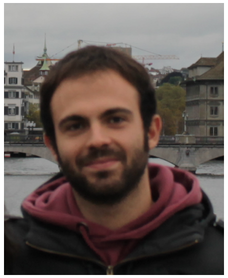
A novel method to estimate the impact parameter on a drift cell by using the information of single ionization clusters



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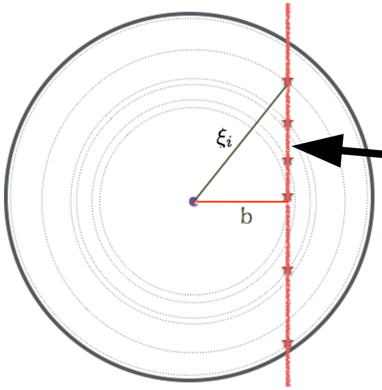
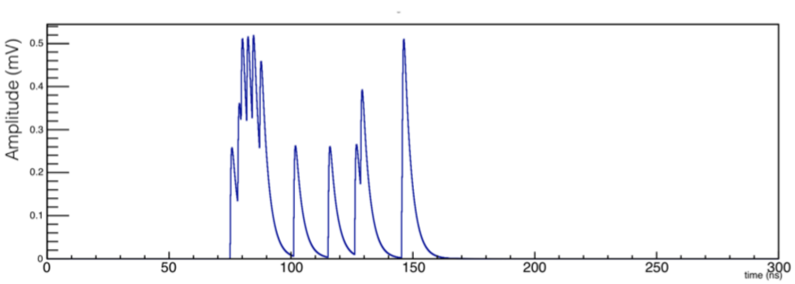
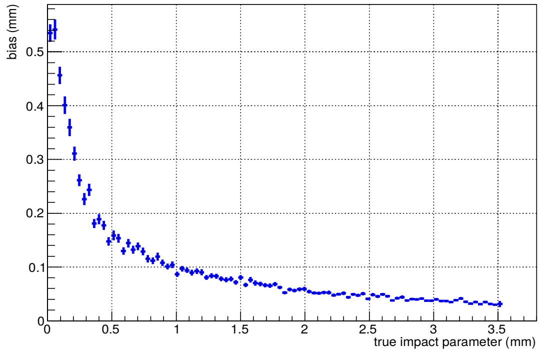
A particle crossing a drift chamber cell produces several **ionization clusters**, but only the signal coming from the **cluster** which is **closest** to the anode wire (with a drift distance ξ_1) is usually exploited to extract the **impact parameter** *b*.



Since $\xi_1 > b$ the estimate of the impact parameter by using the closest cluster is biased. The bias is larger for light gas mixtures (low number of clusters, ~10/cm) and for tracks close to the anode.





Clusters contain the **full information** on the track parameters: with fast electronics one can reconstruct and exploit single clusters, *e.g.* in the MEG II drift chamber.

Maximum Likelihood & Maximum Product of Spacing

The problem of exploiting the information of multiple clusters is similar to the problem of estimating the **upper limit of a flat distribution** (between 0 and *M*) by making *N* measurements $\{x_1 \ldots x_N\}$.

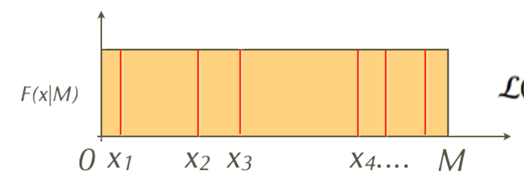
The **Maximum Likelihood** (ML) estimator in this case is $x_{max}=\max_N x_i$, but it is pathological, since the limits of the distribution depend on a parameter!

A **different estimator** for the parameter *M* is the **Maximum Product of Spacing** (MPS) estimator [1,2]. The idea is that if the observables come from a flat distribution their relative spacing $D_i = x_i - x_{i-1}$ should be **uniform**. Since the sum of all spacings is fixed and equals *M*, this reduces to **maximizing the geometrical mean** of the D_i's (or, alternatively, its logarithm).

