Charged Lepton Flavour Physics

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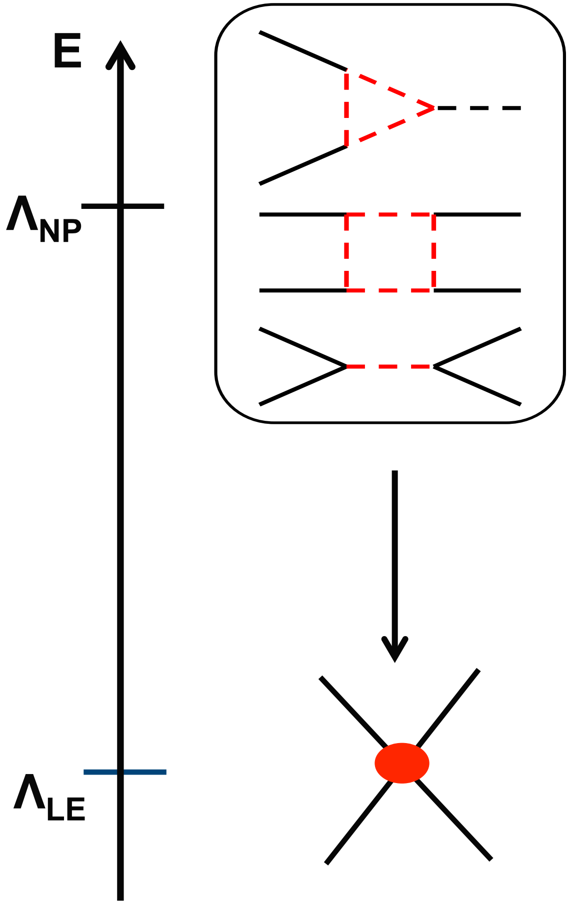
The Landscape of Flavour Physics towards the high intensity era

Pisa, December 9, 2014

1. Introduction and Motivation

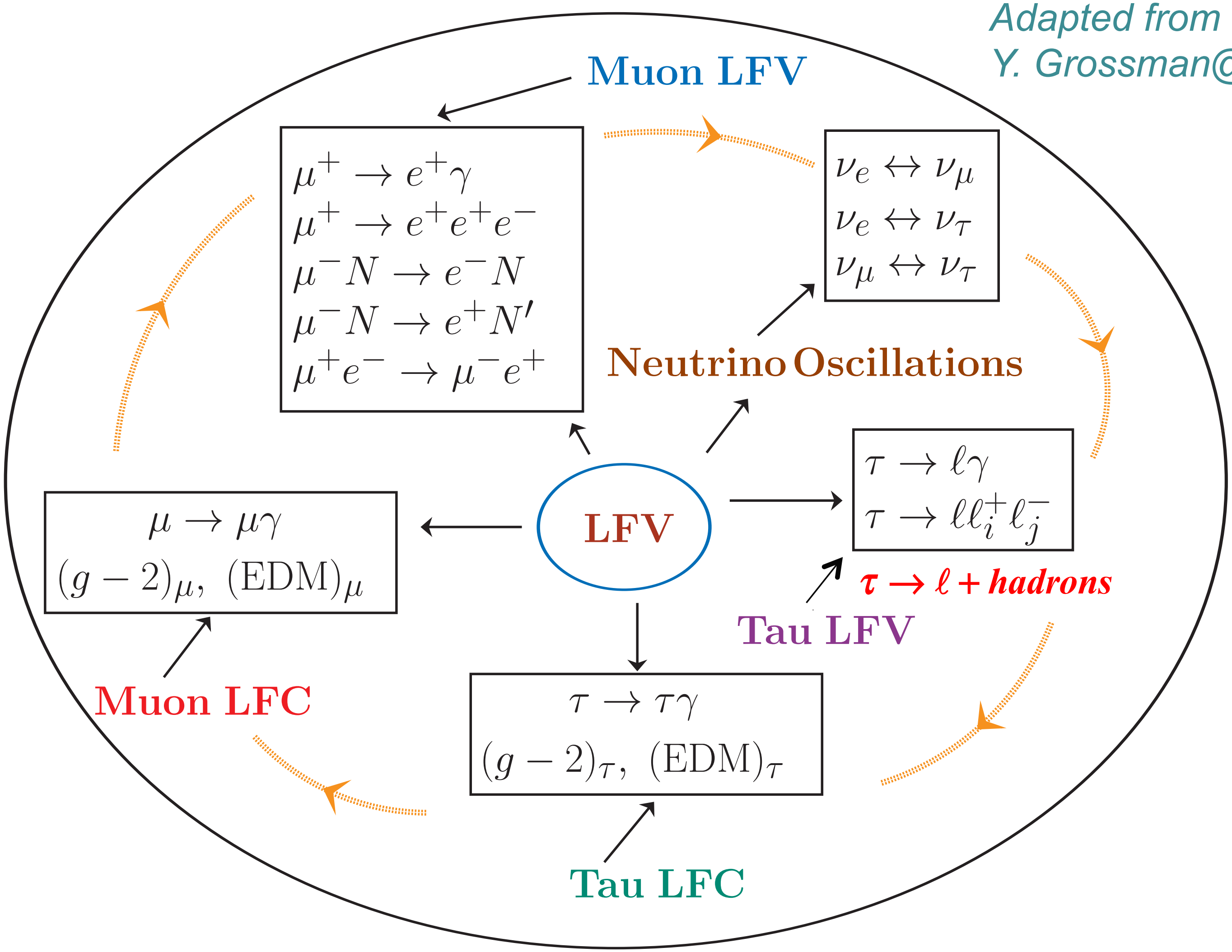
1.1 Why study charged leptons?



- In the quest of New Physics, can be sensitive to very high scale:
  - Kaon physics: $\boxed{\frac{s\bar{d}s\bar{d}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^5 \text{ TeV}}$
    [ε_K]
  - Charged Leptons: $\boxed{\frac{\mu\bar{e}f\bar{f}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^4 \text{ TeV}}$
    [$\mu \to e\gamma$]
- At low energy: lots of experiments e.g., *MEG*, *COMET*, *Mu2e*, *E-969, BaBar*, *BelleI-II*, *BESIII, LHCb* ⇨ huge improvements on measurements and bounds obtained and more expected
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential

⇨ Charged leptons very important to look for *New Physics*!

1.2 The Program



Adapted from Talk by Y. Grossman@CLFV2013

LFV

Muon LFV

$\mu^+ \to e^+\gamma$

$\mu^+ \to e^+e^+e^-$

$\mu^- N \to e^- N$

$\mu^- N \to e^+ N'$

$\mu^+ e^- \to \mu^- e^+$

Neutrino Oscillations

$\nu_e \leftrightarrow \nu_\mu$

$\nu_e \leftrightarrow \nu_\tau$

$\nu_\mu \leftrightarrow \nu_\tau$

Tau LFV

$\tau \to \ell\gamma$

$\tau \to \ell \ell_i^+ \ell_j^-$

$\boldsymbol{\tau \to \ell + hadrons}$

Tau LFC

$\tau \to \tau\gamma$

$(g-2)_\tau,\ (\mathrm{EDM})_\tau$

Muon LFC

$\mu \to \mu\gamma$

$(g-2)_\mu,\ (\mathrm{EDM})_\mu$

2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

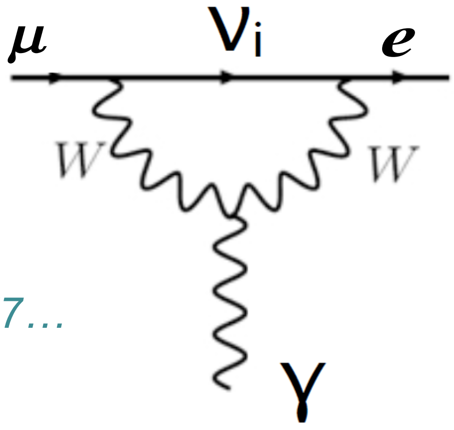
- Neutrino oscillations are the first evidence for lepton flavour violation

- How about in the charged lepton sector?

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br\left(\mu \to e\gamma\right) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br\left(\tau \to \mu\gamma\right) < 10^{-40}\right]$$

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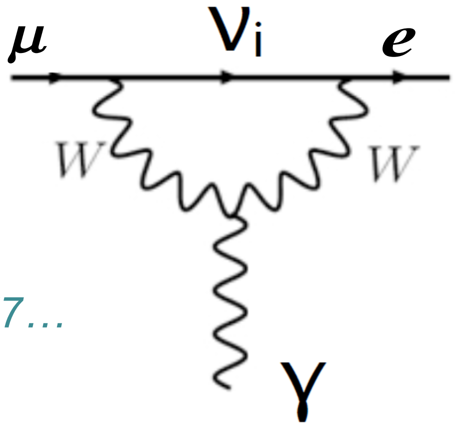

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Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

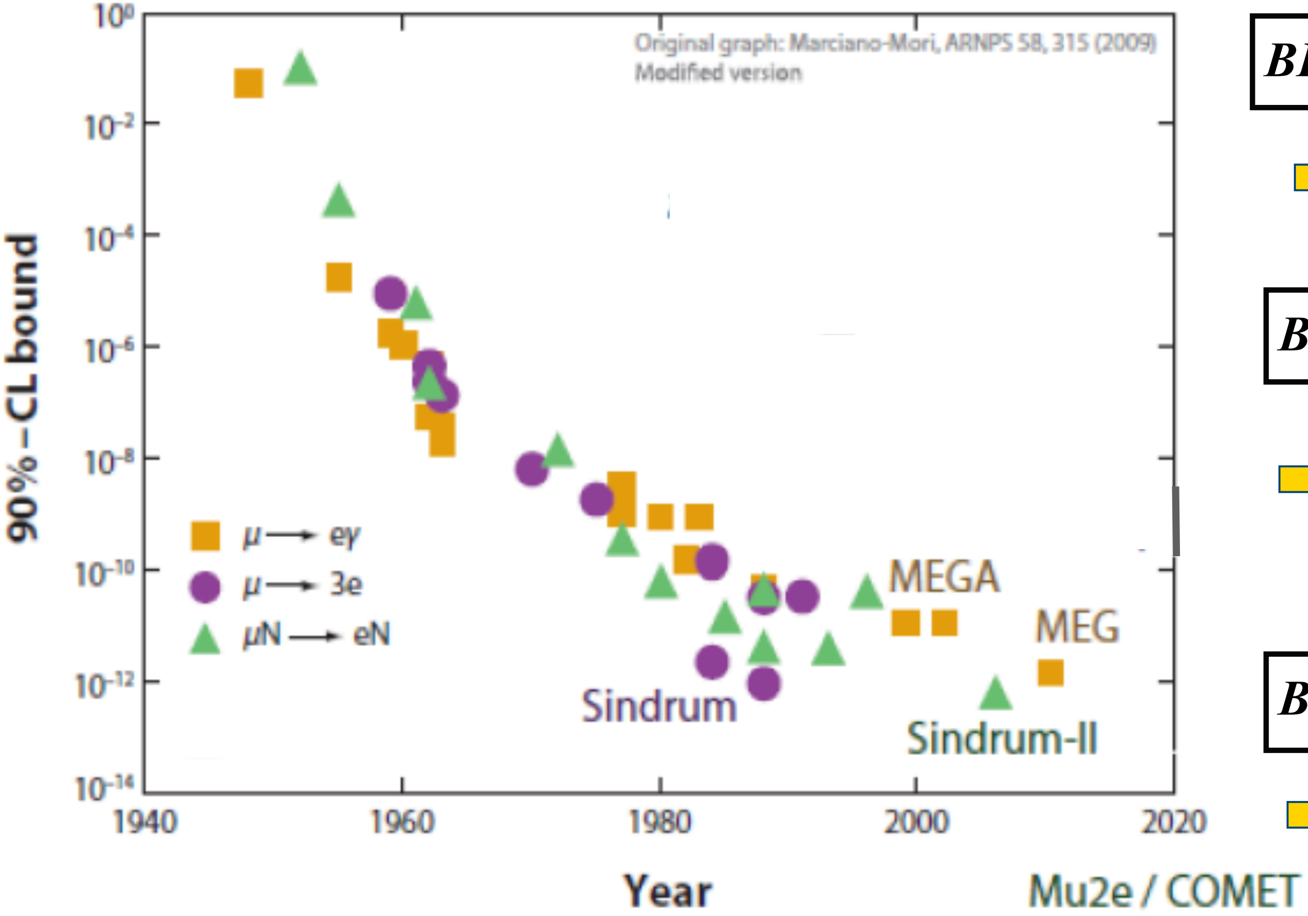
Talk by D. Hitlin @ CLFV2013

		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: muon decays

- Several processes: $\mu \rightarrow e\gamma,\ \mu \rightarrow e\bar{e}e,\ \mu(A,Z) \rightarrow e(A,Z)$



MEG'13

$$\boxed{BR(\mu \rightarrow e\gamma) < 5.7\ 10^{-13}}$$

⇨ 10^{-14}

Sindrum

$$\boxed{BR(\mu \rightarrow eee) < 1.0\ 10^{-12}}$$

⇨ $10^{-15} - 10^{-16}$

Mu3e

Sindrum II

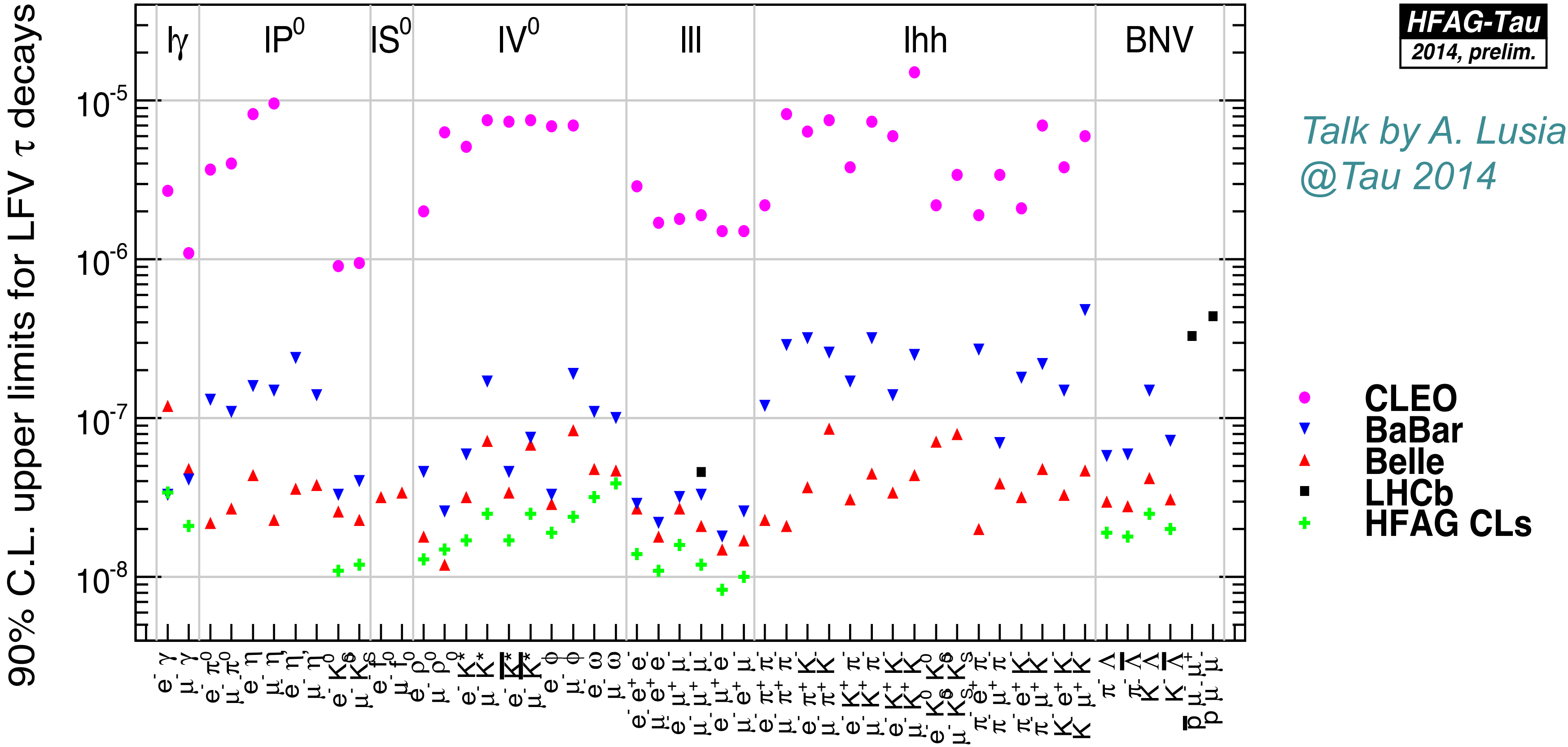
$$\boxed{BR^{Ti}_{\mu-e} < 4.3\ 10^{-12}}$$

⇨ $10^{-16} - 10^{-17}$

Mu2e/COMET

2.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \overline{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ $\nwarrow$ $P,\ S,\ V,\ P\overline{P},\ldots$



Talk by A. Lusiani @Tau 2014

- 48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

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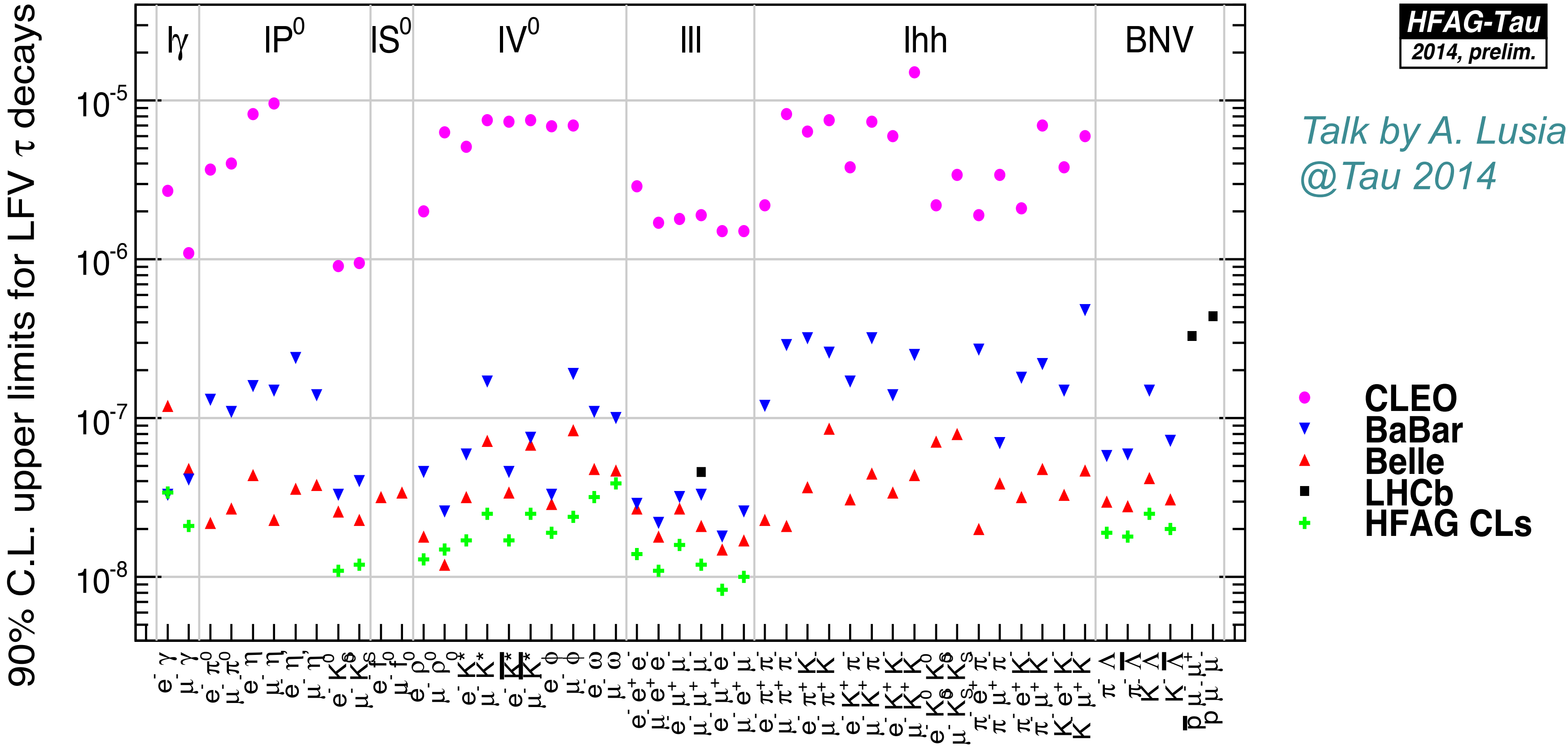


Talk by A. Lusiani @Tau 2014

- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \boxed{\sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)}} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

