Future of Lattice QCD

The landscape of Flavour Physics towards the high intensity era

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Summary

Introduction

1. Why lattice QCD?
2. Overview of a typical lattice QCD calculation & systematic errors
3. Lattice community: main goal & state of the art

Recent progresses in Heavy flavour physics

1. Verification of unitarity of second row of CKM matrix
2. Flavour puzzle from B neutral meson mixings
3. B decays: $B_s \to \mu^+\mu^-$, $B \to D^{(*)}\ell\nu$, $B \to K^{(*)}\ell^+\ell^-$
4. Radiative decays of charmonium
5. Perspectives

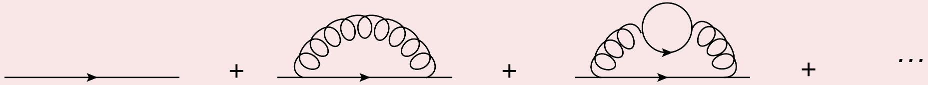
Why lattice QCD

Lattice QCD = QCD

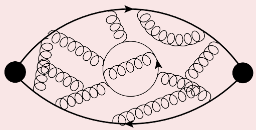
- Only method to solve non-perturbative QCD from the first theory principles
- No need of <u>any</u> parameters apart from those originally present in QCD lagrangian
- Precision, in principle, only limited by available computing power
- All sources of systematic errors can be eliminated

Perturbative vs. nonperturbative

- Perturbative: compute order by order and then summing



- Non-perturbative: take into account directly all contribution (up to cut-off scales)

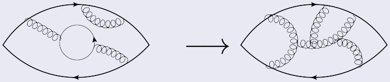


Why Lattice QCD is so computationally demanding?

Quark masses dependency

Simulation cost: rapidly grows as quark masses are lowered

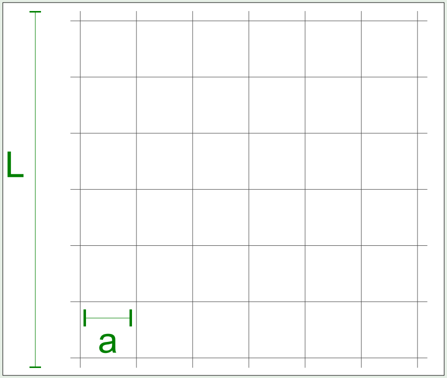
Early solution: quenching = drop virtual/dynamical $q\bar{q}$-loops from partition function



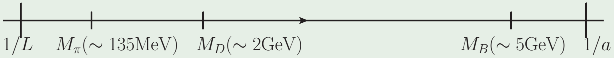
Intermediate solution: consider unphysical light quarks ($M_\pi \sim 300 \div 500$ MeV)

Nowadays: many collaborations (CP-PACS, BMW, RBC/UKQCD ...) use M_π^{phys}

Lattice size dependence



- **Simulation cost:** $[\#points]^{k>1} = \left[(L/a)^4\right]^k$
  (scales: $a \ll 1/M_H$, $L \gg 1/M_\pi$)



- **Early solution:** $\#points = 4^4$
- **Nowadays:** $\#points = 48^3 \times 96 \div 64^3 \times 128$
  - D physics: $M_D/M_\pi \sim 15$, $M_{J/\psi}/M_\pi \sim 22$
  - B physics: $M_B/M_\pi \sim 40$, $M_\Upsilon/M_\pi \sim 70$

Lattice QCD forty years after its introduction

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has

Wilson prophecy

"*Fifty years will be necessary for computational resources and algorithms to reach proper maturity*"

Forty years passed since Lattice methods invention...

Where do we stand?

Main computational problem & State of the art

State of the art

1. Physical light quarks and large volumes ($\gtrsim (6\,\mathrm{fm})^3$)
2. Simulations performed at several lattice spacings
3. Isospin and electromagnetic corrections start to be accounted for

What helped these improvements?

Increase in computing power

Conceptual developments

- Improved regularizations of LQCD
- Better understanding of behavior of Monte Carlo w.r.t (m_0, g_0)

Algorithm breakthroughs

- Multiple timescale Molecular Dynamic integrators
- Deflation, Multigrid, Domain Decomposition solvers, etc.

