A novel method to estimate the impact parameter on a drift cell by using the information of single ionization clusters

G. Signorelli (1), A. D'Onofrio (1,2), M. Venturini (1,3)

(1) INFN Sezione di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

(2) Dipartimento di Fisica, Universita degli studi di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

(3) Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy

SUMMARY: Measuring the time of each ionization cluster in drift chambers has been proposed to improve the single hit resolution, especially for very low mass tracking systems. *Ad-hoc* formulae have been developed to combine the information from the single clusters. We show that the problem falls in a wide category of problems that can be solved with an algorithm called Maximum Possible Spacing (MPS) which has been demonstrated to find the optimal estimator. We show that the MPS approach is applicable and gives the expected results. Its application in a real tracking device, namely the MEG-II cylindrical drift chamber, is discussed.

1. In drift chambers energy is released in discrete clusters

Gaseous wire drift chambers are usually divided in cells made of a central anode wire (at positive potential) surrounded by field shaping or ground wires that isolate one cell from the others. A particle crossing a drift chamber cell produces several ionization clusters, but only the signal coming from the cluster which is closest to the anode wire is usually exploited to extract the impact parameter.

The crossing time of the particle through the cells is usually negligible (order of nanoseconds) compared to the electron drift time in each cell (generally in the few cm/μs for common gas mixtures) therefore the position of the track segment within each cell is reconstructed by referring all the times of the measured signals (t_i) to a common time t_0 given, *e.g.*, by a plastic scintillator placed at the end of the tracking volume.

The signal reaching the anode wire is composed of a succession of pulses corresponding to each ionization cluster. Given the typical drift velocities (4 cm/μs) in a few mm cell the clusters are separated by 10s of nanoseconds and distinguishable by using appropriately fast electronics.

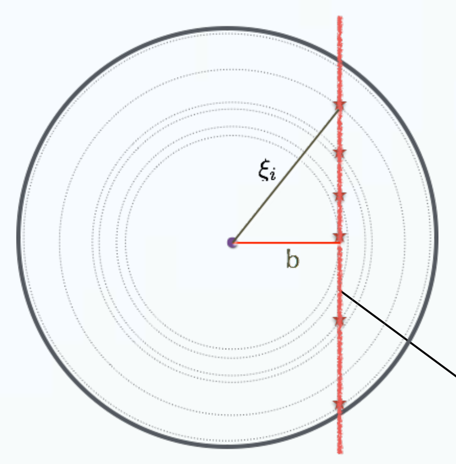
When retaining the information on the first cluster only (i.e. the time of the anode pulse) the impact parameter b is given by the time t_1 of the cluster that is closest to the anode wire converted to a distance ξ_1 by making use of the x-t relations, proper of each cell configuration (electric potential, magnetic field).

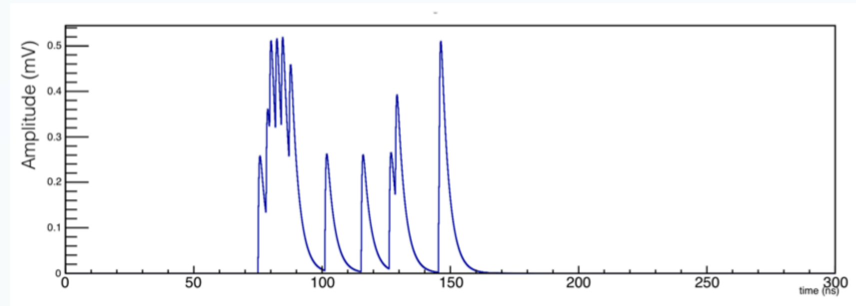
Since ξ_1> true b, the estimate of the impact parameter by using the closest cluster is biased. The bias is larger for light gas mixtures (low number of clusters, ~10/cm) and for tracks close to the anode.

There is more information embedded in the multiple clusters produced by a single track: measuring the time of each ionization cluster has been proposed in the past to improve the impact parameter reconstruction [1], reducing the bias and the single hit resolution, and ad-hoc formulae have been developed to combine information on the single clusters.

The problem of finding an estimator of the impact parameter can be solved with an algorithm called Maximum Possible Spacing (MPS), which provides the optimal estimator in this case. The MPS performs well in comparison with other proposed algorithms at a simulation level, in realistic situation.

A better determination of the impact parameter translates in a better determination of the track parameters, including momentum.

