Aspects of Gauge-Strings Duality

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Outline

1. This talk is in the context of AdS/CFT. I will comment on recent work–started in January 2013. I will try to avoid a bit of the heavy technical side of things (that is interesting!) and focus on the general ideas and outcomes.
2. A summary of the results I will try to present: A geometric perspective on some QFT phenomena (as operations on G-structures). Extend this logic to non-Abelian T-duality and use this as a generating technique to find new string backgrounds.
3. All I will discuss will be in the context of $N = 1$ SUSY gauge theories in four dimensions.
4. This seminar draws from results obtained with: G. Itsios, N. MacPherson, C. Whitting, J. Gaillard, A. Barranco, K. Sfetsos, D. Thompson, E. Caceres, V. Rodgers, L. Pando Zayas.

There are various examples of $\mathcal{N} = 1$ SUSY gauge theories in 4 dimensions with a String dual.

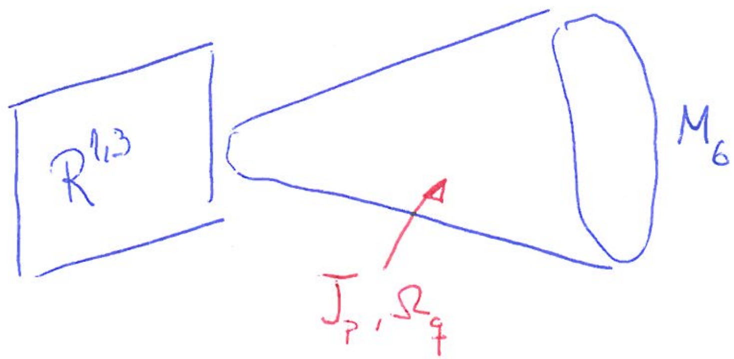
The dual space, typically is of the form $ds^2 \sim dx_{1,3}^2 + ds_6^2$

A set of 'forms' can be defined in the six-space. They 'know' everything about global symmetries, SUSY, BPS objects in the dual QFT.

For example, when the background has $SU(3)$-structure, the forms are J_2 and Ω_3.

It was shown in some examples, that a simple operations on these forms *generates* a new solution. This new background is dual to a new QFT, related to the first one by a field theoretical operation!

G-structure	SU(3	SU(2)
forms	J_2, Ω_3	J_2, ω_2, z_1, v_1



Hence; a well understood operation in 4-d QFT translates into a different operation with 6-d objects defining the G-structure of the dual space-time.

This suggests, that different operations with the forms defining the G-structures, might have a concrete meaning in QFT.

This may be thought of as a way of *constructing* new QFTs.

This will be the 'logic' advocated in this talk. The 'operation' we will perform will be 'non Abelian T-duality'

I will keep this, to be a not-too-technical presentation, trying to emphasise the ideas.

Let me first comment a bit about *non-Abelian T-duality.*

About what I will present now: my understanding of this topic is more solid on the geometry side, than on the QFT-side.

In other words, I will present new geometries, all well defined and smooth. They should be dual to 4-d field theories.

But I will not have a precise Lagrangian description of the dual QFT. May be, there is not such description.

Actually, what I would like to propose, is to *define* the gauge theories, using the healthy dual backgrounds.

There seems to be– at least superficially, some relation with theories recently discussed by Gaiotto, Maldacena, Benini, Tachikawa, Wecht, Bah, Beem, Bobev and many others. Also with 'long quivers' discussed by Aharony, Berdichevsky and Berkooz

Let me *briefly* motivate the topic of Non-Abelian T-duality, by discussing characteristics of duality in two-dimensional systems.

As we know, duality transformations play an important role in:Statistical Mechanics, Quantum Field Theory and String Theory. One can consider for example, the 2-dim Ising model, the two-dimensional abelian bosonisation and Buscher T-duality. They follow a pattern:

- Detect a global symmetry.
- Gauge it, but introduce a Lagrange multiplier so that this gauge field is non-dynamical.
- If we integrate the Lagrange multiplier, we are back to the starting point. But if we instead, integrate 'the other way around' keeping the dynamics of the multiplier, we have a 'dual theory'. I will present an example soon.

The duality that in Statistical Mechanics is exemplified by the Ising Model (Kramers-Wannier, 1940). In 2-d QFT is exemplified by Bosonisation (Coleman, Mandelstam, 1975). In String Theory is exemplified by T-duality (Buscher, 1985).

