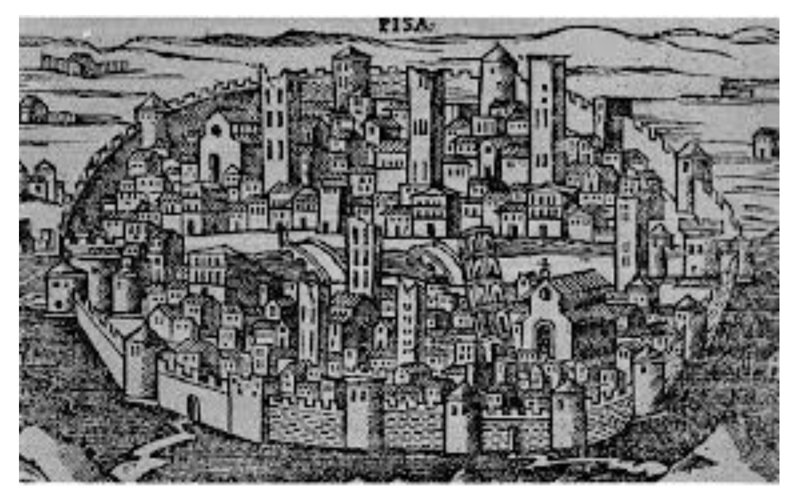
The landscape of Flavour Physics towards the high intensity era

Wednesday, 10 December 2014

The Future of Kaon Physics

Giancarlo D'Ambrosio

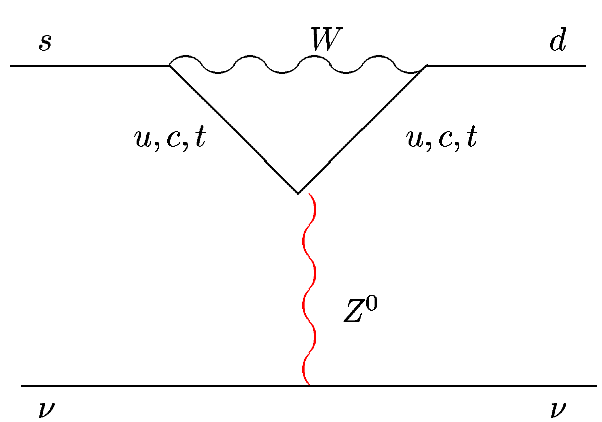
(CERN and INFN-Napoli)



$K \to \pi\nu\overline{\nu}$

Why we need to the experiments KOTO and NA62

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\ (1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM $\quad \underbrace{V-A\otimes V-A}$ $\quad$ Littenberg

$$\Downarrow$$

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

$K^+ \to \pi^+ \nu \bar{\nu}$

Brod,CKM2010, Straub, Gorbhan

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_\mathrm{c}}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 \right]$$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%$\pm$2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^\pm) = (8.22 \pm 0.27 \pm 0.29) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^\pm) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

K_L

$B(K_L) = (2.43 \pm 0.25 \pm 0.06) \times 10^{-11}$ vs

E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman-Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\rm E949} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

Generic Flavor structures strongly constrained

Operator	Bounds on Λ in TeV ($c_{NP}=1$) Re	Im	Bounds on c_{NP} ($\Lambda = 1$ TeV) Re	Im	Observables
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert_D$, ϕ_D
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	
$(\bar{b}_L\gamma^\mu d_L)^2$	6.6×10^2	9.3×10^2	2.3×10^{-6}	1.1×10^{-6}	Δm_{B_d}; $\sin(2\beta)$ from $B_d\to\psi K$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	2.5×10^3	3.6×10^3	3.9×10^{-7}	1.9×10^{-7}	
$(\bar{b}_L\gamma^\mu s_L)^2$	1.4×10^2	2.5×10^2	5.0×10^{-5}	1.7×10^{-5}	Δm_{B_s}; $\sin(\phi_s)$ from $B_s\to\psi\phi$
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	4.8×10^2	8.3×10^2	8.8×10^{-6}	2.9×10^{-6}	

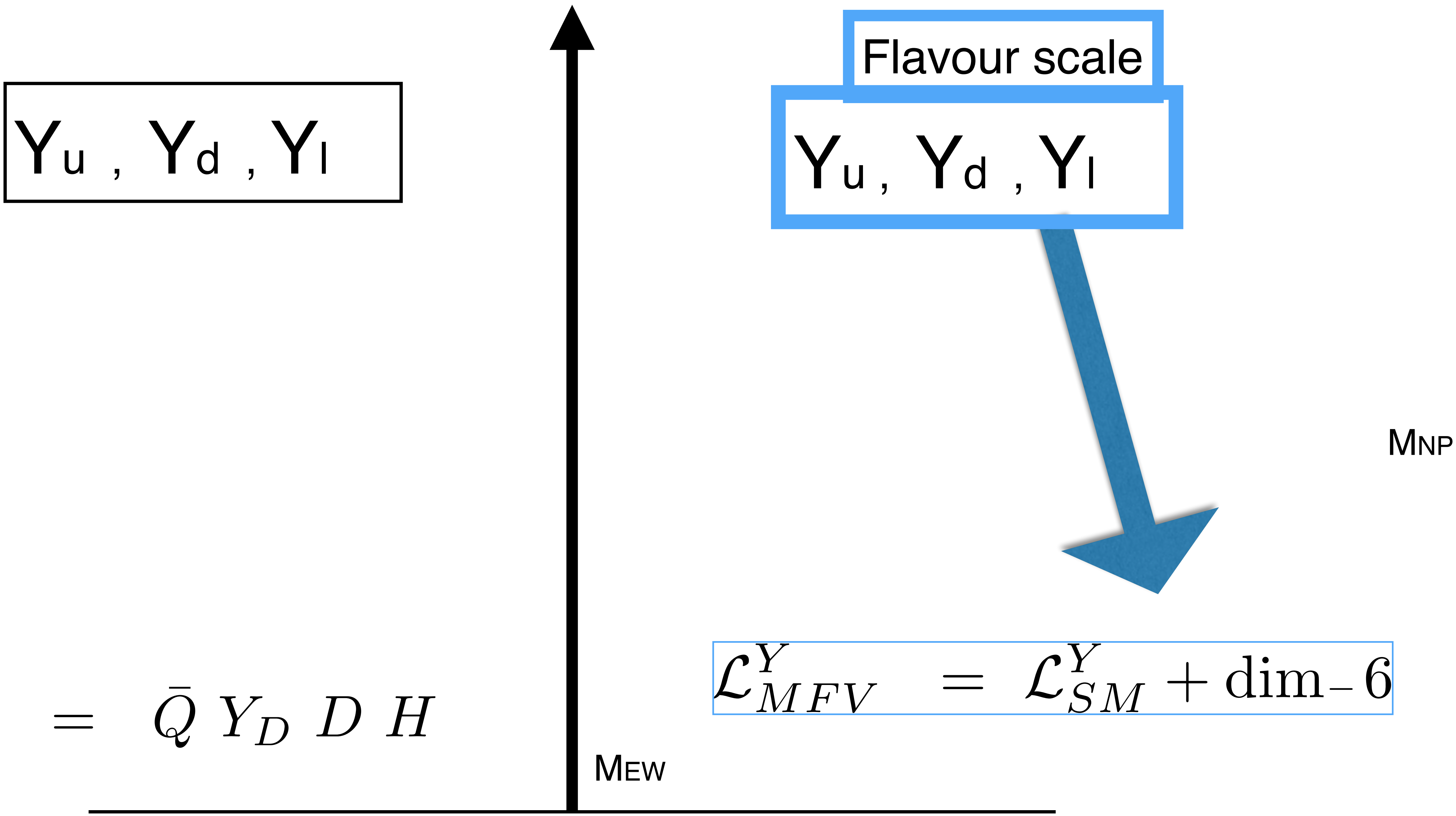
Isidori Nir Perez 10

Problem already known since ’86 technicolour (Chivukula Georgi) susy (Hall Randall) extra dimensions (Rattazzi Zafferoni)

Maybe there is an energy gap between the theory of flavor and the EW scale , ameliorating also a clash from the scale of the bounds in the table above and the requirement of solving the hierarchy problem