Growth of community (Lattice 2014 attended by more than 400 physicists)

Approaching B mesons

Challenge: $1/m_b$ is close to the cut-off given by $1/a$

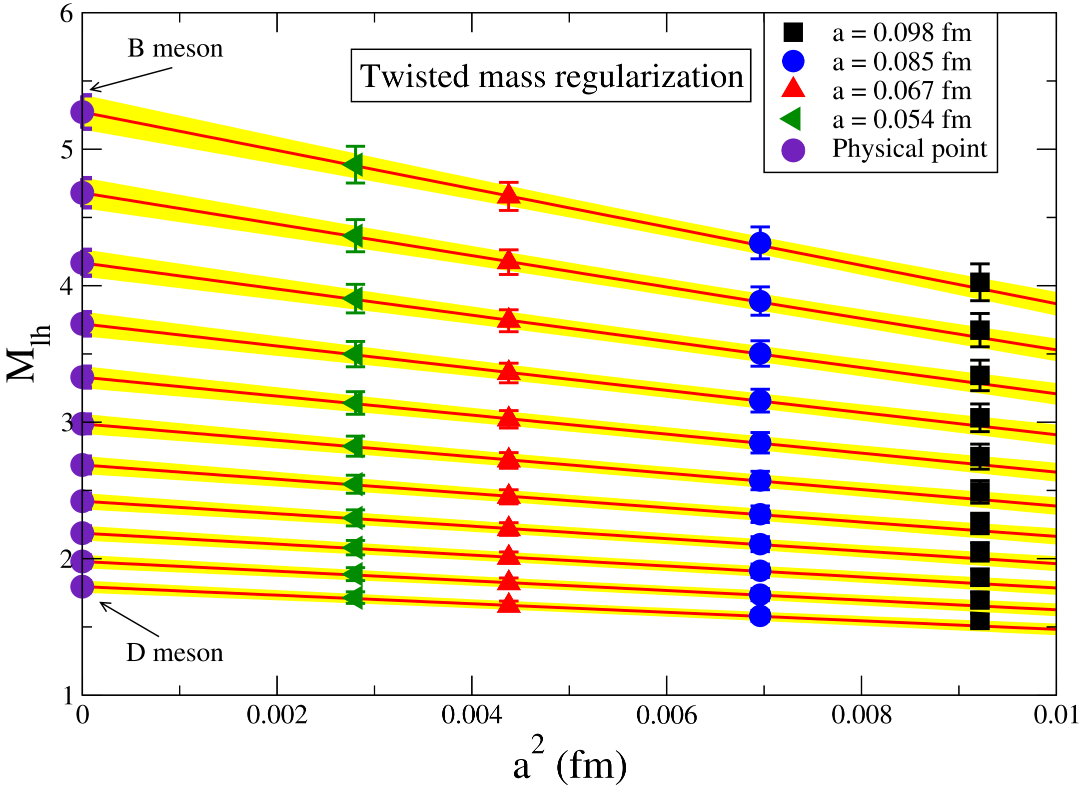
Effective theories

- Heavy Quark Effective Theory (continuum expansion in Λ_{QCD}/m_b)
- Nonrelativistic QCD (expansion in quark velocity and in $1/am_b$)
- Propagating Heavy Quarks (reinterpretation in terms of $1/m_b$ expansion)

Extrapolate results from the charm to the bottom region

- Scaling laws often known in effective theories
- Use numerical (or sometimes exact) results in the static limit
- Results become more reliable as lattice spacings gets smaller
- Use of step scaling function to separate various scales (a, m_b, L)
- Special actions have been proved able to **deal** with b quarks (HISQ, Twisted Mass...)

Can we do heavy physics on current lattices?



CKM unitarity

LQCD helped to check Unitarity of the first row of CKM matrix

$$\underbrace{|V_{ud}|}_{\text{well known}}{}^2 + |V_{us}|^2 + \underbrace{\cancel{|V_{ub}|}}_{\text{negligible}}{}^2 = 1$$

- V_{us}: obtained from $K_{\ell 2}$ and $K_{\ell 3}$
- Needed f_K and $f_+^{K\to\pi}(0)$ from lattice

Similar check of the second row (work in progress)

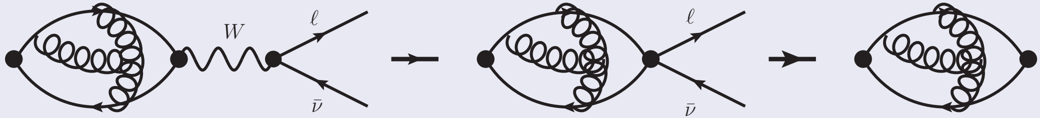
- Need to extract all three CKM entries:

$$\underbrace{|V_{cd}|^2}_{\substack{D\to e\nu \\ \text{or} \\ D\to\pi e\nu}} + \underbrace{|V_{cs}|^2}_{\substack{D_s\to\ell\nu \\ \text{or} \\ D_s\to K\ell\nu}} + \underbrace{|V_{cb}|^2}_{\substack{B_{(s)}\to D_{(s)}\ell\nu \\ \text{or} \\ B^*_{(s)}\to D^*_{(s)}\ell\nu}} = 1$$

- Hadronic quantities entering theoretical expressions:
  - leptonic decay constants f_D, f_{D_s} for $D\to e\nu$, $D_s\to\ell\nu$
  - form factors $f_+^{D\to\pi}(q^2)$, $f_+^{B\to D}(q^2)$ ... for $D\to\pi e\nu$, $B\to D\ell\nu$...