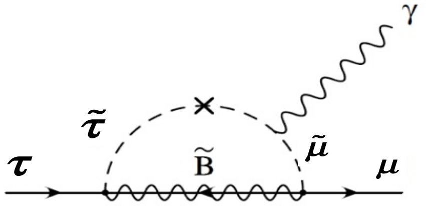
- Build all D=6 LFV operators:
  - Dipole
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector)
  - Lepton-gluon (Scalar, Pseudo-scalar)
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector)
- Each UV model generates a *specific pattern* of them

2.3 Effective Field Theory approach

- Dipole:

$$\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$$

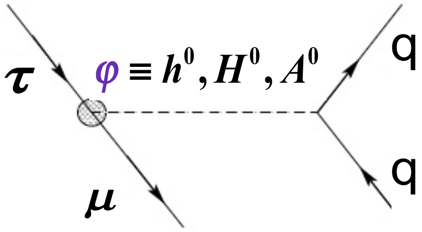
$$O_D$$



- Scalar (Pseudo-scalar) :

$$\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_S}{\Lambda^2} m_\tau m_q G_F \bar{\mu} P_{L,R}\tau\, \bar{q}q}$$

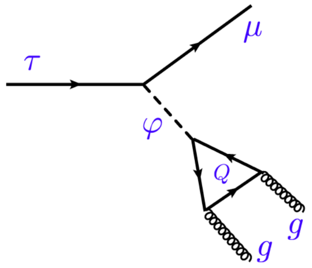
$$O_S^q$$



Integrating out heavy quarks generates *gluonic operator:*

$$\boxed{\frac{1}{\Lambda^2}\bar{\mu}P_{L,R}\tau Q\bar{Q}} \to \boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\, G^a_{\mu\nu} G_a^{\mu\nu}}$$

$$O_{GG}$$



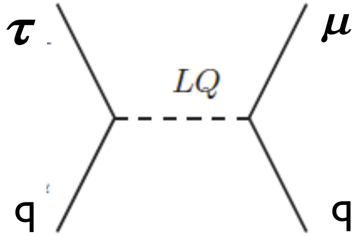
Importance of this operator emphasized in *Petrov & Zhuridov'14*

2.3 Effective Field Theory approach

- Vector (Axial-vector) :

$$\boxed{\mathcal{L}_{eff}^{V} \supset -\frac{C_V^q}{\Lambda^2}\, \bar{\mu}\gamma^{\mu} P_{L,R}\tau\; \bar{q}\gamma_{\mu} q}$$

↖ O_V^q

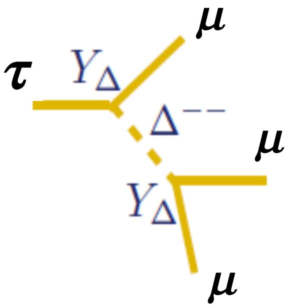


- 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector) :

$$\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2}\, \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{\mu}\, \Gamma P_{L,R}\mu}$$

↖ $O_{S,V}^{4\ell}$

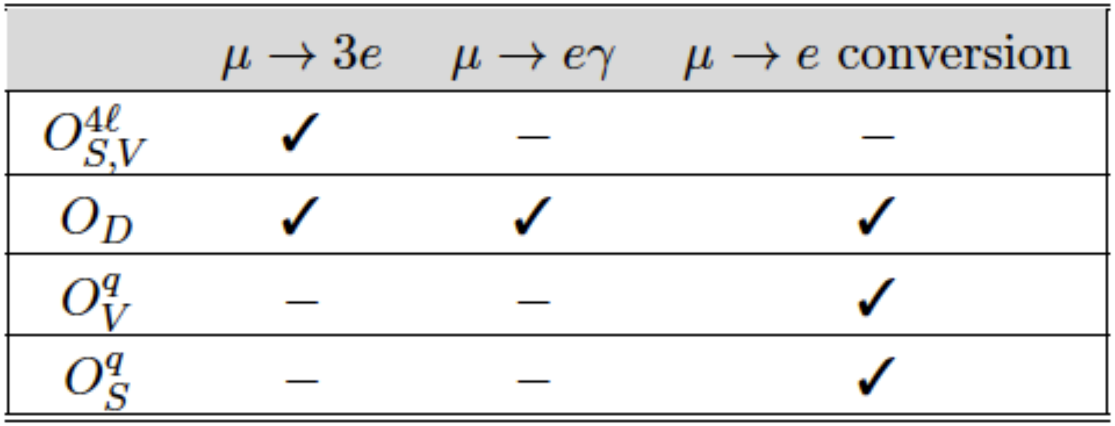
$$\boxed{\Gamma \equiv 1\ ,\gamma^{\mu}}$$



2.4 Model discriminating power of muon processes

- Summary table:

Cirigliano@Beauty2014



	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^q_V	–	–	✓
O^q_S	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes
  ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

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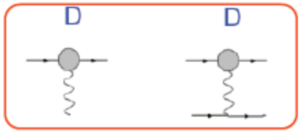
	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O_{S,V}^{4\ell}$	✓	–	–
O_D	✓	✓	✓
O_V^q	–	–	✓
O_S^q	–	–	✓

- $\mu \to e\gamma$ vs. $\mu \to 3e$ ⇨ relative strength between *dipole* and *4L* operators

$$\frac{\Gamma_{\mu\to 3e}}{\Gamma_{\mu\to e\gamma}} = \frac{\alpha}{4\pi} I_{\text{PS}} \left(1 + \sum_i \frac{c_i^{(\text{contact})}}{c^{(\text{dipole})}} \right)$$

6×10^{-3}



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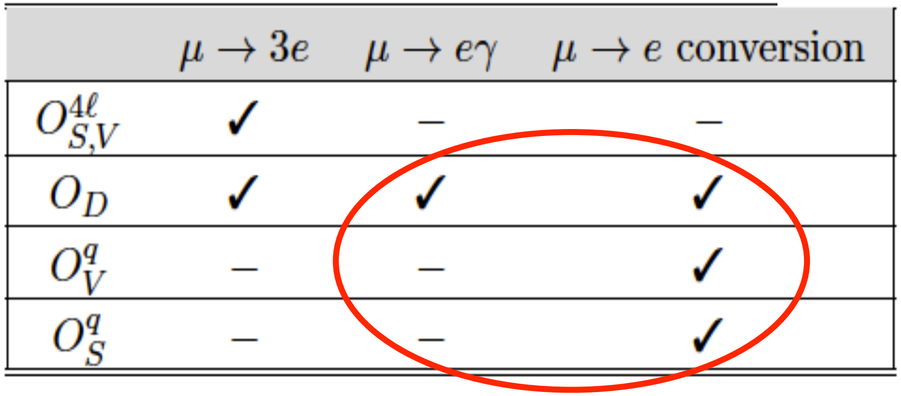
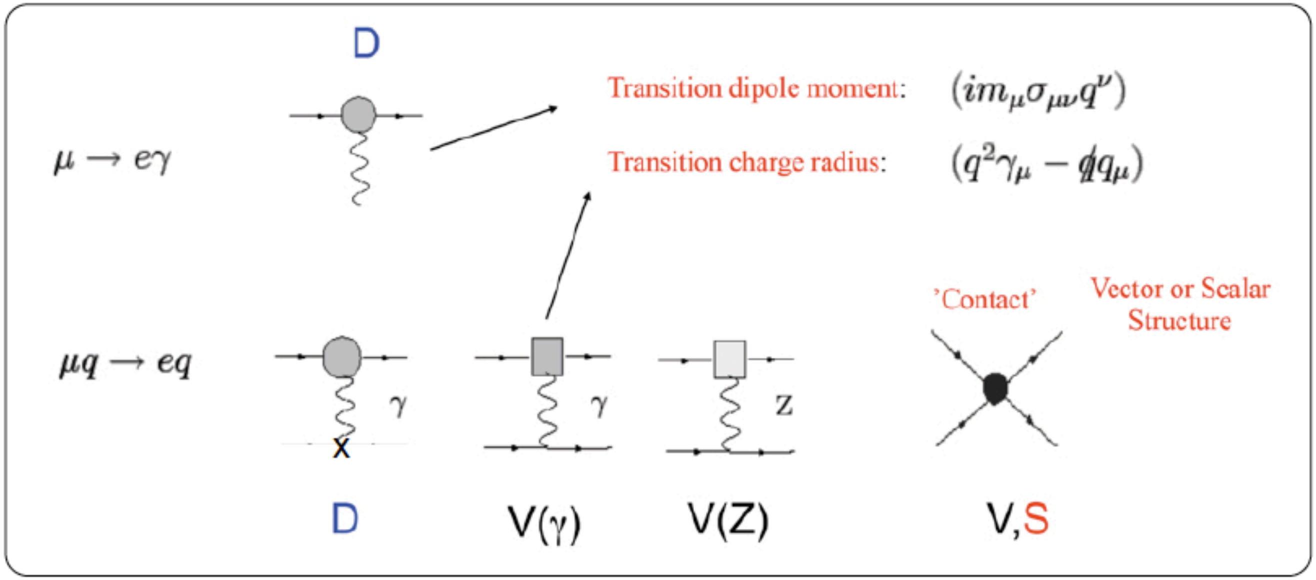


	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^q_V	–	–	✓
O^q_S	–	–	✓

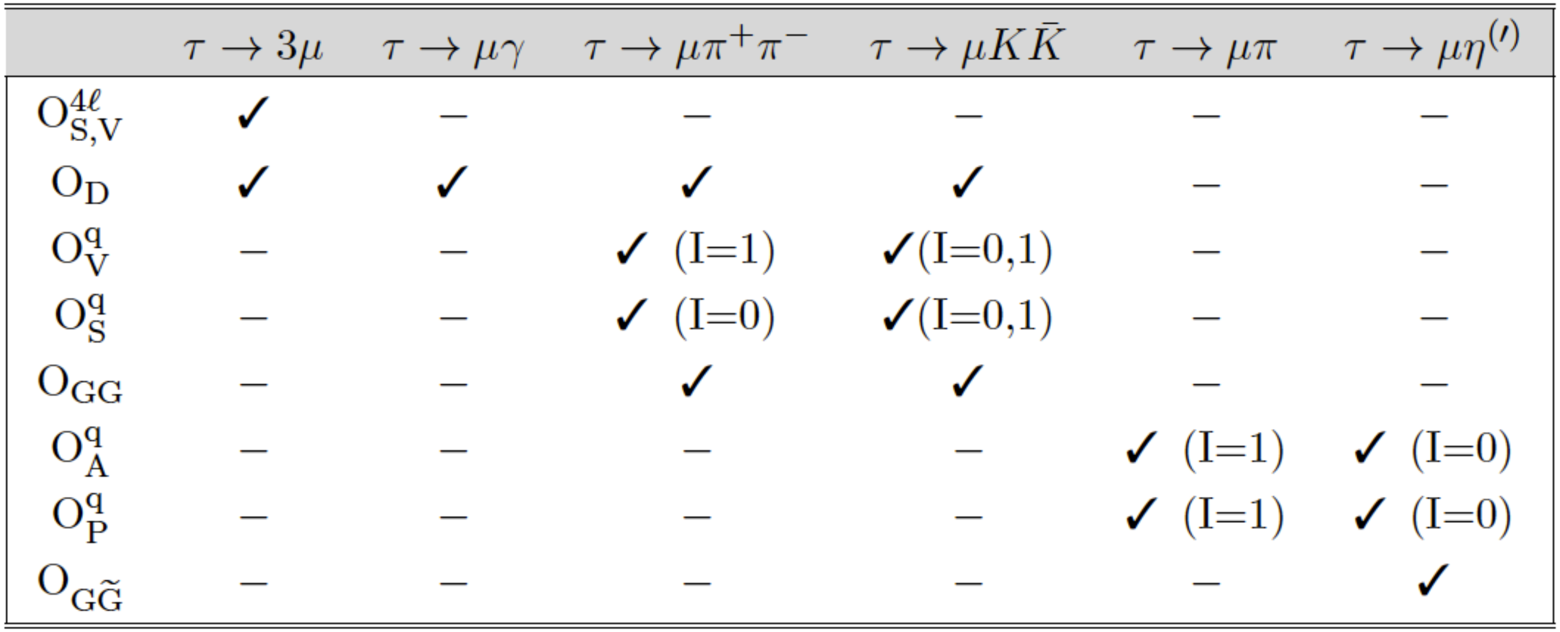
- *$\mu \to e\gamma$* vs. *$\mu \to e$ conversion* ⇨ relative strength between *dipole* and *quark* operators



2.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:



	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important → sensitive to large number of operators!

2.5 Model discriminating power of Tau processes

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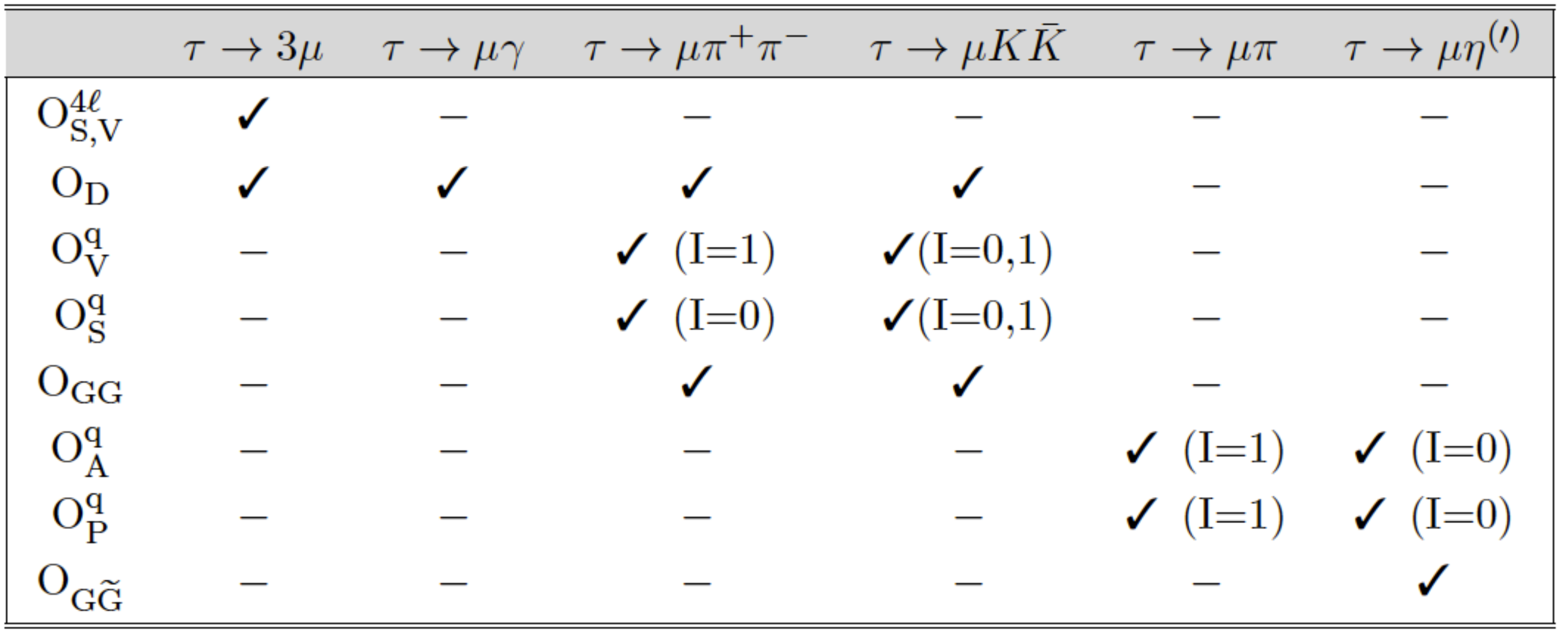

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important ⇨ sensitive to large number of operators!

- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. f_η, $f_{\eta'}$)

2.5 Model discriminating power of Tau processes

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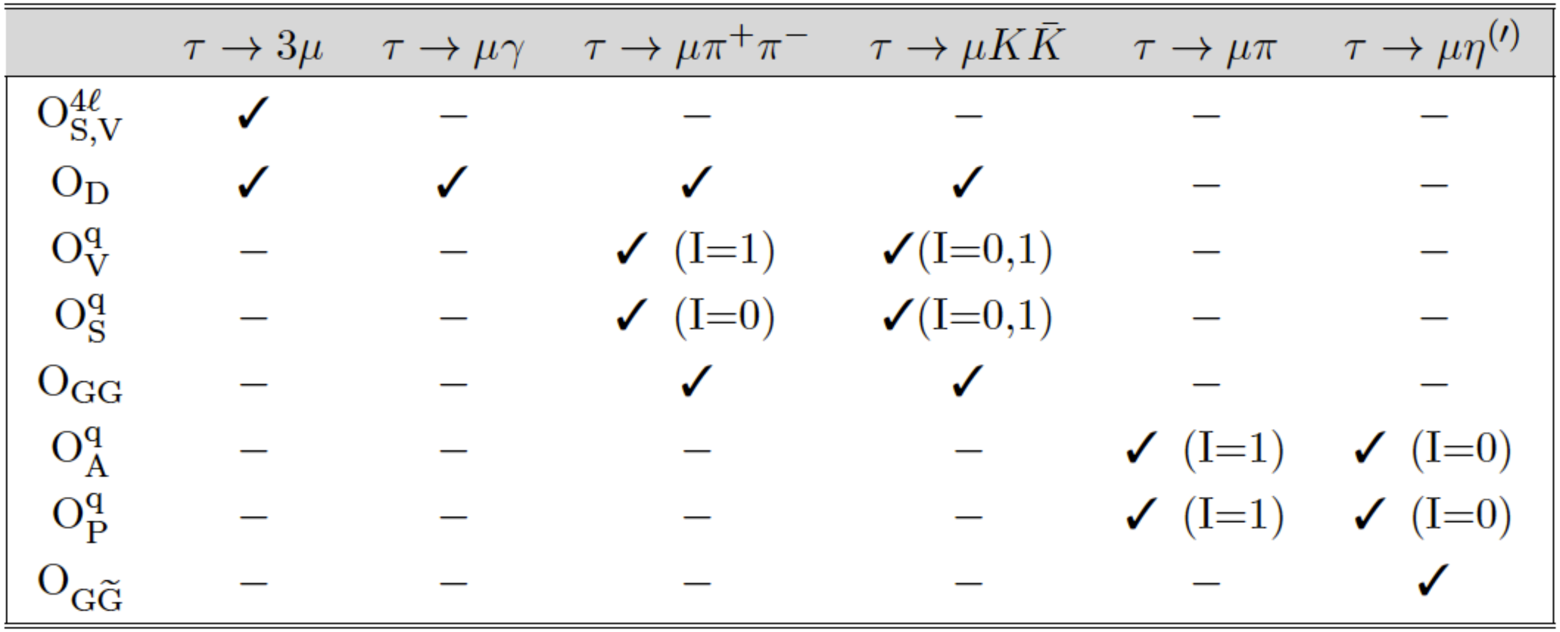
Celis, Cirigliano, E.P.'14



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$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Recent progress in $\tau \to \mu(e)\pi\pi$ using *dispersive techniques*

Daub et al'13
Celis, Cirigliano, E.P.'14

- Hadronic part: $\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(Lorentz\ \mathrm{struct.}\right)^i_\mu F_i(s)}$ with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave $\pi\pi$ and KK scattering data as input

$$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$n = \pi\pi, K\bar{K}$

2.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

2.6 Model discriminating of BRs

- Studies in specific models

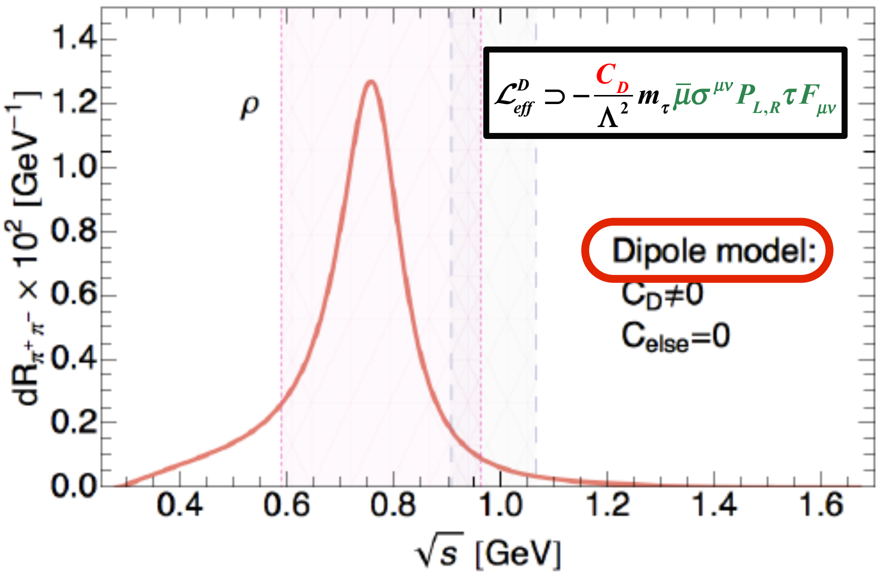
Buras et al.'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^- \to e^- e^+ e^-)}{\mathrm{Br}(\mu \to e\gamma)}$	$0.02 \dots 1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06 \dots 2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04 \dots 0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.07 \dots 2.2$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04 \dots 0.4$	$\sim 2 \cdot 10^{-3}$	$0.06 \dots 0.1$	$0.06 \dots 2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04 \dots 0.3$	$\sim 2 \cdot 10^{-3}$	$0.02 \dots 0.04$	$0.03 \dots 1.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04 \dots 0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.04 \dots 1.4$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8 \dots 2$	~ 5	$0.3 \dots 0.5$	$1.5 \dots 2.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}$	$0.7 \dots 1.6$	~ 0.2	$5 \dots 10$	$1.4 \dots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti} \to e\mathrm{Ti})}{\mathrm{Br}(\mu \to e\gamma)}$	$10^{-3} \dots 10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08 \dots 0.15$	$10^{-12} \dots 26$