Ising Model $\longleftrightarrow$ Bosonization $\longleftrightarrow$ T duality

Let us see this briefly in the case of bosonisation. This is taken from Burgess+Quevedo, 1995

Suppose that you consider the Lagrangian (in two dimensions)

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi, \rightarrow Z[j] = \int D\bar{\psi} D\psi e^{-\int d^2x (L + j_\mu \bar{\psi}\gamma^\mu \psi)}$$

In the following and for clarity, allow me to ignore the source terms

One notices a vectorial global symmetry $\psi \rightarrow e^{i\alpha}\psi$ and $\bar{\psi} \rightarrow \bar{\psi}e^{-i\alpha}$.

We will *gauge* this global symmetry, with a gauge field A_μ. But we will do so, in a way that we do not introduce new degrees of freedom. Basically, imposing that $F_{\mu\nu} = 0$. The generating functional will read,

$$Z = \int DA_\mu D\Lambda D\psi D\bar{\psi} \exp\left[- \int d^2x \bar{\psi}\gamma^\mu (i\partial_\mu + A_\mu)\psi + \Lambda \epsilon_{\mu\nu} F^{\mu\nu} \right]$$

If we integrate the Lagrange multiplier Λ, we are back to the fermionic Action above.

If we integrate the fermions and then the gauge field, we will then get a partition function

$$Z = \int D\Lambda \exp\left[-\int d^2x(\partial_\mu\Lambda)^2\right],$$

Which is the characteristic bosonisation prescription. Had I kept the two source terms, the more meaningful prescription arises,

$$i\bar{\psi}\gamma_\mu\psi \to \frac{\epsilon_{\mu\nu}\partial_\nu\Lambda}{\sqrt{\pi}}$$
$$i\bar{\psi}\gamma_\mu\gamma_5\psi \to \frac{\partial_\mu\Lambda}{\sqrt{\pi}}$$

In exactly the same way, Buscher presented the T-duality rules. Then this should be complemented by the transformations for RR fields and fermions (Bergshoeff, Hull, Ortin; Benichou, Policastro, Troost; Hassan....)

One may wonder about the non-abelian version of the procedure above

As we know, there is a non-abelian analog of the bosonisation procedure, with two subtleties: some gauge fixing is needed, the WZW Action appears and the Polyakov-Wiegmann identity needs to be used.

Clearly, there are technical subtleties, but the idea is the same.

What in 2-d QFT is exemplified by non-Abelian bosonisation: Witten 1982 (Burgess+Quevedo 1995)

In String Theory is exemplified by non-Abelian T-duality: de la Ossa+Quevedo, 1993.

Statistical Mechanics

?

QFT

Non-Abelian Bosonization (Witten 1982)

String Theory

Non Abelian T-duality (Quevedo de la Ossa 1993)

Non Abelian T-duality in simple terms

$$\left(\omega^a = i\,\mathrm{Tr}\left[T^a g^{-1} dg\right]\right)$$

$$\boxed{g = e^{i\sigma_3 \phi} e^{i\theta \tau_2} e^{i\psi \tau_3}}$$

$$\omega_1 = \cos\psi\, d\theta + \sin\psi \sin\theta\, d\phi$$

$$\omega_2 = -\sin\psi\, d\theta + \cos\psi \sin\theta\, d\phi$$

$$\omega_3 = d\psi + \cos\theta\, d\varphi$$

We have a space
$$ds^2 = -dt^2 + dx^2 + \frac{R^2}{4}\underbrace{(\omega_1^2 + \omega_2^2 + \omega_3^2)}_{(d\theta^2 + \sin^2\theta\, d\varphi^2) + (d\psi + \cos\theta\, d\varphi)^2}$$

The σ-model will read
$$\mathcal{L} = G_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + \underbrace{g_{ab}\,\omega_+^a\,\omega_-^b}_{\omega_\pm^a = \mathrm{Tr}(T^a g^{-1} \partial_\pm g)}$$

$$\left[\omega_\pm^{(1)} = \cos\psi\,\partial_\pm\theta + \sin\psi\sin\theta\,\partial_\pm\phi \quad \text{etc.}\right]$$

- We gauge a symmetry $\quad \partial_\pm g \longrightarrow \partial_\pm g - A_\pm g$

- The Lagrange multiplier
$$i\,\mathrm{Tr}(v^a F^a_{+-}) \equiv i\,\mathrm{Tr}\left(v^a\left(\partial_+ A_-^a - \partial_- A_+^a - \underbrace{f^{abc} A_+^b A_-^c}\right)\right)$$

here is the Non Abelian information

- Integrate by parts
$$i\,\mathrm{Tr}(v_a F^a_{+-}) = -i\,\mathrm{Tr}\left\{\partial_+ v^a A_-^a - \partial_- v^a A_+^a - \underbrace{f^{abc} v^a A_+^b A_-^c}\right\}$$