Leptonic decays of mesons

Full process — **Eff. weak hamiltonian** — **QCD side**



Two point correlation functions

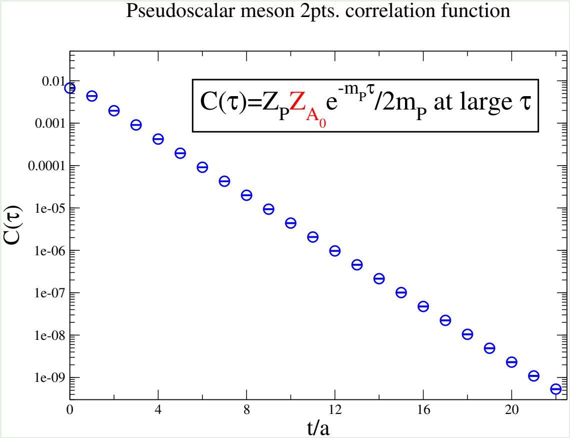
$$\mathcal{B}_{P\to\ell\bar\nu} = \underbrace{|V_{xy}|}_{\text{CKM}}{}^2\,\underbrace{\mathcal{K}(m_\ell,\, m_M)}_{\text{kinematics}}\,|\underbrace{f_P}_{\text{QCD}}|^2$$

$$f_P = \frac{Z_A}{m_P} = \frac{\langle 0|A_0|P\rangle}{m_P}$$

Z: coupling of current inducing decay

From lattice, 2 point correlation functions:

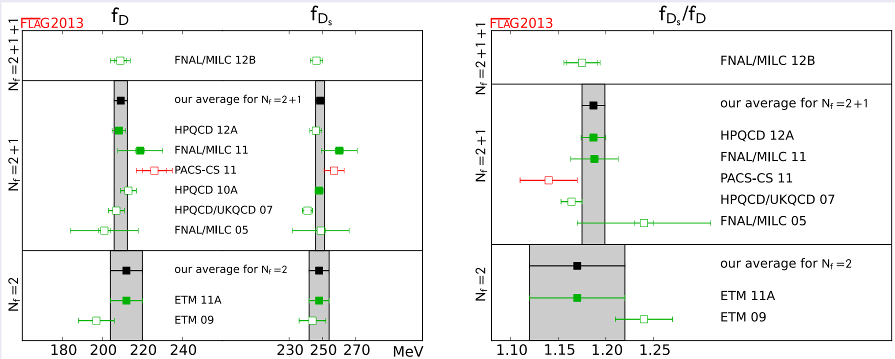
$$C(\tau) = \langle O^\dagger_{A_0}(\tau)\, O_P(0)\rangle,\ \ O = \bar\psi\Gamma\psi$$



D meson decay constants

Pseudsocalar decay constants, f_D f_{D_s}

Use to compute V_{cd}, V_{cs} and check unitarity of 2nd row of CKM matrix



$$|V_{cd}| = 0.2218(35)(95)\,, \quad |V_{cs}| = 1.018(11)(21)\,, \qquad (\text{leptonic decays}, N_f = 2+1)$$
$$|V_{cd}| = 0.2189(83)(94)\,, \quad |V_{cs}| = 1.021(25)(21)\,, \qquad (\text{leptonic decays}, N_f = 2)$$

Flavour Lattice Averaging Group (FLAG) second review published in Oct. '13

- Eur.Phys.J. C74 ('14), 255 pages, 29 Authors from all main lattice collaborations
- Emerging consensus as reference for averages of lattice results
- Good starting point to answer the question: "Which lattice QCD value to use?"

B meson decay constants

Physical motivation for a big question to lattice community: "Can you provide $f_{B_{(s)}}$ at % accuracy?"

