by one-loop effects.
  - ▶ As a result, we expect significant “naive scaling” violations



# Light (pseudo)scalars and $a_\ell$



- In the regions where the  $\Delta a_\mu$  anomaly is accommodated,  $\Delta a_e$  typically exceeds the  $10^{-13}$  level, providing a splendid opportunity to test the  $(g - 2)_\mu$  anomaly.
- $\Delta a_\tau$  can reach values up to the level of  $10^{-3}$ , well within the experimental resolutions expected at Belle II.

- An explanation of  $\Delta a_\mu$  is given by a new hidden  $U(1)_d$  symmetry which kinetically mixes with  $U(1)_Y$  by [Marciano et al., '08]

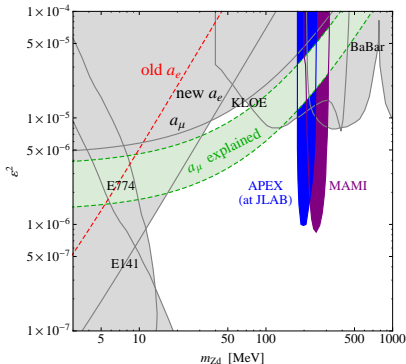
$$\mathcal{L} = \frac{1}{2} \frac{\epsilon}{\cos \theta_W} B_{\mu\nu} Z_d^{\mu\nu},$$

where  $\epsilon$  parametrizes the mixing,  $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ ,  $X = B, Z_d$ , is a  $U(1)$  field strength tensor.

- Upon kinetic diagonalization, one obtains an induced coupling  $e \epsilon Z_d^\mu J_\mu^{em}$ , where  $J_\mu^{em}$  is the electromagnetic current.
- The 1-loop contribution of the  $Z_d$  to  $a_\mu$  is given by

$$a_\mu^{Z_d} = \frac{\alpha}{2\pi} \epsilon^2 F_V(m_{Z_d}/m_\mu)$$

$$F_V(x) \equiv \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}$$



Exclusion region in  $m_{Z_d} - \epsilon^2$  space.

- **Important questions in view of ongoing/future experiments are:**

- ▶ What are the expected deviations from the SM predictions induced by TeV NP?
- ▶ Which observables are not limited by theoretical uncertainties?
- ▶ In which case we can expect a substantial improvement on the experimental side?
- ▶ What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers:**

- ▶ The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes.
- ▶ On general grounds, we can expect any size of deviation below the current bounds.
- ▶ cLFV processes, leptonic EDMs and LFU observables do not suffer from theoretical limitations (clean th. observables).
- ▶ On the experimental side there are still excellent prospects of improvements in several clean channels especially in the leptonic sector:  $\mu \rightarrow e\gamma$ ,  $\mu N \rightarrow eN$ ,  $\mu \rightarrow eee$ ,  $\tau$ -LFV, EDMs and leptonic  $(g - 2)$ .
- ▶ The the origin of the  $(g - 2)_\mu$  discrepancy can be understood testing new-physics effects in the electron  $(g - 2)_e$ . This would require improved measurements of  $(g - 2)_e$  and more refined determinations of  $\alpha$  in atomic-physics experiments.

The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?
- Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?

**Irrespectively of whether the LHC will discover or not new particles, leptonic dipoles (leptonic  $g - 2$ ,  $\mu \rightarrow e\gamma$  and the electron EDM) will teach us a lot...**