SM

MFV



$$\mathcal{L}^{Y}_{SM} = \bar{Q}\, Y_D\, D\, H$$

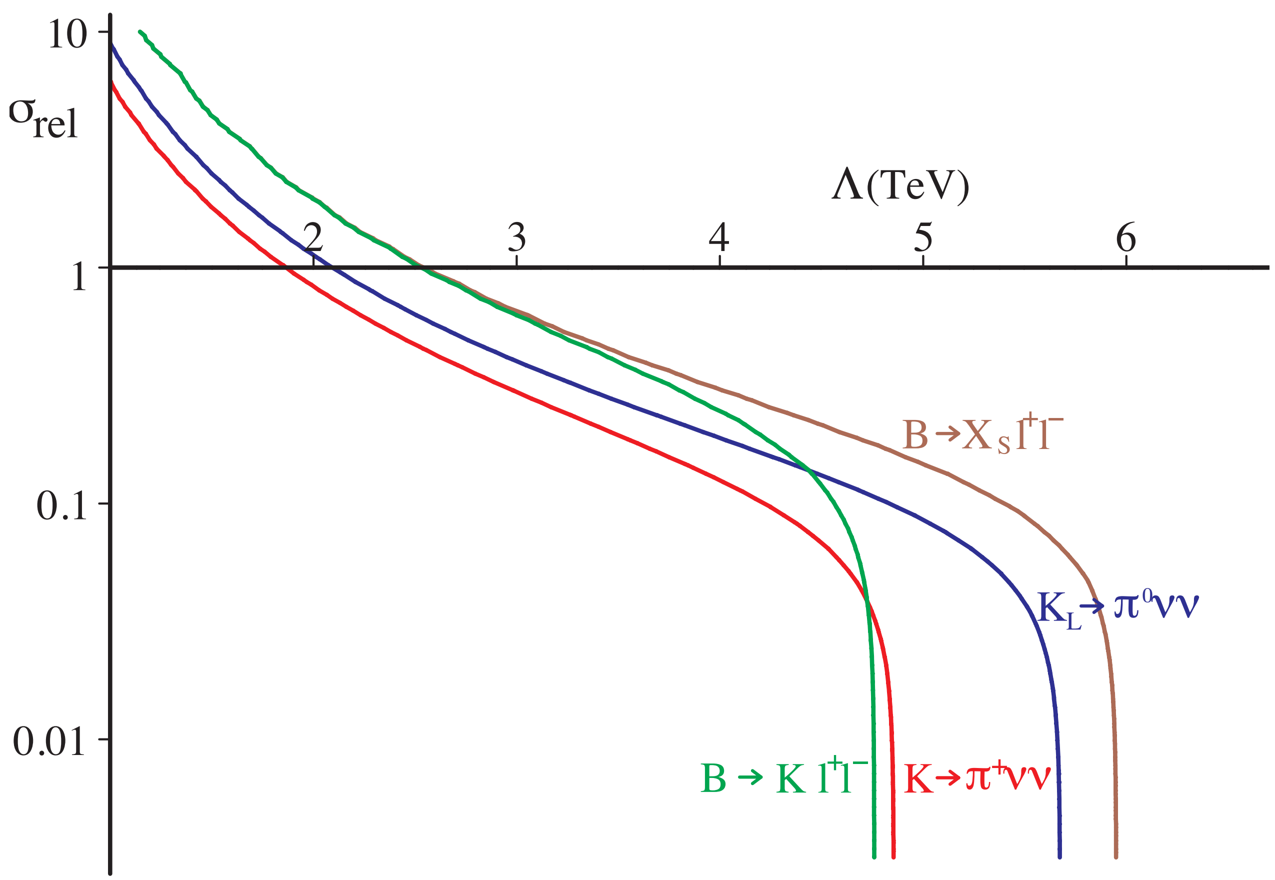
$$\mathcal{L}^{Y}_{MFV} = \mathcal{L}^{Y}_{SM} + \text{dim-}6$$

$$G_F = \overbrace{\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_U \otimes \mathrm{U}(3)_D \otimes \mathrm{U}(3)_L \otimes \mathrm{U}(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

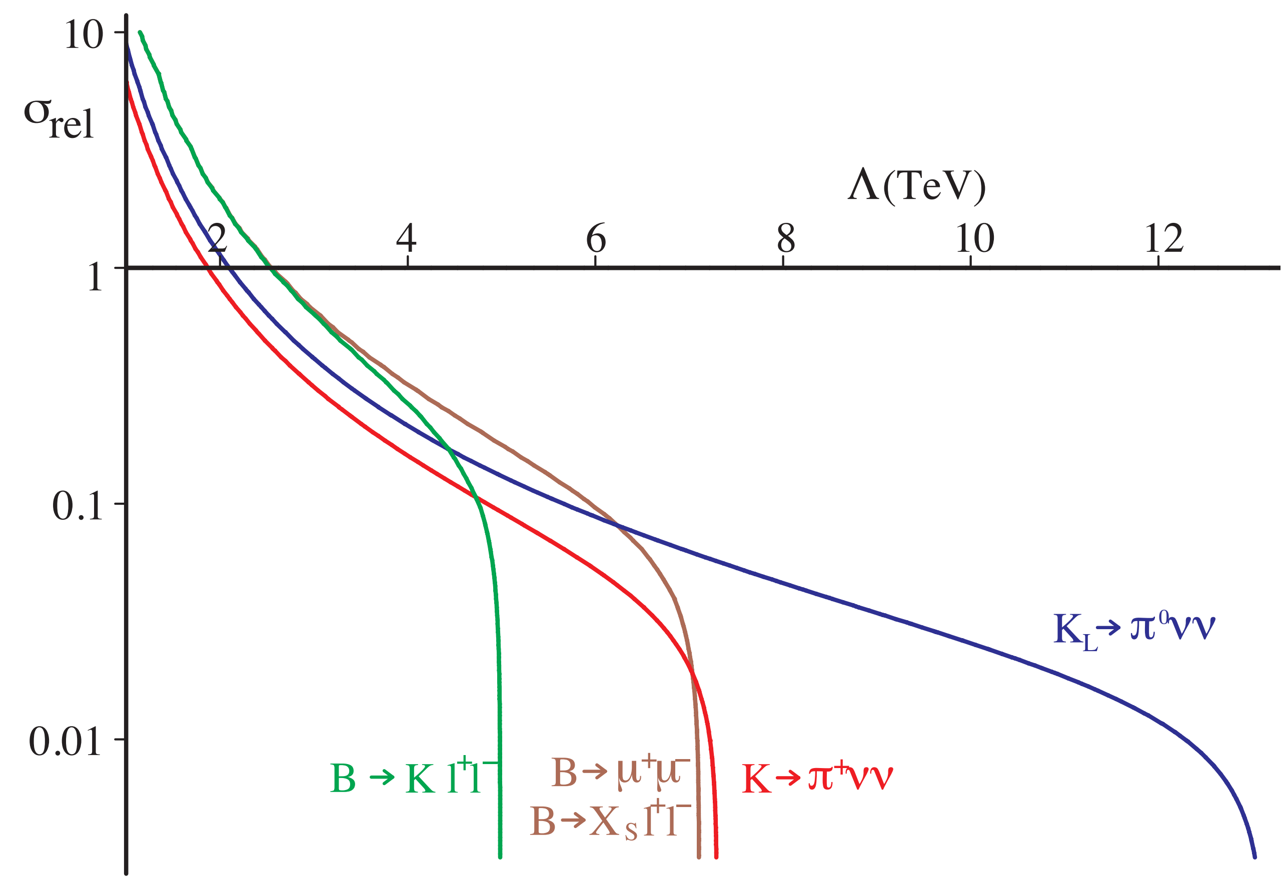
Bounds ameliorated

Minimally flavour violating dimension six operator	main observables	Λ [TeV] −	+	
$\mathcal{O}_0 = \frac{1}{2}(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)^2$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0	
$\mathcal{O}_{F1} = H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} Q_L\right) F_{\mu\nu}$	$B \to X_s \gamma$	8.3	13.4	
$\mathcal{O}_{G1} = H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} T^a Q_L\right) G^a_{\mu\nu}$	$B \to X_s \gamma$	2.3	3.8	
$\mathcal{O}_{\ell 1} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7	*
$\mathcal{O}_{\ell 2} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu \tau^a Q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0	*
$\mathcal{O}_{H1} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(H^\dagger i D_\mu H)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6	*
$\mathcal{O}_{q5} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{D}_R \gamma_\mu D_R)$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1		