- We have a new σ-model

$$\boxed{\mathcal{L} = G_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + g_{ab}\,\omega_+^a\,\omega_-^b - i\,\mathrm{Tr}\left\{\partial_+ v^a A_-^a - \partial_- v^a A_+^a - f^{abc} v^a A_+^b A_-^c\right\}}$$

We now integrate out $A^a_\pm$ $\longrightarrow$ Lagrangian in terms of $[t, x, \theta, \varphi, \psi, v^1, v^2, v_3]$

— gauge choice $\longrightarrow$ "dual" Lagrangian $\mathcal{L}(t, x, v^1, v_2^2, v^3)$

$\downarrow$

from here we read

now $\begin{cases} G_{\mu\nu} \\ B_{\mu\nu} \\ \Phi \end{cases}$

Example : $ds^2 = \omega_1^2 + \omega_2^2 + \omega_3^2$ $\longrightarrow$ follow procedure gauge fix $v^1 = v^2 = \psi = 0$

$$ds^2_{NOW} = dr^2 + \frac{r^2}{1+r^2}(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

$$B_2 = \frac{r^3}{1+r^2} \sin\theta \, d\theta \wedge d\varphi$$

$$e^{2\Phi} = \frac{1}{1+r^2}$$

range of r ?

The analog of non-abelian bosonisation for T-duality, could be thought in this way: consider a background with some $SU(2)$ isometry

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu + 2G_{\mu i}dx^\mu L^i + g_{ij}L^i L^j,$$
$$B_2 = B_{\mu\nu}dx^\mu dx^\nu + B_{\mu i}dx^\mu L^i + B_{i\mu}L^i dx^\mu + \frac{b_{ij}}{2}L^i L^j,$$
$$\Phi(x).$$

where these L^i are given by

$$L^k = itr(T^k g^{-1}dg), \quad g = e^{\frac{i\tau_3\theta}{2}} e^{\frac{i\tau_2\varphi}{2}} e^{\frac{i\tau_3\psi}{2}},$$

One can write the (bosonic) sigma model for the string in this background

$$L = Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + Q_{\mu i}\partial_+ X^\mu L^i + Q_{i\mu}L^i\partial_- X^\mu + E_{ij}L^i_+ L^j_-,$$
$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, Q_{\mu i} = G_{\mu i} + B_{\mu i}, Q_{i\mu} = G_{\mu i} + B_{i\mu}, \ E_{ij} = g_{ij} + b_{ij},$$
$$L^i_\pm = -iTr(T^i g^{-1}\partial_\pm g).$$

One detects the global $SU(2)$ symmetry and gauges it

$$\partial_{\pm} g \to D_{\pm} g = \partial_{\pm} g - A_{\pm} g.$$

Now, follow the same procedure as described above:

- gauge the global symmetry
- but do it without introducing new degrees of freedom, hence add a Lagrange multiplier $v = v^a T^a$ to impose $F_{+-} = 0$
- then partially integrate, gauge fix and integrate out $A_{\pm}$
- One will get $A^j_{\pm}$ in terms of $\partial_{\pm} v^j \pm Q_{\mu j} \partial_{\pm} X^{\mu}$
- One will then obtain a new Action

$$L = Q_{\mu\nu} \partial_+ X^{\mu} \partial_- X^{\nu} + (\partial_+ v^j + Q_{\mu j} \partial_+ X^{\mu})(\partial_- v^i - Q_{i\mu} \partial_- X^{\mu}) M^{-1}_{ij}.$$

$$M_{ij} = E_{ij} + f_{ijk} v^k.$$

here is the Non-Abelian information!

From here, one reads a new metric, a new B_2 field and a new dilation. Write very clean formulas for these quantities

The interesting thing is that there is a similar procedure that one can follow for the RR fields! One can produce a prescription for a transformed of a generic Ramond field. Sfetsos and Thompson did this in 2010.

Objections posed during the 1990's:

A more canonical string theorist could object: is this a symmetry of the full string theory?

The answer is that quite probably it is NOT. The genus expansion will probably make this duality fail. Alvarez-Gaume, Alvarez, Barbon, Lozano, 1995.

This is probably a transformation between different CFTs, not the same CFT before and after the duality. Giveon+Verlinde, 1996.

The view I will take is the following: I will use this as a solution generating technique in Supergravity. If the string genus expansion breaks the generating technique, this I will interpret as telling us that the field theories dual to the backgrounds I am generating, cease to have certain properties—for example conformality—once the $1/N_c$ corrections are taken into account.

For the rest of this talk, I will go with this idea, restrict myself to the sphere-worldsheet (avoiding some problems with global issues).

Let me describe the idea of the projects I was lately involved in.