⇨ Disentangle the *underlying dynamics* of NP

2.7 Model discriminating of Spectra: τ→μππ

Celis, Cirigliano, E.P.'14



$$\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}$$

Dipole model:
$C_D \neq 0$
$C_{else} = 0$

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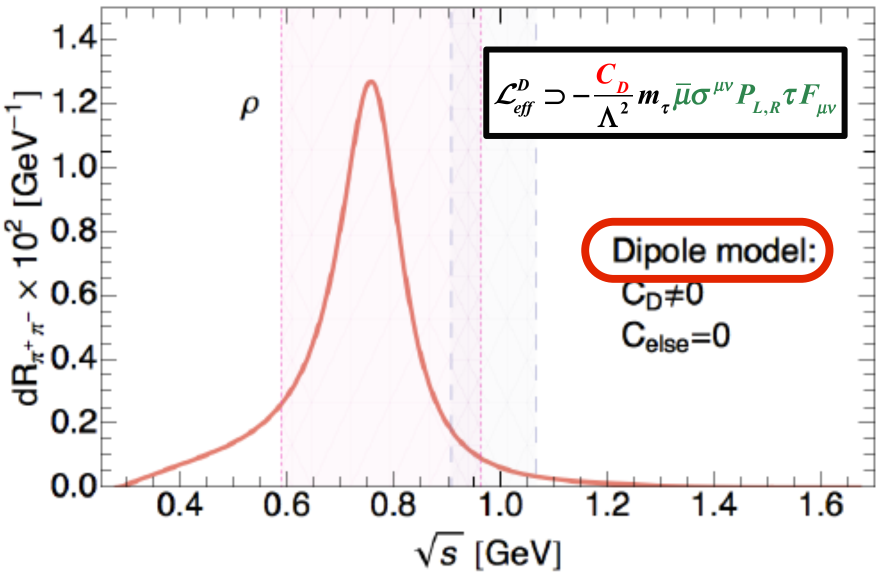
Celis, Cirigliano, E.P.'14



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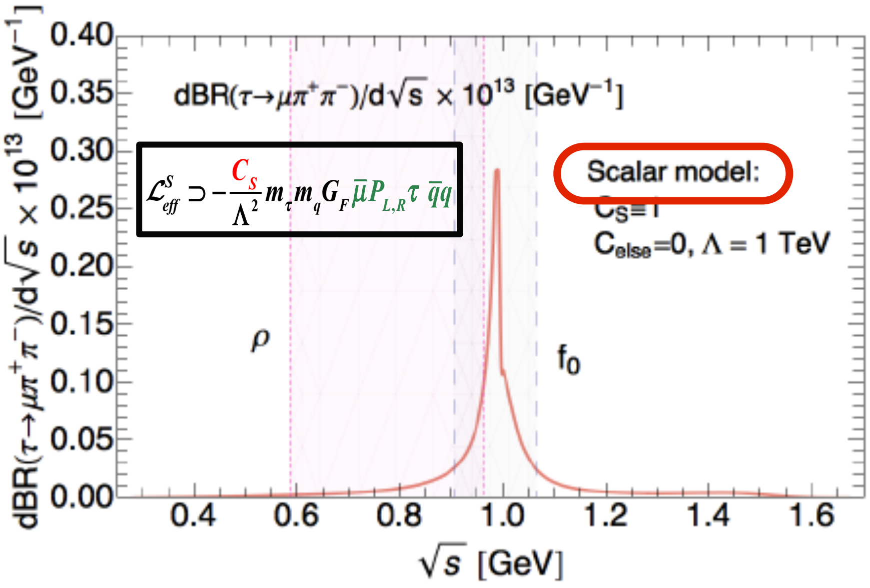

$$\mathcal{L}^S_{eff} \supset -\frac{C_S}{\Lambda^2} m_\tau m_q G_F \bar{\mu} P_{L,R} \tau \bar{q} q$$

Scalar model:
$C_S=1$
$C_{else}=0$, $\Lambda = 1$ TeV

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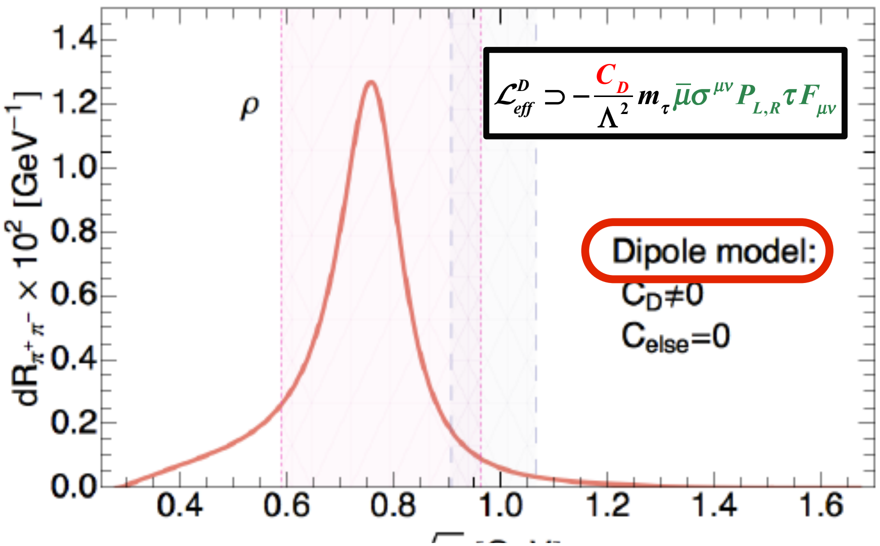
Celis, Cirigliano, E.P.'14



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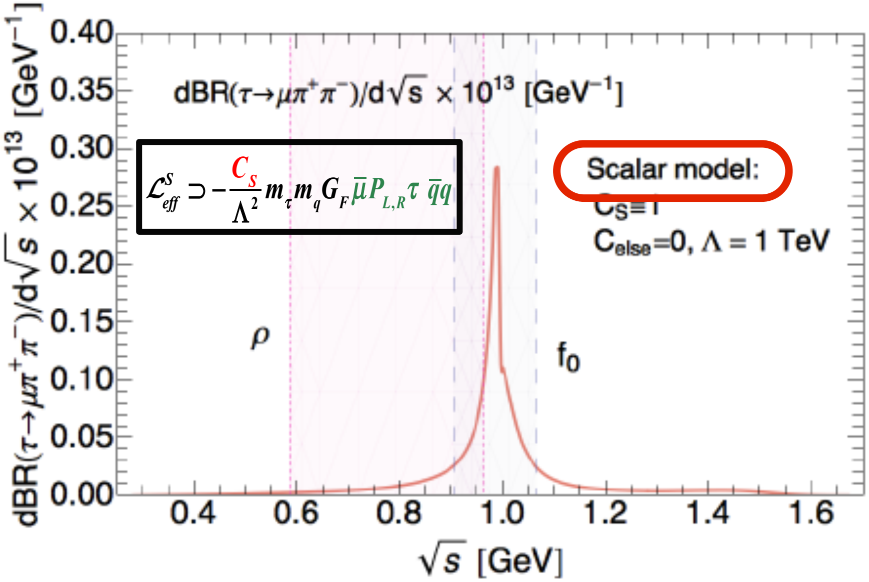
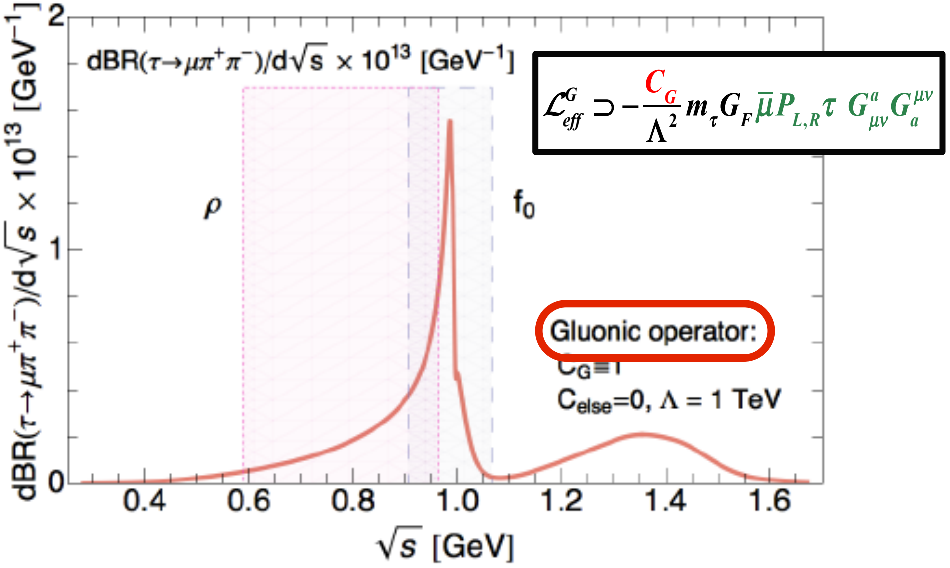
$C_D \neq 0$
$C_{else}=0$



$$\mathcal{L}_{eff}^{S} \supset -\frac{C_S}{\Lambda^2} m_\tau m_q G_F \bar{\mu} P_{L,R}\tau\, \bar{q}q$$

Scalar model:
$C_S=1$
$C_{else}=0$, $\Lambda = 1$ TeV



$$\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\, G^a_{\mu\nu}G^{\mu\nu}_a$$

Gluonic operator:
$C_G=1$
$C_{else}=0$, $\Lambda = 1$ TeV

Very different distributions according to the *final hadronic state!*

3. Charged Lepton-Flavour Violation and Higgs Physics

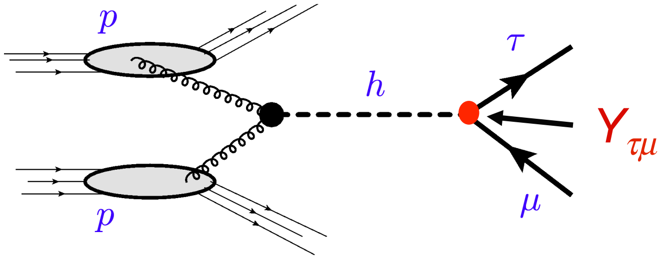
3.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H} \Rightarrow \boxed{-Y_{ij}\left(\bar{f}_L^i f_R^j\right)h}$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

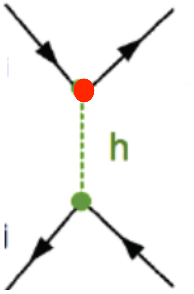
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

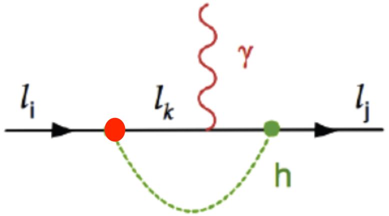
- High energy : LHC

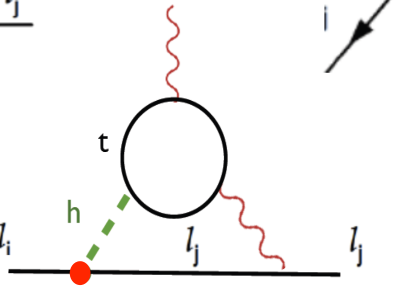


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







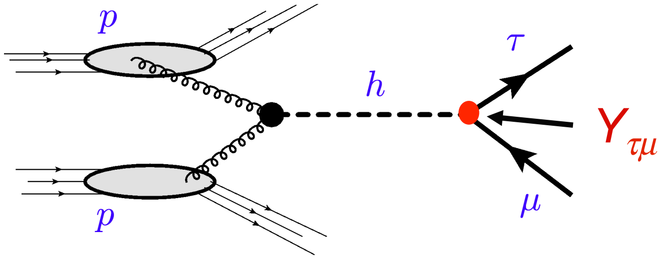
3.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

- High energy : LHC

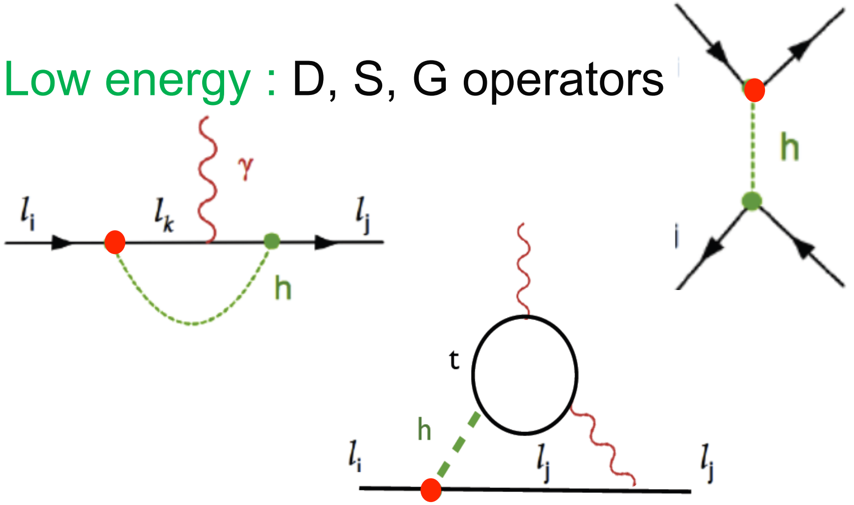
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$



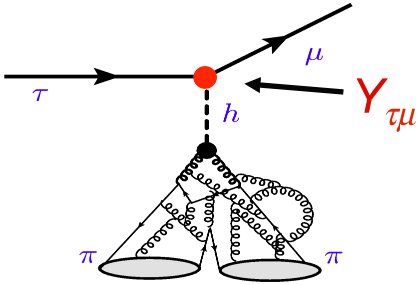
Hadronic part treated with perturbative QCD

Reverse the process

- Low energy : D, S, G operators



+

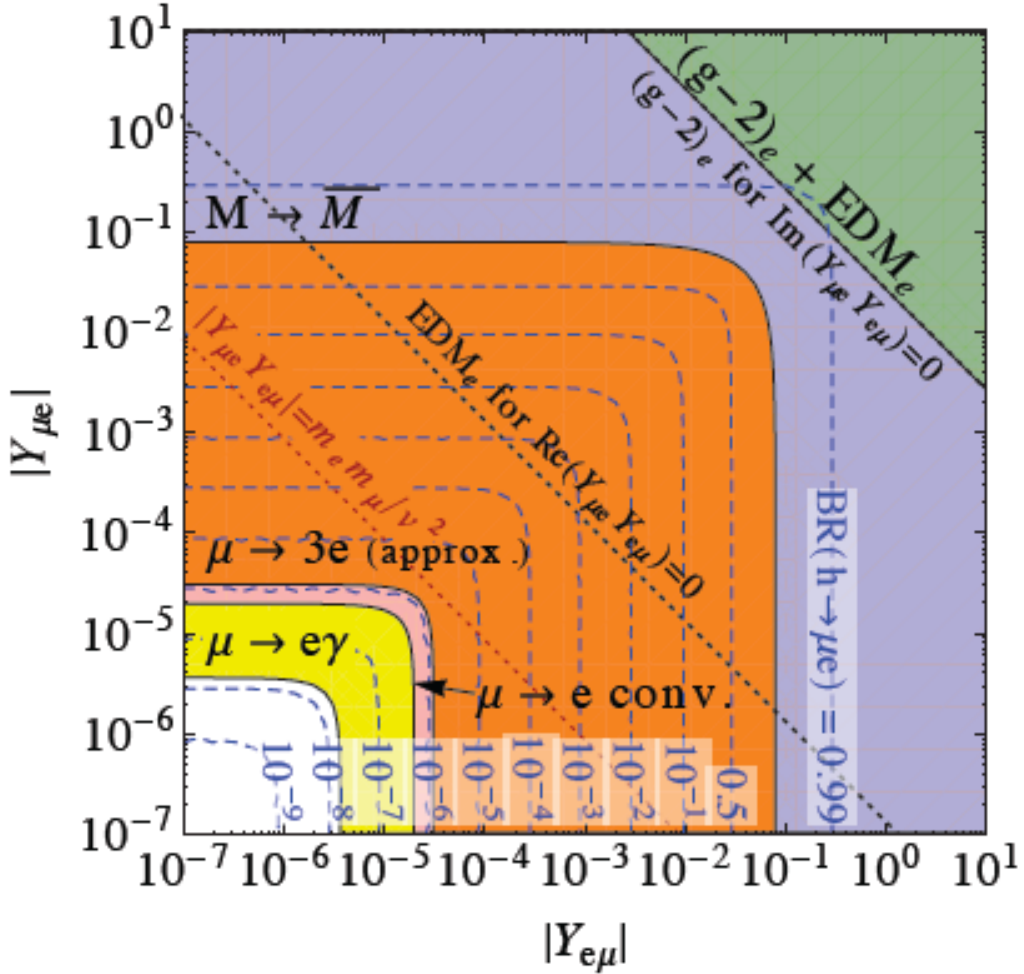


Hadronic part treated with non-perturbative QCD

3.2 Constraints in the μe sector

- Constraints from Higgs decay (LHC) vs. low energy LFV and LFC observables

Harnick, Koop, Zupan'12



- Best constraints coming from low energy: $\mu \to e\gamma$

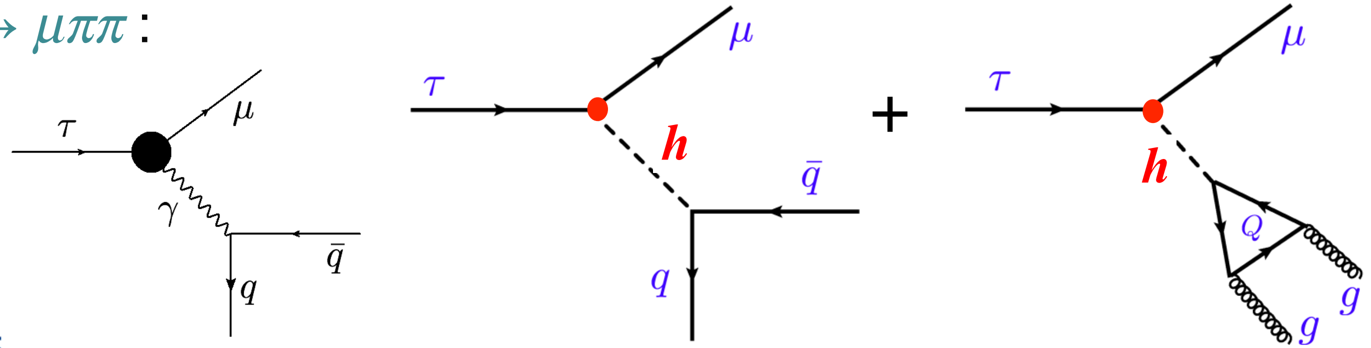
MEG'13

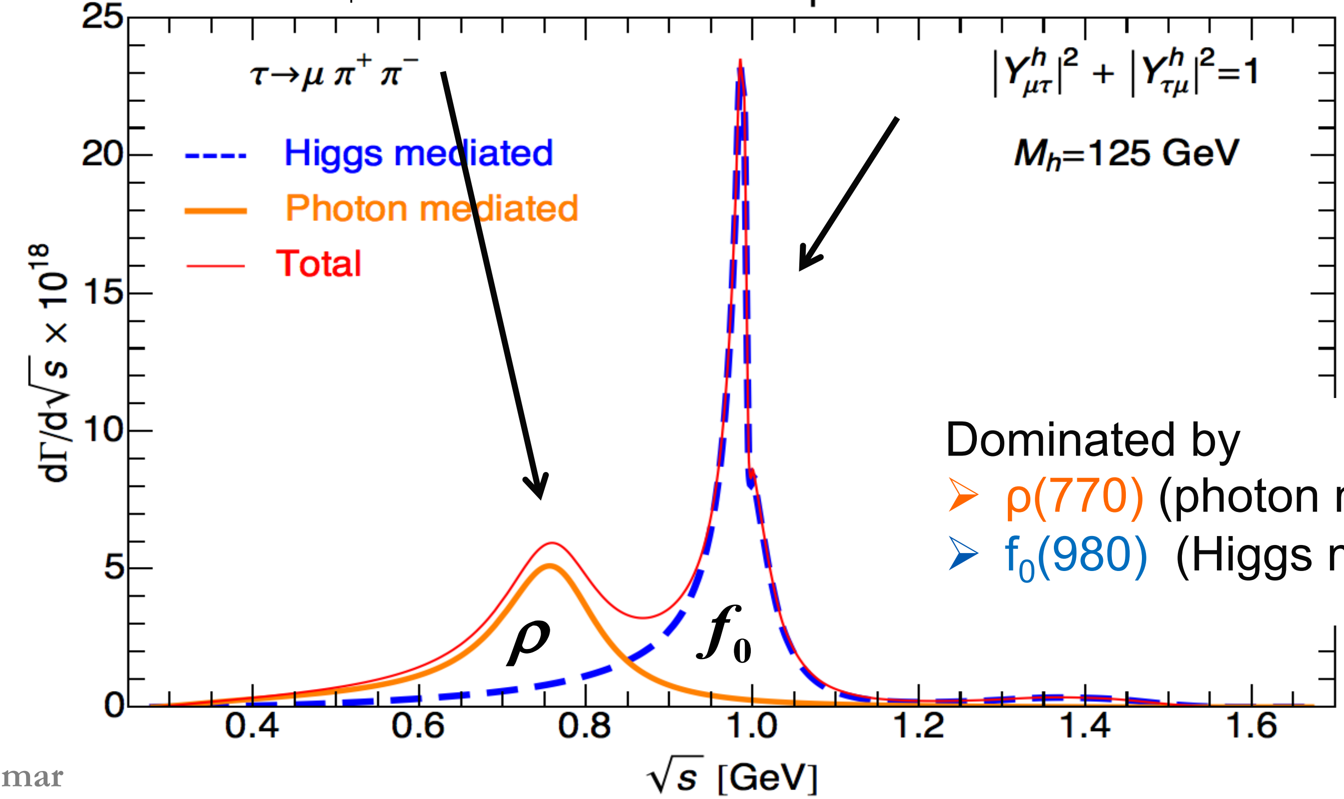
$$\boxed{BR\left(\mu \to e\gamma\right) < 5.7\ 10^{-13}}$$