GD,Giudice,Isidori,Strumia

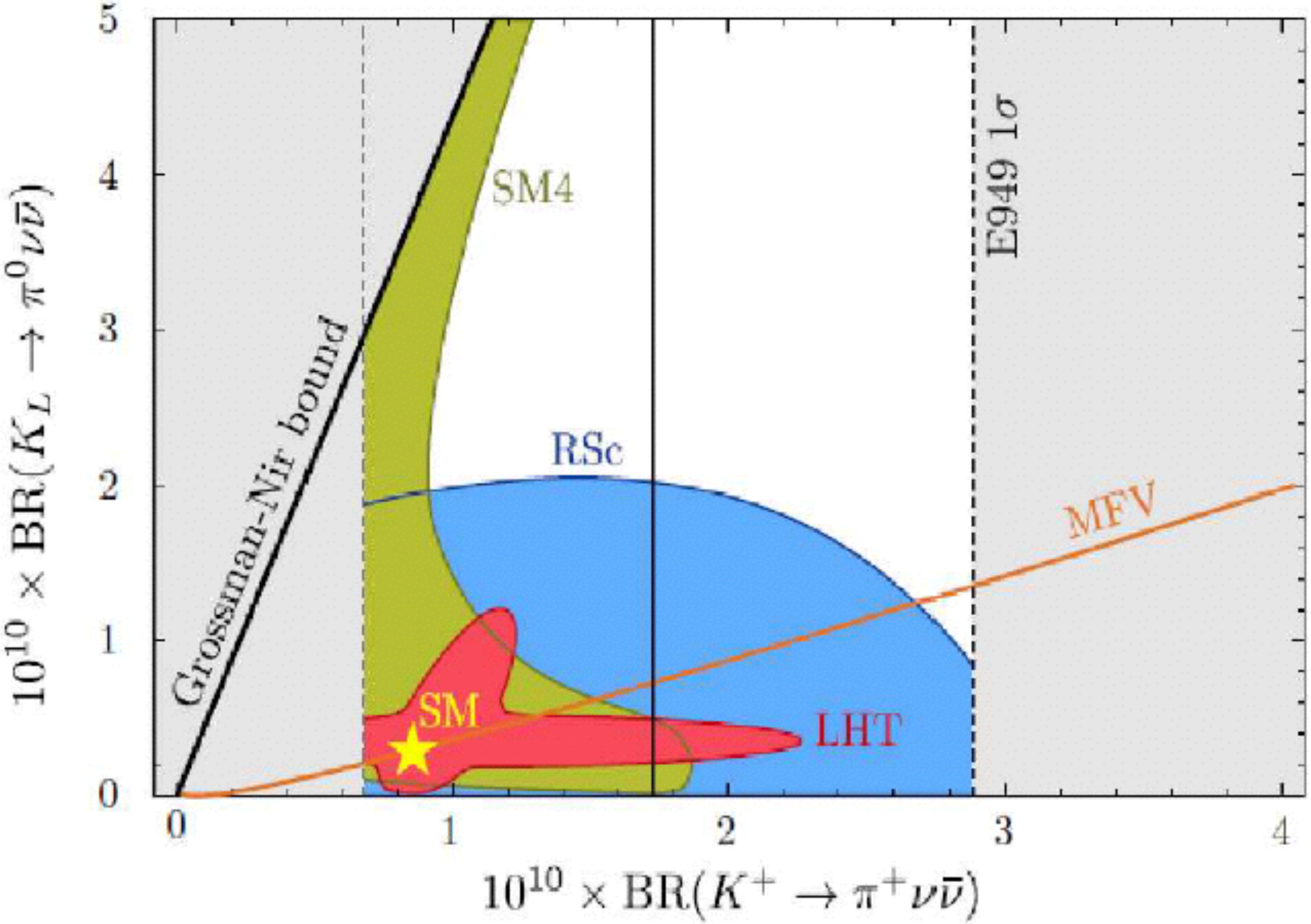


10% Overall CKM error



1% Overall CKM error

NA62 , KOTO



Also Z' Buras et al, see Fulvia's talk, and M.Blanke rev

Straub, CKM 2010 workshop (arXiv:1012.3893v2)

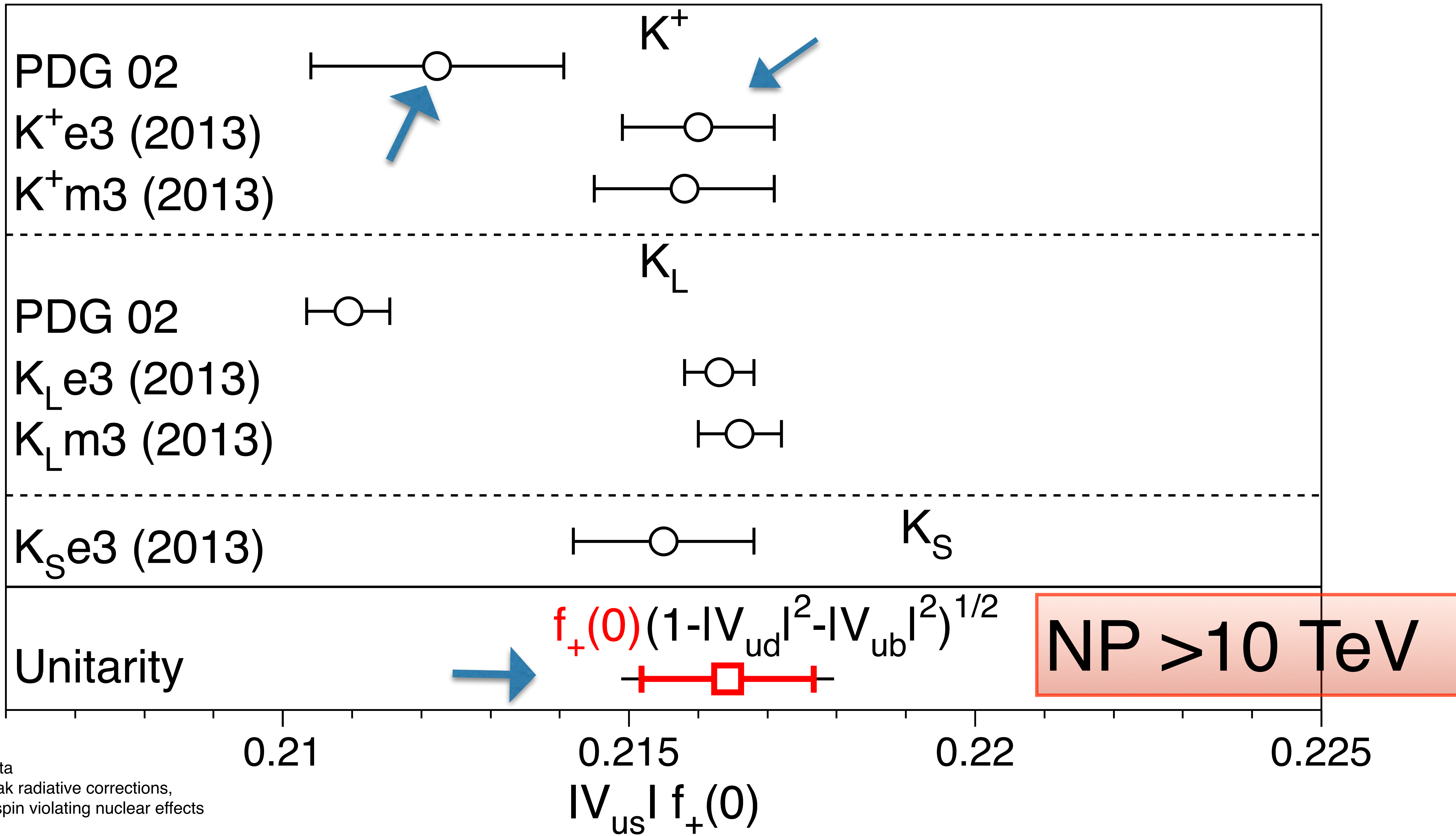
Vus from semileptonic decays

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)



superallowed +neutron beta
error: Rad corr electroweak radiative corrections,
nuclear structure, and isospin violating nuclear effects

High Statistics Measurement of the $K^+ \to \pi^0 e^+ \nu$ (K^+_{e3}) Branching Ratio

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Role of KTeV, KLOE, Istra

Chiral Perturbation theory

χPT effective field theory approach based on two assumptions

- π's Golstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ — $SU(3)_L \times SU(3)_R$ **symm.** $\mathcal{L}_{QCD}$ $\mathbf{m_q = 0}$
- (chiral) power counting There is a small expansion parameter $p^2/\Lambda^2_{\chi SB}$

$\Lambda_{\chi SB} \simeq 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{F_\pi{}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

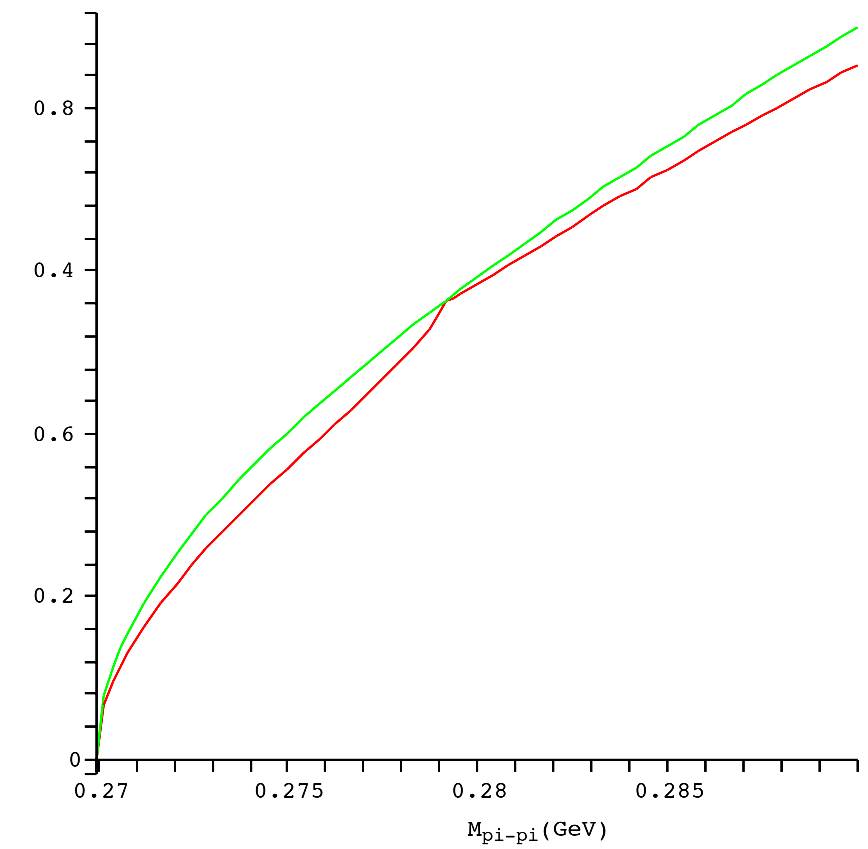
L_i Gasser Leutwyler coeff
expts. O_i p^4

Fantastic chiral prediction

$A_{\pi\pi} \sim (s - m_\pi{}^2)/F_\pi{}^2$

Cusp effect in K->3pi

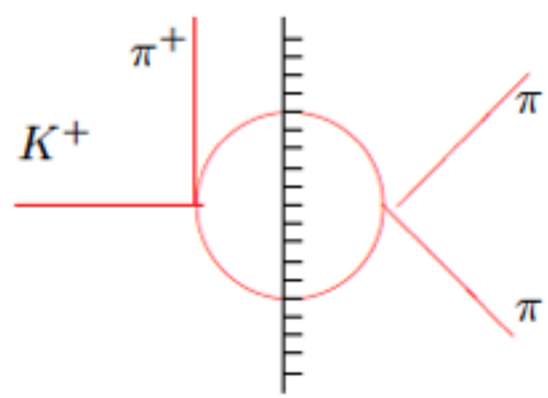
- in 2002 Mannelli at CERN discusses that their incredible energy resolution may lead to pionium discovery in K+ ->pi+ pi0 pi0
- But the plot (expt red curve) on the right was not yet understood



a_0, a_2 **from** $K \to 3\pi$ **rescattering;** Cabibbo,Cabibbo-Isidori

- rescattering generates an absorptive contribution proportional to the scattering lenghts a_0, a_2