- $|V_{ub}|$ from $B \to \tau\nu$, compare with $|V_{ub}|$ from $B \to \pi\ell\nu$ and $B \to X_u \ell\nu$
- f_{B_s} essential for $\mathrm{Br}\,(B_s \to \ell^+\ell^-)$

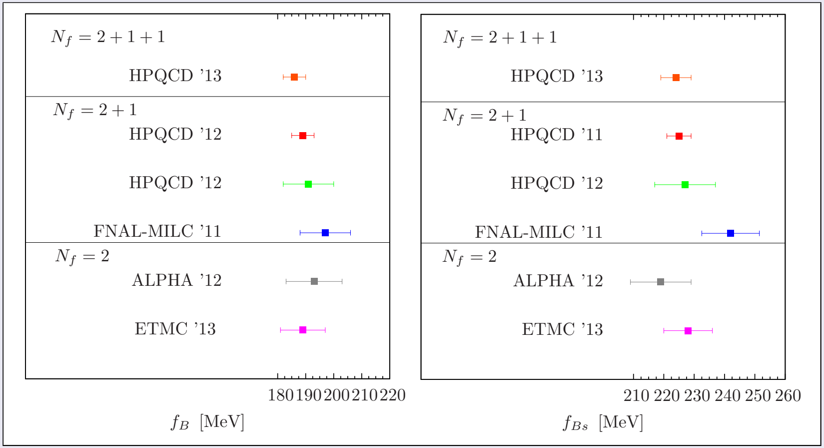
Employed strategies and current situation

FNAL-MILC Fermilab method

HPQCD Non Relativistic QCD, or HISQ

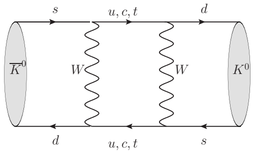
ETMC Ratios with known static limit

ALPHA HQET + Step Scaling



[Comparison taken from N.Carrasco ed al, JHEP 1403 (2014)]

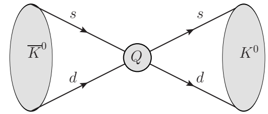
Neutral meson mixing



Integrating out the heavy degrees of freedom

OPE

$$Q_1 = [\bar{s}\gamma^{\mu}(1-\gamma_5)d]\,[\bar{s}\gamma_{\mu}(1-\gamma_5)d]$$



Beyond the Standard Model

$$\underbrace{\langle \overline{B}^0 | H_{eff}^{\Delta F=2} | B^0 \rangle}_{\text{experiments}} = \frac{G_F^2 M_W^2}{16\pi^2} \sum_{i=1}^{5} \underbrace{C_i(\mu)}_{\text{short distance}} \underbrace{\langle \overline{B}^0 | Q_i(\mu) | B^0 \rangle \left(\equiv \boldsymbol{B_i} \langle \overline{B}^0 | Q_i(\mu) | B^0 \rangle_{VIA} \right)}_{\text{long distance}}$$

From the experimental parametrisation of meson oscillation

- and the knowledge of hadronic matrix element computed on the lattice
- we check Standard Model (where $C_{2,..,5} = 0$)
- and gain insight in physics BSM (C_i depend on model details)

See: *UTfit Collaboration and CKM fitter*

B_i from lattice QCD

- Technology pioneered for $\bar{K}^0 - K^0$ system
- Compute from three point functions: $C_{3;i}(\tau) = \langle O^{\dagger}_{\bar{B}^0}(t_{sep})\, Q_i(\tau)\ O_{B^0}(0)\rangle$
- Mixing pattern among operators complicated on the lattice regularisation scheme

Flavour puzzle

Relation to the scale of New Physics

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2} = \begin{cases} \text{couplings \& loop effects of New Physics} \sim 1 \text{ in generic FCNC} \\ \text{scale of New Physics} \end{cases}$$

Computed first time 12 years ago - D.Becirevic et al., JHEP 0204 (2002)

Quenched, no continuum extrapolation, mixing-lattice artifacts, HQET driven...

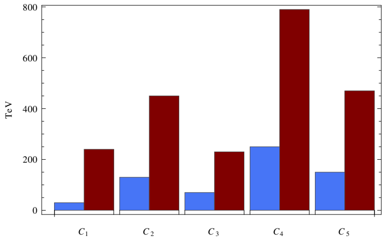
$$\boxed{B_i^{(d)}(m_b)^{\overline{\mathrm{MS}}} = \{0.87(4)(5),\ 0.82(3)(4),\ 1.02(6)(9),\ 1.16(3)(^{+5}_{-7}),\ 1.91(4)(^{+22}_{-7})\}}$$

New results from ETM - N. Carrasco et al., JHEP 1403 (2014) 016

- $N_f = 2$ & extrapolated to m_π^{phys}, 4 lattice spacings, NPR with mixing of continuum

$$\boxed{B_i^{(d)}(m_b)^{\overline{\mathrm{MS}}} = \{0.85(3)(2),\ 0.72(3)(1),\ 0.88(12)(6),\ 0.95(4)(3),\ 1.47(8)(9)\}}$$

- Ongoing joint analysis from Fermilab and MILC Collaborations
- RBC/UKQCD computed $SU(3)$ breaking ratio $\xi^{stat} = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.13(12)$, PRD82,'10