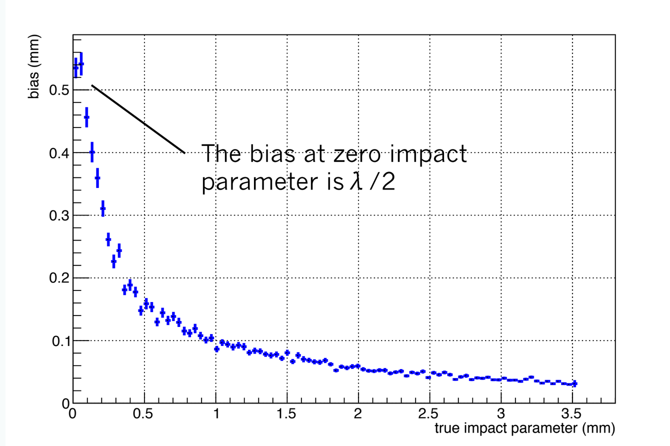




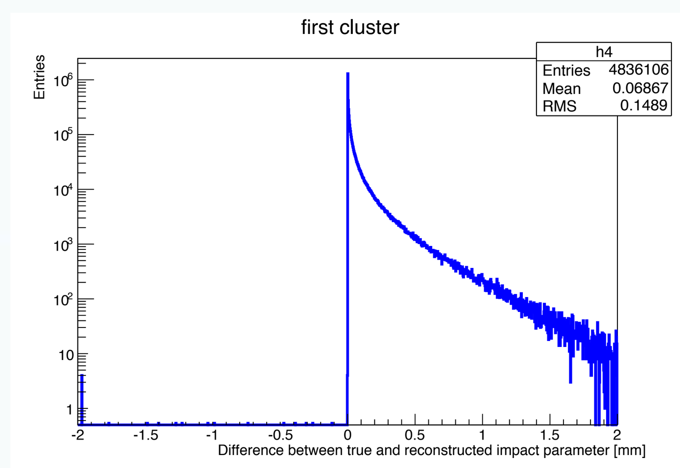
Typical anodic waveform with 800 MHz band electronics

A ionizing particle crossing a drift chamber cell releases energy in multiple clusters spaced by λ on average

Frist cluster only
Bias on the reconstruction of the impact parameter by using only the information from the first cluster

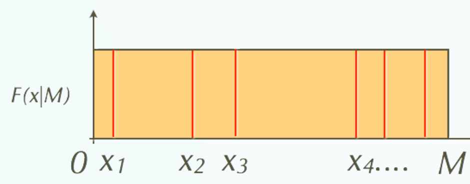


Projected distribution of the bias: always positive



2. A typical case in which the Maximum Likelihood estimator fails triggers the idea of MPS

$$\mathcal{L}(M|x_0) = \frac{1}{M}\theta(M > x_0)$$



$$\mathcal{L}(M|\{x_i\}) = \frac{1}{M^n}\theta(M > x_{\max})$$

$$x_0 = 0 \quad x_1 < x_2 < \ldots < x_n \quad x_{n+1} = M$$

$$D_i = x_i - x_{i-1} \quad G = \left\{\prod_{i=1}^{n+1} \frac{D_i}{M}\right\}^{1/(n+1)}$$

$$M_{\mathrm{MPS}} = x_{\max}\frac{n+1}{n}$$

The problem of exploiting the information of multiple clusters is similar to the problem of estimating the upper limit of a flat distribution (between 0 and M) by making N measurements $\{x_1 \ldots x_N\}$.

The maximum likelihood (ML) estimator in this case is $x_{max}=\max_N x_i$, but it is pathological (i.e. not unbiased, not sufficient nor efficient), since the limits of the distribution depend on a parameter, spoiling the statistical hypotheses that make ML estimators so appreciated.

A different estimator for the parameter M has been proposed independently by Cheng *et al.* [2] and by Ranneby [3] and it is named maximum product of spacing (MPS) estimator. The idea is that if the observables come from a flat distribution their relative spacing $D_i = x_i - x_{i-1}$ should be uniform. Since the sum of all spacings is fixed and equals M, this reduces to maximizing the geometrical mean of the D_i's (or, alternatively, its logarithm).

It is straightforward to see that, in this case, $M_{\mathrm{MPS}} = x_{max}(n+1)/n$ leading to the intuitive result that the MPS estimator finds the expected value a little bit larger than $x_{\max}$, and this little bit is just the average spacing between two consecutive x_i's.

It can be shown that MPS estimators have the same properties of ML estimators, or even better. In particular they are asymptotically sufficient, consistent and asymptotically efficient [2].