$\Rightarrow$ $$\boxed{BR\left(h \to \mu e\right) < 10^{-7}}$$

3.3 Constraints in the τμ sector

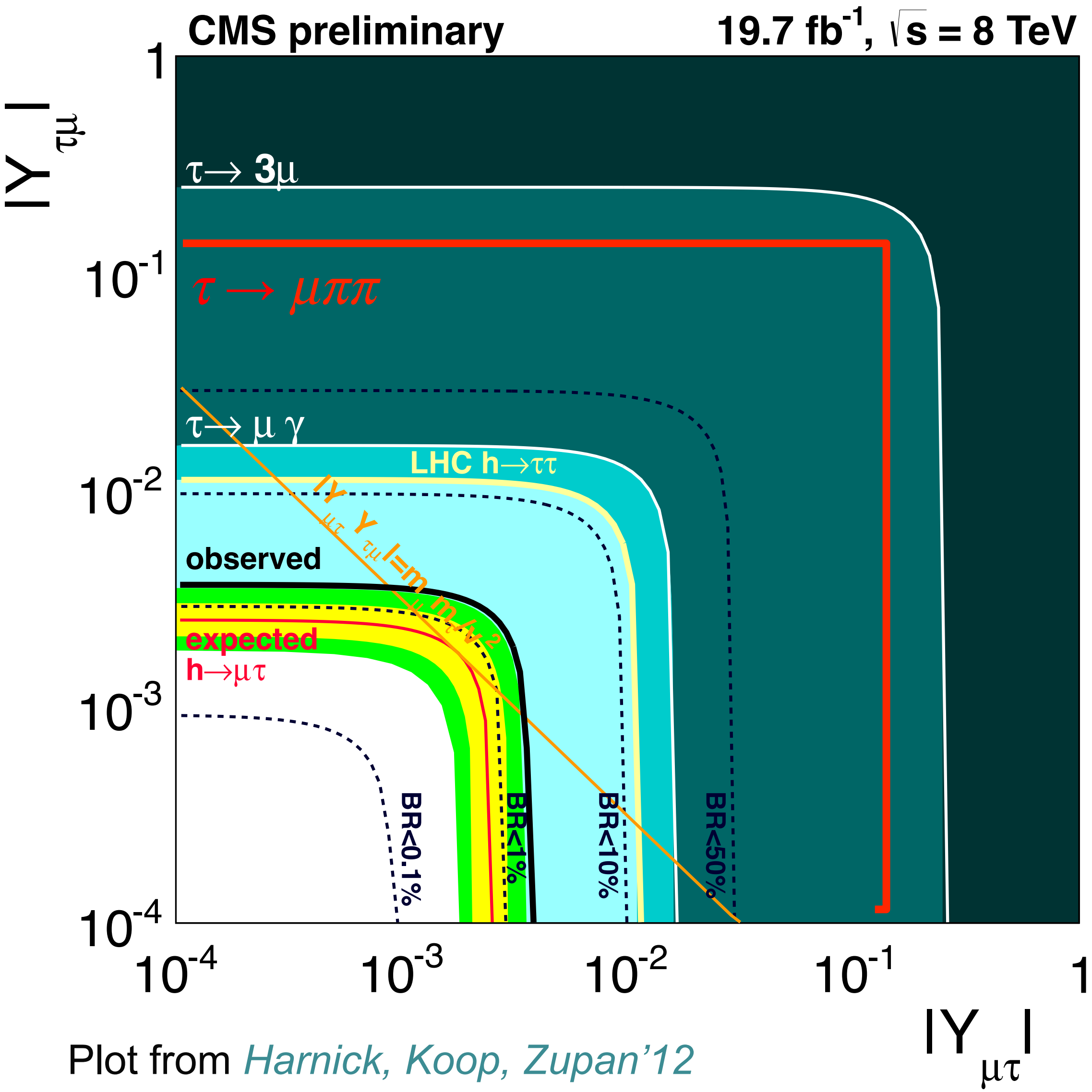
- At low energy
  - $\tau \to \mu\pi\pi$:





Dominated by
- ρ(770) (photon mediated)
- $f_0(980)$ (Higgs mediated)

3.3 Constraints in the τμ sector



Plot from *Harnick, Koop, Zupan'12*
updated *by CMS*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

$$BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}$$

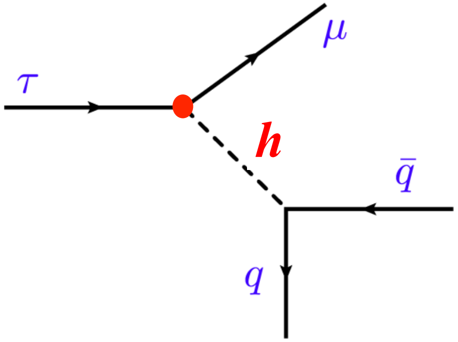
$$BR(\tau \to \mu\pi\pi) < 1.5 \times 10^{-11}$$

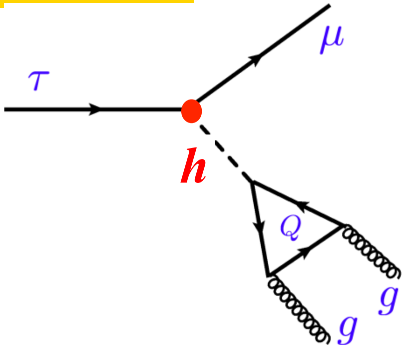
3.4 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan
@ KEK-FF2014FALL

- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$





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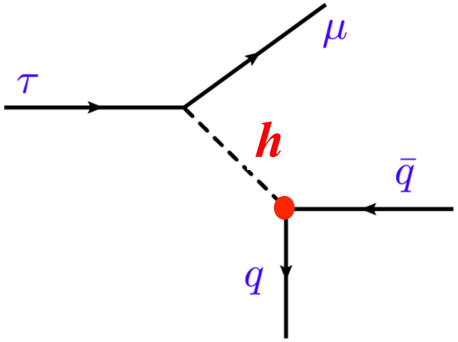
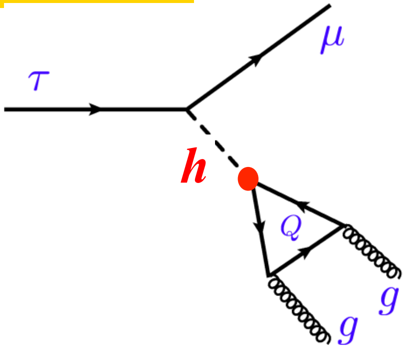
- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!





3.4 What if $\tau \to \mu(e)\pi\pi$ observed?

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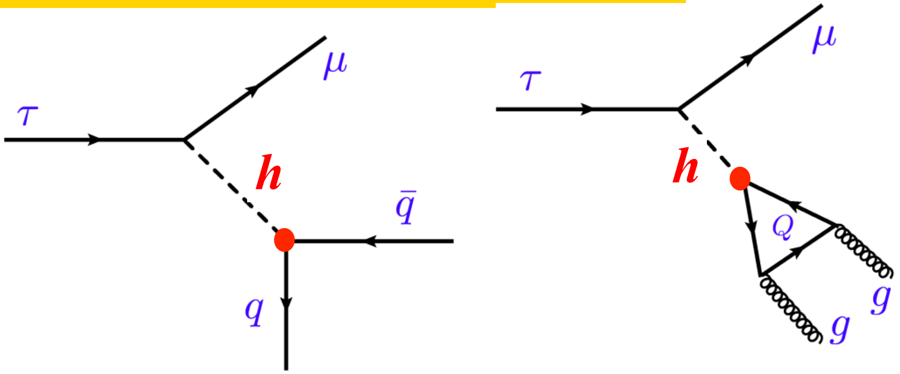


- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!

- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

below present experimental limits!

- If discovered ⇨ among other things *upper limit* on $Y_{u,d,s}$*!*

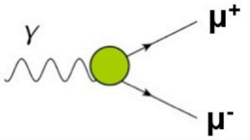
⇨ Interplay between high-energy and low-energy constraints!

4. LFC processes: anomalous magnetic moment of the muon

4.1 Introduction

$$a_\mu = \frac{(g-2)_\mu}{2}$$

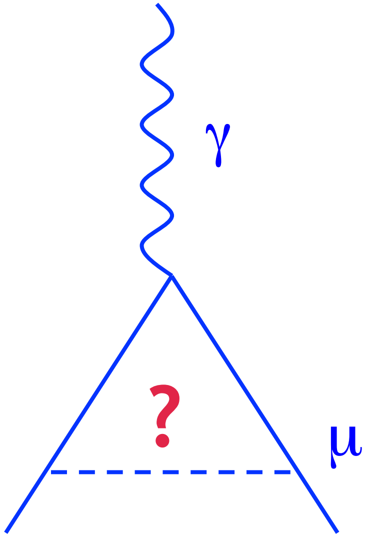
Anomalous magnetic moment



- The gyromagnetic factor of the muon is modified by loop contribution

- We can also study a_e with better experimental precision but if new physics heavy then more sensitivity in a_μ

$$\boxed{a_\ell^{\text{NP}}(\Lambda_{\text{NP}}) \propto \mathcal{O}\left(\frac{m_\ell^2}{\Lambda_{\text{NP}}^2}\right)} \Rightarrow \boxed{\frac{a_\mu^{\text{NP}}}{a_e^{\text{NP}}} \propto \mathcal{O}\left(\frac{m_\mu^2}{m_e^2}\right) \approx 43{,}000}$$



a_τ even more sensitive but insufficient experimental accuracy *Eidelman, Giacomini, Ignatov, Passera'07*

- But a_e important if NP is light

  ⇒ Important constraints on NP scenarios

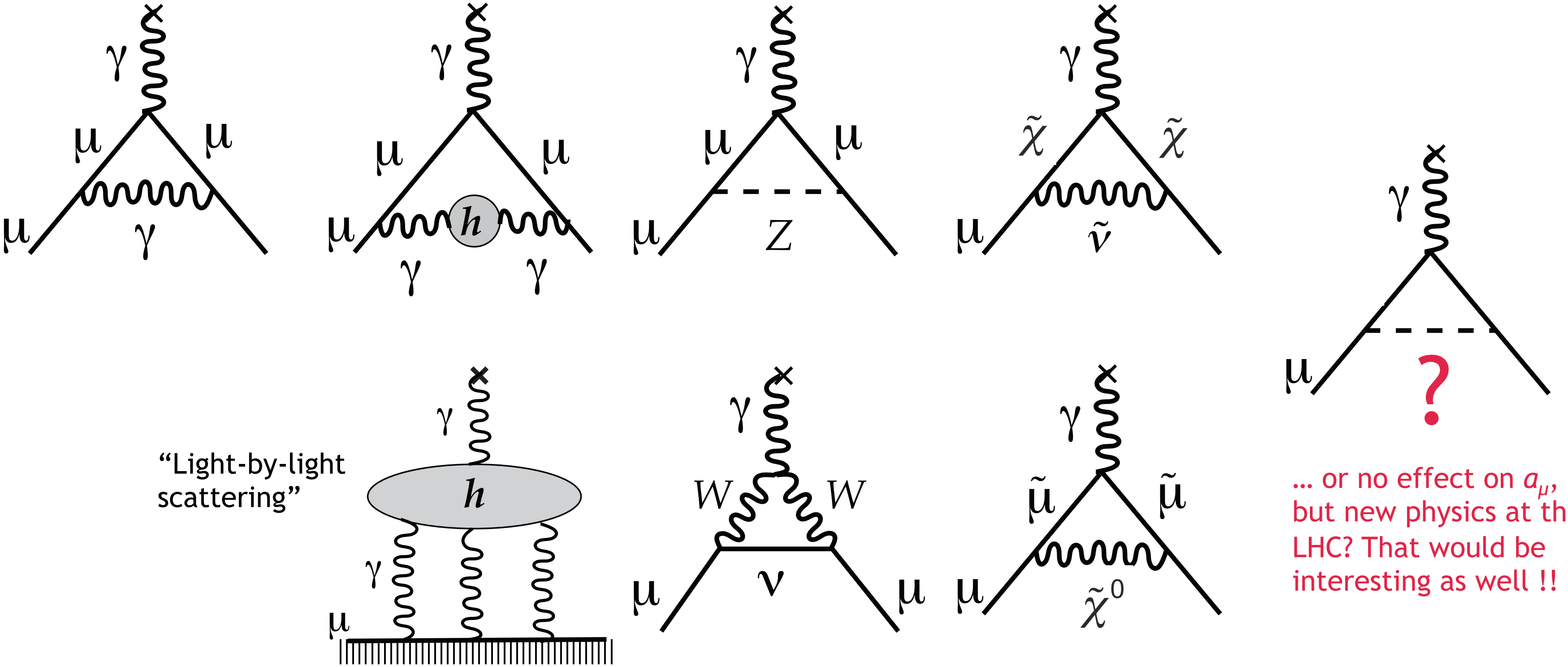
Giudice, Paradisi, Passera'12

4.2 Contribution to $(g-2)_\mu$

Hoecker'11

QED **Hadronic** **Weak** **SUSY… ?**

… or some unknown type of new physics ?



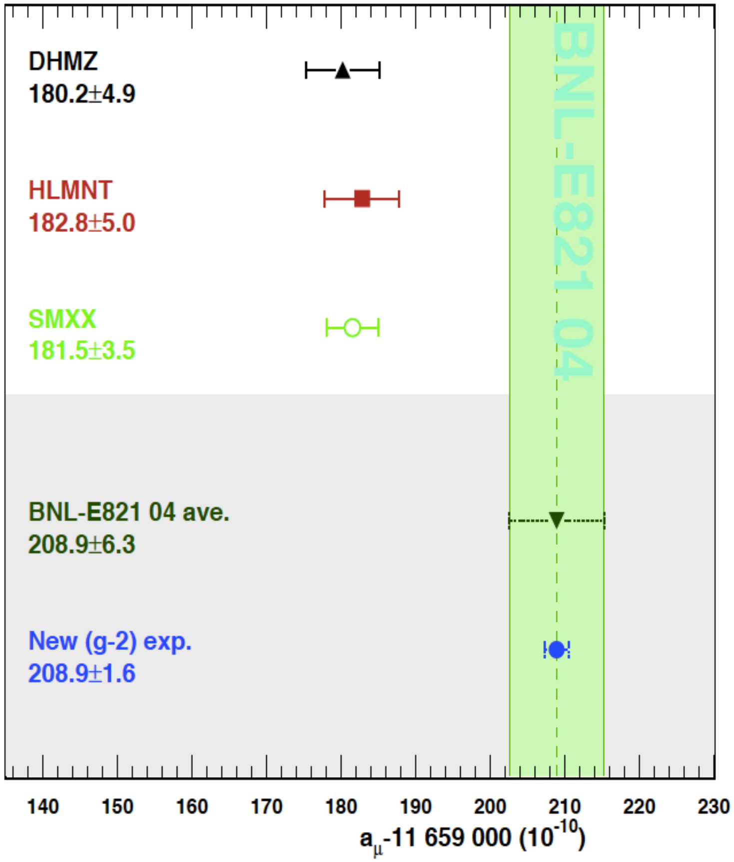
"Light-by-light scattering"

… or no effect on a_μ, but new physics at the LHC? That would be interesting as well !!

Need to compute the SM prediction with high precision! ⇒ *Not so easy!*

4.3 Confronting measurement and prediction

Blum et al.'13



$$a_\mu|_{\rm exp} = 0.001\ 165\ 920\ 80(54)(33)[63]$$

$$a_\mu|_{\rm theo} = 0.001\ 165\ 918\ 40(59)$$

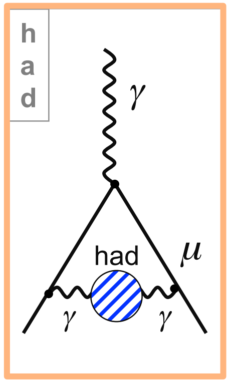
$$a_\mu|_{\rm exp} - a_\mu|_{\rm theo} = 240 \times 10^{-11}$$ 2.9σ diff.