FLAVOUR PUZZLE

Assuming arbitrary flavour structure: $\Lambda_{NP} > 10^5$ TeV
For New Physics at 1 TeV one needs to:

- forbid FCNC or
- have non trivial flavour structure

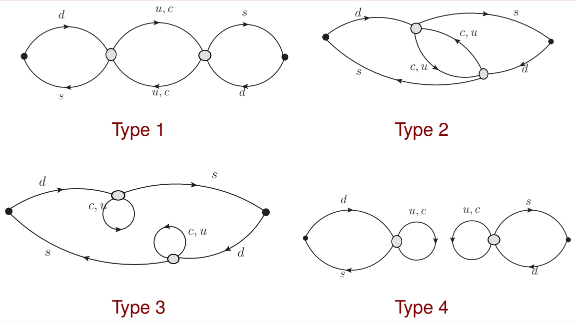
$D\bar{D}$ mixing and Long distance effects

Neutral D meson

- Short distance contribution recently computed on the lattice [N.Carrasco et al., PRD90,'14]
- In the Standard Model the process is dominated by **Long Distance** effects

Long distance effects

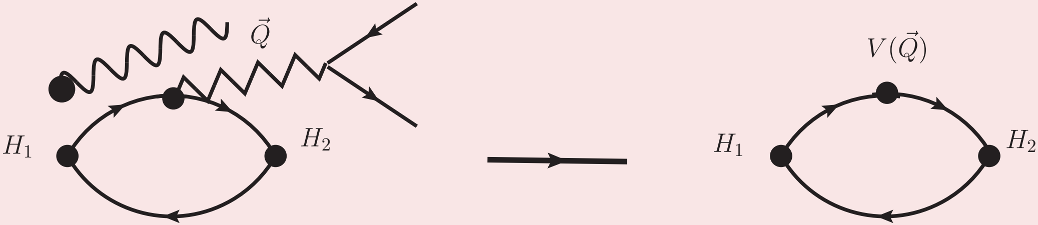
- Framework to compute long distance effects has been set up
- Pioneering study was carried out [Z.Bai et al., RBC/UKQCD coll, PRL113 '14]



- Also **rare kaon** decay amplitudes are being explored [See C.Sachrajda talk at Lattice'14]

Semileptonic & radiative meson decays

Accessible through three point functions



Prototype: $K\ell_3$ decays, now computable at the physical point

Semileptonic decays

- $D \to K\ell\nu$, $\pi\ell\nu$: V_{us}, V_{ud}
- $B \to D^{(*)}\ell\nu$: V_{cb}
- $B \to K\ell\nu$, $\pi\ell\nu$: V_{ub},
  limited to region of large q^2

Useful for model-independent studies

Radiative decays

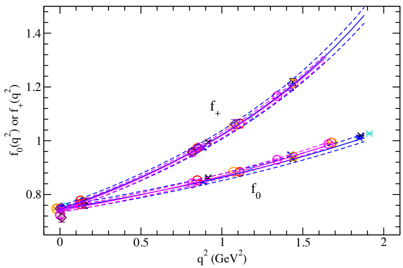
Independents from CKM matrix

- J/ψ, $h_c \to \eta_c\gamma$
- $\eta' \to J/\psi\gamma$
- $\Upsilon \to \eta_b\gamma$

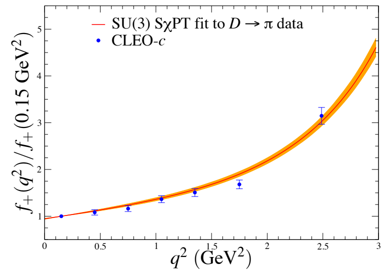
$D \to P\ell\nu$ partial width in terms of Form Factors

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = |V_{cx}|^2 \left[K_+(q^2)\left|f_+^{D\to P}(q^2)\right|^2 + K_0(q^2)\left|f_0^{D\to P}(q^2)\right|^2 + K_T(q^2)\left|f_T^{D\to P}(q^2)\right|^2\right]$$

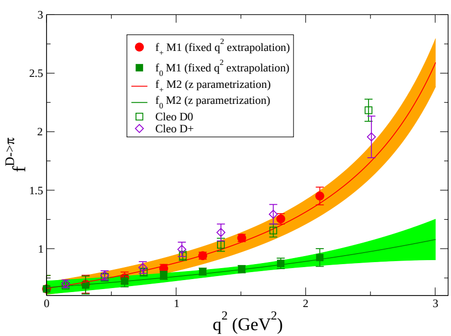
HPQCD, $D \to K\,(\pi)$, arXiv:1305.1462



Fermilab + MILC; Latt. '12



D.Becirevic, F.S, V.Lubicz; Latt.'13



- **Historically** QCD prediction at $q^2 = 0$ used to normalize experimental data fits
- **Now:** q^2-bin comparison

