$B_{d,s}$ Mixing

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Landscape of Flavour Physics towards the high intensity era
Pisa – 2014

Outline

- Introduction

  Formalism, observables, measurements . . .

- The Standard Model (SM)

  Predictions, uncertainties . . .

- Beyond the SM

  Fits, . . .

Introduction to B mixing

Formalism of time-evolution for neutral B_q-meson mixing

Flavour eigenstates: $\overline{B}_q = (b\bar{q})$ and $B_q = (\bar{b}q)$ with $q = d, s$ (PDG-convention)

Time evolution (omitting index $q = d, s$)

$$i \frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\overline{B}(t)\rangle \end{pmatrix} = \left[\hat{M} - \frac{i}{2} \hat{\Gamma} \right] \begin{pmatrix} |B(t)\rangle \\ |\overline{B}(t)\rangle \end{pmatrix}$$

2×2-matrices: $\hat{M} = \hat{M}^\dagger, \quad \hat{\Gamma} = \hat{\Gamma}^\dagger$

Assuming CPT-invariance

$$M_{11} = M_{22}, \qquad M_{12} = (M_{21})^*, \qquad \Gamma_{11} = \Gamma_{22}$$

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Heavy & light mass eigenstates: $|B_{L,H}\rangle = p|B\rangle \pm q|\overline{B}\rangle$ from diagonalisation of $(\hat{M} - i/2\,\hat{\Gamma})$

$\Rightarrow$ eigenvalues $M_{L,H}$ and $\Gamma_{L,H}$ (all positive)

$$\overline{M} \equiv M_{11} = M_{22} = \frac{M_H + M_L}{2}$$

$$\overline{\Gamma} \equiv \Gamma_{11} = \Gamma_{22} = \frac{\Gamma_L + \Gamma_H}{2}$$

- average mass $\overline{M}$ and decay width $\overline{\Gamma}$

$$\Delta M \equiv M_H - M_L \qquad \geq 0$$

$$\Delta\Gamma \equiv \Gamma_L - \Gamma_H \qquad \gtrless 0$$

- ΔM determines oscillation frequency
- $\Delta\Gamma$ corresponds to “damping”

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$$\Delta M \equiv M_H - M_L = 2|M_{12}| + \ldots \geq 0$$

$$\Delta\Gamma \equiv \Gamma_L - \Gamma_H = 2|\Gamma_{12}|\cos(\zeta) + \ldots \gtrless 0$$

- ΔM determines oscillation frequency
- $\Delta\Gamma$ corresponds to “damping”

$\Rightarrow \overline{\Gamma}, \Delta M$ and $\Delta\Gamma$ determined by weak interactions (in the SM)

Phenomenology ...

(4 + 1) real-valued parameters

$$\left.\begin{array}{c} M_{12} \\ \Gamma_{12} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} |M_{12}| \\ |\Gamma_{12}| \\ \zeta = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \\ \phi_M = \arg(M_{12}) \end{array}\right.$$

$$\Gamma_{11} = \Gamma_{22}$$

are related to

$$\Delta M = 2\,|M_{12}| + \ldots$$

$$\Delta\Gamma = 2\,|\Gamma_{12}|\cos(\zeta) + \ldots$$

$$a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\zeta) + \ldots$$

$$\phi_M = \phi_f + (\text{final state } f \text{ specific})$$

$\Rightarrow$ interference of mixing and decay amplitudes in decays of neutral $B \to f$

$$\overline{\Gamma} = \frac{1}{\overline{\tau}} = \Gamma_{11}$$

or separate measurements of Γ_H and Γ_L

ΔM, $\Delta\Gamma$, ϕ_f and $\overline{\Gamma}$ determined in time-dependent studies of neutral B-meson decays

Flavour-specific CP-asymmetries (semi-leptonic CP asy's)

. . . usually measured in semi-leptonic decays $B_q \to X\ell\nu_\ell$

$$a_{\rm fs} = a_{\rm sl}^q = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \bar{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \bar{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\zeta)$$

$\Rightarrow$ measures of CP-violation in mixing: $1 - a_{\rm fs} = |q/p|^2 \neq 1$

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Relation to like-sign dimuon asymmetry $A_{\rm CP}$

Example of DØ: $p\bar{p} \to b\bar{b} + X$ with mixing gives $\bar{b} \to B_q \to \overline{B}_q \to \mu^-$ "wrong sign" μ

- Measure "raw asymmetries" including not just b

$$A = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)}$$

- subtract all non-b background $A_{{\rm bkg},\not b}$ measured with same data

$$A_{\rm CP} = A - A_{{\rm bkg},\not b}$$

- $A_{\rm CP}$ receives contribution from mixing $A_{\rm sl}^b = C_d a_{\rm fs}^d + C_s a_{\rm fs}^s$

$$A_{\rm CP} = A_{\rm sl}^b + C_{\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d} + C_{\Gamma_s} \frac{\Delta\Gamma_s}{\Gamma_s}$$

  and interference of mixing with decays (example: $B_d(\overline{B}_d) \to D^{(*)+} D^{(*)-}$)