Idea in these projects

Take $AdS_5 \times S^5/\mathbb{Z}_2$ $\xrightarrow{\text{NATD}}$ IIA geometry $g_{\mu\nu}, F_2, F_4, \Phi, B_2$ [Singular] $\xrightarrow{\text{lift to 11 dim}}$ Gaiotto-Maldacena-like geometry

$$\begin{cases} AdS_5 \times S^2 \times M_4 \\ F_4 \end{cases}$$

$\mathcal{N}=2$ CFT

T_N

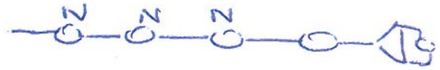
This was done by Sfetsos-Thompson (2010)

Idea in these projects

Take $AdS_5 \times S^5/\mathbb{Z}_2$ $\xrightarrow{\text{NATD}}$ IIA geometry $g_{\mu\nu}, F_2, F_4, \Phi, B_2$ [Singular] $\xrightarrow{\text{lift to 11 dim}}$ Gaiotto-Maldacena-like geometry

$$\begin{cases} AdS_5 \times S^2 \times M_4 \\ F_4 \end{cases}$$

$\mathcal{N}=2$ CFT

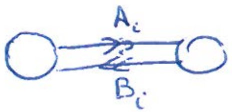


T_N theory

$$\Big\downarrow \; m^2(\Phi_1^2 - \phi_2^2)$$

$AdS_5 \times T^{1,1}$ $\xrightarrow{\text{NATD}}$ IIA geometry $g_{\mu\nu}, F_2, F_4, B_2\,\Phi$ [NON-SINGULAR] $\xrightarrow{\text{lift to 11 dim}}$

$$\begin{cases} AdS_5 \times S^2 \times \tilde{M}_4 \\ F_4 \end{cases}$$



$+ W \sim ABAB$.

$\mathcal{N}=1$ T_N theory "Sicilian"
(Benini Tachikawa Wecht)
2011
[massive chiral multiplet in vector $\mathcal{N}=2$ multiplet]

"obvious" (in retrospective!)

Idea in these projects

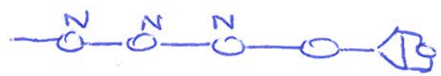
Take $AdS_5 \times S^5/\mathbb{Z}_2$ $\xrightarrow{\text{NATD}}$ IIA geometry $\xrightarrow{\text{lift to 11 dim}}$ Gaiotto-Maldacena-like geometry

$g_{\mu\nu}$, F_2, F_4, Φ, B_2

[Singular]

$$\begin{cases} AdS_5 \times S^2 \times M_4 \\ F_4 \end{cases}$$

$\mathcal{N}=2$ CFT



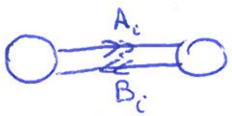
T_N theory

$\downarrow \; m^2(\Phi_1^2 - \Phi_2^2)$

$AdS_5 \times T^{1,1}$ $\xrightarrow{\text{NATD}}$ IIA geometry $\xrightarrow{\text{lift to 11 dim}}$ $\begin{cases} AdS_5 \times S^2 \times \tilde{M}_4 \\ F_4 \end{cases}$

$g_{\mu\nu}$, F_2, F_4, B_2, Φ

[NON-SINGULAR]



$+ W \sim ABAB$

$\mathcal{N}=1$ T_N theory, "Sicilian"

(Benini Tachikawa Wecht) 2011

[massive chiral multiplet in vector $\mathcal{N}=2$ multiplet]

$\downarrow$ Break conformality

Baryonic Branch of the KS Theory $\xrightarrow{\text{NATD}}$ Massive IIA

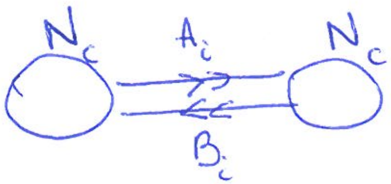
$g_{\mu\nu}$, F_2, F_4, B_2, Φ, F_0 $\longrightarrow$ Suggests new QFTs

[NON-SINGULAR]

To give one 'simple' example—the formulas may give you a taste of the sort of technicalities involved

let me discuss how the non abelian transformed of the Klebanov-Witten background looks like and remind a couple of things about Klebanov-Witten's theory and its dual description

The field theory is conformal–strongly coupled fixed point–of a quiver with two gauge groups and two sets of bifundamental fields