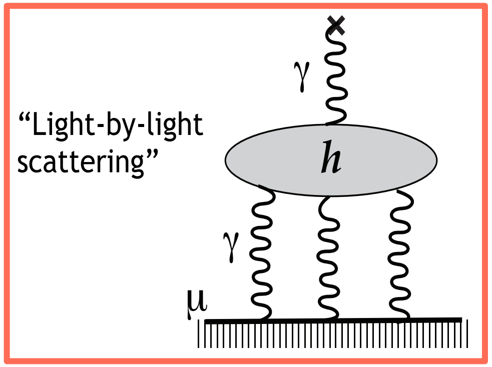
Theoretical Prediction:

Lafferty, summary talk@Tau2014

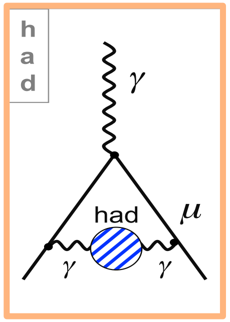
Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	690.8 ± 4.7
HVP(higher order) (*)	-10.0 ± 0.02
HLBL	11.6 ± 4.0
EW	15.4 ± 0.1
Total	11659179.7 ± 6.2

Jegerlehner and Nyffeler '09



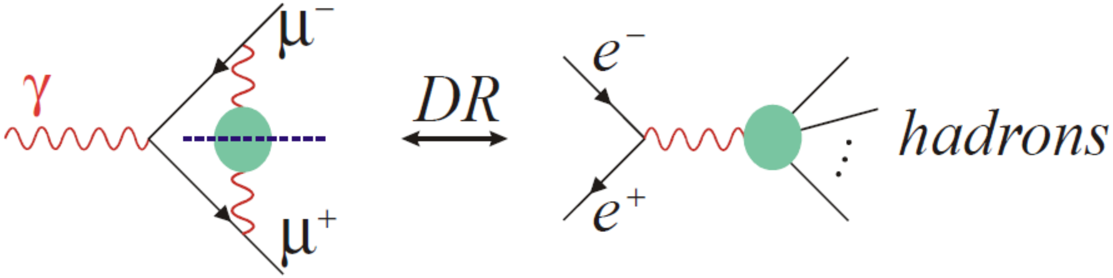


4.4 Towards a model independent determination of HVP and LBL



- Hadronic contribution cannot be computed from first principles due to low-energy hadronic effects

- Use analyticity + unitarity ➡ real part of photon polarisation function from *dispersion relation* over *total hadronic cross section data*



$$R_V(s) = \frac{\sigma\left(e^+e^- \to hadrons\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

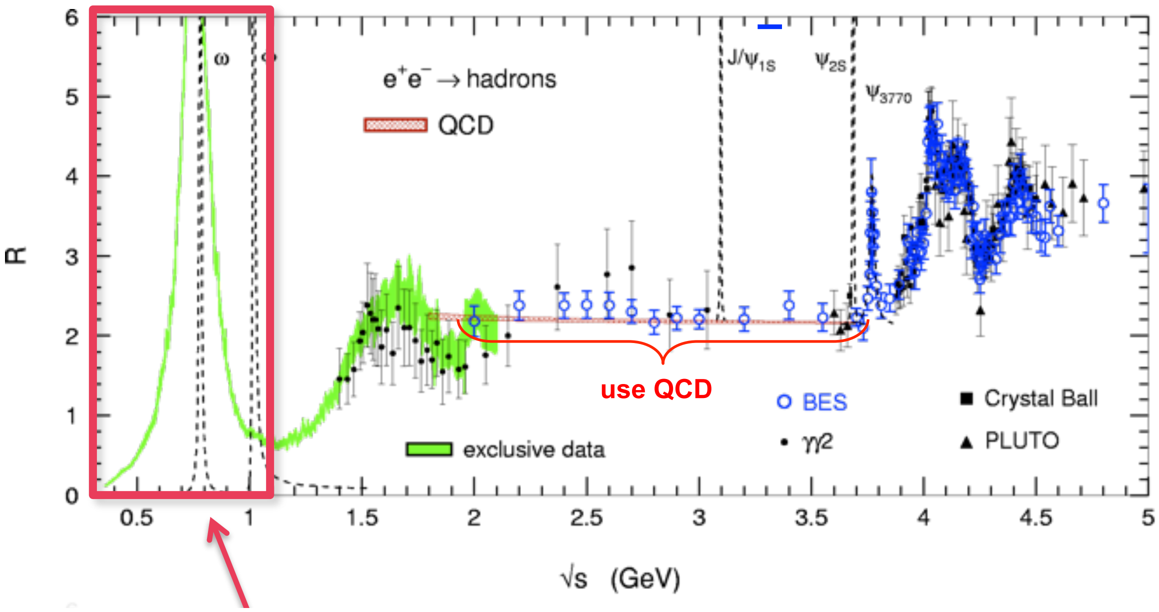
- Leading order hadronic vacuum polarization :

$$a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_V(s)$$

- Low energy contribution dominates : ~75% comes from s < (1 GeV)^2
  ➡ ππ contribution extracted from data

4.4 Towards a model independent determination of HVP and LBL

- Huge 20-years effort by experimentalists and theorists to reduce error on lowest-order hadronic part
  - Improved e^+e^- cross section data from Novisibirsk (Russia)
  - More use of perturbative QCD
  - Technique of "*radiative return*" allows to use data from Φ and *B* factories
  - Isospin symmetry allows us to also use τ hadronic spectral functions



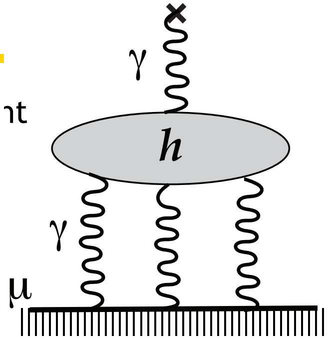
Dominant Region

But still some progress need to be done

- Inconsistencies τ vs. e+e-: *Isospin corrections?*
- Inconsistencies between ISR and direct data: *Radiative corrections?*
- Lattice Calculation?

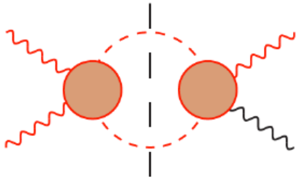
New data expected from KLOE2, Belle-II, BES-III?

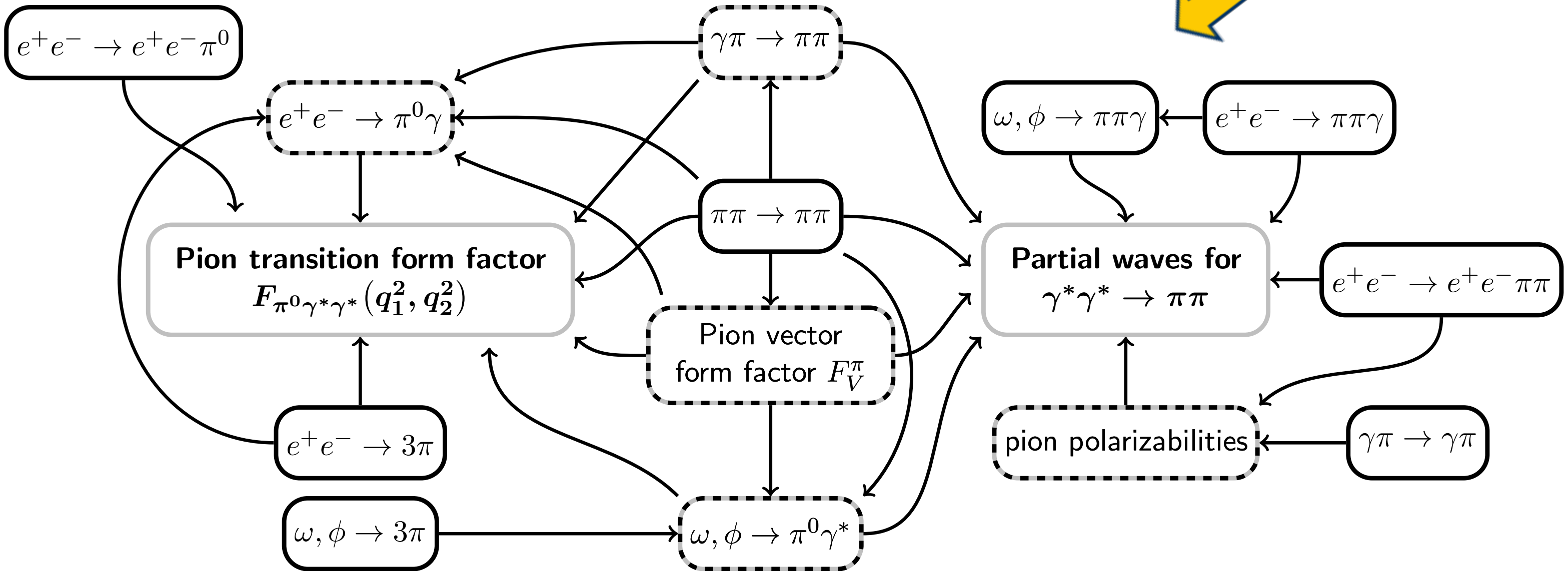
4.4 Towards a model independent determination of HVP and LBL



- For light-by-light scattering: until recently it was believed that dispersion relation approach not possible (4-point function)
  ⇨ only model dependent estimates

- But recent progress from Bern group: *Colangelo, Hoferichter, Procura, Stoffer'14*
  ⇨ *Data driven* estimate possible using *dispersion relations!*





$e^+e^- \to e^+e^-\pi^0$

$\gamma\pi \to \pi\pi$

$e^+e^- \to \pi^0\gamma$

$\omega, \phi \to \pi\pi\gamma$

$e^+e^- \to \pi\pi\gamma$

$\pi\pi \to \pi\pi$

Pion transition form factor
$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

Partial waves for
$\gamma^*\gamma^* \to \pi\pi$

$e^+e^- \to e^+e^-\pi\pi$

Pion vector form factor F_V^π

$e^+e^- \to 3\pi$

pion polarizabilities

$\gamma\pi \to \gamma\pi$

$\omega, \phi \to 3\pi$

$\omega, \phi \to \pi^0\gamma^*$

4.5 What could a 3σ discrepancy tell us?

- Amount of discrepancy in ballpark of SUSY with mass scale of several 100 GeV

$$\Delta a_\mu^{\text{SUSY}} \approx +13 \cdot 10^{-10}\, \text{sgn}(\mu) \left(\frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \tan\beta$$

- Dark photon scenarios?

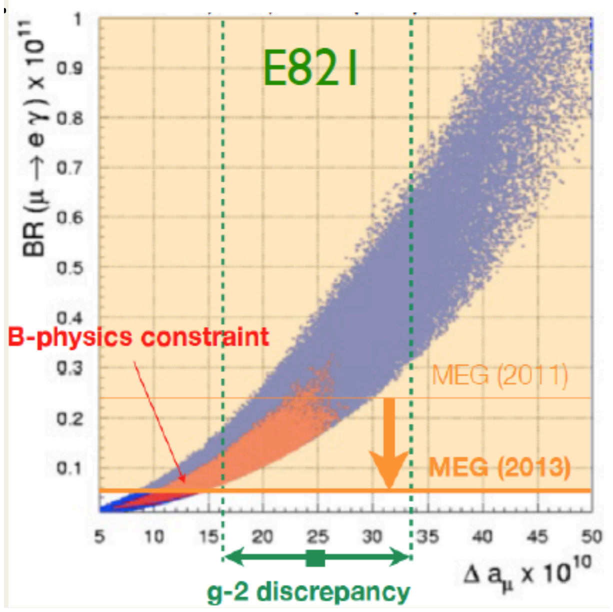
$$\Delta a_\mu^{\text{dark }\gamma} \approx \frac{\alpha}{2\pi} \varepsilon^2 \cdot F\left(\frac{m_{\text{dark }\gamma}}{m_\mu} \right)$$

- Correlations with LFV modes to study ⇨

- Before interpreting this discrepancy as new physics ⇨ need to be sure the hadronic background is under control:
  - Theoretical efforts needed
  - New experimental measurements

Lafferty, summary talk@Tau2014

MSSM with large tanβ and heavy squarks



Isidori, Mescia, Paradisi, Temes'06

5. Conclusion and Outlook

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS → energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

- *Charged leptons* offer an important spectrum of possibilities:
    - LFV measurement has SM-free signal
    - Current experiments and mature proposals promise orders of magnitude sensitivity improvements
    - The muon g–2 **may** already show a deviation from the SM
    - Progress towards a better knowledge of hadronic uncertainties
    - New physics models usually strongly correlate these sectors

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

- *Charged leptons* offer an important spectrum of possibilities:
  - We show how CLFV decays offer an excellent model discriminating tools giving indications on
    - the *mediator* (operator structure)
    - the *source of flavour breaking* (comparison $\tau\mu$ vs. τe vs. μe)

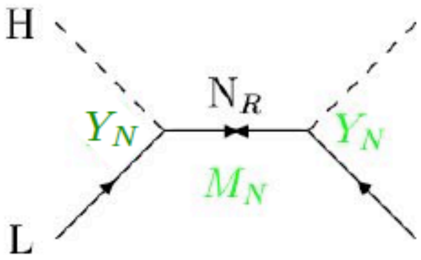
- Interplay low energy and collider physics: LFV of the Higgs boson

7. Back-up

CLFV in see-saw models

Type I:
Fermion singlet

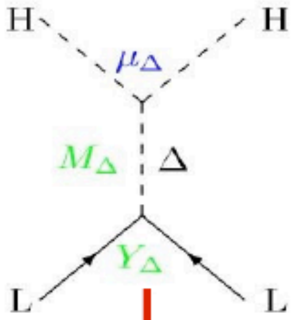
N_{R_i}



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Type II:
Scalar triplet

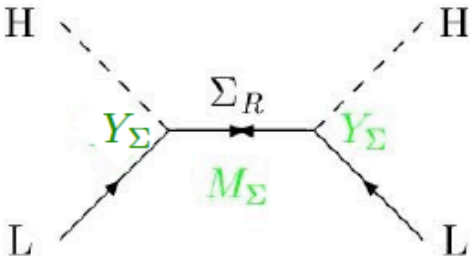
$\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Type III:
Fermion triplet

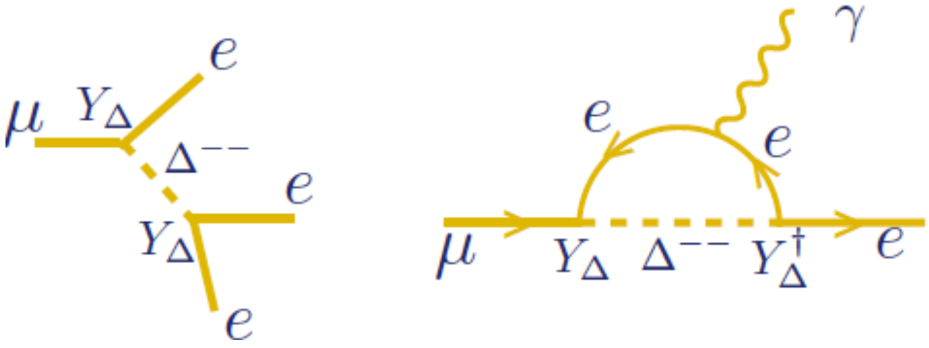
$\Sigma_i \equiv (\Sigma_i^{+}, \Sigma_i^{0}, \Sigma_i^{-})$



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

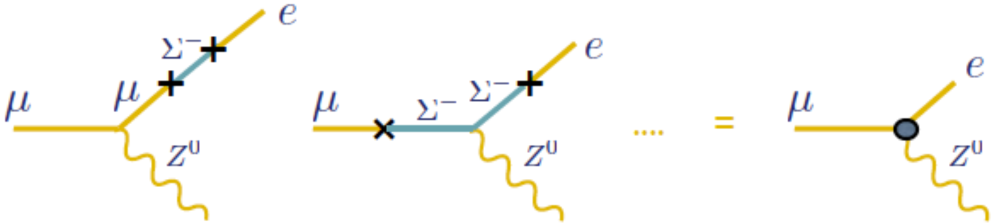
- Observable CLFV if see-saw scale low (with protection of LN)
- Each model leads to specific CLFV pattern

- CLFV in Type II seesaw: tree-level 4L operator (D,V at loop) → 4-lepton processes most sensitive



- CLFV in Type III seesaw: tree-level LFV couplings of Z ⇒

μ →3e and μ →e conversion at tree level, μ →eγ at loop



Abada-Biggio-Bonnet-Gavela-Hambye '07, '08

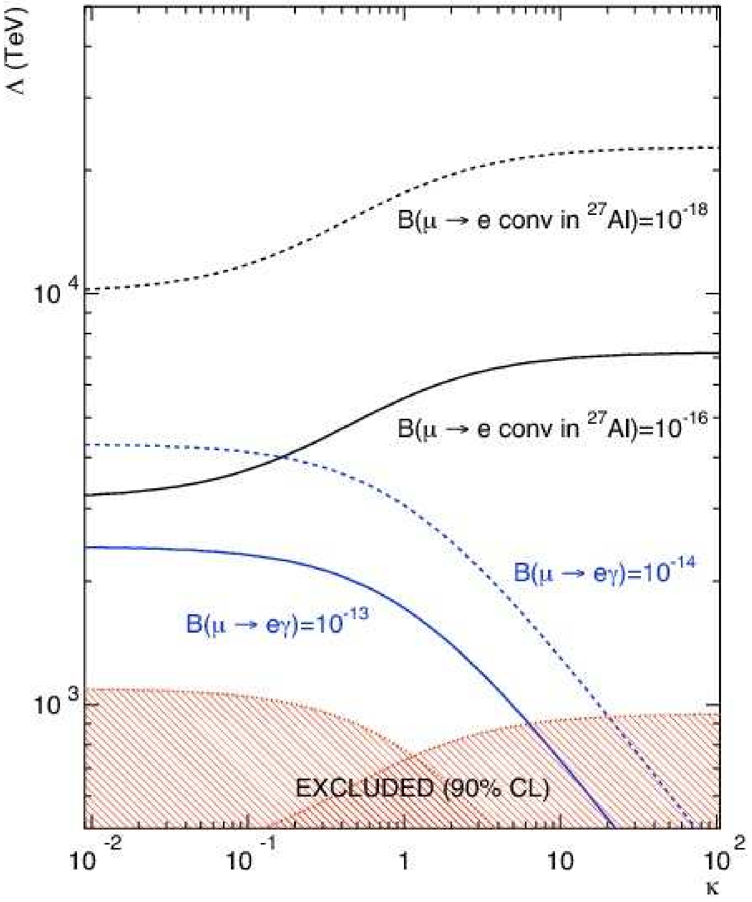
- Ratios of 2 processes with same flavor transition are fixed

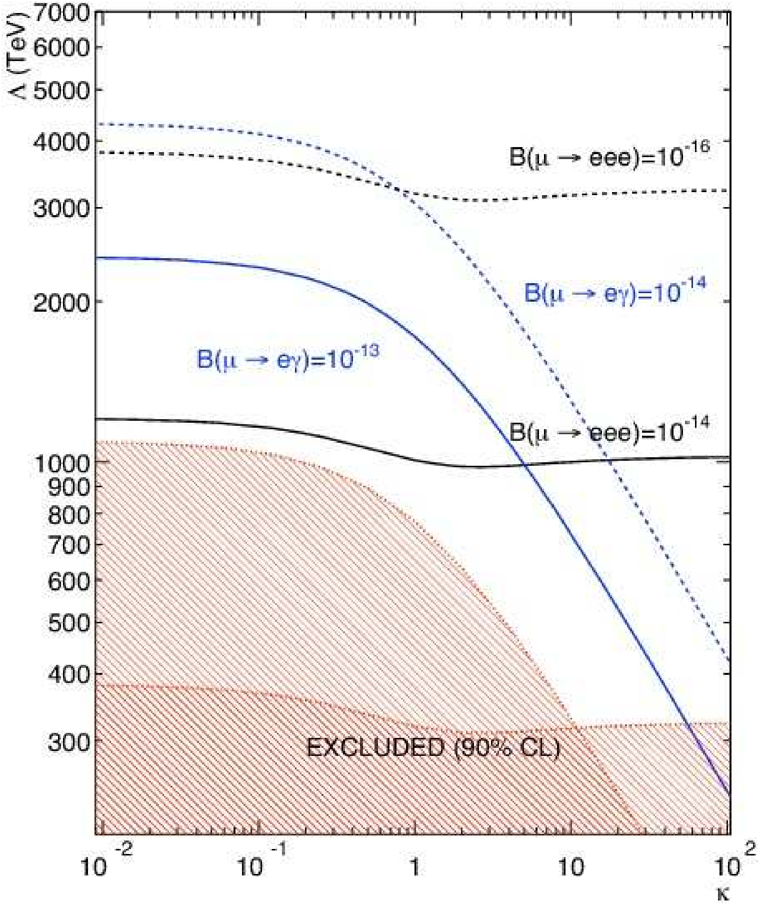
$$\begin{aligned}
Br(\mu\to e\gamma) &= 1.3\cdot 10^{-3}\cdot Br(\mu\to eee) = 3.1\cdot 10^{-4}\cdot R^{\mu\to e}_{Ti}\\
Br(\tau\to \mu\gamma) &= 1.3\cdot 10^{-3}\cdot Br(\tau\to \mu\mu\mu)\\
Br(\tau\to e\gamma) &= 1.3\cdot 10^{-3}\cdot Br(\tau\to eee)
\end{aligned}$$

2.4 Model discriminating power of muon processes

- Dependence: NP scale Λ versus ratio of two operators $\kappa = \frac{C_1}{C_2}$

DeGouvea & Vogel'13





2.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

- Benchmarks:
  - Dipole model: $C_D \neq 0$, $C_{else} = 0$
  - Scalar model: $C_S \neq 0$, $C_{else} = 0$
  - Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
  - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