[Borissov/Hoeneisen arXiv:1303.0175, DØ arXiv:1310.0447, Nierste @ CKM Workshop 2014]

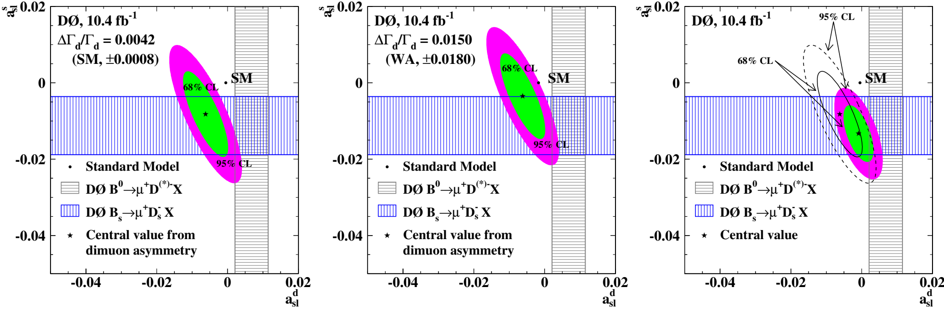
- $C_d = F(f_{d,s}, \chi_{d,s})$, $C_s = 1 - C_d$ $(C_d \approx C_s \approx 0.5)$ with χ_q mean mixing prob's, f_q production fractions of B_q
- C_{Γ_d} (see arXiv:1310.0447 and Nierste @ CKM 2014) contribute destructive to $A_{\rm CP}$, C_{Γ_s} negligible contribution to $A_{\rm CP}$

Status of $A_{\rm CP}$ and $a_{\rm fs}^{d,s}$

Final analysis of DØ 10.4 fb^{-1} [DØ arXiv:1310.0447]

- complex analysis of $A_{\rm CP}$ and $a_{\rm CP}$ in several bins of μ-impact pmr. (IP)
- allows for combined fit of $a_{\rm fs}^d$, $a_{\rm fs}^s$ and $\Delta\Gamma_d$ $\Rightarrow$ 3.0σ away from SM

$$a_{\rm fs}^d = (0.62 \pm 0.43)\%, \quad a_{\rm fs}^s = (-0.82 \pm 0.99)\%, \quad \Delta\Gamma_d/\Gamma_d = (0.50 \pm 1.38)\%$$



fixed $\Delta\Gamma_d/\Gamma_d$ = SM fixed $\Delta\Gamma_d/\Gamma_d$ = world ave. fitting also $\Delta\Gamma_d/\Gamma_d$

$\Rightarrow$ How well can we constrain NP to $\Gamma_{12}^d \longrightarrow \Delta\Gamma_d$ and $a_{\rm fs}^d$?

Status of A_{CP} and $a_{\mathrm{fs}}^{d,s}$

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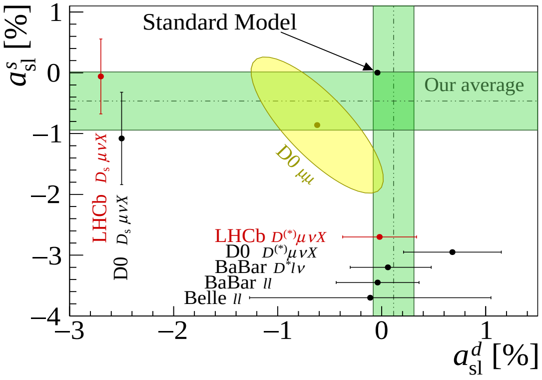
[LHCb (Grillo) arXiv.1411.6198]

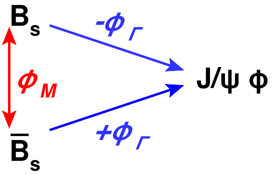
Latest measurements LHCb (red)

with previous DØ, BaBar and Belle



Time-dependent CP asy's for $\overline{B}_q, B_q \to f_{CP}$ to CP eigenstates

Mixing also interferes with CP violation in neutral B-meson decays
$\Rightarrow$ further information on $M_{12} = |M_{12}| \exp(i\,\phi_M)$



$$a_{CP}(t) = \frac{\Gamma(\overline{B}(t) \to f_{CP}) - \Gamma(B(t) \to f_{CP})}{\Gamma(\overline{B}(t) \to f_{CP}) + \Gamma(B(t) \to f_{CP})}$$

$$= \frac{A_{\rm mix} \sin(\Delta M\, t) - A_{\rm dir} \cos(\Delta M\, t)}{\cosh(\Delta\Gamma/2\, t) - A_{\Delta\Gamma} \sinh(\Delta\Gamma/2\, t)}$$

with 3 CP asymmetries $\quad A_{\rm dir} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \qquad A_{\rm mix} = \frac{2\,{\rm Im}\lambda_f}{1 + |\lambda_f|^2}, \qquad A_{\Delta\Gamma} = \frac{2\,{\rm Re}\lambda_f}{1 + |\lambda_f|^2},$

and $\quad \lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \frac{q}{p} \approx -\frac{(M_{12})^*}{|M_{12}|}, \qquad 1 = |A_{\rm dir}|^2 + |A_{\rm mix}|^2 + |A_{\Delta\Gamma}|^2$