Remarks:

- f_+ not described by single pole VMD model [D.Becirevic et al, arXiv:1407.1019]
- f_T/f_+ deviates from constant behavior in q^2 predicted by HQET

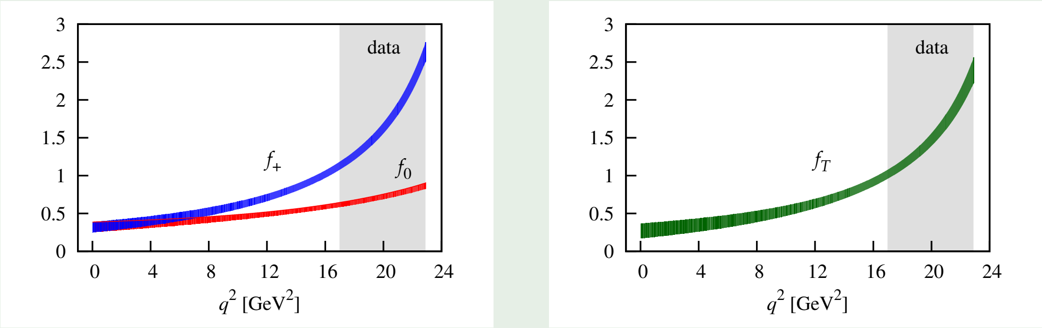
Form factors for $B \to K^{(*)}\ell^+\ell^-$ decays

$B \to K\ell^+\ell^-$ - three form factors

$$\langle K(k)|\bar{b}\gamma_\mu s|B(p)\rangle \propto f_+(q^2), f_0(q^2) \qquad \langle K(k)|\bar{b}\sigma_{\mu\nu}q^\nu s|B(p)\rangle \propto f_T(q^2)$$

HPQCD, C. Bouchard et al, PRD88(2013)054509, PRL111(2013)162002

- Staggered light quarks and the non-relativistic expansion on the lattice



- Lattice data available only in the shaded area

Remarks

- Major improvement over the quenched results [cf. PRD86(2012)034034]
- New and old values for $f_{0,T}(q^2)$ consistent, new value for $f_+(q^2)$ lower then before
- The new $f_+(m^2_{J/\psi})/f_0(m^2_{\eta_c})$ suggests a sizable violation of the factorization approximation in $B(B \to \eta_c K)/B(B \to J/\psi K)$ [cf. Nuc.Phys.B, 883]

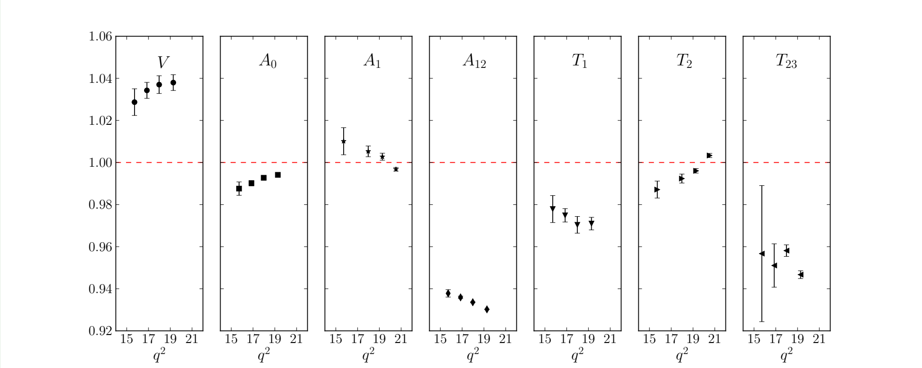
Form factors for $B \to K^{(*)}\ell^+\ell^-$ decays

$B \to K^*\ell^+\ell^-$ - seven form factors

$$\langle K^*(k,\varepsilon)|\bar{b}\gamma_\mu s|B(p)\rangle \propto V(q^2) \qquad \langle K^*(k,\varepsilon)|\bar{b}\gamma_\mu\gamma_5 s|B(p)\rangle \propto A_1(q^2), A_2(q^2), A_0(q^2)$$

$$\langle K^*(k,\varepsilon)|\bar{b}\sigma_{\mu\nu}q^\nu s|B(p)\rangle \propto T_1(q^2),\ T_2(q^2),\ T_3(q^2)$$

The case of $B_s \to \phi\ell^+\ell^-$, R.R.Horgat et al, PRD89.094501



- Also here the results restrained to large q^2's (small recoils)
- Where $A_{12} = f[A_1(q^2), A_2(q^2)]$ and $T_{23} = f[T_2(q^2), T_3(q^2)]$
- Need results with different approach to heavy quark and light quarks other than staggered