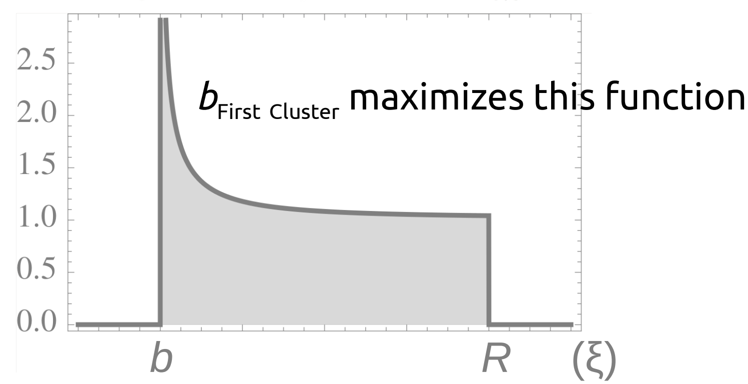
[1] R.C.H. Cheng and N.A.K. Amin, J.R. Statist. Soc. **B45** (1983) 394–403

[2] B. Ranneby, Scand. J. Statist. **11** (1984) 93–112 458-469.

$$\mathcal{L}(M|x_0) = \frac{1}{M}\theta(M > x_0)$$



$$\mathcal{L}(M|\{x_i\}) = \frac{1}{M^n}\theta(M > x_{\max})$$



$$x_0 = 0 \quad x_1 < x_2 < \ldots < x_n \quad x_{n+1} = M$$

$$D_i = x_i - x_{i-1} \qquad G = \left\{\prod_{i=1}^{n+1} \frac{D_i}{M}\right\}^{1/(n+1)}$$

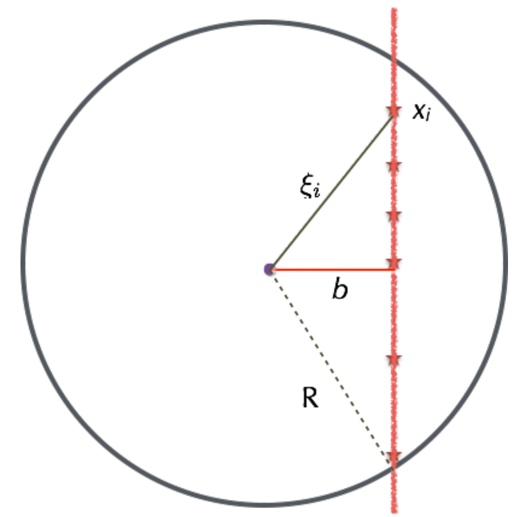
$$M_{\text{MPS}} = x_{\max}\frac{n+1}{n}$$

Idea: Applying MPS to tracking

Clusters are randomly distributed along the cord

$$F(x|b)\mathrm{d}x = \frac{\mathrm{d}x}{2\sqrt{R^2-b^2}}\theta(-\sqrt{R^2-b^2} < x < \sqrt{R^2-b^2})$$

$$G(\xi|b)\mathrm{d}\xi = \frac{\xi\mathrm{d}\xi}{\sqrt{R^2-b^2}\sqrt{\xi^2-b^2}}\theta(b < \xi < R)$$

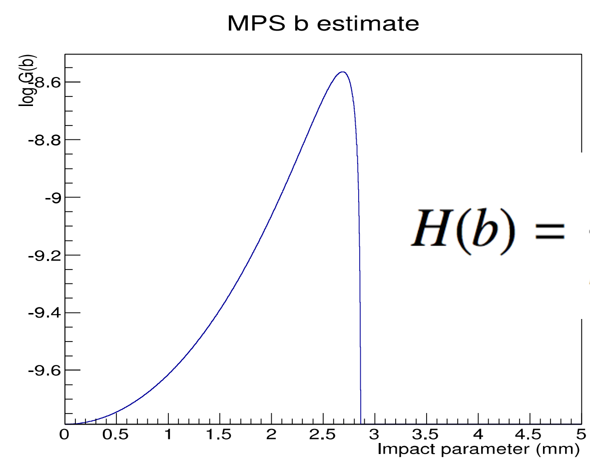


To obtain the MPS estimator we need to compute the D_i's from its cumulative distribution:

$$y_i = \int_b^{\xi_i} \frac{\xi}{\sqrt{R^2-b^2}\sqrt{\xi^2-b^2}}\mathrm{d}\xi = \frac{\sqrt{\xi_i^2-b^2}}{\sqrt{R^2-b^2}}$$

$$D_i = y_i - y_{i-1} = \frac{\sqrt{\xi_i^2-b^2} - \sqrt{\xi_{i-1}^2-b^2}}{\sqrt{R^2-b^2}}$$

b_{MPS} maximizes this function:

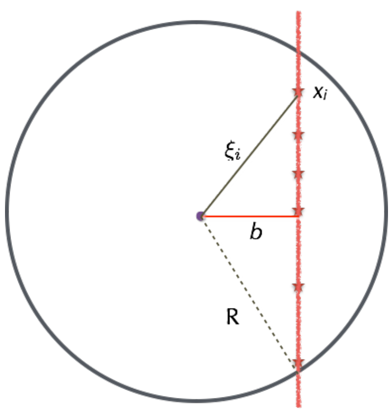


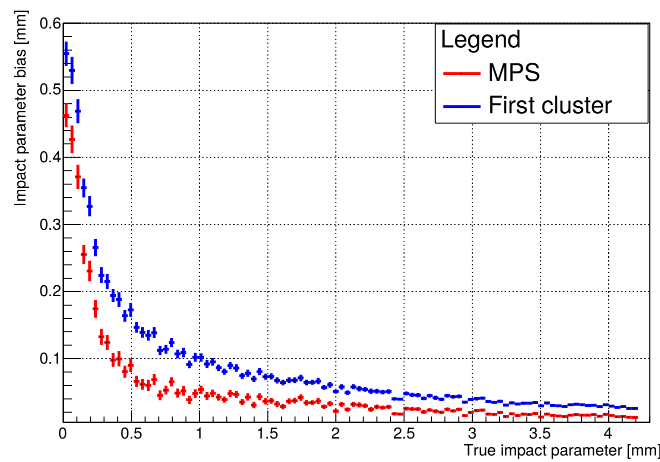
$$H(b) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log D_i$$

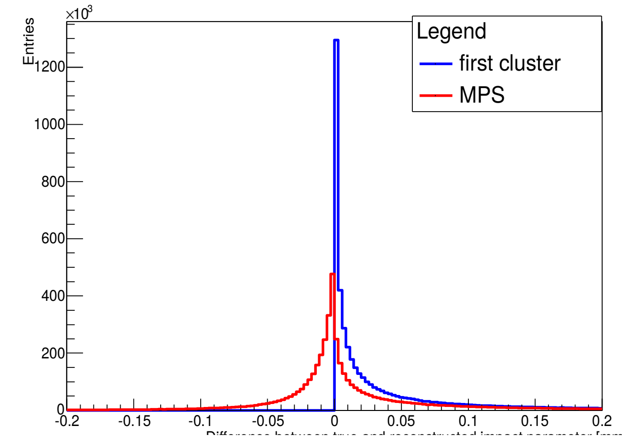
Results

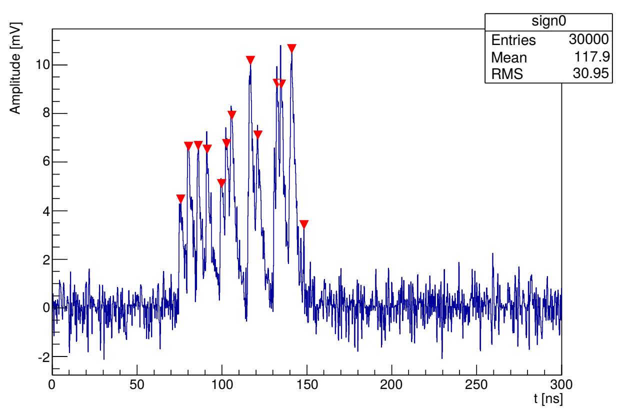
We simulated a number of tracks crossing a R=5mm drift cell, filled with a light He:isobutane (85:15) mixture (λ=13 clusters/cm) to simulate the MEG II drift chamber cell.

MPS reduces the bias and the hit position uncertainty!

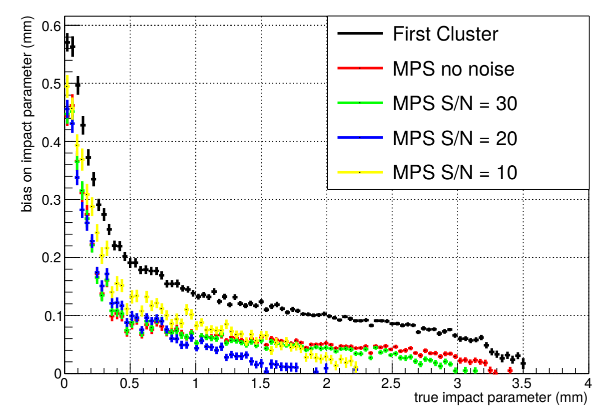








Waveform simulation with realistic noise conditions, combined with a refined version of the ROOT TSpectrum class.



MPS works also in realistic conditions
In progress: application to real data and improvement on track parameters.