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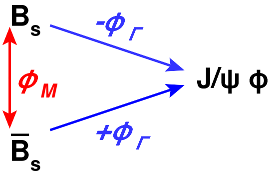

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$\Rightarrow$ Combined fits of Γ, $\Delta\Gamma$ and $\lambda_f = |\lambda_f| \exp(i\,\phi_f)$

Golden-plated modes $|\lambda_f| \approx 1$, $A_{\rm dir} = 0$ and $A_{\rm mix} = {\rm Im}\lambda_f$: for example $B_s \to J/\psi + (KK \text{ or } \pi\pi)$

- $\phi_s|_{\rm SM} = \phi_M - 2\phi_\Gamma \approx -2 \arg\left(\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*}\right) = -2\beta_s$ [LHCb arXiv:1112.3183 . . .]
- $2\phi_s^{J/\psi\phi}|_{\rm SM} \approx 2\beta_s = (2.1 \pm 0.1)^\circ = (0.0363 \pm 0.0013)$ rad [Lenz/Nierste/CKMfitter arXiv:1203.0238 . . .]
  neglecting penguin pollution

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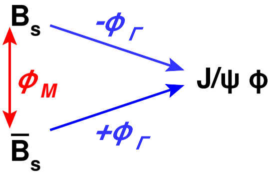

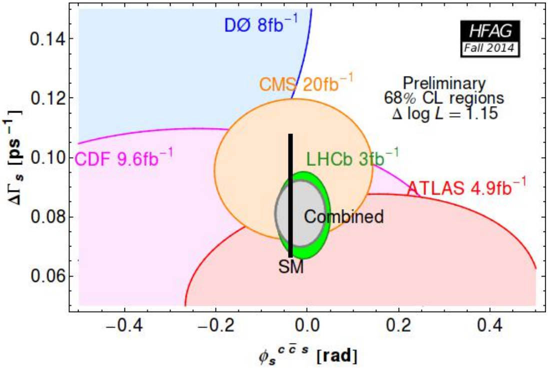

[LHCb arXiv:1411.3104 3/fb]

From $B_s \to J/\psi\,(K^+K^-)$ alone
(neglecting different polarizations)

$$\phi_s = (-0.058 \pm 0.050)\ \text{rad}$$

$$\Delta\Gamma_s = (0.0805 \pm 0.0096)\ \text{ps}^{-1}$$

and for the 1st time independently for each polarization $k = 0, \perp, \parallel, S$

$$\phi_s^0 = (-0.045 \pm 0.054)\ \text{rad}$$

$$\phi_s^\perp - \phi_s^0 = (-0.018 \pm 0.044)\ \text{rad}$$

$$\phi_s^\parallel - \phi_s^0 = (-0.014 \pm 0.036)\ \text{rad}$$

$$\phi_s^S - \phi_s^0 = (+0.015 \pm 0.065)\ \text{rad}$$

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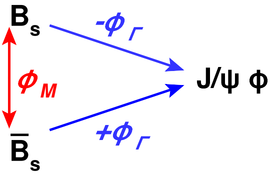

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LHCb combination of $B_s \to J/\psi\,(K^+K^-)$ and $B_s \to J/\psi\,(\pi^+\pi^-)$
(with some assumptions) [LHCb arXiv:1411.3104 3/fb]

LHCb:	$\phi_s = (-0.010 \pm 0.039)$ rad	$\lvert\lambda\rvert = 0.957 \pm 0.017$
theory (neglecting penguins):	$-2\beta_s = (-0.0363 \pm 0.0013)$ rad	$\lvert\lambda\rvert_{\rm SM} \simeq 1$

$\Rightarrow$ Need to consider "penguin pollution" in future

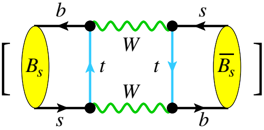
Theory strategies with "control modes", using $SU(3)_{\rm flavour}$ sum's [Faller/Fleischer/Mannel arXiv:0810.4248]

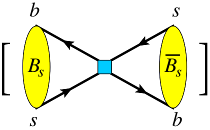
First results from $B_d \to J/\psi\,\rho^0(770)$ [LHCb arXiv:1411.1634 3/fb]

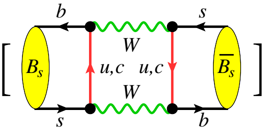
- half of the uncertainty on ϕ_s assuming approximate $SU(3)_{\rm flavour}$
- consistent with theory estimates of penguin pollution
- need theory input for $SU(3)_{\rm flavour}$ breaking

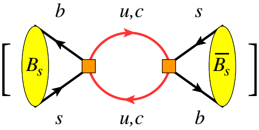
Standard Model

In the SM . . .