	$SU(N_c)$	$SU(N_c)$	$SU(2)$	$SU(2)$	$\times U(1)_R$	$\times U(1)_B$
A_α	N_c	$\bar{N}_c$	2	1	$\frac{1}{2}$	1
B_α	$\bar{N}_c$	N_c	1	2	$\frac{1}{2}$	-1

$$W = \mathrm{Tr}\,\varepsilon_{\alpha\beta}\,\varepsilon_{ij}\,A_\alpha B_i A_\beta B_j$$

$\longrightarrow$ strongly coupled CFT : $\beta_{g_1} = \beta_{g_2} = \Delta\theta_1 = \Delta\theta_2 = 0 \iff \underline{\gamma_A = \gamma_B = -\frac{1}{2}}$

One of these $SU(2)$'s will be used in the Non-Abelian T-duality

The dual description of this strongly coupled CFT is in terms of a smooth background in Type IIB string theory,

$$
\begin{aligned}
ds^2 &= \overbrace{\frac{r^2}{L^2}dx_{1,3}^2 + \frac{L^2}{r^2}dr^2}^{CFT} + L^2 ds^2_{T^{1,1}} \;, \\
F_{(5)} &= \frac{4}{g_s L}\left(\mathrm{Vol}(AdS_5) - L^5 \mathrm{Vol}(T_{1,1})\right) \rightsquigarrow U(1)_B
\end{aligned}
$$

$$
ds^2_{T(1,1)} = \lambda_1^2 \overbrace{(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)}^{SU(2)} + \lambda_2^2 \overbrace{(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2)}^{SU(2)} + \lambda_3^2 \underbrace{(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2}_{U(1)_R} \;.
$$

$$
\lambda_1^2 = \lambda_2^2 = \frac{1}{6}, \quad \lambda_3^2 = \frac{1}{9}, \quad \frac{1}{2\kappa_{10}^2 T_{D3}} \int F_5 = N_{D3}.
$$

we perform NATD in $(\theta_2, \phi_2, \psi) \rightsquigarrow (w_1, w_2, w_3)$

We will transform this configuration, using the $SU(2)$ in (θ_2, ϕ_2, ψ) use a particular gauge, where the new coordinates are (x_1, x_2, x_3), follow the lengthy procedure, we notice that this looks quite ugly. Then, we change to spherical coordinates

$$x_1 = \rho \sin\chi \cos\xi, \quad x_2 = \rho \sin\chi \sin\xi, \quad x_3 = \rho \cos\chi.$$

To obtain a not-so-ugly looking background, consisting on a metric, dilaton, B-field and RR-forms F_2 and F_4

$$d\hat{s}^2 = \overbrace{\frac{r^2}{L^2}dx_{1,3}^2 + \frac{L^2}{r^2}dr^2}^{\text{CFT}} + L^2\lambda_1^2\overbrace{(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)}^{SU(2)}$$

$$+\frac{\alpha'^2}{QL^2}\Big[(\lambda_2^4L^4(d\rho\cos\chi - \rho d\chi\sin\chi)^2 + \lambda^2\lambda_2^2L^4\Big(\rho^2 d\chi^2\cos^2\chi + \rho d\rho d\chi\sin 2\chi$$

$$+\sin^2\chi\left(d\rho^2 + \rho^2\underbrace{(d\xi + d\phi_1\cos\theta_1)}_{U(1)_R}{}^2\right)\Big) + \rho^2\alpha'^2 d\rho^2\Big],$$

$$QL^2\hat{B}_2 = \frac{1}{2}\rho^2\alpha'^3\sin\chi\left(\left(\lambda^2 - \lambda_2^2\right)\sin 2\chi d\xi\wedge d\rho + 2\rho N d\xi\wedge d\chi\right)$$

$$-\lambda^2\alpha'\cos\theta_1\left(\cos\chi\left(\rho^2\alpha'^2 + \lambda_2^4L^4\right)d\rho\wedge d\phi_1 - \lambda_2^4L^4\rho\sin\chi d\chi\wedge d\phi_1\right),$$

$$e^{-2\hat{\Phi}} = \frac{QL^2}{\alpha'^3},$$

with, $Q = (\lambda^2\lambda_2^4L^4 + \rho^2\alpha'^2(\lambda^2\cos^2\chi + \lambda_2^2\sin^2\chi))$.

- range of (ρ, χ, ξ)?

$\chi \in [0, \pi]$

$\xi \in [0, 2\pi]$

The RR fields are given by

$$\hat{F}_2 = \frac{4\lambda\lambda_1^2\lambda_2^2 L^4 \sin\theta_1 d\theta_1 \wedge d\phi_1}{\alpha'^{3/2}}, \quad F_4 = B_2 \wedge F_2$$

So, we see that we have 'lost' one of the $SU(2)$ global symmetries in the QFT/isometries in the background

$\underbrace{\text{Conformality}}_{AdS_5}$, the other $\underbrace{SU(2)}_{(\theta_1, \phi_1, \xi)}$ and the $\underbrace{\text{R-symmetry}}_{\xi}$ are still there.