The result for the flat distribution is readily generalized to any distribution $f(x|M)$ (where M is a parameter, not necessarily the maximum of the domain) by using the cumulative *pdf* defining x_i: $D_i = y_i - y_{i-1} = \int_{x_{i-1}}^{x_i} f(x|M)\mathrm{d}x \quad (i = 1, 2, \ldots, n+1)$

3. Application of the MPS estimator to particle tracking

In the following we will restrict to the case of cylindrical cells of radius R. This is easily generalized for square or arbitrary shaped cells by discarding late clusters. For a track passing at a true impact parameter b the N clusters are distributed randomly on the cord, i.e. according to the probability density:

$$F(x|b)\mathrm{d}x = \frac{\mathrm{d}x}{2\sqrt{R^2 - b^2}}\theta(-\sqrt{R^2-b^2} < x < \sqrt{R^2-b^2})$$

x = 0 being the point of closest approach. Electrons drift at known velocity towards the anode, arriving at times $\{t_i\}$ (we shall assume, without loosing in generality, t_0 = 0 the time of the passage of the particle). We shall call $\{\xi_i\}$ the corresponding distances obtained by the time-drift tables. The normalized *pdf* for the ξ_i's is obtained by a change in variable:

$$G(\xi|b)\mathrm{d}\xi = \frac{\xi \mathrm{d}\xi}{\sqrt{R^2-b^2}\sqrt{\xi^2-b^2}}\theta(b < \xi < R)$$

To obtain the MPS estimator we need to compute the D_i's from its cumulative distribution:

$$y_i = \int_b^{\xi_i} \frac{\xi}{\sqrt{R^2-b^2}\sqrt{\xi^2-b^2}}\mathrm{d}\xi = \frac{\sqrt{\xi_i^2-b^2}}{\sqrt{R^2-b^2}}$$

using $\xi_0 = b$ and $\xi_{n+1} = R$. Hence:

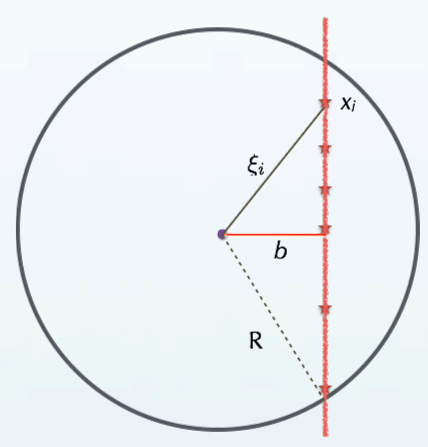
$$D_i = y_i - y_{i-1} = \frac{\sqrt{\xi_i^2-b^2} - \sqrt{\xi_{i-1}^2-b^2}}{\sqrt{R^2-b^2}}$$

We define the MPS estimator of the track-anode point of closest approach, the b that maximizes the (logarithm of) the geometric mean of the D_i's:

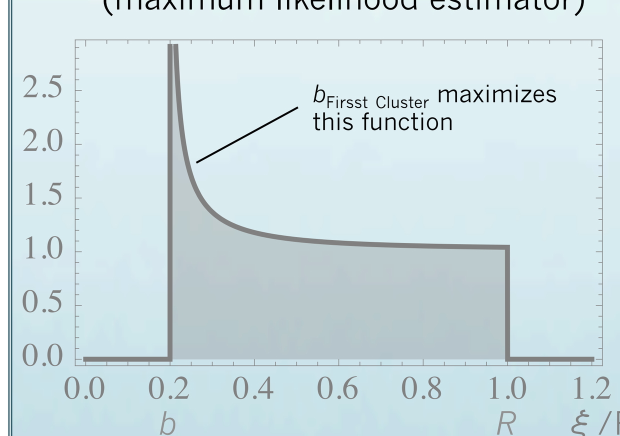
$$H(b) = \frac{1}{n+1}\sum_{i=1}^{n+1} \log D_i$$

We will show how this estimator of the impact parameter improves track reconstruction by removing the bias.

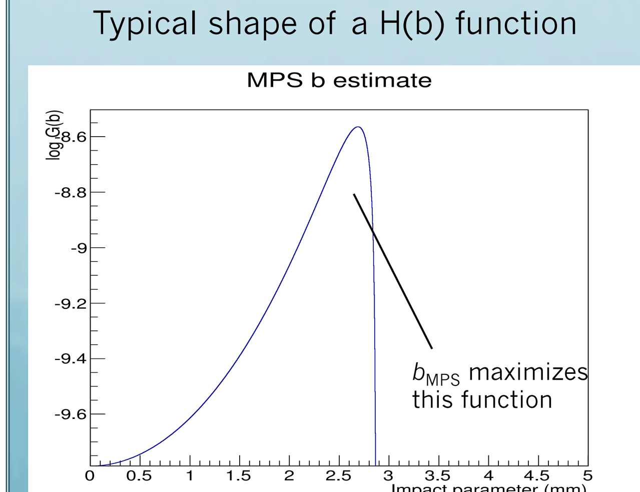
A track crossing a cell: definitions



Shape of G(ξ|b): it diverges for ξ=b (maximum likelihood estimator)