$M_{12}^s \simeq$ 

$\Rightarrow$ $C_i \times$ 

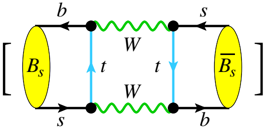
$\Gamma_{12}^s \simeq \text{Im}$ 

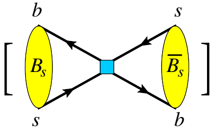
$\Rightarrow$ $C_a C_b^* \times \text{Im}$ 

For heavy new physics ($M_W \lesssim \Lambda_{\rm NP}$) $\Rightarrow$ can use local dim-6 op's a la Fermi-theory

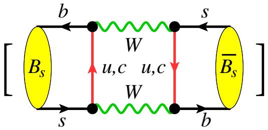
- $\Delta B = 2 \to [\bar{s}\Gamma b][\bar{s}\Gamma' b]$ with hadronic matrix elements from lattice
- $\Delta B = 1 \to [\bar{s}\Gamma b][\bar{f}_1\Gamma' f_2]$ with "$(m_{f_1} + m_{f_2}) \lesssim M_{B_s}$" $\Rightarrow f = (u, d, s, c)$ and (e, μ, τ)

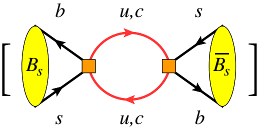
In the SM … Beyond SM … ⇒ $\Delta B = 2$ most senstive to NP

$M_{12}^s \simeq$ 

$\Rightarrow$ $C_i \times$ 

$$\Delta B = 2 \sim \left|A^{\rm SM}_{\Delta B=2}\right| \times \left(1 + \frac{(V_{\rm NP})^4}{(V_{tb}V_{ts}^*)^2}\frac{M_W^2}{\Lambda_{\rm NP}^2}\right)$$

$\Gamma_{12}^s \simeq \mathrm{Im}$ 

$\Rightarrow$ $C_a C_b^* \times \mathrm{Im}$ 

$$(\Delta B = 1)^2 \sim \left|A^{\rm SM}_{\Delta B=1}\right|^2 \times \left(1 + \frac{(V_{\rm NP})^2}{(V_{tb}V_{ts}^*)}\frac{M_W^2}{\Lambda_{\rm NP}^2} + \ldots\right)$$

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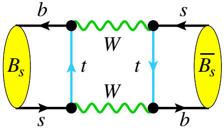
Theory predictions for $M_{12}^q \longrightarrow \Delta M_q = 2\,|M_{12}^q|$

Short-distance (decoupling of W's and top's in box diagrams) + local matrix element

$$M_{12}^q = \frac{G_F^2}{12\pi^2}\left(V_{tb}V_{tq}^*\right)^2 M_W^2 S_0(x_t)\hat{\eta}_B B_{B_q} f_{B_q}^2 M_{B_q}$$



- Short-distance under control

  1-loop result $S_0(x_t = m_t^2/m_W^2)$ [Inami/Lim Prog.Theor.Phys. 65 (1981) 297]
  2-loop QCD corrections $\hat{\eta}_B$ [Buras/Jamin/Weisz Nucl.Phys. B347 (1990) 491]
  2-loop EW corrections tiny (usually neglected) [Gambino/Kwiatkowski/Pott hep-ph/9810400]

- Hadronic matrix element $\langle \overline{B}_q|(\bar{b}q)_{V-A}(\bar{b}q)_{V-A}|B_q\rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}$

  $\Rightarrow$ preciser Lattice results become available [averages from FLAG arXiv:1310.8555]

[MeV]	$N_f = 2+1$	$\delta(\Delta M_q)$		$N_f = 2+1$	$\delta(\Delta M_q)$
f_{B_s}	227.7 ± 4.5	4.0%	$\hat{B}_{B_s}$	1.33 ± 0.06	4.5%
f_{B_d}	190.5 ± 4.2	4.4%	$\hat{B}_{B_d}$	1.27 ± 0.10	7.9%

- and CKM we would like to determine from experiment …
  … and confront with tree-fit (semileptonic, $B \to D^{(*)}K$) results

  $\Rightarrow$ however, when taking UTfit tree-fit (pre-Moriond 2013): [www.utfit.org]
  rel. err. on V_{ts} (V_{td}) about 2.5% (6%) induces 5% (12%) uncertainty on ΔM_s (ΔM_d)