$B \to D\ell\nu$ decays

Popular test of New Physics

$$R(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu)}, \qquad R(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu)}, \ (\ell = e,\ \mu)$$

Ratios useful to cancel/reduce theoretical uncertainties in V_{cb}/f.f

BaBar ('12)

$$\boxed{\begin{array}{ll} R(D) = 0.440 \pm 0.058 \pm 0.042, & R(D)^{SM} = 0.31 \pm 0.02 \\ R(D^*) = 0.332 \pm 0.024 \pm 0.018, & R(D^*)^{SM} = 0.252 \pm 0.003 \end{array}}$$

- Larger than the SM expectations! New Physics?
- $B \to D\ell\nu$ needs form factors $f_{+,0,T}^{B\to D}$ to check SM and constraint the NP contribution

Form factors for $B_{(s)} \to D_{(s)}$

- Convenient parametrisation (HQET motivated) in terms of $\mathcal{G}(w)$

$$\frac{1}{\sqrt{m_{B_{(s)}} m_{D_{(s)}}}} \langle D_{(s)}(k) | V_\mu | B_s(p) \rangle \propto \mathcal{G}(w) + corr.$$

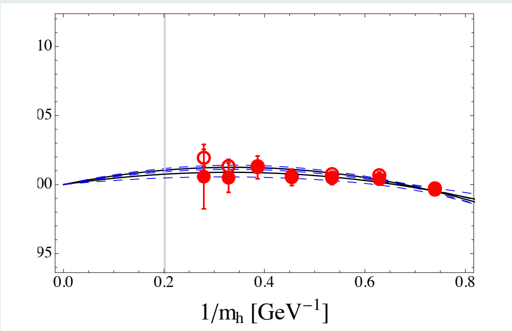
- $\mathcal{G}(1) = 1$ up to radiative and $1/m_h$ correction
- Compute the true $\mathcal{G}(1)$ on the lattice

$B \to D\ell\nu$ decays

New results, M.Atoui, V.Morénas, D.Bečirevic, FS., Eur.Phys.J. C74 (2014)

- Define: $\mathcal{G}(1,\ m_b,\ m_c) = \sigma_n\sigma_{n-1}...\sigma_1\sigma_0 \cancelto{1}{\mathcal{G}\left(1,\ m_c,\ m_c\right)}$, $\qquad \sigma_i = \dfrac{\mathcal{G}(1, \lambda m_h, m_c)}{\mathcal{G}(1, m_h, m_c)}$
- In the elastic case $D_{(s)} \to D_{(s)}$ by definition $\mathcal{G}(1) = 1$
- Towards the static h-quark: $\lim_{m_h \to \infty} \sigma(m_h) = 1$
- Extrapolate constrained σ from c to b, reconstructing $\mathcal{G}\left(1\right)$ from the chain of products

Results & comparison with previous studies



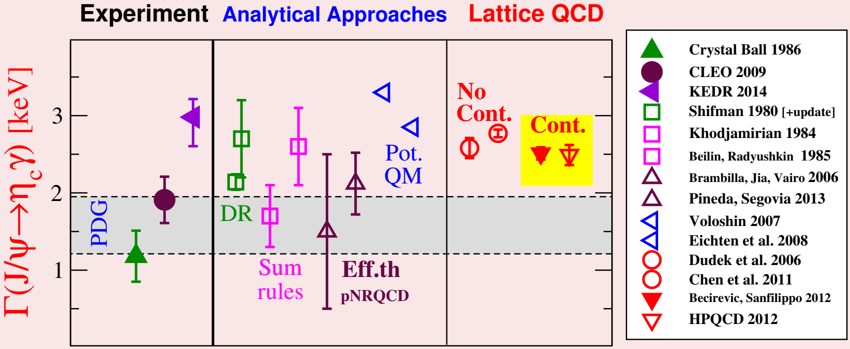
- Final Results: $\mathcal{G}^{B_s \to D_s}\left(1\right) = 1.052(46)$
  Unquenched, full QCD heavy quark
- De Divitiis et al. (Phys.Lett.B '07): 1.026(17)
  - ✓ Step scaling method
  - ✗ Quenched
- MILC+Fermilab: 1.074(24), Lattice '04

The case of $B \to D^*\ell\nu$

Very recently Fermilab + MILC reported: $\mathcal{F}\left(1\right) = 0.906(4)(12)$, PRD 89, 114504

Radiative decays of charmonia

$J/\psi \to \eta_c\gamma$ puzzle solved?



Tension in $h_c \to \eta_c\gamma$?