We impose that in the new background, charges are quantised.

$$Q_{Page,D6} = \frac{1}{2\kappa_{10}^2 T_{D6}} \int F_2 - F_0 B_2 = N_{D6}$$
$$Q_{Page,D4} = \frac{1}{2\kappa_{10}^2 T_{D2}} \int F_4 - B_2 \wedge F_2 = N_{D4} = 0.$$

This implies a relation $\frac{L^4}{\alpha'^2} = \frac{27}{2} N_{D6}$. One can also define a two-cycle

$$\Sigma_2 = [\chi, \xi = 2\pi - \phi_1], \quad \theta_1 = 0, \rho = \rho_0.$$

and calculate the quantity $b_0 = \frac{1}{4\pi^2\alpha'} \int_{\Sigma_2} B_2$.
We observe that $b_0 = \frac{\rho_0}{\pi}$. This *suggests* that the ρ-coordinate should be taken between $[0, \pi]$. Beyond that, some transformation in the CFT should 'change description'.

We could also study large gauge transformations of the B-field,

$$B_2[new] = B_2[old] + n\alpha'\pi \sin\chi d\chi \wedge d\xi.$$

To realise that the Page charges change as,

$$Q_{Page,D6} = N_{D6}, \quad Q_{Page,D4} = 0$$
$$\Delta Q_{Page,D6} = 0, \quad \Delta Q_{Page,D4} = -nN_{D6}$$

Some of you may recognise that this is the same structure we observed long ago in the duality cascade of the Klebanov-Strassler background-QFT dual pair.

In this case, this is happening but in motions in the ρ-coordinate; not an Energy coordinate. The QFT is conformal.

Another interesting observable to calculate is the central charge.
In the end is just the calculation of 'volumes' of the internal space.
We will then 'bound' the ρ-coordinate in $[0, \pi]$—sort of a cell where a CFT lives, once passed, the description must change.
We obtain

$$c_{KW} = \frac{27}{8}\pi^5 N_{D3}^2, \quad c_{NATDKW} = \frac{9}{8}\pi^5 N_{D6}^2,$$

$$\frac{c_{KW}}{c_{NATDKW}} = 3\left(\frac{N_{D3}}{N_{D6}}\right)^2.$$

We have observed the same relation $\frac{c_{before}}{c_{after}} = 3(\frac{N_{D3}}{N_{D6}})^2$, occurs for the cases of S^5, $Y^{p,q}$.

Geometrically, there is a nice way of 'organising' this information.
It uses the so called $SU(2)$-structures; a topic on which I am 'just a user'. But it is a very useful way of encoding the information of SUSY backgrounds.
Let me describe, what this $SU(2)$-structure is in this particular case of the non-abelian transformed of Klebanov-Witten. A quite similar structure appears for the whole baryonic branch mentioned before.
It is useful to define vielbeins, before and after the non-abelian T-duality

Before | After

$e^{\theta_1} = L\lambda_1\, d\theta_1 \quad e^{\phi_1} = L\lambda_1 \sin\theta_1\, d\phi_1\, ; \; e^r = \frac{L}{r} dr \longrightarrow$ same.

$e^1 = L\lambda_2\, \omega_1 \; ; \; e^2 = L\lambda_2\, \omega_2 \; ; \; e^3 = L\lambda_3(\omega_3 + \cos\theta_1\, d\phi_1) \longrightarrow \left.\begin{array}{l}\hat{e}^1\\ \hat{e}^2\\ \hat{e}^3\end{array}\right\}$ Complicated, but can be written in terms of $(\rho, \chi, \xi), (L, \lambda_1, \lambda_2, \lambda_3)$

With these vielbein basis, one can define a two form and a three form J_2, Ω_3 for the original KW background and forms j_2, ω_2, v_1, w_1 for the transformed background. They read,

<u>Before</u> $\xrightarrow{\text{NATD}}$ <u>After</u>

$$\underbrace{J_2 = e^{\theta_1 \varphi_1} - e^{12} + e^{r3} \qquad \Omega_3 = (e^2 + i e^1) \wedge (e^\theta + i e^\varphi) \wedge (e^3 + i e^r)}_{\text{This defines an } SU(3)\text{-structure}}$$

$$\underbrace{z_1 = v_1 + i w_1 = \hat{e}^r + i \hat{e}^3 \qquad j_2 = e^{\theta\varphi} - \hat{e}^1 \wedge \hat{e}^2 \qquad \omega_2 = (e^\theta + i e^\varphi) \wedge (\hat{e}^2 + i \hat{e}^1)}_{\text{This defines an } SU(2)\text{-structure}}$$