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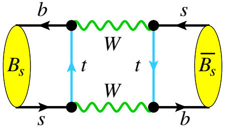

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- Hadronic matrix element $\langle \overline{B}_q|(\bar{b}q)_{V-A}(\bar{b}q)_{V-A}|B_q\rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}$

  $\Rightarrow$ preciser Lattice results become available [averages from FLAG arXiv:1310.8555]

[MeV]	$N_f = 2+1$	$\delta(\Delta M_q)$		$N_f = 2+1$	$\delta(\Delta M_q)$
f_{B_s}	227.7 ± 4.5	4.0%	$\hat{B}_{B_s}$	1.33 ± 0.06	4.5%
f_{B_d}	190.5 ± 4.2	4.4%	$\hat{B}_{B_d}$	1.27 ± 0.10	7.9%

- and CKM we would like to determine from experiment ...
  ... and confront with tree-fit (semileptonic, $B \to D^{(*)}K$) results

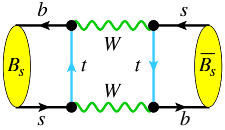
  $\Rightarrow$ however, when taking UTfit tree-fit (pre-Moriond 2013): [www.utfit.org]

  rel. err. on V_{ts} (V_{td}) about 2.5% (6%) induces 5% (12%) uncertainty on ΔM_s (ΔM_d)

Theory predictions for $M_{12}^q \longrightarrow \Delta M_q = 2\,|M_{12}^q|$

Short-distance (decoupling of W's and top's in box diagrams) + local matrix element

$$M_{12}^q = \frac{G_F^2}{12\pi^2}\left(V_{tb}V_{tq}^*\right)^2 M_W^2 S_0(x_t)\hat{\eta}_B B_{B_q} f_{B_q}^2 M_{B_q}$$



- Short-distance under control

  1-loop result $S_0(x_t = m_t^2/m_W^2)$ [Inami/Lim Prog.Theor.Phys. 65 (1981) 297]
  2-loop QCD corrections $\hat{\eta}_B$ [Buras/Jamin/Weisz Nucl.Phys. B347 (1990) 491]
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Theory predictions for Γ_{12}^q

Based on Heavy Quark Expansion (HQE) = local OPE, used also for $\Delta B = 0$

$\lambda = \Lambda_{\text{QCD}}/m_b$

$$\Gamma_{12}^q = \lambda^3\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)}\right) + \lambda^4\left(\Gamma_4^{(0)} + \ldots\right) + \lambda^5\left(\Gamma_5^{(0)} + \ldots\right) + \ldots$$

[Beneke/Buchalla hep-ph/9605259; Beneke/Buchalla/Greub/Lenz/Nierste 9808385; Beneke/Buchalla/Lenz/Nierste 0307344; Ciuchini/Franco/Lubicz/Mescia/Tarantino 0308029; Lenz/Nierste 0612167; Badin/Gabiani/Petrov arXiv:0707.0294]

- individual contributions show convergent behaviour
- HQE works well for total width ($\overline{\Gamma}_q$) and their ratios ($\overline{\Gamma}_d/\overline{\Gamma}_s$) ... assuming no NP

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Error budget $\Delta\Gamma_s$

[Lenz/Nierste arXiv:1102.4274]

- $1\times$ decay constant f_{B_q} — $\delta(f_{B_s}) \sim 13.2\%$
  $2\times$ dim-6 matrix elements: B_{B_q}, $\tilde{B}_S^q$ — $\delta(B_{B_s}, \tilde{B}_S^s) \sim 4.8\%$
  $\Rightarrow$ improved Lattice results already available
- dim-7 matrix elements: $R_{0,1,2,3}$ and $\widetilde{R}_{0,1,2,3}$ — $\delta(\widetilde{R}_2) \sim 17.2\%$, $\delta(\widetilde{R}_0) \sim 3.4\%$
  $\Rightarrow$ No lattice predictions yet, but QCD sum rules [Mannel/Pecjak/Pivovarov hep-ph/0703244]
- renormalisation scale μ_b — $\delta(\mu_b) \sim 7.8\%$
- CKM V_{cb} — $\delta(\mu_b) \sim 3.4\%$

$\Rightarrow$ total $\delta(\Delta\Gamma_s) \sim 25\%$

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SM prediction [Lenz/Nierste arXiv:1102.4274]

Experimental results [HFAG 2014]

$$\Delta\Gamma_s^{\rm SM} = (0.087 \pm 0.021)\,\mathrm{ps}^{-1}$$
$$\Delta\Gamma_s^{\rm Exp} = (0.091 \pm 0.008)\,\mathrm{ps}^{-1}$$

$$\Delta\Gamma_d^{\rm SM} = (0.0029 \pm 0.0007)\,\mathrm{ps}^{-1}$$
$$\Delta\Gamma_d^{\rm Exp} = (0.0059 \pm 0.0079)\,\mathrm{ps}^{-1}$$