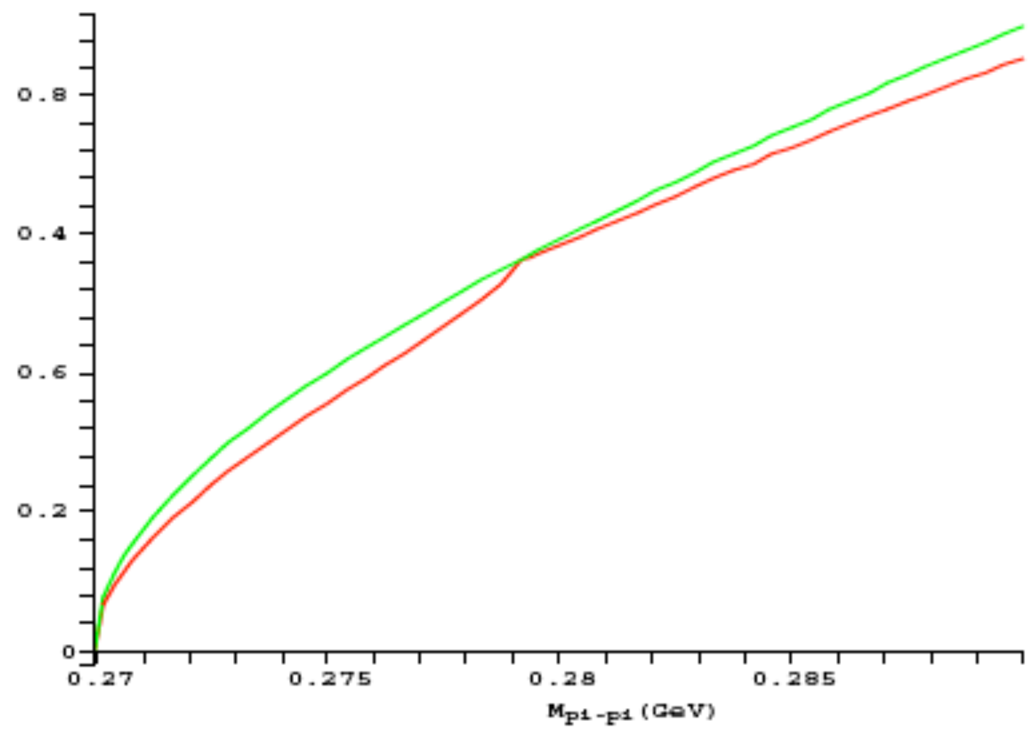
Final State Interaction



Zeldovich,Grinstein et al
Isidori,Maiani,Pugliese

The amplitude T(s) has a critical behaviour near $\pi\pi$ threshold: NA48 good energy resolution$\Longrightarrow a_0, a_2$

a_0, a_2 Cabibbo,Cabibbo-Isidori



- No cusp with cusp
- cusp: opening of the $\pi^+\pi^-$-thresho
- Rescattering $\pi^+\pi^-$ $\rightarrow$ π proportional to $a_0 - a_2 \Longrightarrow$

$$\left.\frac{d\Gamma(K^+ \to \pi^+\pi^0\pi^0)}{dM_{\pi^0\pi^0}}\right|_{\mathrm{NA48}} \Rightarrow \text{cusp for} M_{\pi^0\pi^0} = M_{\pi^+\pi^-}$$

$$\stackrel{\text{cusp}}{\Rightarrow} a_0 - a_2.$$

Ke4, K->3pi, Dirac, CHPT

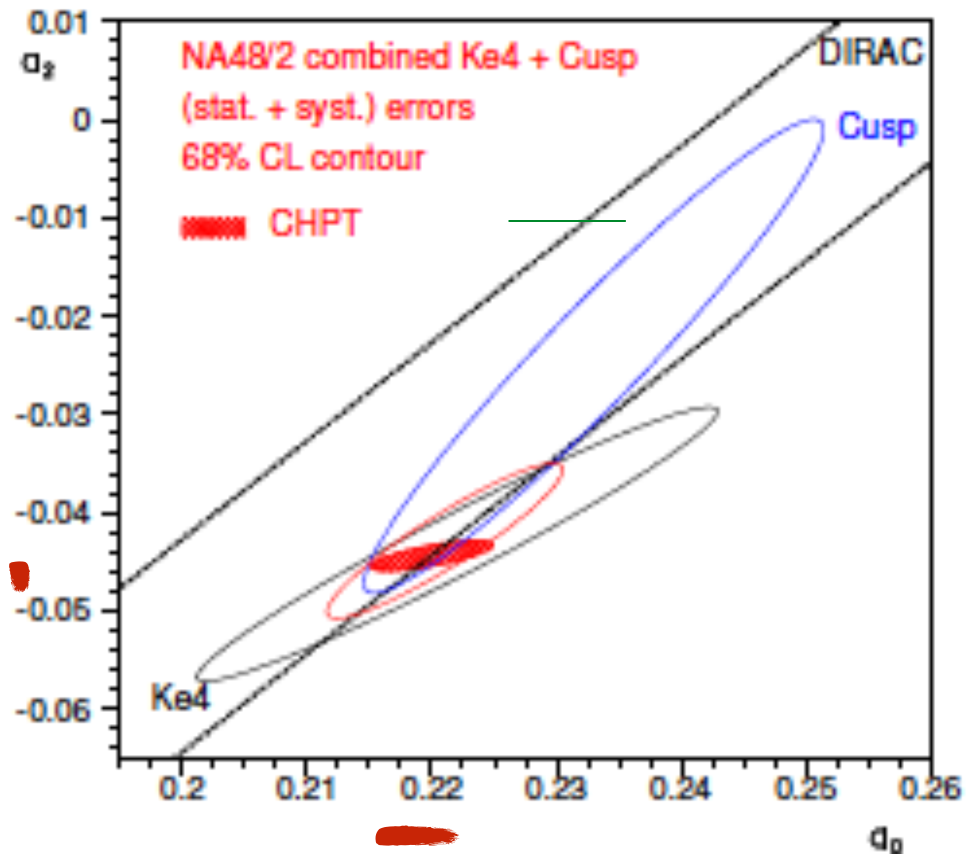
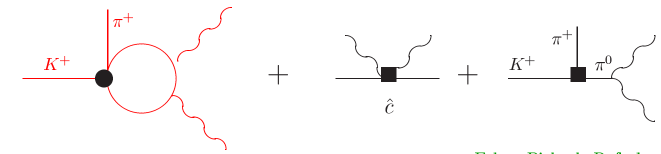


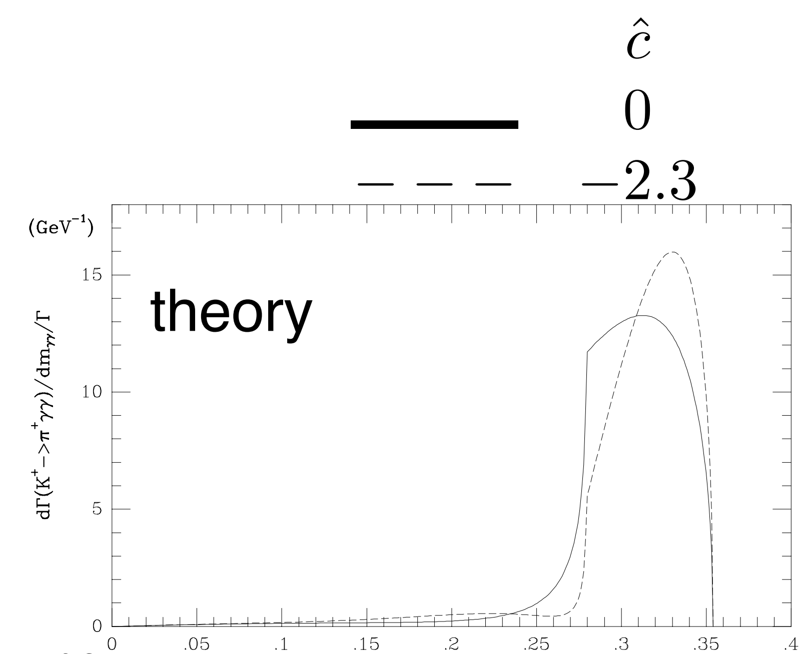
FIG. 8 NA48/2 K_{e4} and cusp results from two-parameter fits in the (a_0, a_2) plane. The smallest contour corresponds to the combination of NA48/2 results. The cross-hatched ellipse is the CHPT prediction (4.92) of Colangelo *et al.* (2001a,b). The dash-dotted lines correspond to the recent result from DIRAC (Adeva *et al.*, 2011). We thank Brigitte Bloch-Devaux for updating the original figure from Batley *et al.* (2010c).