- D.Becirevic, F.S (2012): $\Gamma_{h_c} = \frac{\Gamma^{LAT}(h_c\to\eta_c\gamma)}{\mathrm{Br}^{\mathrm{Bes\ III}}(h_c\to\eta_c\gamma)} = 1.37(23)\,\mathrm{MeV}$, JHEP 1301 (2013)
- BES III: $\Gamma^{incl}_{h_c} = 0.73(45)(28)\,\mathrm{MeV}$, $\Gamma^{excl}_{h_c} = 0.70(28)(22)\,\mathrm{MeV}$, X Confinement 2012

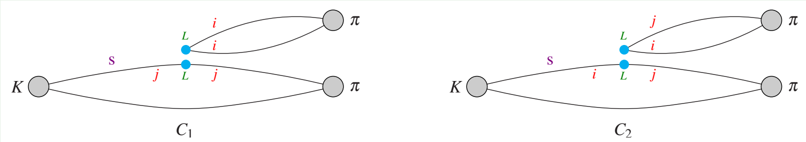
$\eta_c(2S) \to J/\psi\gamma$ (unobserved process)

- Recent lattice QCD prediction, D.Becirevic, M.Kruse and FS, arXiv:1411.6426
- $\mathrm{Br} = 1.4(6) \times 10^{-3}$ about **40 times larger** than naive expectation
- Suggestion for experimentalists: could it be detected **experimentally**?

Hadronic decays

"Emerging understanding of the $\Delta I = 1/2$ rule from Lattice QCD"

RBC-UKQCD Collaboration, PRL110, '13



Observed cancellation between diagrams in one isospin channel, and not in the other

Prediction for other hadronic decays

- We observed for some time an unexpected excess of CP violation in $D \to \pi\pi/KK$
- Excess **evaporated**, but in the future experiments could... trip over other unexpected features of hadronic decays (see e.g. perspectives of LHCb upgrade)
- Are we ready to make precise predictions for **other fully hadronic processes**?

Decays beyond inelastic threshold

- Method used for $K \to \pi\pi$ works only for a single final state containing two particles
- Recent progress by Hansen and Sharpe:
  - PRD86 (2012): Inclusion of channels with multiple two-particle states e.g. $\to \pi\pi \to KK$
  - arXiv:1408.5933 (52 pages!): **Three** scalar particles in the final state
- For typical process (e.g. $D \to KK$) we would need to build a more general framework (need to consider many hadrons in the final state)

Future perspectives

Current status forty years after the formulation of LQCD

Lattice Calculations in the precision era

1. Many simulations include $N_f = 2 + 1 + 1$ physical quarks
2. Continuum limit extrapolation under control for charm physics
3. Many different methods to study b-physics allow for crosschecks
4. Emerging consensus on Lattice averages

Next steps

1. Include Isospin Breaking/Electromagnetic effects
2. Simulate at lattice spacing small enough to treat directly the physical b quark
3. $g - 2$, rare kaon decay amplitudes

Long time perspectives

1. Hadronic decays above inelastic thresholds & full understanding of resonance spectrum
2. Calculate $K_L - K_S$ mass difference and make
   one more lattice QCD attempt to compute ϵ'/ϵ

THANKS!!

BACKUP SLIDES

Isospin breaking

Ab initio calculation of the neutron-proton mass difference

- BMW collaboration, arXiv:1406.4088, to appear on Science
- Massive QCD+QED simulations (4 lattice spacings, many volumes, dynamical QED)
- Main results: $[(M_n - M_p)_{EXP} = 1.2933322(4)\,\mathrm{MeV}]$

$$M_n - M_p = 1.51(16)(23)\,\mathrm{MeV} = 2.52(17)(24)\,\mathrm{MeV}_{QCD} - 1.00(07)(14)\,\mathrm{MeV}_{QED}$$

Correction to decay process

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	
130.2(1.4)	156.3(0.8)	209.2(3.3)	248.6(2.7)	190.5(4.2)	227.7(4.5)	[MeV]

[FLAG Collaboration, Eur.Phys.J. C74 (2014)]

- Precision quoted at $\mathcal{O}(1\%)$: need to include Isospin Breaking effects
- Need to go **beyond factorisation** approximation and consider the full decay process
- Need to include also real emission process together (to cancel IR divergence)
- At first order in α_{QED}: $\Gamma\left[P^+ \to \ell^+\nu_\ell(\gamma)\right]$ with $E_\gamma < \Delta \sim 10\,\mathrm{MeV}$
- Proposed framework:

$$\boxed{\Gamma_0 + \Gamma_1(\Delta) = \lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{pt}\right) + \lim_{V\to\infty}\left(\Gamma_0^{pt} + \Gamma_1(\Delta)\right)}$$

[See C.Sachrajda talk at Lattice'14]