It is then possible, to write a set of equations for these forms, that are the BPS equations and equations that define the different fluxes

Before SU(3) structure of KW

$$ds^2 = e^{2\Delta}(dx_{1,3}^2 + ds_6^2)$$

$$F_5 = e^{4\Delta+\phi} V_4 \wedge f_1 \, (1+*_{10})$$

where

$$f_1 = -e^{-4\Delta+\phi} d(e^{4\Delta})$$

$$\phi = F_3 = H_3 = F_1 = 0$$

fluxes

$$d(e^{6\Delta+\phi/2}\Omega_3) = 0$$

$$d[e^{8\Delta} J_2 \wedge J_2] = 0$$

BPS eqs

After SU(2) structure of NATD KW

$$ds^2 = e^{2A} dx_{1,3}^2 + ds_6^2, \quad \hat{\Phi}_{new}, \quad H_3 = dB_2$$

fluxes

$$F_2 = e^{-4A} d[e^{4A-\hat{\Phi}} \omega_1]$$

$$F_4 = e^{-4A} d[e^{4A-\hat{\Phi}} J_2 \wedge v_1] - e^{-\hat{\Phi}} H_3 \wedge \omega_1$$

$$*F_0 = \frac{e^{-4A}}{2} d[e^{4A-\hat{\Phi}} J_2 \wedge J_2 \wedge \omega_1] + e^{-\hat{\Phi}} H_3 \wedge J_2 \wedge v_1$$

$$*_6 F_6 = 0$$

BPS eqs.

$$d[e^{2A-\hat{\Phi}} J_2 \wedge J_2 \wedge v_1] = e^{2A-\hat{\Phi}} H_3 \wedge J_2 \wedge \omega_1$$

$$d[e^{3A-\hat{\Phi}} \omega_2] = 0; \quad d[e^{2A-\hat{\Phi}} v_1] = 0$$

$$\omega_2 \wedge \{ d(v_1 \wedge \omega_1) + i H_3 \} = 0$$

Once we generated the NS-part of the background,
we may think it is more effective, to first recognise the $SU(2)$ structure and then write the explicit expressions for the RR-fluxes (without going over the usual non-abelian T-duality procedure.)

$$F_6 = d\left[e^{4A-\phi} vol_4 \wedge w_1\right],$$
$$F_8 = d\left[e^{4A-\phi} vol_4 \wedge j_2 \wedge v_1\right] - e^{4A-\phi} H \wedge vol_4 \wedge w_1,$$
$$F_{10} = -\frac{1}{2} d\left[e^{4A-\phi} Vol_4 \wedge j_2 \wedge j_2 \wedge w_1\right] + e^{4A-\phi} H \wedge vol_4 \wedge j_2 \wedge v_1;$$

where the remaining fluxes can be obtained from the duality condition $F_{2n} = (-)^n \star F_{10-2n}$. The following potentials can be derived:

$$C_5 = e^{4A-\phi} vol_4 \wedge w_1, \quad C_7 = e^{4A-\phi} vol_4 \wedge j_2 \wedge v_1,$$
$$C_9 = -\frac{1}{2} e^{4A-\phi} vol_4 \wedge j_2 \wedge j_2 \wedge w_1.$$

- Let us move into something more dynamical!

Some more interesting or dynamical results are obtained when we apply these techniques to field theories with a RG flow:
For example, applying this to the solution of D5 branes, the Klebanov-Tseytlin-Strasler solution or the whole baryonic branch.
One can see that many phenomena, like the cascade of Seiberg dualities, the presence of domain walls, the presence of confinement (Wilson loops), symmetries breaking, quotients of central charges, etc; are not modified.
But some other quantities, like other correlators in the QFT, can indeed be very different.
A way of thinking about this is: all correlation functions -uncharged- under the $SU(2)$ symmetry to be dualised will remain invariant.
This is some pattern that we saw also in previous solution-generating/new field theory pairs.
The different observables are defining a QFT at strong coupling.

Let me describe something interesting that occurs when one applies this logic to the Baryonic Branch of Klebanov-Strassler. I will use the beautiful solution found by Butti, Graña, Minasian, Petrini and Zaffaroni in 2005.