Typical shape of a H(b) function

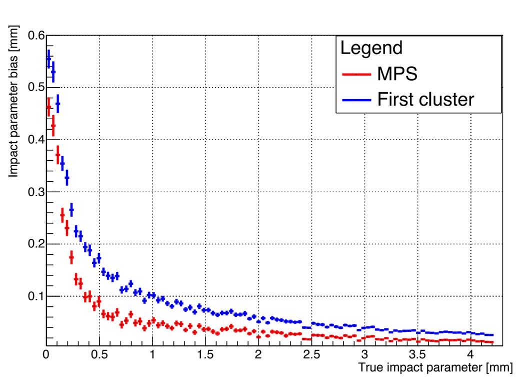


4. Results: MPS reduces the bias and the hit position uncertainty

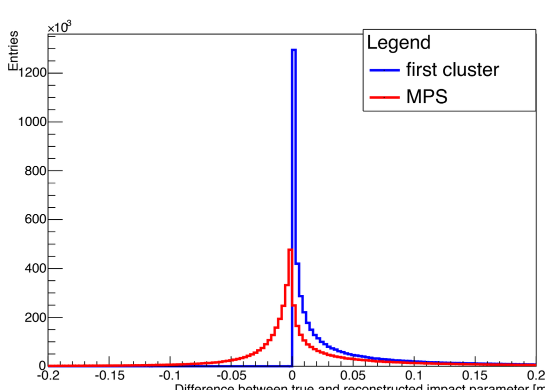
We simulated a number of tracks crossing a R=5mm drift cell, filled with a light He:isobutane (15:85) mixture (λ=13 clusters/cm) to simulate the MEG-II drift chamber cell [4].

The information of all clusters is used to evaluate the MPS estimate of the impact parameter. As can be seen from the picture below b_{MPS} reduces considerably the bias on the impact parameter present when only the first cluster is used.

Improvement of MPS algorithm with respect to first cluster only

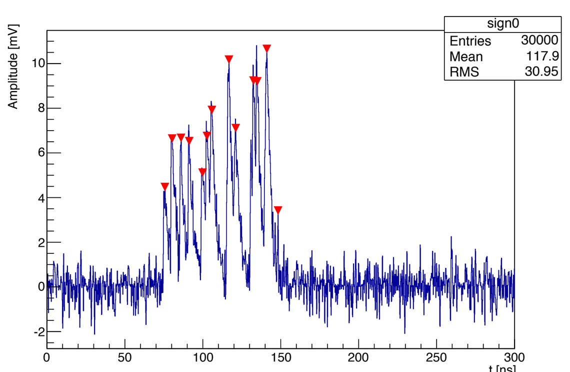


Projection for all impact parameters showing the bias reduction

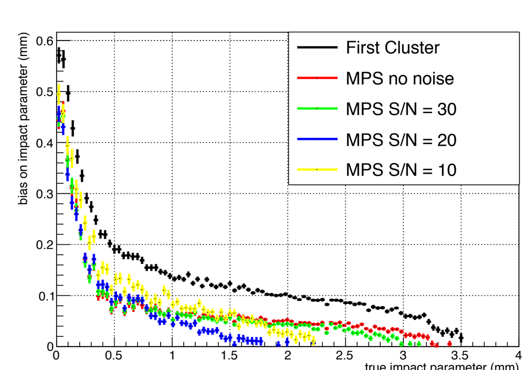


A waveform simulation with realistic noise conditions, combined with a refined version of the ROOT TSpectrum class, shows that we can reconstruct clusters with an efficiency>80% without spoiling the algorithm performance:

Waveform with simulated clusters and S/N = 10



Performance of MPS for various S/N levels



The algorithm is being applied to data collected from a drift cell prototype at the center of a silicon cosmic ray test facility [5] which measures independently the "true" impact parameter to confirm the improvement induced by the MPS reconstruction algorithm [6].

References:

[1] F. Grancagnolo *et al.*, Nucl. Instr. Meth. A**386** (1997) 458-469.

[2] R.C.H. Cheng and N.A.K. Amin, J. R. Statist. Soc. **B45** (1983) 394–403

[3] B. Ranneby, Scand. J. Statist. **11** (1984) 93–112

[4] M. Venturini *et al.*, poster at this conference.

[5] L. Galli *et al.*, poster at this conference.

[6] A.D'Onofrio, MS Thesis, Università di Pisa, 2015

Contacts: Giovanni.Signorelli@pi.infn.it