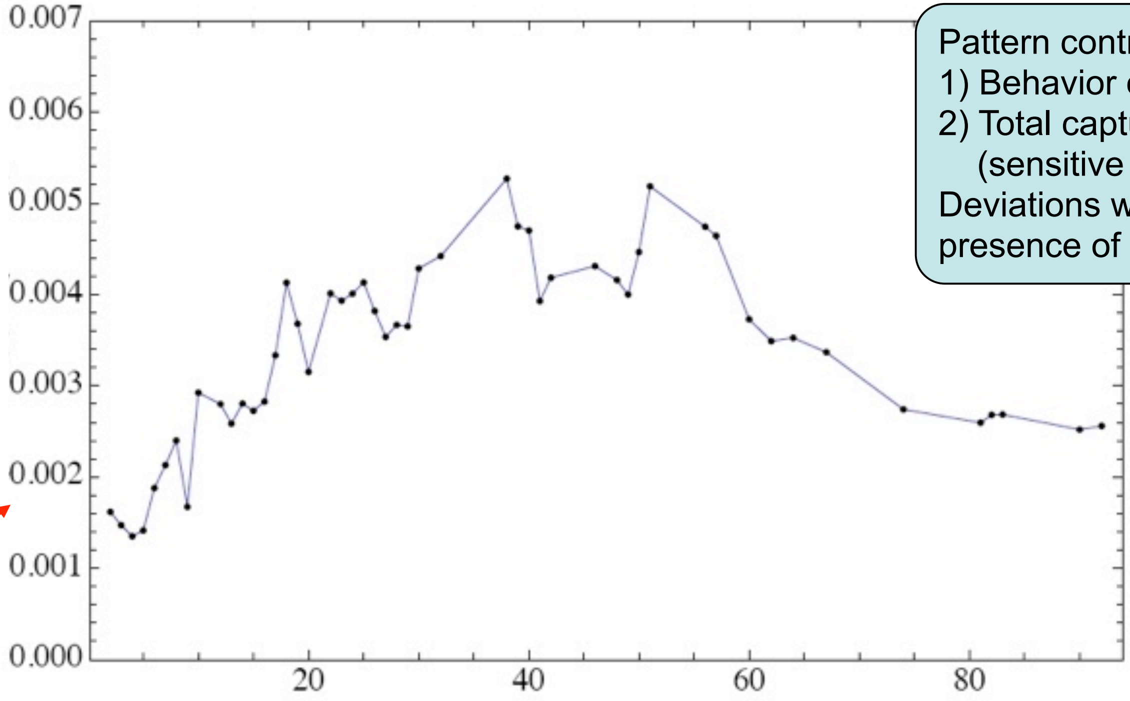
μ→e vs μ→eγ

- Assume dipole dominance:

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

Kitano-Koike-Okada '02
VC-Kitano-Okada-Tuzon '09

$$\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}$$



Z

O(α/π)

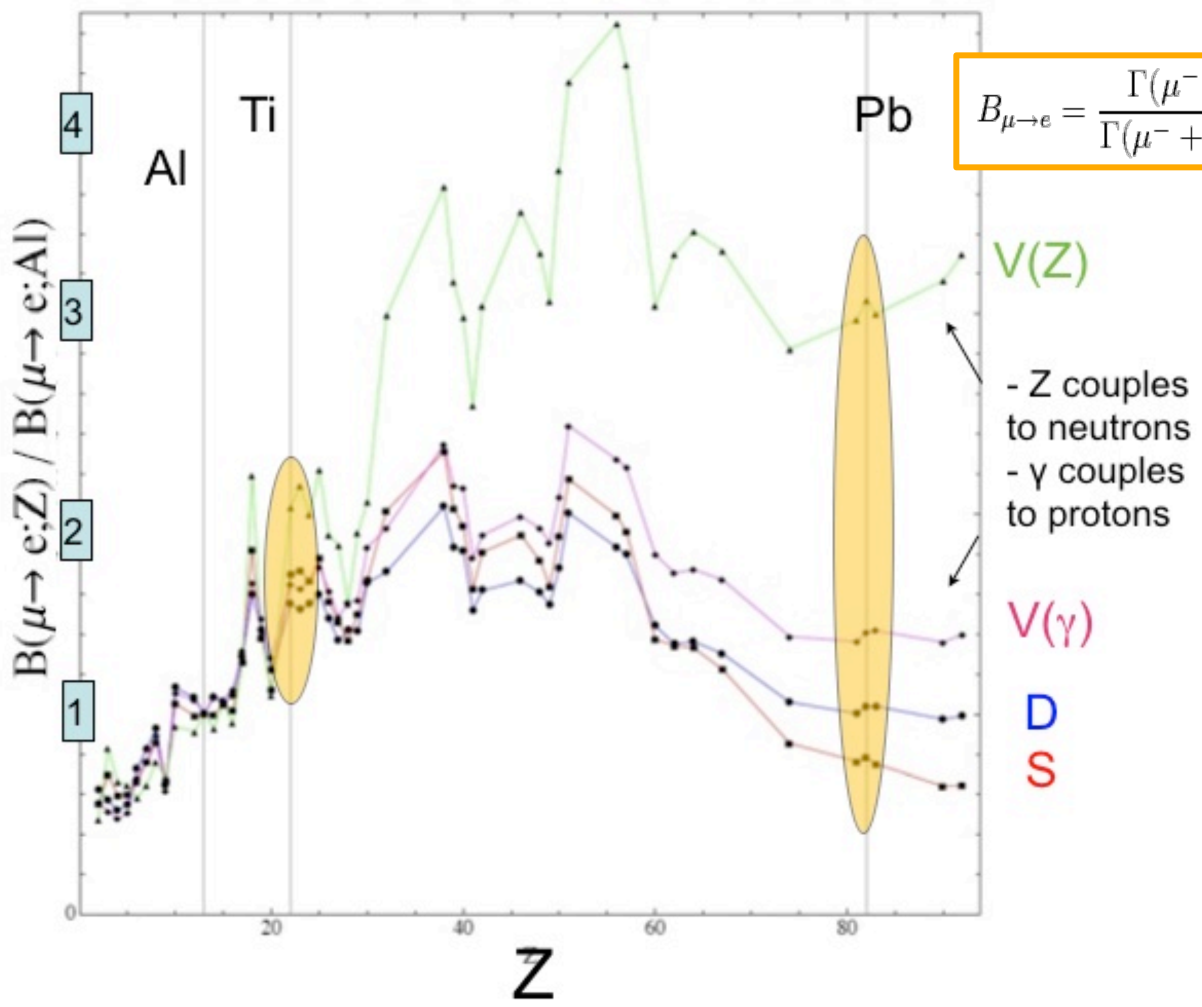
Pattern controlled by:
1) Behavior of overlap integrals
2) Total capture rate
(sensitive to nuclear structure)
Deviations would indicate presence of scalar / vector terms

2.6.1 BR for $\mu \rightarrow$ e conversion

- For *μ →e conversion,* target dependence of the amplitude is different for V,D or S models

Cirigliano, Kitano, Okada, Tuzon'09

$$\boxed{B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}}$$



- Z couples to neutrons
- γ couples to protons

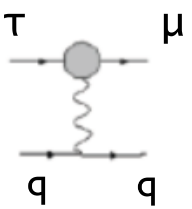
2.5 Model discriminating power of Tau processes

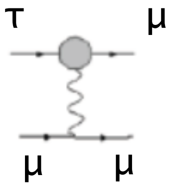
Celis, Cirigliano, E.P.'14

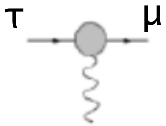
- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$

Benchmark







- ρ (770) resonance ($J^{PC}=1^{--}$): cut in the $\pi^+\pi^-$ invariant mass:

$$587 \text{ MeV} \leq \sqrt{s} \leq 962 \text{ MeV}$$

- f_0 (980) resonance ($J^{PC}=0^{++}$): cut in the $\pi^+\pi^-$ invariant mass:

$$906 \text{ MeV} \leq \sqrt{s} \leq 1065 \text{ MeV}$$

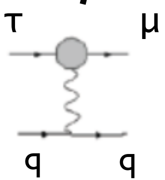
2.5 Model discriminating power of Tau processes

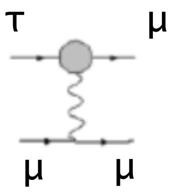
Celis, Cirigliano, E.P.'14

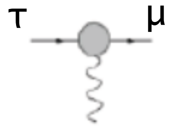
- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

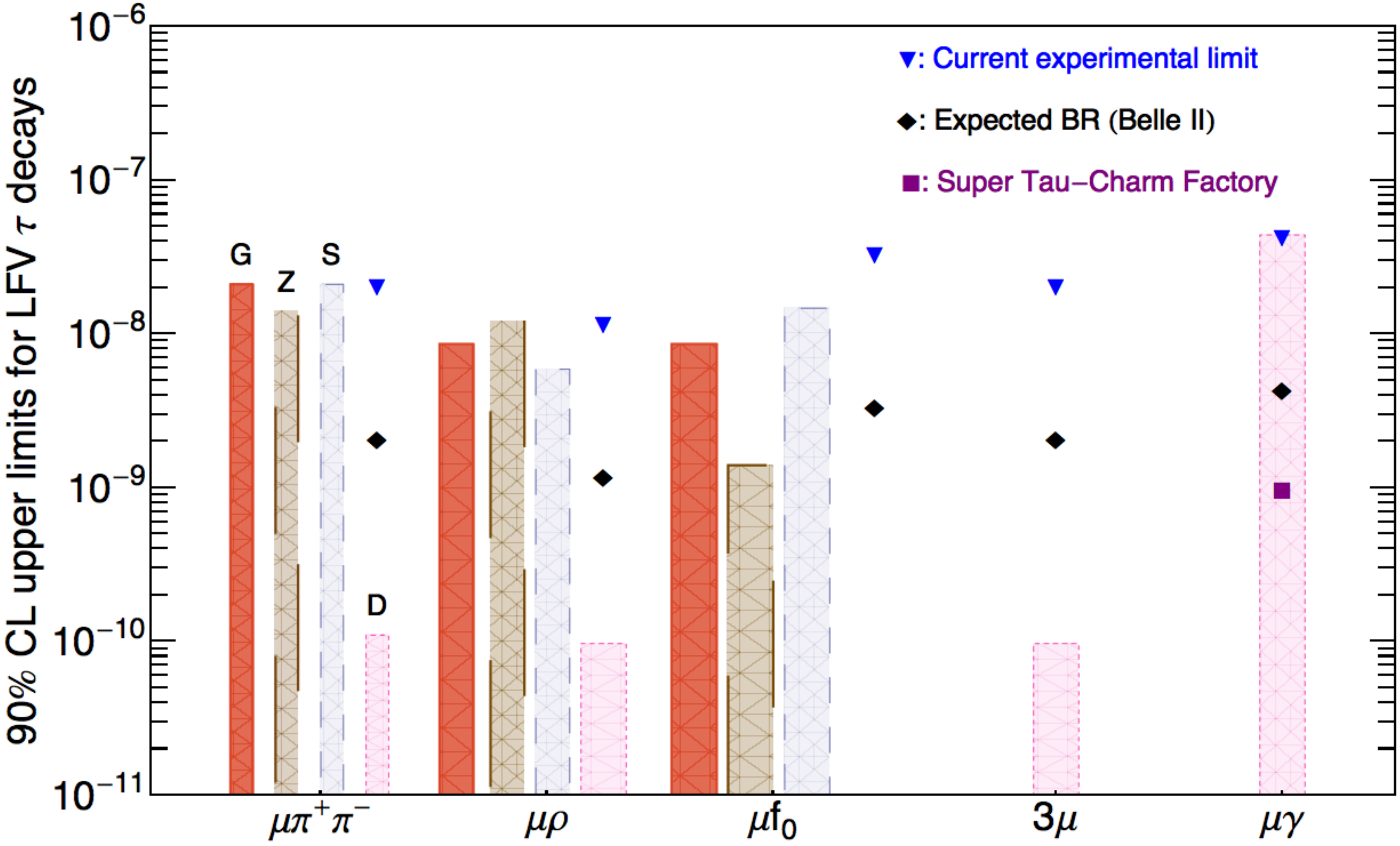
Benchmark







4.2 Prospects:

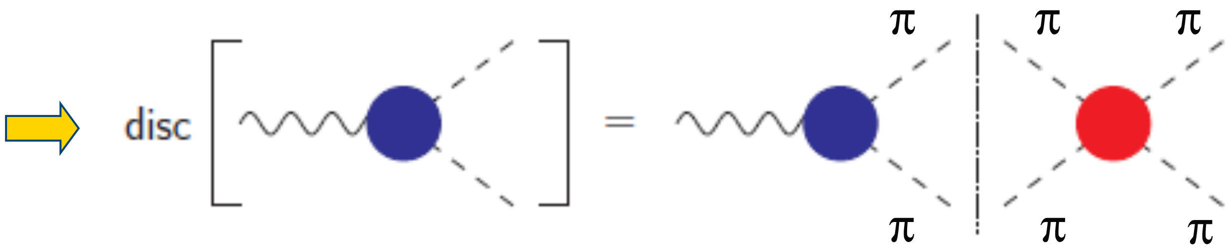


3.4.2 Dispersion relations: Method

- Unitarity ⇒ the discontinuity of the form factor is known

$$\boxed{\frac{1}{2i}\,disc\; F_{\pi\pi}(s) = \mathrm{Im}\,F_{\pi\pi}(s) = \sum_n F_{\pi\pi\to n}\left(\mathrm{T}_{n\to\pi\pi}\right)^*}$$

- Only one channel n = ππ



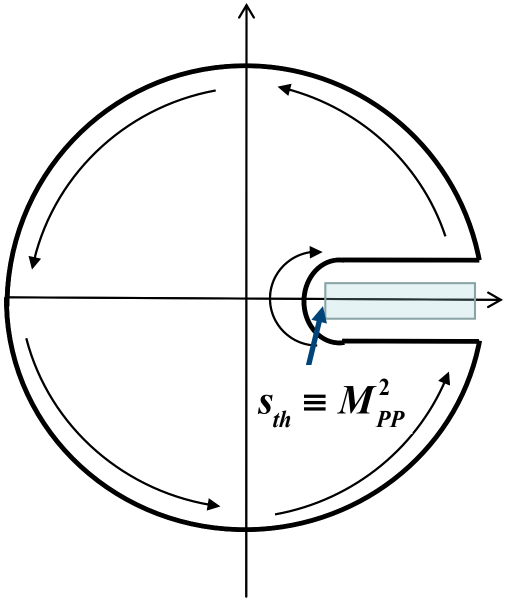
$$\boxed{\frac{1}{2i}\,disc\; F_I(s) = \mathrm{Im}\,F_I(s) = F_I(s)\;\sin\delta_I(s)e^{-i\delta_I(s)}}$$

ππ scattering phase
known from experiment

Watson's theorem

3.4.2 Dispersion relations: Method

- Knowing the discontinuity of F ⇨ write a dispersion relation for it
- Cauchy Theorem and Schwarz reflection principle



$$F(s) = \frac{1}{\pi}\oint \frac{F(s')}{s'-s}ds' \Rightarrow \frac{1}{2i\pi}\int_{M_{PP}^2}^{\infty}\frac{disc\left[F(s')\right]}{s'-s-i\varepsilon}ds'$$

- If F does not drop off fast enough for $|s| \to \infty$
  ⇨ subtract the DR

$$\boxed{F(s) = P_{n-1}(s) + \frac{s^n}{\pi}\int_{M_{PP}^2}^{\infty}\frac{ds'}{s'^n}\frac{\mathrm{Im}\left[F(s')\right]}{\left(s'-s-i\varepsilon\right)}}$$

$P_{n-1}(s)$ polynomial

3.4.2 Dispersion relations: Method

- Solution: Use analyticity to reconstruct the form factor in the entire space

⇨ Omnès representation : $F_I(s) = \underset{\nwarrow\ \text{polynomial}}{P_I(s)}\ \underset{\nwarrow\ \text{Omnès function}}{\Omega_I(s)}$

- Omnès function : $\boxed{\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right]}$

- Polynomial: $P_I(s)$ not known but determined from a matching to experiment or to ChPT at low energy

3.4.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

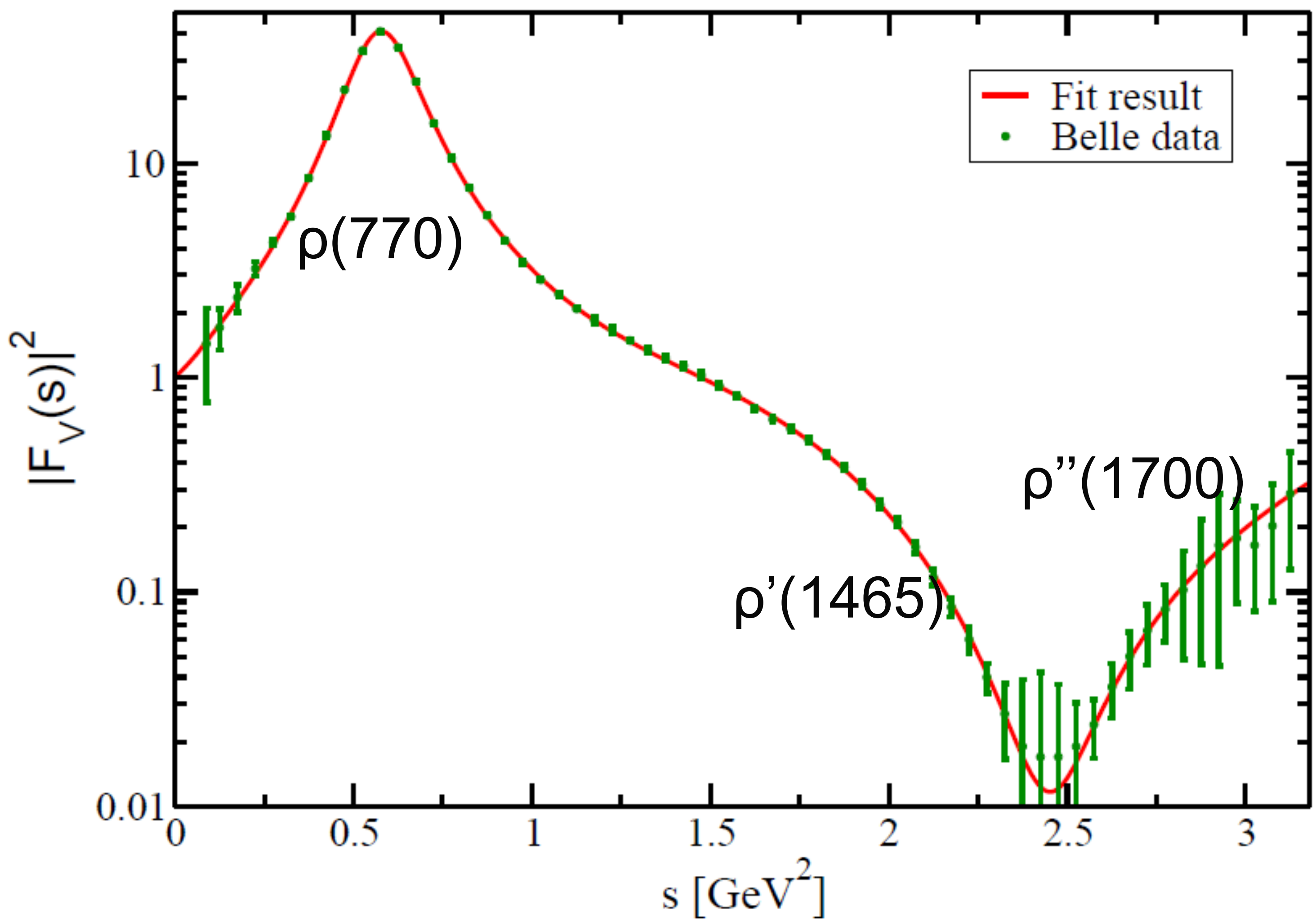
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_V(s')}{\left(s'-s-i\varepsilon\right)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

3.4.3 Determination of $F_V(s)$



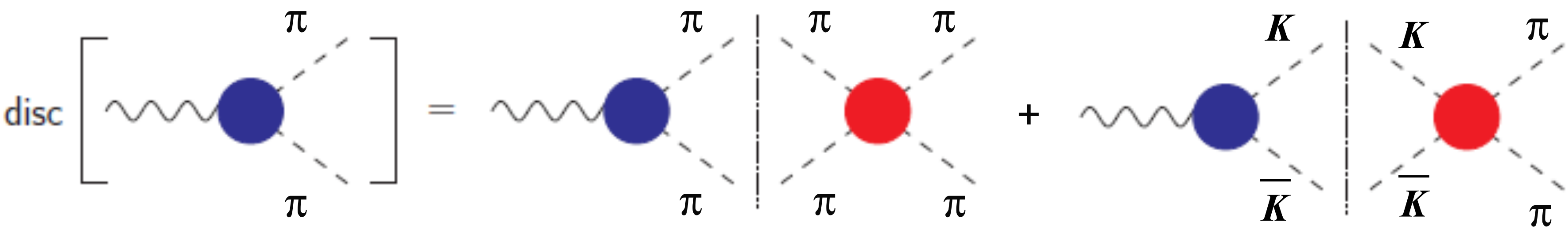
Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs up to √s~1.4 GeV → *Coupled channel analysis*
  Inputs: I=0, S-wave ππ and KK data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:



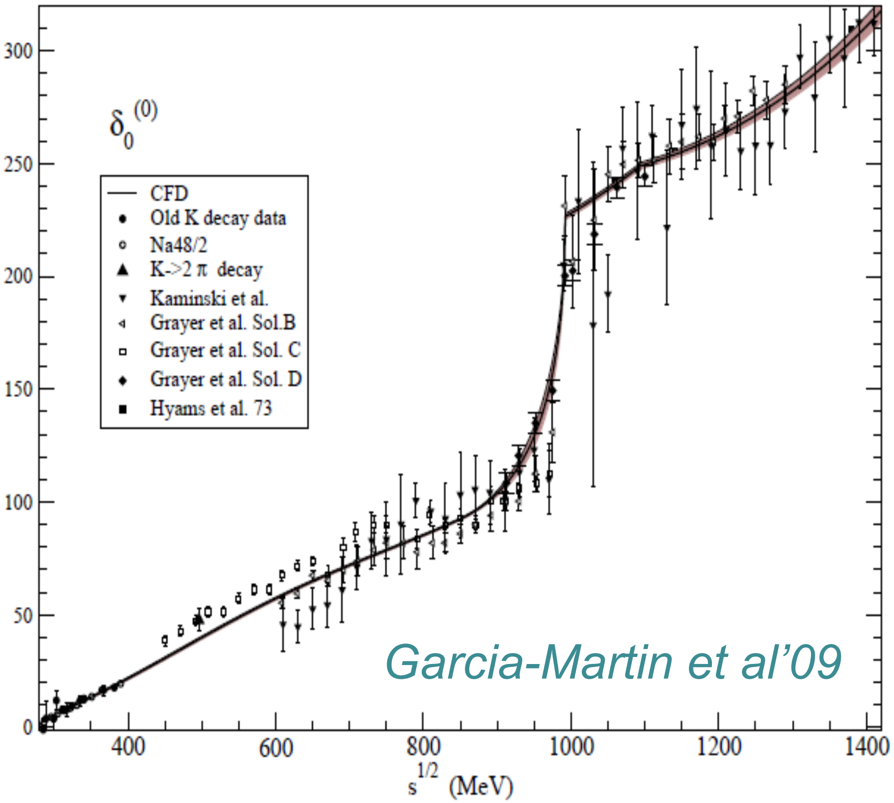
$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

$$n = \pi\pi, K\bar{K}$$

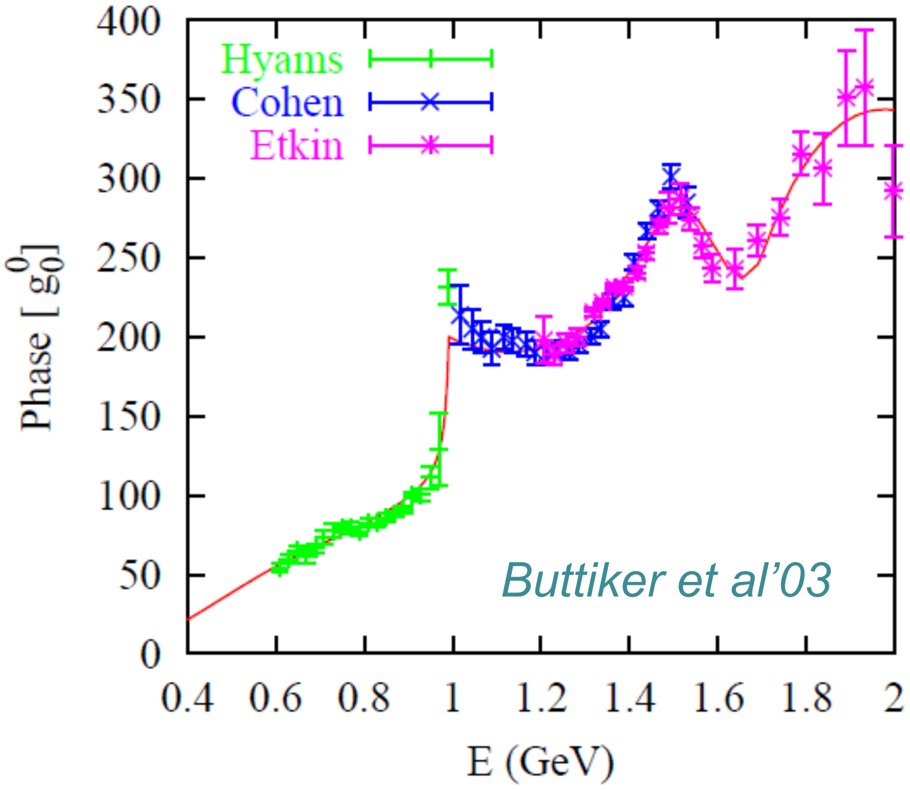
3.4.4 Determination of the form factors : $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$



Garcia-Martin et al'09



Buttiker et al'03

- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\boxed{\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T_{nm}^* \sigma_m(s) X_m^{(N)}\right\}} \longrightarrow \boxed{\mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \rightarrow 1/s$ (*Brodsky & Lepage*) + ChPT:

Feynman-Hellmann theorem: ⇨

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2)$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$\begin{aligned} M_{\pi^+}^2 &= (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2) \\ M_{K^+}^2 &= (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2) \\ M_{K^0}^2 &= (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2) \end{aligned}$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!

  ⇨ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\ M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13

Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

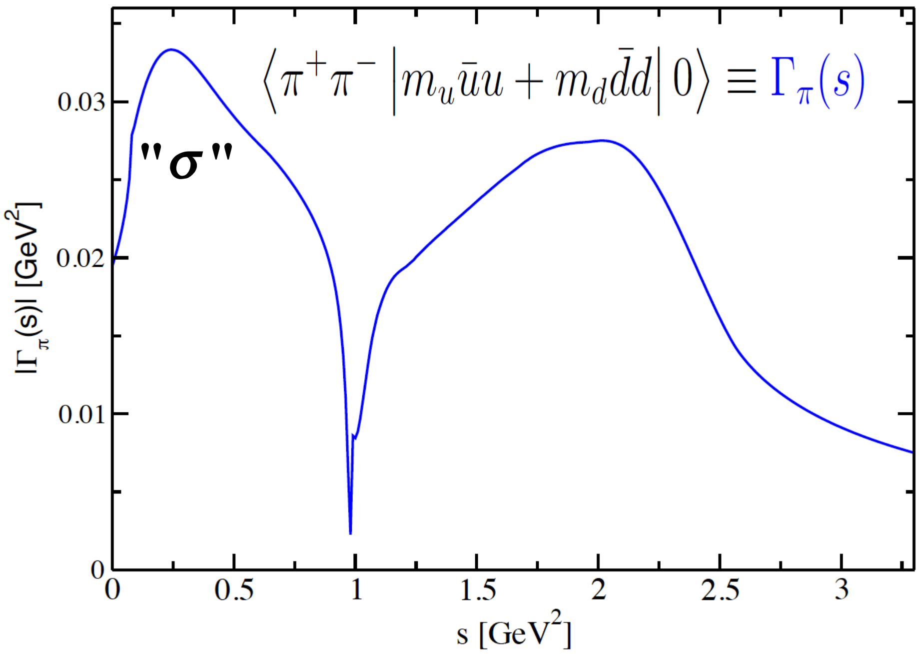
- General solution

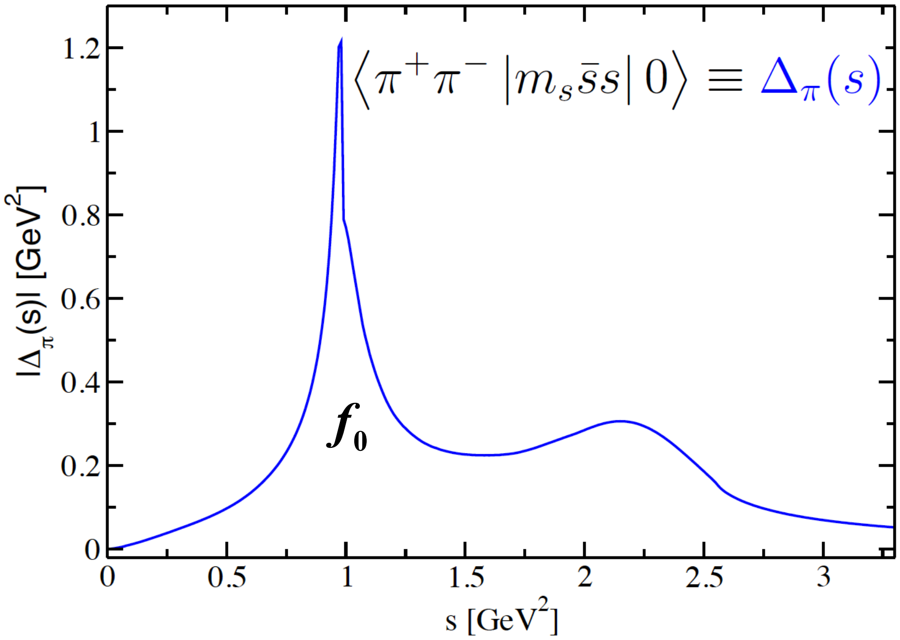
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

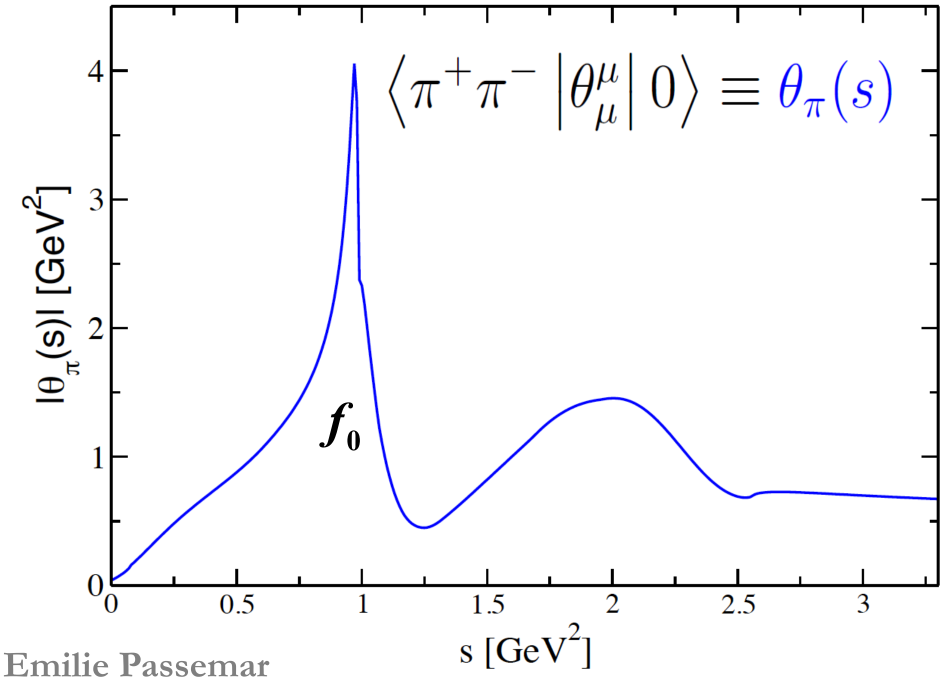
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT The unsubtracted DR is not saturated by the 2 states

⇨ Relax the constraints and match to ChPT

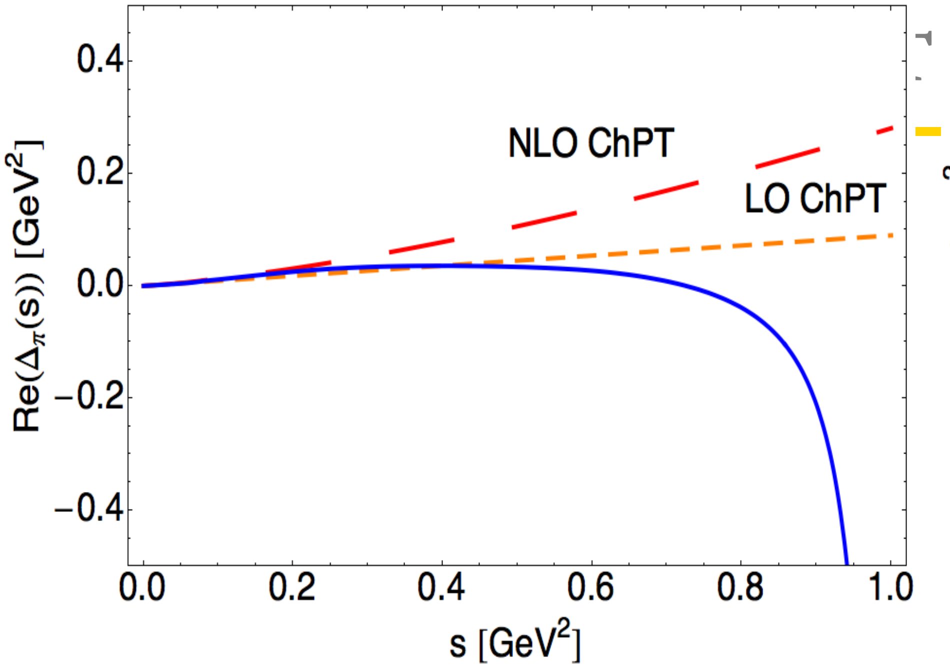
$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1 \right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2 \right) s \end{aligned}$$

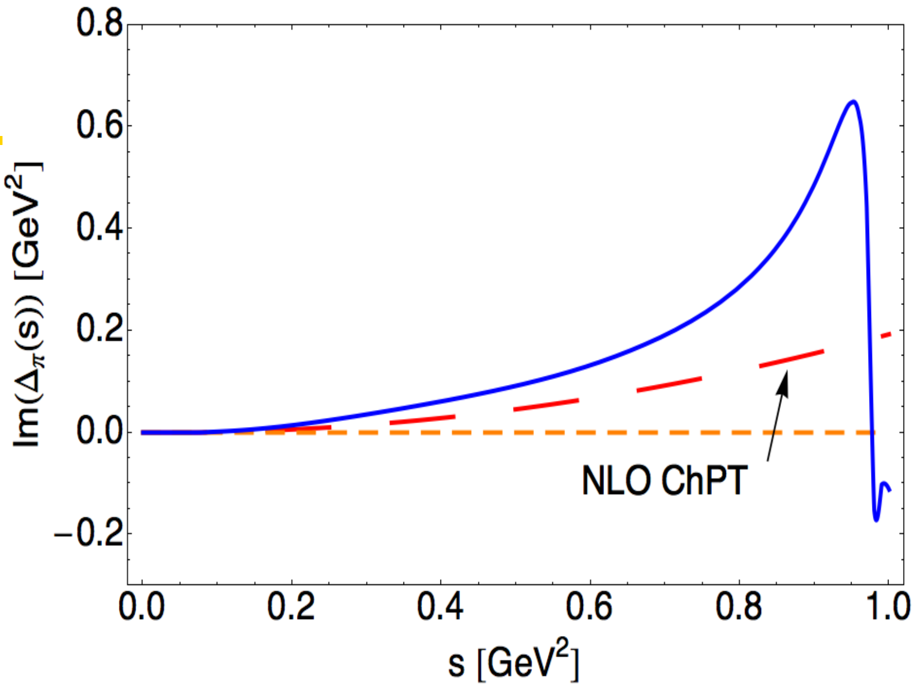


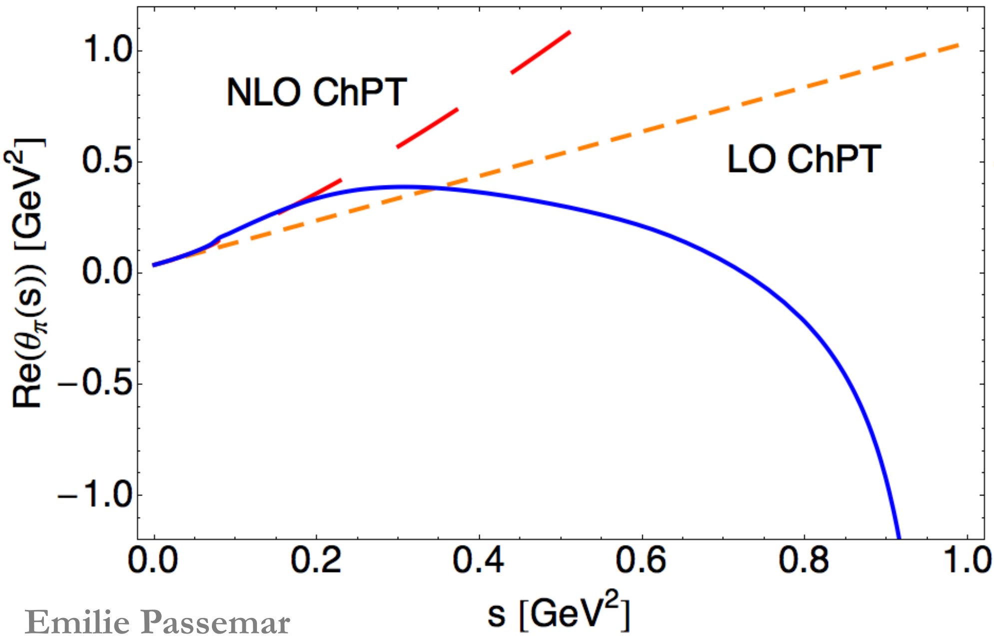


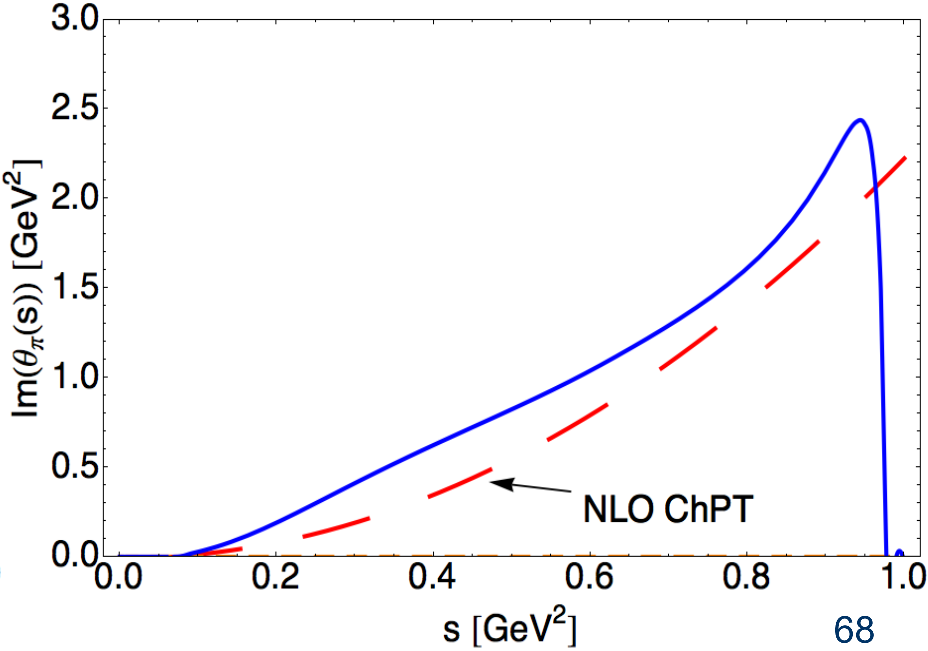


Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data

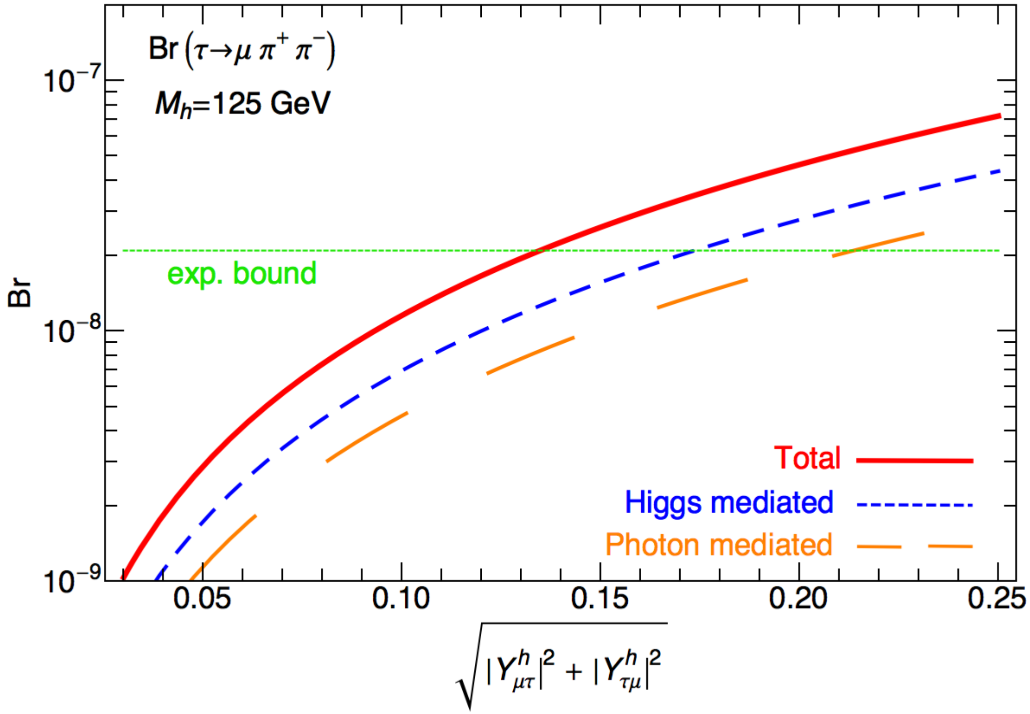

NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
0.4
0.2
0.0
−0.2
−0.4
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]