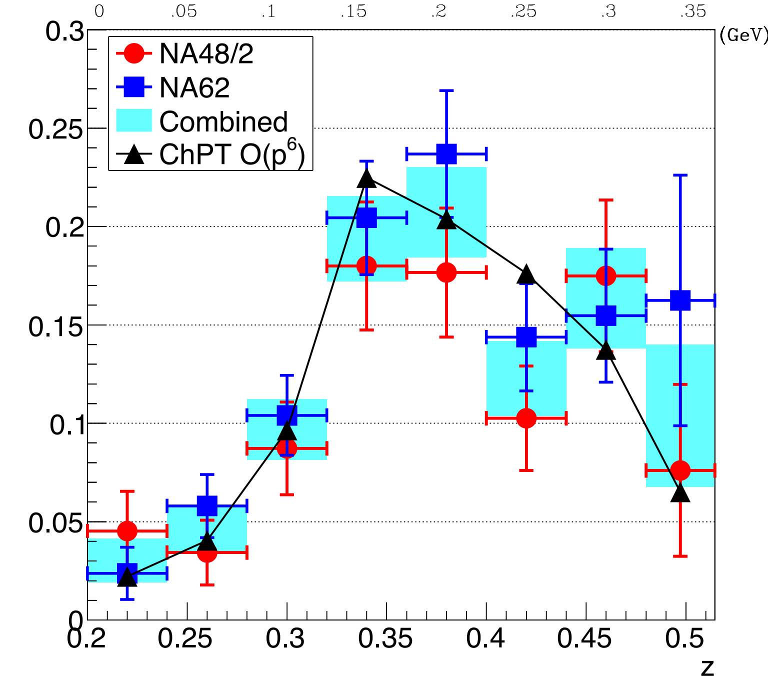
$K^+ \to \pi^+\gamma\gamma$ NA48/2 + NA62 ('14)



Auxiliary channel useful to assess the CP conserving contribution to $K_L \to \pi^0 ee$

Final 381 evts NA48/2 + NA62
during a 3-day special NA48/2 run in
2004 and a 3-month NA62 run in 2007

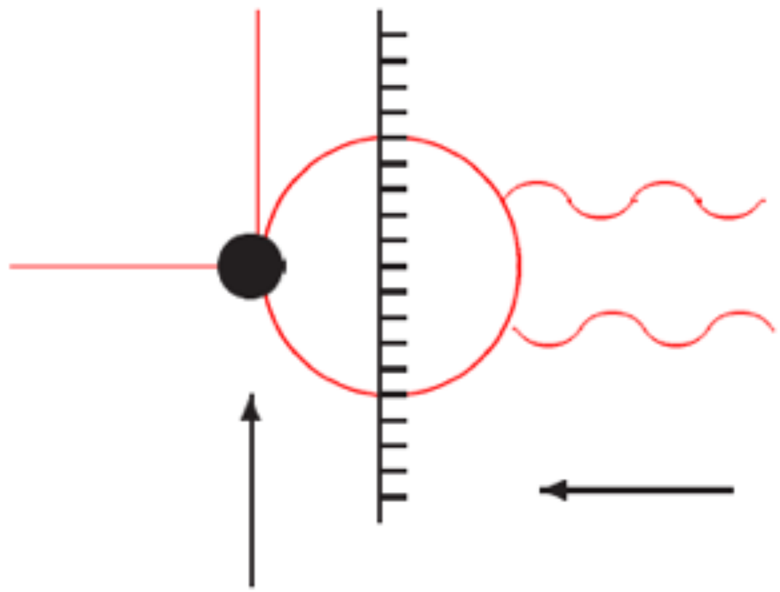




$$B = (1.003 \pm 0.051_{\rm stat} \pm 0.024_{\rm syst}) \cdot 10^{-6}$$

$$\boxed{\hat{c} = 1.86 \pm 0.26}$$

$K^+ \to \pi^+\gamma\gamma$ NA62 sensitivity



Full description of unitarity cut

$$A(K \to 3\pi) = a + b\,Y + c\,Y^2 + d\,X^2$$

This decay $K^+ \to \pi^+\gamma\gamma$: The error obtained in the form factor ($\hat{c}$) is dominated by the expt K-> 3pi error in the quadratic slope !

KTeV and NA48 not only ϵ'

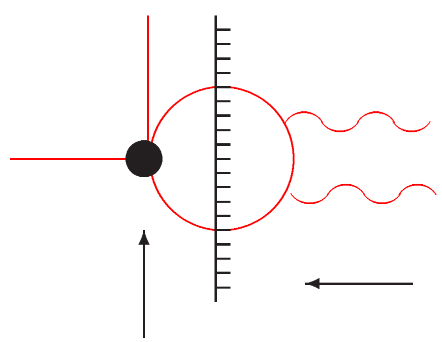
Actually the study of unit. cut was crucial to i) to bring agreement expt vs Theory in $K_L \to \pi^0\gamma\gamma$ and ii) show that $K_L \to \pi^0 ee$ CP conserving was negligible

3 CT's

$F_{\mu\nu}F^{\mu\alpha}\partial_\alpha K_L \partial^\nu \pi^0$

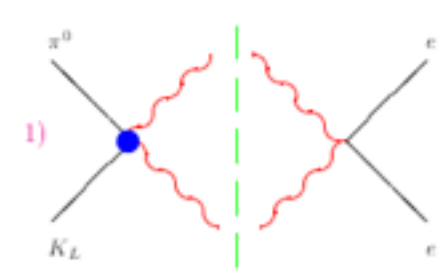
$F^2 \; \partial K_L \partial \pi^0$

$F^2 \; m_K^2 K_L \pi^0$



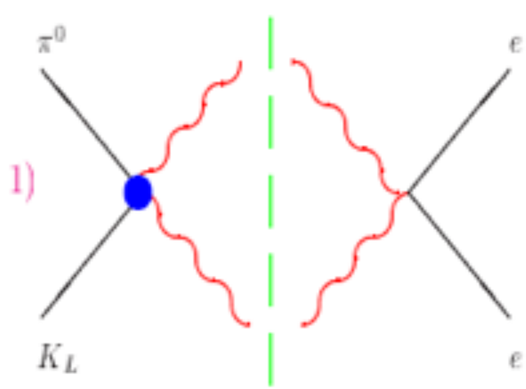
Full description of unitarity cut

$A(K \to 3\pi) = a + b\,Y + c\,Y^2 + d\,X^2$



$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ at 90% CL KTeV

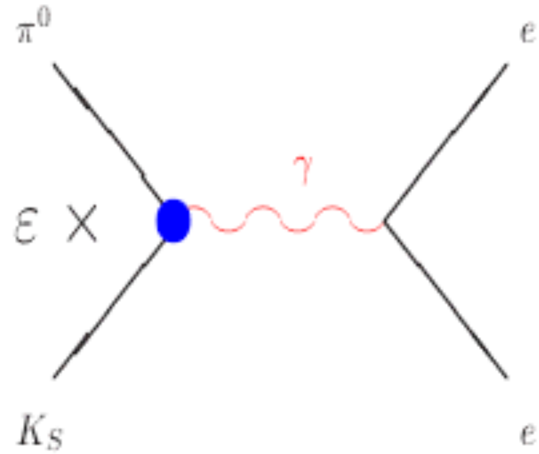


CP conserving NA48

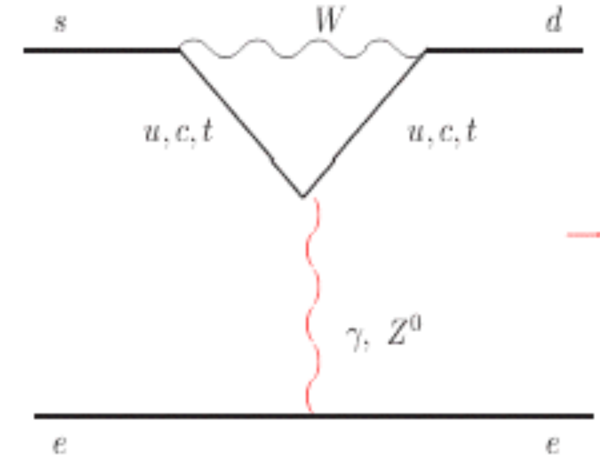
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon \times$



+ 3)



$\longrightarrow$ Im λ_t

$\lambda_t = V_{td} V_{ts}^*$

$\uparrow$ $\mathrm{B}(K_s \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\, a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\, a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \cdot 10^{-12}$$

$$[17.7 \pm \quad 9.5 + \quad 4.7] \quad \cdot 10^{-12}$$

PRIN studies: $K_L \to \pi^0 \ell^+ \ell^-$

$K_L \to \pi^0 \ell^+ \ell^-$ vs $K \to \pi \nu \bar{\nu}$:

- Measurements are complementary and can help to discriminate among NP models

  Different operators contribute to $K_L \to \pi^0 \ell^+ \ell^-$ and $K \to \pi \nu \bar{\nu}$
- Nominally easier experimental signatures for $\pi^0 \ell^+ \ell^-$, but some irreducible backgrounds (esp. for $\pi^0 e^+ e^-$)
- Larger theoretical uncertainties, need progress on ancillary measurements such as BR($K_S \to \pi^0 \ell^+ \ell^-$)

Modifications to NA62 needed for $K_L \to \pi^0 \ell^+ \ell^-$ are straightforward

- Removal of CEDAR, Gigatracker
- Realignment of straws, RICH; new IRC
- Possibly new SAC to handle higher rates

Potential for $K_L \to \pi^0 \ell^+ \ell^-$ experiment was studied by NA48

$K_L \to \pi^0 \ell^+ \ell^-$ with NA62 setup?