$\Rightarrow$ need preciser measurement of $\Delta\Gamma_d$

Overall SM compares well to data

$$\frac{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm Exp}}{\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\rm SM}} = 1.02 \pm 0.09 \pm 0.19$$

Dominant uncertainty from NNLO QCD and Lattice

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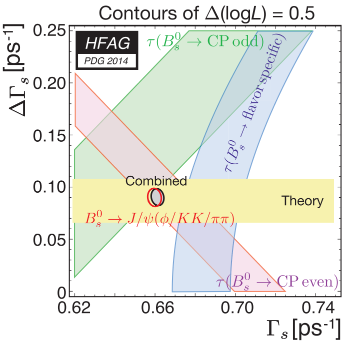

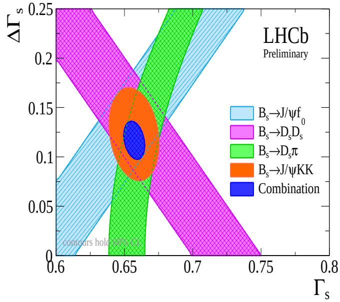

only LHCb [LHCb (Dordei) arXiv:1411.4247]

Theory predictions for $a_{\rm fs}^q$

SM predictions are highly sensitive to CKM input:

V_{ub}, V_{cb} and γ (of which errors not included) from global CKM fit

$a_{\rm fs}^d = -(4.1 \pm 0.6) \cdot 10^{-4}$ $\quad a_{\rm fs}^s = (1.9 \pm 0.3) \cdot 10^{-5}$ [Lenz/Nierste arXiv:1102.4274]

Experimental results

$a_{\rm fs}^d \cdot 10^4$		$a_{\rm fs}^s \cdot 10^5$	
$68 \pm 45 \pm 14$	[DØ 1208.5813]	$-(1120 \pm 740 \pm 170)$	[DØ 1207.1769]
$6 \pm 17^{+38}_{-32}$	[BaBar 1305.1575]	$-(60 \pm 500 \pm 360)$	[LHCb 1308.1048 1/fb]
$-(2 \pm 19 \pm 30)$	[LHCb 1409.8586 3/fb]		

Prospects for LHCb 3 fb^{-1}: $\sigma(a_{\rm fs}^s) \sim (200 - 300) \cdot 10^{-5}$ [talk K. Kreplin at Beauty 2014]

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Like-sign dimuon asymmetry [Lenz/Nierste arXiv:1102.4274]

$$A_{\rm sl}^b = 0.406\, a_{\rm fs}^d + 0.594\, a_{\rm fs}^s = -(2.3 \pm 0.4) \cdot 10^{-4}$$

Experimental result from DØ assuming $\Delta\Gamma_d/\Gamma_d$ = SM [DØ arXiv:1310.0447]

$$A_{\rm sl}^b = -(49.6 \pm 15.3 \pm 7.2) \cdot 10^{-4}$$

$\Rightarrow$ 2.8σ from theory

Beyond the Standard Model

New physics in M_{12}^q

- perform global CKM-fit
- use parametrisation of New Physics (NP)

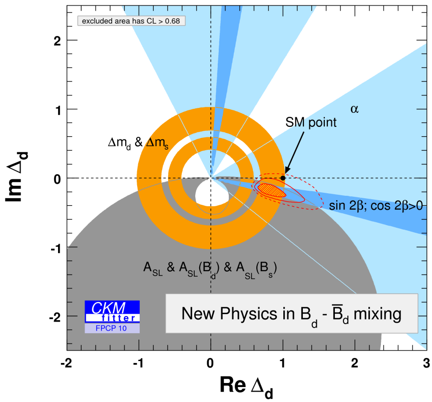
$$M_{12}^q = M_{12}^{\text{SM},q} \cdot \Delta_q, \qquad \Delta_q = |\Delta_q| e^{i\phi_q^\Delta}, \qquad q = d, s$$

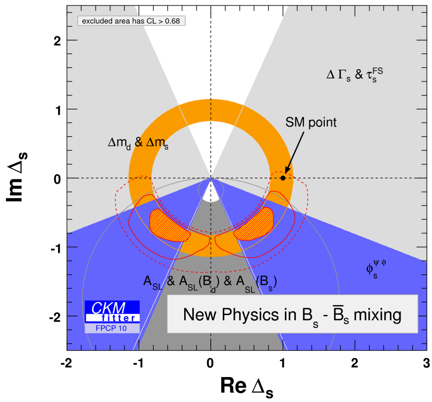
- 2 complex NP parameters → 4 dimensional NP parameter space
- B_d- and B_s-sector connected via A_{sl}^b

New physics in M_{12}^q

year 2010

[Lenz/Nierste/CKMfitter arXiv:1008.1593]