$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
0.8
0.6
0.4
0.2
0.0
−0.2
NLO ChPT
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]


NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]


$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
3.0
2.5
2.0
1.5
1.0
0.5
0.0
NLO ChPT
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]

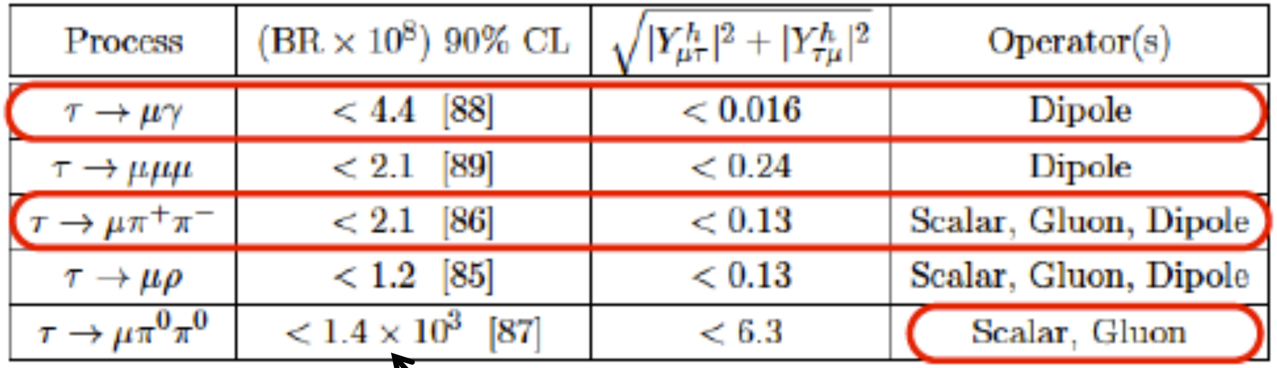
3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

Less stringent but more robust handle on LFV Higgs couplings

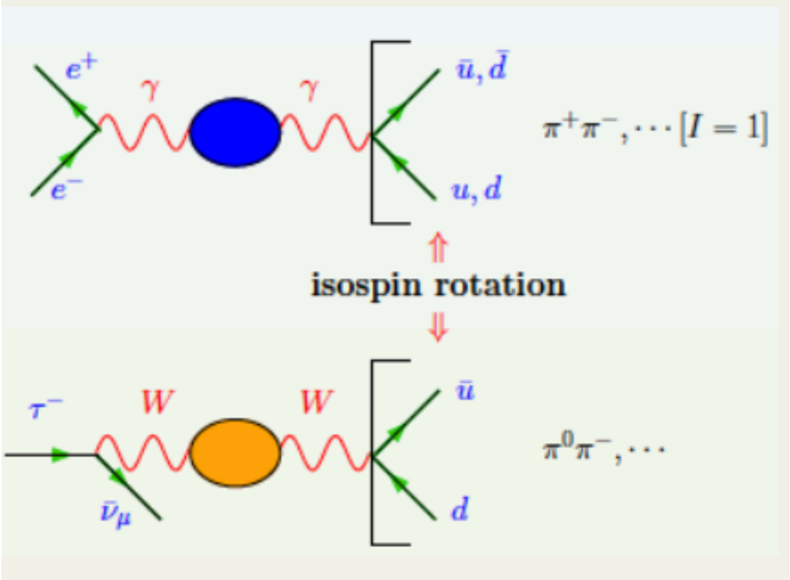


Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{|Y^h_{\mu\tau}|^2 + |Y^h_{\tau\mu}|^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

?

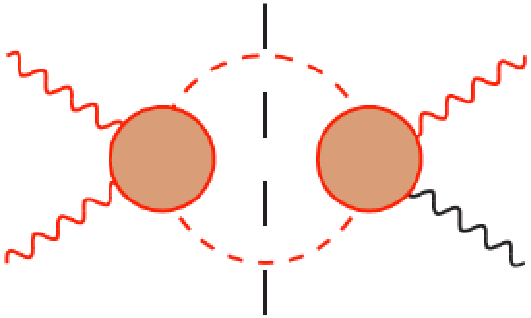
Belle'08'11'12 except last from *CLEO'97*

Isospin relation muon g-2



dispersion relation approaches for a_μ (I)

Hoferichter, Colangelo, Procura, Stoffer (2014)



- **dispersion formalism** for $\gamma^* \gamma^* \to \gamma^* \gamma$

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$

- **master formula** for a_μ

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s((p+q_1)^2 - m^2)((p-q_2)^2 - m^2)},$$

$$I^{\pi\pi} = \sum_{i \in \{1,2,3,6,14\}} \left(T_{i,s} I_{i,s} + 2 T_{i,u} I_{i,u}\right) + 2 T_{9,s} I_{9,s} + 2 T_{9,u} I_{9,u} + 2 T_{12,u} I_{12,u}$$

with $I_{i,(s,u)}$ dispersive integrals and $T_{i,(s,u)}$ integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \left(\frac{1}{s'-s} - \frac{s' - q_1^2 - q_2^2}{\lambda(s', q_1^2, q_2^2)} \right) \mathrm{Im}\bar{h}^{0}_{++,++}(s'; q_1^2, q_2^2; s, 0),$$

$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s' - q_1^2 - q_2^2)(s'-s)^2} \mathrm{Im}\bar{h}^{2}_{+-,+-}(s'; q_1^2, q_2^2; s, 0) \left(\frac{75}{8}\right)$$

Helicity amplitudes contribute up to $J = 2$ (S and D waves)

Slide by M. Vanderhaeghen, talk at “Lepton Moments 2014”, July 2014

2.5 Model discriminating power of Tau processes

- Studies in specific models

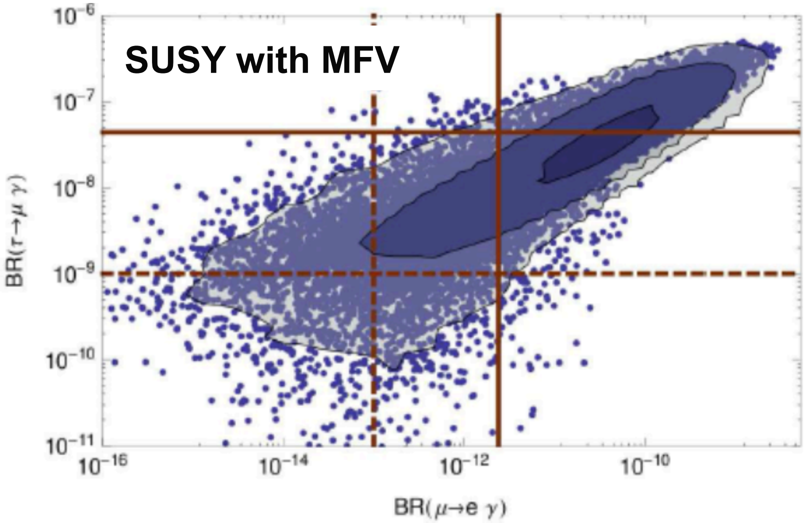
Buras et al..'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^- \to e^- e^+ e^-)}{\mathrm{Br}(\mu \to e\gamma)}$	$0.02\ldots1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04\ldots0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.07\ldots2.2$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04\ldots0.4$	$\sim 2 \cdot 10^{-3}$	$0.06\ldots0.1$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04\ldots0.3$	$\sim 2 \cdot 10^{-3}$	$0.02\ldots0.04$	$0.03\ldots1.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04\ldots0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.04\ldots1.4$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8\ldots2$	~ 5	$0.3\ldots0.5$	$1.5\ldots2.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}$	$0.7\ldots1.6$	~ 0.2	$5\ldots10$	$1.4\ldots1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti} \to e\mathrm{Ti})}{\mathrm{Br}(\mu \to e\gamma)}$	$10^{-3}\ldots10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08\ldots0.15$	$10^{-12}\ldots26$

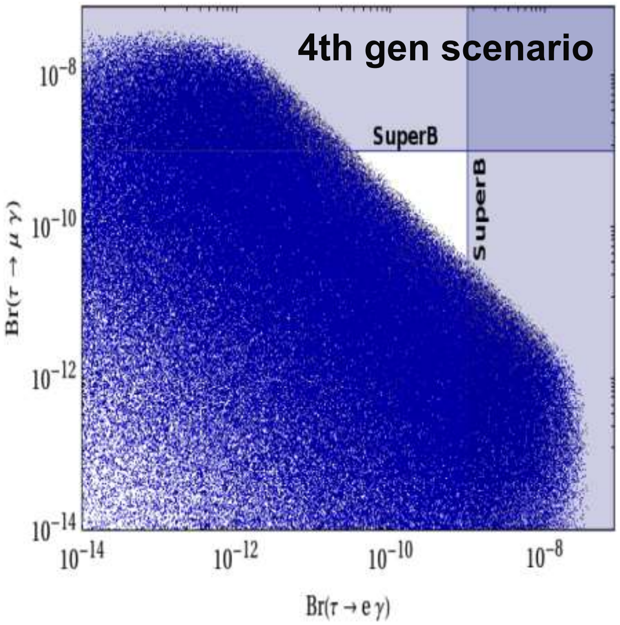
2.5 Model discriminating power of Tau processes

- Depending on the UV model different correlations between the BRs

Blankerburg et al.'12



Buras et al.'10

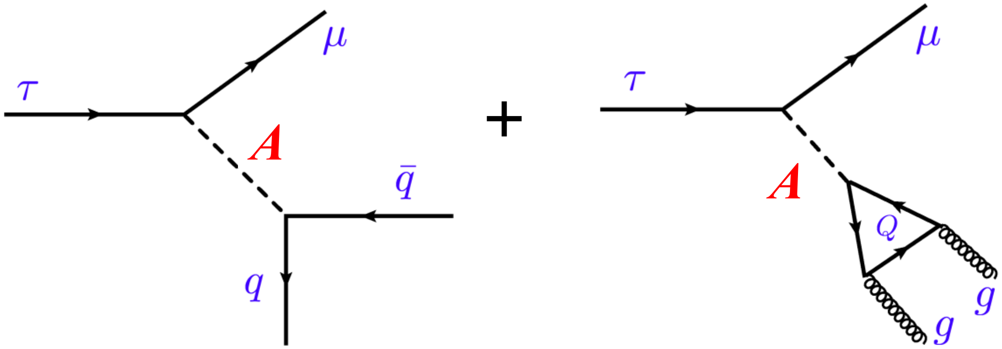


⇨ Interesting to study to determine the underlying dynamics of NP

4. CP-odd Higgs with LFV

4.1 Constraints from τ → 1P

- Tree level Higgs exchange



- L_Y ⇨ $$\boxed{\mathcal{L}_{eff}^{A} \simeq -\frac{A}{v}\left(\sum_{q=u,d,s} y_q^A\, m_q\, \bar{q} i\gamma_5 q - \sum_{q=c,b,t} y_q^A \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \widetilde{G}_{\mu\nu}^a\right)}$$

$$\widetilde{G}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}^a$$

- Mediate only one pseudoscalar meson ⇨ very characteristic!

4.1 Constraints from τ → 1P

- Tree level Higgs exchange
  - η, η’

$$\Gamma\left(\tau \to \ell\eta^{(\prime)}\right) = \frac{\bar\beta\,(m_\tau^2 - m_\eta^2)\left(|Y^A_{\mu\tau}|^2 + |Y^A_{\tau\mu}|^2\right)}{256\,\pi\, M_A^4\, v^2\, m_\tau}\left[(y_u^A + y_d^A)h^q_{\eta'} + \sqrt{2}y_s^A h^s_{\eta'} - \sqrt{2}a_{\eta'}\sum_{q=c,b,t} y_q^A\right]^2$$

with the decay constants :

$$\langle\eta^{(\prime)}(p)|\bar q\,\gamma_5\, q|0\rangle = -\frac{i}{2\sqrt{2}m_q}h^q_{\eta^{(\prime)}} \qquad \langle\eta^{(\prime)}(p)|\bar s\,\gamma_5\, s|0\rangle = -\frac{i}{2m_s}h^s_{\eta^{(\prime)}}$$

$$\langle\eta^{(\prime)}(p)|\frac{\alpha_s}{4\pi}\,G_a^{\mu\nu}\widetilde{G}^a_{\mu\nu}|0\rangle = a_{\eta^{(\prime)}}$$

- π :

$$\Gamma(\tau \to \ell\pi^0) = \frac{f_\pi^2\, m_\pi^4\, m_\tau}{256\pi M_A^4\, v^2}\left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2\right)\left(y_u^A - y_d^A\right)^2$$

4.2 Results

- $\tau \rightarrow \mu P$

Process	BR 90% CL	$M_A = 200$ GeV	$M_A = 500$ GeV	$M_A = 700$ GeV
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	$Z < 0.018$	$Z < 0.040$	$Z < 0.055$
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	$Z < 0.28$	$Z < 0.60$	$Z < 0.85$
$^{(*)}$ $\tau \to \mu\pi$	$< 11 \times 10^{-8}$	$Z < 41$	$Z < 257$	$Z < 503$
$^{(*)}$ $\tau \to \mu\eta$	$< 6.5 \times 10^{-8}$	$Z < 0.52$	$Z < 3.3$	$Z < 6.4$
$^{(*)}$ $\tau \to \mu\eta'$	$< 13 \times 10^{-8}$	$Z < 1.1$	$Z < 7.2$	$Z < 14.1$
$\tau \to \mu\pi^+\pi^-$	$< 2.1 \times 10^{-8}$	$Z < 0.25$	$Z < 0.54$	$Z < 0.75$
$\tau \to \mu\rho$	$< 1.2 \times 10^{-8}$	$Z < 0.20$	$Z < 0.44$	$Z < 0.62$

BaBar'06'10 , Belle'10'11'13

$$\boxed{Z = \sqrt{\left|Y^A_{\mu\tau}\right|^2 + \left|Y^A_{\tau\mu}\right|^2}}$$

(*) : No contribution from effective dipole operator or CP-even Higgs

N.B.: Diagonal couplings $\left|y^A_f\right| = 1$

4.2 Results

- $\tau \longrightarrow eP$

Process	BR 90% CL	$M_A = 200$ GeV	$M_A = 500$ GeV	$M_A = 700$ GeV
$\tau \to e\gamma$	$< 3.3 \times 10^8$	$Z < 0.016$	$Z < 0.034$	$Z < 0.05$
$\tau \to eee$	$< 2.7 \times 10^8$	$Z < 0.14$	$Z < 0.30$	$Z < 0.42$
$^{(*)}\ \tau \to e\pi$	$< 8 \times 10^8$	$Z < 35$	$Z < 219$	$Z < 430$
$^{(*)}\ \tau \to e\eta$	$< 9.2 \times 10^8$	$Z < 0.6$	$Z < 3.9$	$Z < 7.6$
$^{(*)}\ \tau \to e\eta'$	$< 16 \times 10^8$	$Z < 1.3$	$Z < 8$	$Z < 15.6$
$\tau \to e\pi^+\pi^-$	$< 2.3 \times 10^8$	$Z < 0.26$	$Z < 0.56$	$Z < 0.80$
$\tau \to e\rho$	$< 1.8 \times 10^8$	$Z < 0.25$	$Z < 0.54$	$Z < 0.76$

BaBar'06'10 , Belle'10'11'13

$$\boxed{Z = \sqrt{\left|Y^A_{e\tau}\right|^2 + \left|Y^A_{\tau e}\right|^2}}$$

(*) : No contribution from effective dipole operator or CP-even Higgs

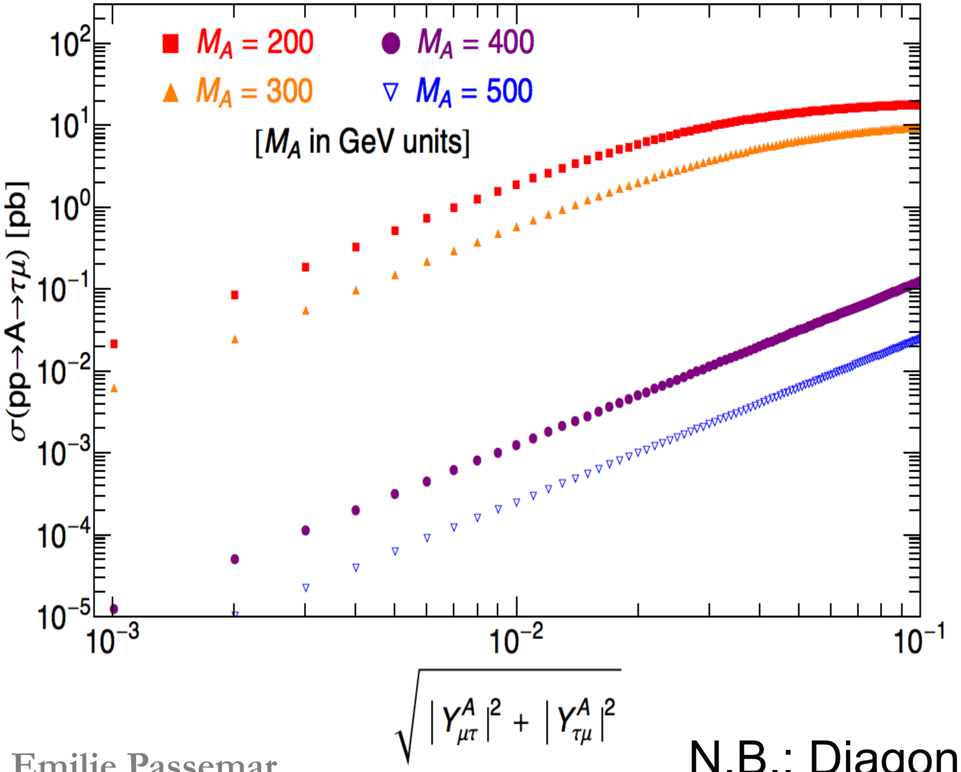
N.B.: Diagonal couplings $\left|y^A_f\right| = 1$

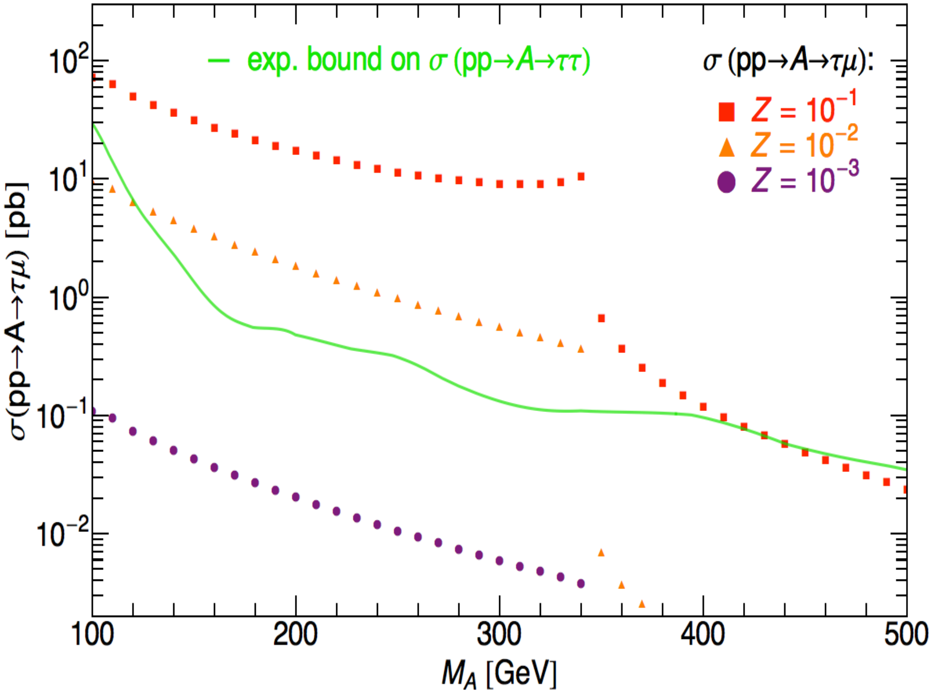
4.3 Prospects at LHC

see also e.g., Assamagan, Deandrea, Delsart'03
Davidson & Verdier'12
Arana-Catania, Arganda, Herrero'14

- Decay width : $\Gamma(A \to \tau^+\mu^- + \tau^-\mu^+) \equiv \Gamma(A \to \tau\mu) = \dfrac{M_A\left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2\right)}{8\pi}$

  Assumption : only SM channels ($A \to gg, b\bar{b}, c\bar{c}, \tau\tau\ldots$) are important
- Large BR for $A \to \tau\mu$ can be expected since A does not couple to WW, ZZ at tree level. Results :





N.B.: Diagonal couplings $\left|y^A_f\right| = 1$