Extrapolated from studies for NA48

Assuming 1 sly at 2.4 × 10^{13} → 3 × 10^{12} K_L decays in FV

	$K_L \to \pi^0 e^+ e^-$	$K_L \to \pi^0 \mu^+ \mu^-$
SM BR	3.5×10^{-11}	1.4×10^{-11}
Acceptance	3%	18%
SM signal events	~3	~8
S/B	~1/10	~1/6

$K_L \to \pi^0 e^+ e^-$ channel is plagued by $K_L \to e^+ e^- \gamma\gamma$ background

- Like $K_L \to \gamma\gamma$ with internal conversion + bremsstrahlung
- 3% acceptance for $K_L \to \pi^0 e^+ e^-$ reflects tight cuts on Dalitz plot to reject
- Need to explore other strategies: statistical separation, kinematic fitting
- NA62 has better 2-3× better mass resolution on $\ell\ell$ vertex than NA48

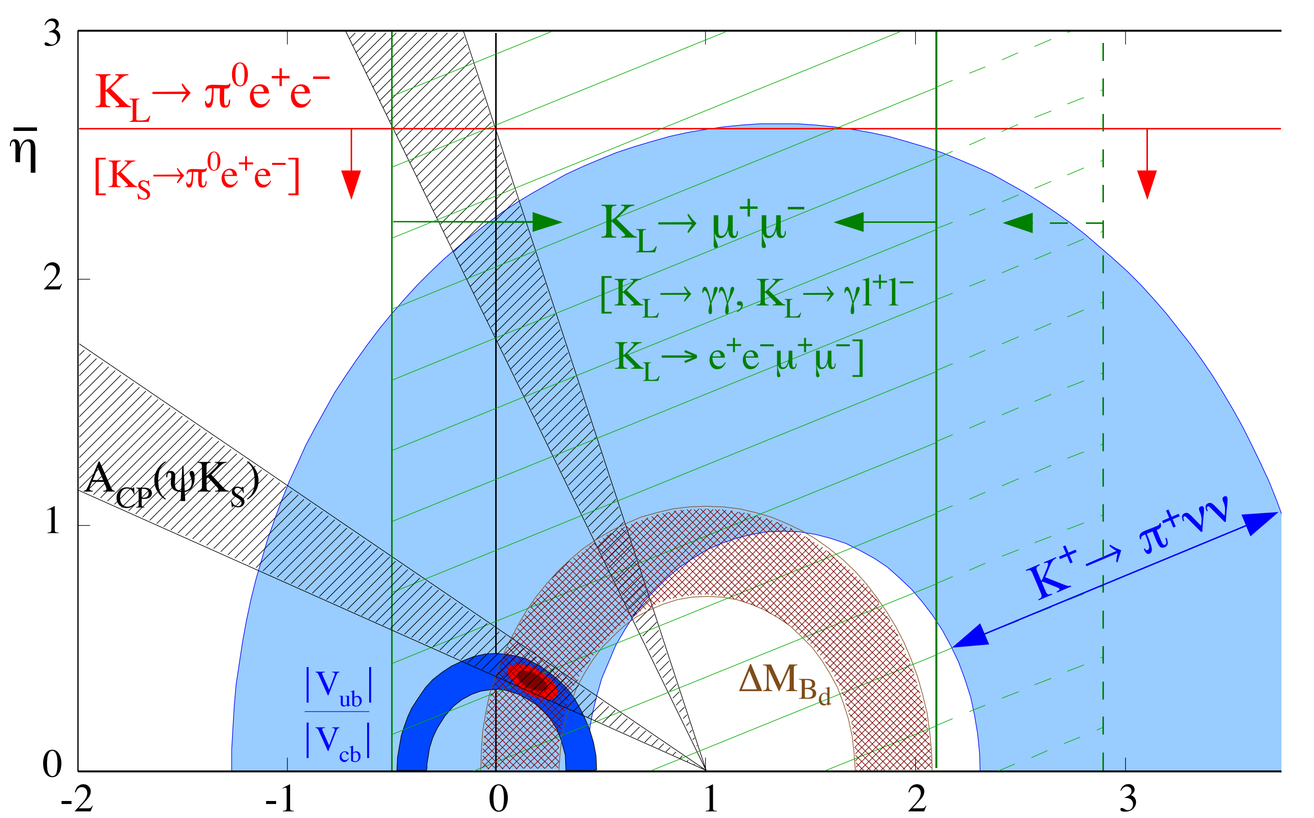
Continuing to study in context of PRIN project

Highlights $K_S \to \mu\overline{\mu}$ LHCB

After 40 years improvement by 3 orders of magnitudes from LHCB

$$B(K_S \to \mu\overline{\mu}) < 11 \times 10^{-9}$$

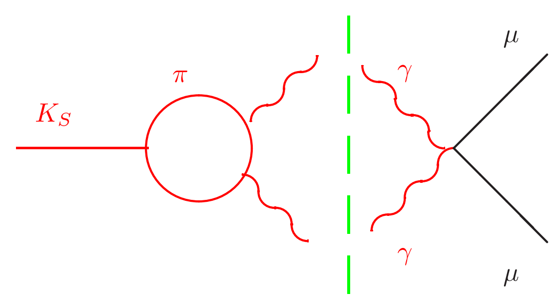
95% CL



Isidori Underdorfer

SM

$\sim 5 \times 10^{-12}$



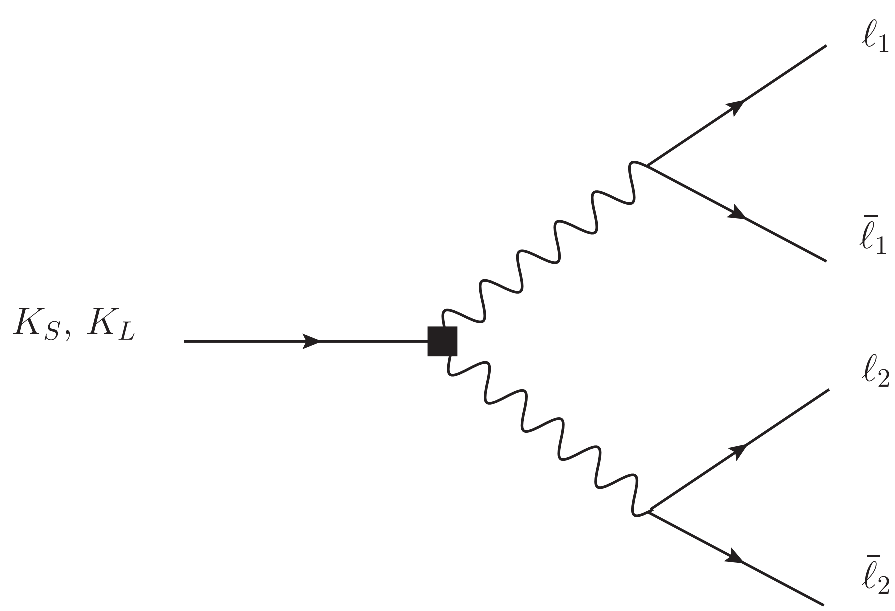
SD 1.5 10^-12

NP 1.5 10^-11
Allowed

NP Limits from CPviol in $K_L \to \mu\mu$

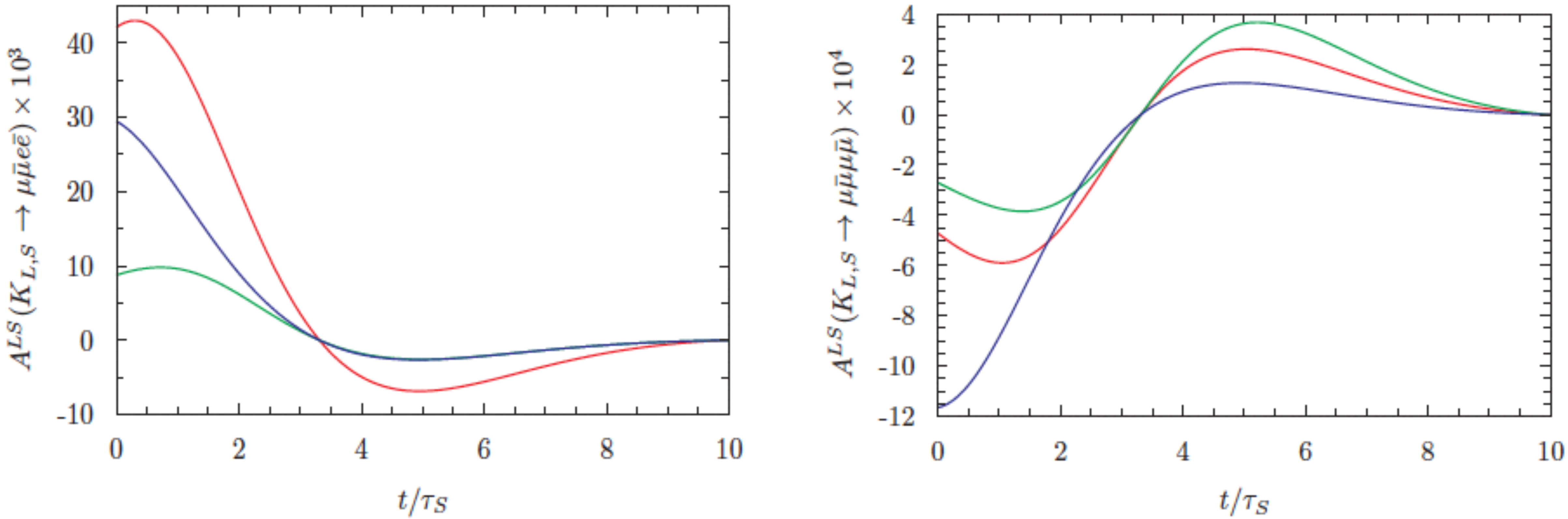
Other interesting channels

$$K_S \to \mu\mu\mu\mu \quad - \quad \text{SM LD} \; \sim 2 \times 10^{-14}$$
$$K_S \to ee\mu\mu \quad - \quad \sim 10^{-11}$$
$$K_S \to eeee \quad - \quad \sim 10^{-10}$$