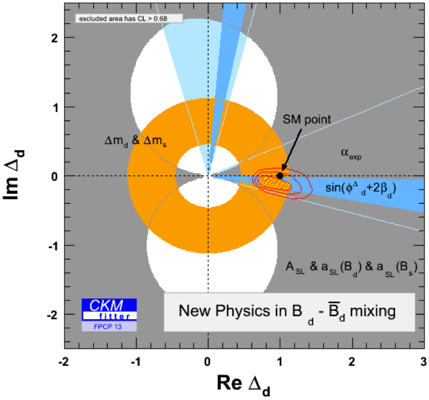


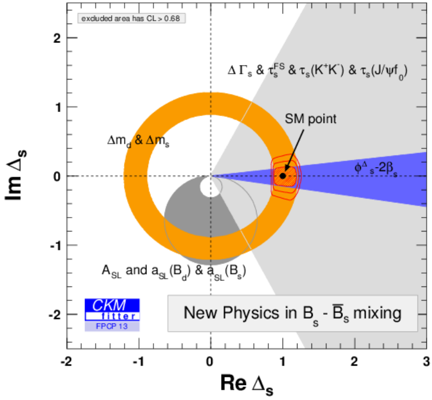
significance of SM $\Delta_d = 1$ (2D) $\rightarrow p = 2.5\sigma$ $\Delta_s = 1$ (2D) $\rightarrow p = 2.7\sigma$

New physics in M_{12}^q

year 2010 $\longrightarrow$ year 2013

[Lenz/Nierste/CKMfitter 1203.0238v2 and update FPCP 2013]





significance of SM $\Delta_d = 1$ (2D) $\rightarrow p = 1.5\sigma$ $\Delta_s = 1$ (2D) $\rightarrow p = 0.0\sigma$

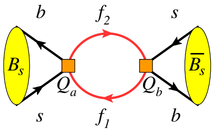
$A_{\rm sl}^b$ and $Br(B \to \tau\nu_\tau)$ prefer $\phi_d^\Delta < 0$:

$\Rightarrow$ SM-hypothesis $\Delta_d = \Delta_s = 1$ (4D) has significance 1σ (compared to 3.6σ in 2010)

$\Rightarrow$ pull of $A_{\rm sl}^b$ is 3.4σ

New physics in Γ_{12}^q ...

... using $\Delta B = 1$ operators $Q_i = [\bar{q}\Gamma b][\bar{f}_1 \Gamma' f_2]$ with light $f_{1,2}$

$$\Gamma_{12}^q = \Gamma_{12}^{q,\mathrm{SM}} + C_a C_b^* \times \mathrm{Im}\Big[$$  $$\Big]$$

⇒ subject to constraints since contribute to: Γ_q, $\tau(B_s)/\tau(B_d)$, $b \to q\,\bar{f}_1 f_2$ decays ...

- $\Delta\Gamma_s$ dominated by $b \to s\,c\bar{c}$
- $\Delta\Gamma_d$ also sizeable contributions from $b \to d + (u\bar{u},\, c\bar{u})$ with partial cancellations
- comparing $Br(b \to s\,c\bar{c}) = (23.7 \pm 1.3)\%$ with $Br(b \to d\,c\bar{c}) = (1.31 \pm 0.07)\%$
  ⇒ NP in $b \to s\,c\bar{c}$ more severely constrained than $b \to d\,c\bar{c}$ [Krinner/Lenz/Rauh arXiv:1305.5390
- $b \to s\,\tau\bar{\tau}$ change $\Delta\Gamma_s$ at most by 30% from SM

[Dighe/Kundu/Nandi 0705.4547, Bauer/Dunn 1006.1629, CB/Haisch 1109.1826]

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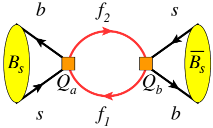
$$\Gamma_{12}^q = \Gamma_{12}^{q,\mathrm{SM}} + C_a C_b^* \times \mathrm{Im}\Big[$$  $$\Big]$$

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⇒ Improve knowledge on NP in Γ_{12}^d to progress with A_{sl}^b:

- improve current bounds on $\Delta\Gamma_d = (0.006 \pm 0.008)\,\mathrm{ps}^{-1}$
- improve current bounds on a_{fs}^d

blue = $B \to X_d \gamma$, green $a_{\rm sl}^d$, red = dim-8 $\sin 2\beta$

New physics in $\Delta\Gamma_d$

[CB/Haisch/Lenz/Pecjak/Tetlamatzi-Xolocotzi arXiv:1404.2531]

Model-independently, $\Delta B = 1$ dim-6 operators:

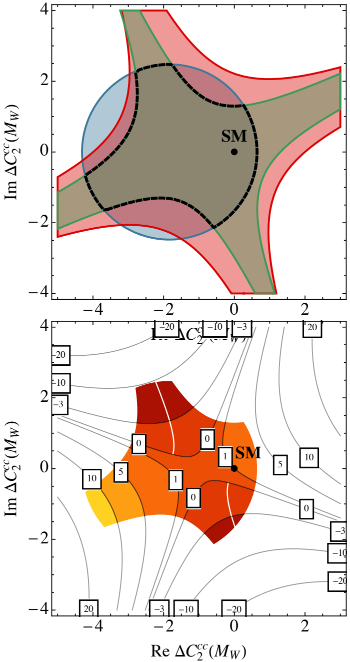
$$b \to d + (u\bar{u},\ c\bar{u},\ c\bar{c})$$

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{p,p'=u,c} V_{pd}^* V_{p'b} \sum_{i=1,2} C_i^{pp'} O_i^{pp'}$$

$$O_1^{pp'} = (\bar{d}^\alpha \gamma^\mu P_L p^\beta)(\bar{p}'^\beta \gamma_\mu P_L b^\alpha)$$
$$O_2^{pp'} = (\bar{d} \gamma^\mu P_L p)(\bar{p}' \gamma_\mu P_L b)$$

$$\Rightarrow C_i^{pp'} = C_i^{\rm SM} + \Delta C_i^{pp'}$$

- used experimental constraints from $B \to \pi\pi,\ \rho\pi,\ \rho\rho,\ D^*\pi,\ X_d\gamma$ and $\sin 2\beta$
- $\Delta\Gamma_d/\Delta\Gamma_d^{\rm SM} \in [-1.0, 1.4]$ for $b \to d + (u\bar{u},\ c\bar{u})$
- huge effects of several 100% in $b \to d\, c\bar{c}$ can not be ruled out



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[CB/Haisch/Lenz/Pecjak/Tetlamatzi-Xolocotzi arXiv:1404.2531]

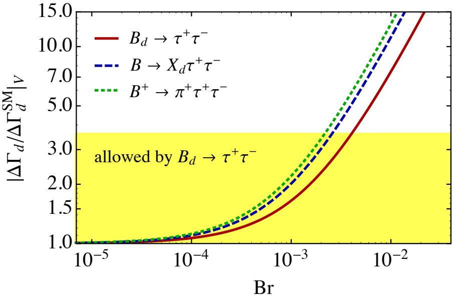
$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i,j} C_{i,j} Q_{i,j}$$

Model-independently, $\Delta B = 1$ dim-6 operators:

$$b \to d\,\tau\bar{\tau}$$

$$Q_{S,AB} = \left(\bar{d} P_A b\right)\left(\bar{\tau} P_B \tau\right)$$
$$Q_{V,AB} = \left(\bar{d}\gamma^\mu P_A b\right)\left(\bar{\tau}\gamma_\mu P_B \tau\right)$$
$$Q_{T,A} = \left(\bar{d}\sigma^{\mu\nu} P_A b\right)\left(\bar{\tau}\sigma_{\mu\nu} P_A \tau\right)$$

- direct constraints from $Br(B_d \to \bar{\tau}\tau)$
- indirect (operator mixing) constraints from $Br(B \to X_d\gamma)$, $Br(B^+ \to \pi^+\bar{\tau}\tau)$
- large effects of 270% in $C_{V,AB}$
  smaller effects of 60% in $C_{S,AB}$
  can not be ruled out



Summary

Conclusion

- Theoretical methods (OPE, HQE) work well
  ($\Delta B = 0$: lifetimes and ratios of lifetimes $\Rightarrow$ should also for $\Delta B = 2$: $\Delta\Gamma_q$ and $a_{\rm fs}^q$)

- SM and CKM picture describe data (ΔM_q, $\Delta\Gamma_q$)

  $\Rightarrow$ soon improved lattice results of f_{B_q} and bag factors
  from several lattice groups (ideally all $\Delta B = 2$ op's, so far only [ETM arXiv:1308.1851])

  ??? however, lacking lattice results for dim-7 operators, needed for $\Delta\Gamma_q$

- huge NP effects not seen, current data still permits sizeable effects

  $\Rightarrow$ except no satisfactory explanation of DØ measurement of
  like-sign dimuon asymmetry

  ??? NP in Γ_{12}^d

  $\Rightarrow$ better measurements needed for $a_{\rm fs}^s$, $a_{\rm fs}^d$ and $\Delta\Gamma_d$ $\Rightarrow$ LHCb and Belle II

- "penguin pollution" in $B_s \to J/\psi(K^+K^-)$ requires improved understanding of $SU(3)_{flav}$
  breaking from theory and/or measurements of "control modes"

  ??? extract $SU(3)_{flav}$ breaking also from data $\Rightarrow$ LHCb and Belle II

B-mixing will provide also in the future stringent constraints on flavour sector of the SM
and constrain non-standard interactions