Let me briefly remind how the story goes:

CFT

$+ W_{tree} = ABAB \rightsquigarrow AdS_5 \times T^{1,1} \;;\; g_{\mu\nu}, F_5$

deform with quasi marginal couplings g_1, g_2, add "M" $\rightsquigarrow$ Klebanov-Tseytlin

$g_{\mu\nu}, F_5, H_3, F_3 \;;\; (\Phi)$

flow down

Cascade of Seiberg dualities

flow down

If $N = KM$

$W = W_{tree} + \hat{\Lambda}(\det M - B\tilde{B} - \Lambda^{2M})$

"deformation" of geometry

→ confinement +

Z_{2M} R-Symmetry breaking

- Baryonic
- Mesonic
- Mixed

branches → Baryonic branch

In terms of G-structures, we are 'rotating' the pure spinors or the defining forms J_2, Ω_3 by an angle ξ.

Depending on the angle ξ, we are at different points in the Baryonic Branch VEVs.

If we non-Abelian T-dualise the background, we get a solution in Massive IIA.

$\{ g_{\mu\nu}, F_2, F_4, H_3, \hat{\Phi}, F_0 \} \longrightarrow$ Smooth solution

for its G-structure

Radius = Energy

ψ_1, ψ_2 "Static" orthogonal SU(2)

ψ_1, ψ_2 "dynamic" SU(2)

This solution is SUSY, smooth and with a bounded dilaton.

Presents the following characteristic: the $SU(2)$-structure changes with the RG-evolution.

This is one of the few known examples of dynamical $SU(2)$-structure in Massive IIA.

This suggests ways to find these strange / uncommon G-structures' situations!

Other interesting things about the QFT, learnt from the background are:

- The UV of the field theory still behaves like 'logarithmically approaching to a CFT'. A cascade of Seiberg dualities is present.
- The R-symmetry can be geometrically identified. Anomalies can be understood using D0-branes. The R-symmetry can be seen to be broken
- A nice interplay between domain walls, confinement, R-symmetry breaking and the dynamical character of the $SU(2)$-structure.
- Is possible to find the fluctuation associated with the Goldstone mode of the $U(1)_B$ global symmetry breaking.
- One can find in the background the branes that compute for us: the baryon vertex and the baryon VEV.

Plus other pieces of interesting dynamics, of the strongly coupled QFT. Understood using a gravitational perspective.

Let me briefly discuss one of the points above.

Let us comment about Domain Walls.
These are realised in the non-Abelian duals of the baryonic branch, as D2 branes extended on (t, x_1, x_2). If we also turn on a gauge field on the world-colume of the D2 branes, we get an action

$$S_{BIWZ} = -T_{D2}\int d^{2+1}x e^{\Phi/2}\hat{h}^{-3/4}\sqrt{1-\alpha' F_{\mu\nu}F^{\mu\nu}}$$
$$+T_{D2}\int d^{2+1}x F_0 A_1 \wedge F_2.$$

A special WZ-like term appears in Massive IIA (Green-Hull-Townsend, 1996). The Chern-Simons term is quantised, being proportional to $T_{D2}N_c$
Supersymmetry gives support to this. One can see that these defects are calibrated. Again, being in massive IIA is crucial for this to be the case
This ties up nicely with the presence of confinement, the 'dynamical' character of the $SU(2)$-structure and the SUSY preservation of these defects.

One last thing for the 'geometrically minded' in the audience

It is possible to find new solutions in Type IIB, containing an AdS_5 factor. These are 'new' in the sense that they are analytic solutions that avoid all known classifications.

This is OK. These classifications [Gauntlett, Martelli, Sparks, Waldram+....] start by force, from some assumptions. For example, the existence of F_5, a holomorphic axion-dilaton, etc.

One can do the following: Perform a NATD on the $SU(2)$ of the background (for example) $AdS_5 \times X^5$, with $X^5 = S^5, T^{1,1}, Y^{p,q}$.

Then, 'mix' coordinates, for example using a parameter γ as in, $\phi \to \phi + \gamma\xi$. Then perform for example a usual T-duality on one of these periodic directions.

This is obviously inspired on Lunin-Maldacena's TsT. One can even perform NATD-s-NATD. These typically leads to new -families- of solutions, SUSY, but characteristically singular.

Brief summary

There is a nice interplay between different backgrounds dual to field theories with minimal SUSY in four dimensions.

This is the strong-coupling version of some perturbative relations between apparently different field theories.

The relation extends with nice subtleties when you add flavors to the initial QFT's. Or when you consider subtle variations of the dynamics: the addition of operators that dominate the dynamics at different scales (making it a theory with various tunable scales).

One may discover new geometries, dual to N=1 SUSY models, inspired by perturbative connections between the field theories.

On the other hand, new QFT's can be discovered by starting with non-perturbative description (smooth string background) and operating on it with generating techniques. We have seen an example in this talk, with non-abelian T-duality. The job now is to clarify the QFT description of this operation.