GD,Greynat, Vulvert

Time interference effects



Interferences between K_L and $K_S \to \ell_1\bar{\ell}_1\ell_2\bar{\ell}_2$. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. As explained in the text we assume the sign $K_L \to \gamma\gamma$. For 4μ's 10^{14} K_S needed , $ee\mu\mu$ 10^{12}

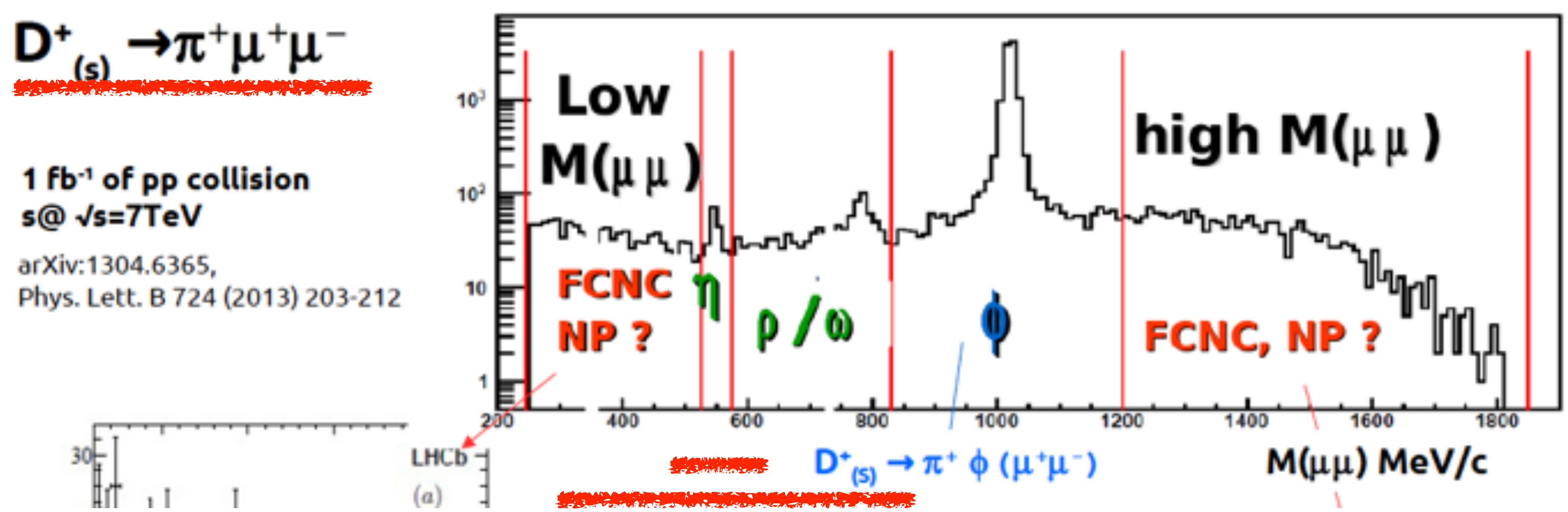
Conclusion I

Theorists had a good idea: $B_s \to \mu\mu$

Experimentalists did better

SM $B_s \to \mu\mu$

$K_S \to \mu\mu$ bound



Conclusion II

Theorists had a good idea: ϵ'

Experimentalists did better KTeV and NA48!

see Rare Kaon decays

Conclusion III

NP maybe HIDDEN but still present (see GIM)

Weak interaction

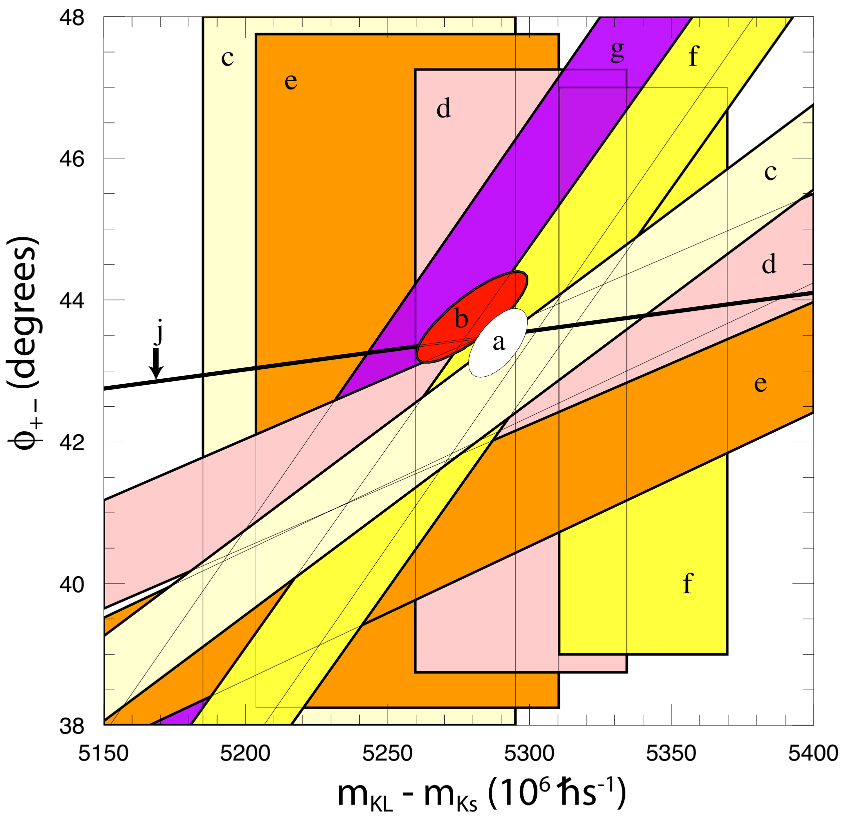
The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$

described in CHPT

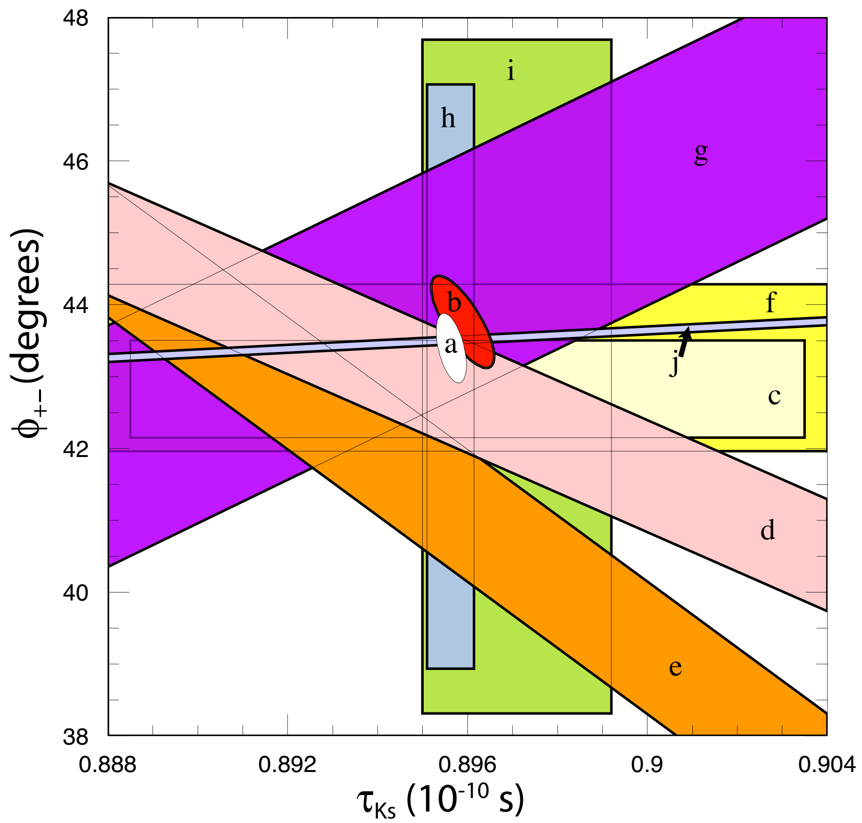
$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

VMD not as successful, in particular for K->3pi, where in principle large VMD important

CP-Violation in KL Decays Wolfenstein, Lin Trippe



do not assume CPT invariance



assume CPT invariance

$$\phi_{+-} - \phi_{00} \sim 0.006^{\circ} \pm 0.008^{\circ} \quad \tau_{K_S} = 0.8954 \pm 0.0004 \cdot 10^{-10} s$$

CPT Invariance Tests in Neutral Kaon Decay

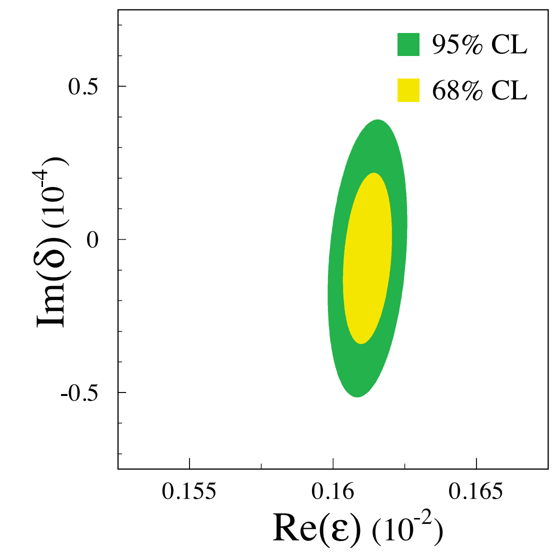
Antonelli, G.D.

$\Re(\epsilon)$ $\Im(\delta)$

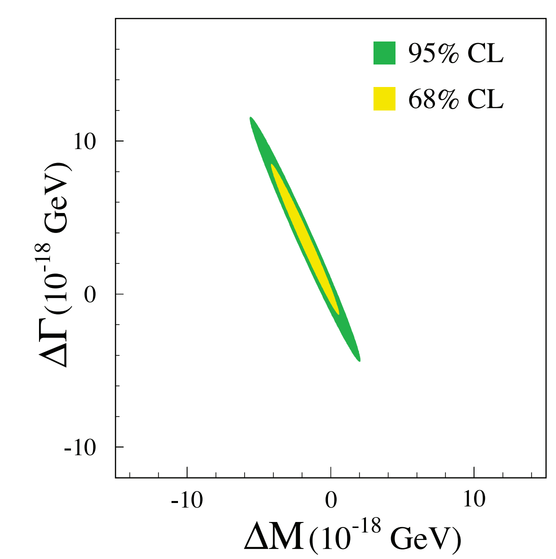
Review Bell-Steinberger relations: unitarity determines CP and CPT violating in terms of $A_L(f)A_S^*(f)$

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2}-i\Im(\delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f A_L(f)A_S^*(f)$$

CPLEAR, NA48, KLOE, PDGfit, KTEV



$$|m_{K^0}-m_{\overline{K}^0}|<4.0\times10^{-19}\ \text{GeV}\quad\text{at 95 \% C.L.}$$



Issues

- Still to improve: maybe some form factors can be removed
- Do we need a mini review for CHPT?

$$\mathcal{L}_{SM}^{Y} \quad = \quad \bar{Q} Y_D D H + \bar{Q} Y_U U H_c + \bar{L} Y_E E H$$

$$\mathcal{H}_{\Delta F=2}^{SM} \sim \frac{G_F^2 M_W^2}{16\pi^2} \left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2 \right] + \text{charm}$$

$$\mathcal{L}_{soft} = \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \bar{\tilde{\bar{U}}} a_u \tilde{Q} \ H_u$$

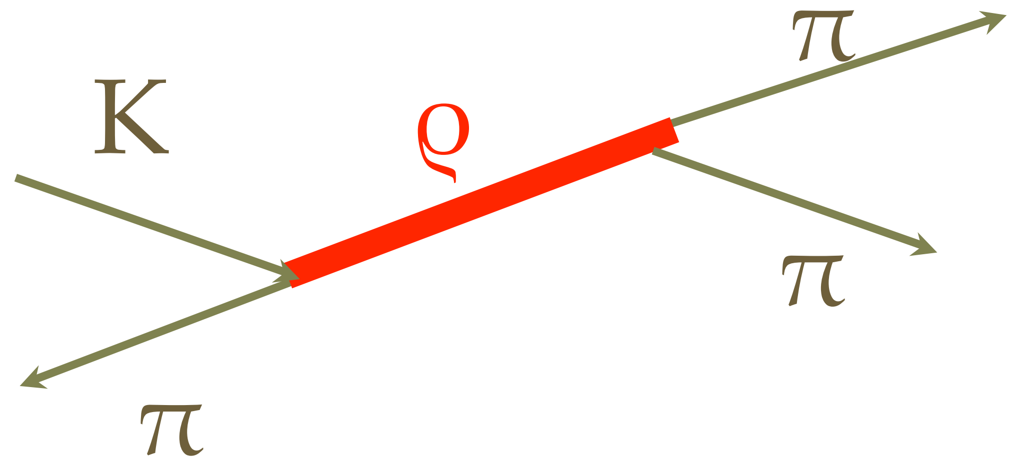
$$\mathcal{L}_{\rm soft} = \tilde{Q}^\dagger m^2_{\tilde{Q}} \tilde{Q} + \tilde{L}^\dagger m_{\tilde{L}}{}^2 \tilde{L} + \bar{\tilde{\bar{U}}} a_u \tilde{Q}\ H_u$$

$$G_F = \overbrace{\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_U \otimes \mathrm{U}(3)_D \otimes \mathrm{U}(3)_L \otimes \mathrm{U}(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

Hard Wall weak interactions: K->3pi

Luigi Cappiello, Oscar Cata and G.D.

In this channel there is a large VMD in the phenomenological slope

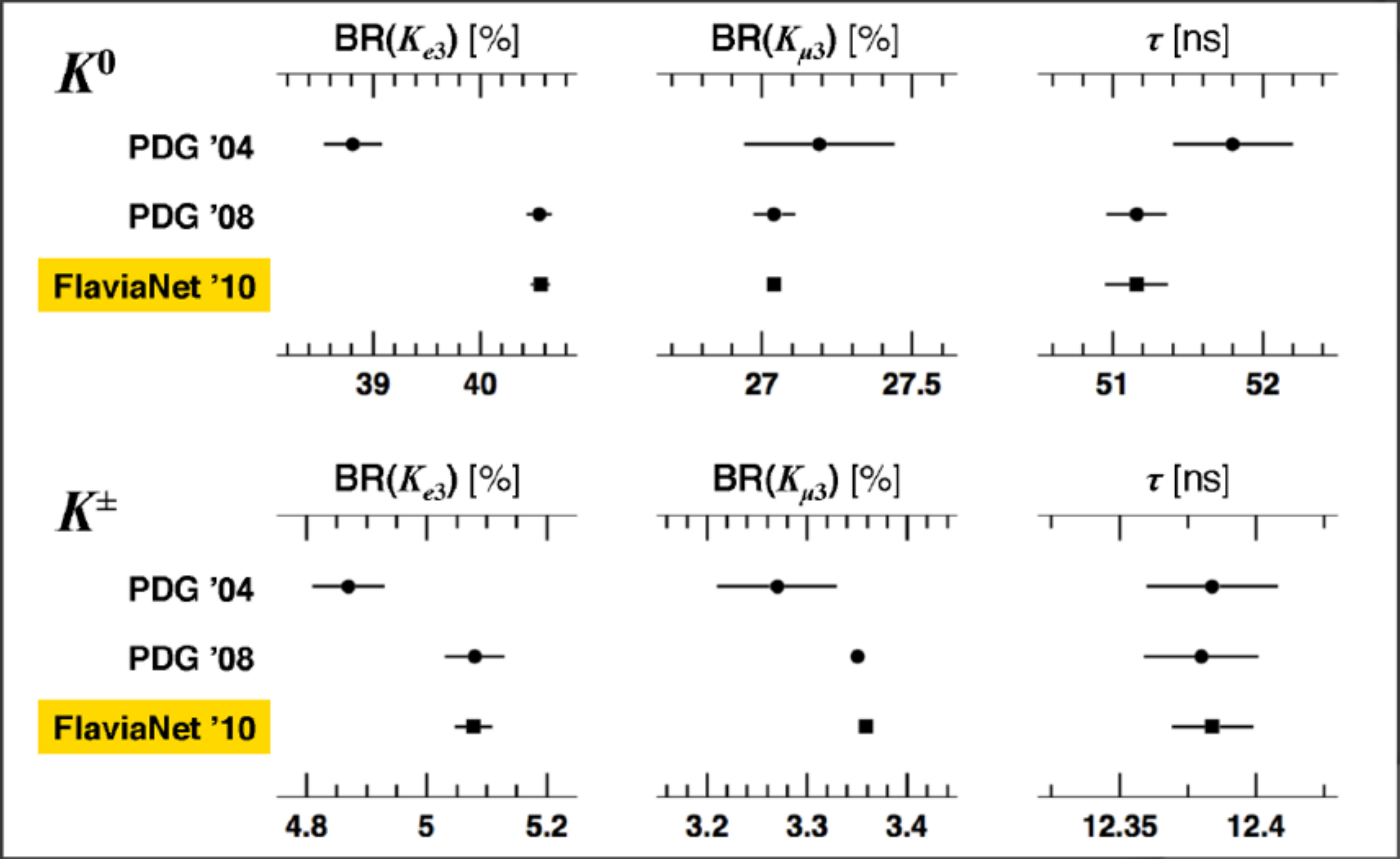


However this is proportional to $L_3 + 3/4\, L_9$

4D $L_3 + 3/4\, L_9 = 0$

5D $L_3 + 3/4\, L_9 \neq 0$ and in agreement with phenomenology

Evolution of Experimental Input…



"V_{us} Revolution" with experimental input changing ~ 5% in some cases…..

Input from many experiments: **BNL865, KTeV, ISTRA+, KLOE, NA48, NA48/2**

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L	L	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L	0.4 ± 0.3	0,6	0	0,6	0,9
L	1.4 ± 0.3	1,2	0	1,2	1,8
L	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L	-0.3 ± 0.5	0	0	0	0
L	1.4 ± 0.5	0	0	1,4	1,4
L	-0.2 ± 0.3	0	0	0	0
L	-0.4 ± 0.2	0	0	-0,3	-0,3
L	0.9 ± 0.3	0	0	0,9	0,9
L	6.9 ± 0.7	6,9	0	6,9	7,3
L	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$ determined by dominance of pion, V,A to recover QCD short distance constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)$$