Università del Salento and INFN Lecce

Research Activity of the TAsP Group – 1

Estimating orbital period of exoplanets in microlensing events

Mosè Giordano (PhD student)

Section Council (G4)

July 21, 2014

Theoretical Astroparticle Physics Group

- National Coordinator: Eligio Lisi
- Local Sections: Bari, Cagliari, Ferrara, Lecce, LNF, LNGS, Napoli, Padova, Pavia, Pisa, Roma1, Torino, Trieste

Section of Lecce

- Composition of the participants to the Section of Lecce
  - Francesco De Paolis (University Researcher, Local Coordinator)
  - Mosè Giordano (PhD Student)
  - Gabriele Ingrosso (University Associate Professor)
  - Luigi Manni (PhD Student)
  - Daniele Montanino (University Researcher)
  - Achille A. Nucita (University Researcher)
- Research Fields
  - Astrophysics
    - studies of galaxy rotation phenomena in the context of cosmic microwave background data
    - polarization in microlensing events
    - X-ray source population in selected galaxies
  - Astroparticle physics
    - analysis of neutrino oscillations and of photon-axion oscillations
    - supernova neutrino physics

Microlensing

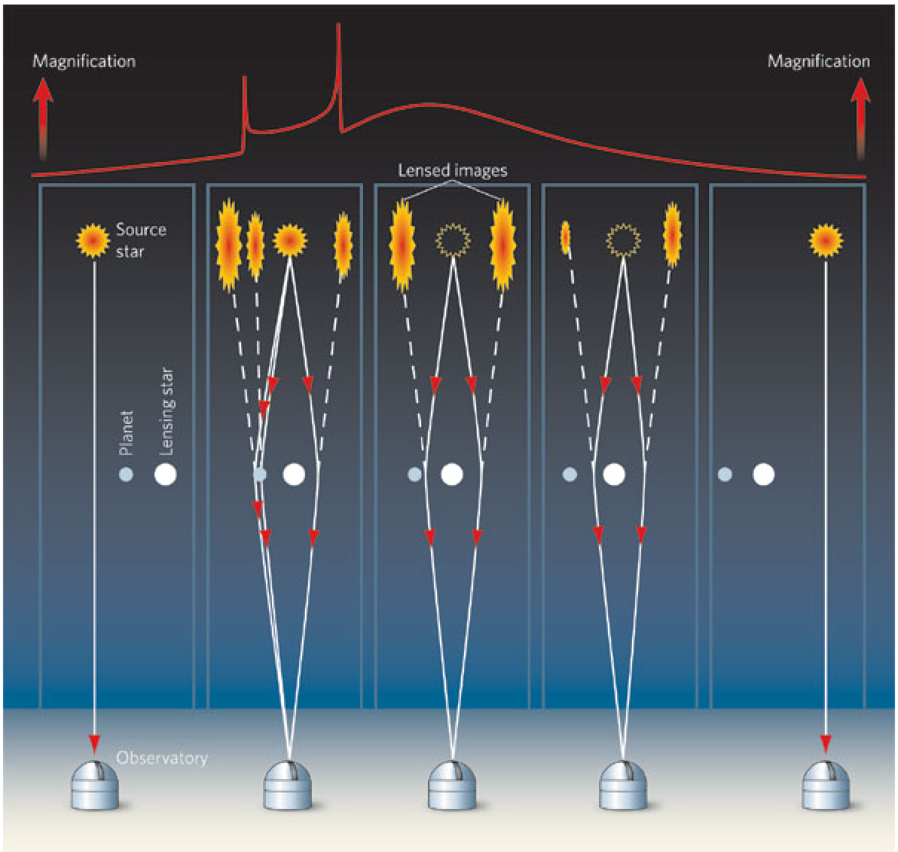


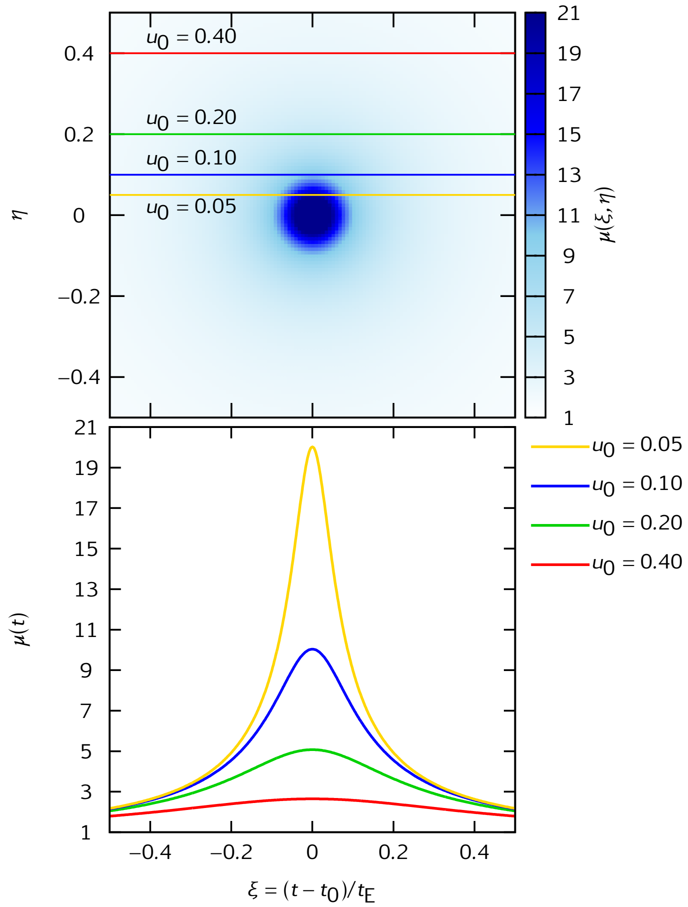
Figure: Credits: Didier Queloz, *Nature* **439**, 400–401. DOI: 10.1038/439400a

Schwarzschild Lens

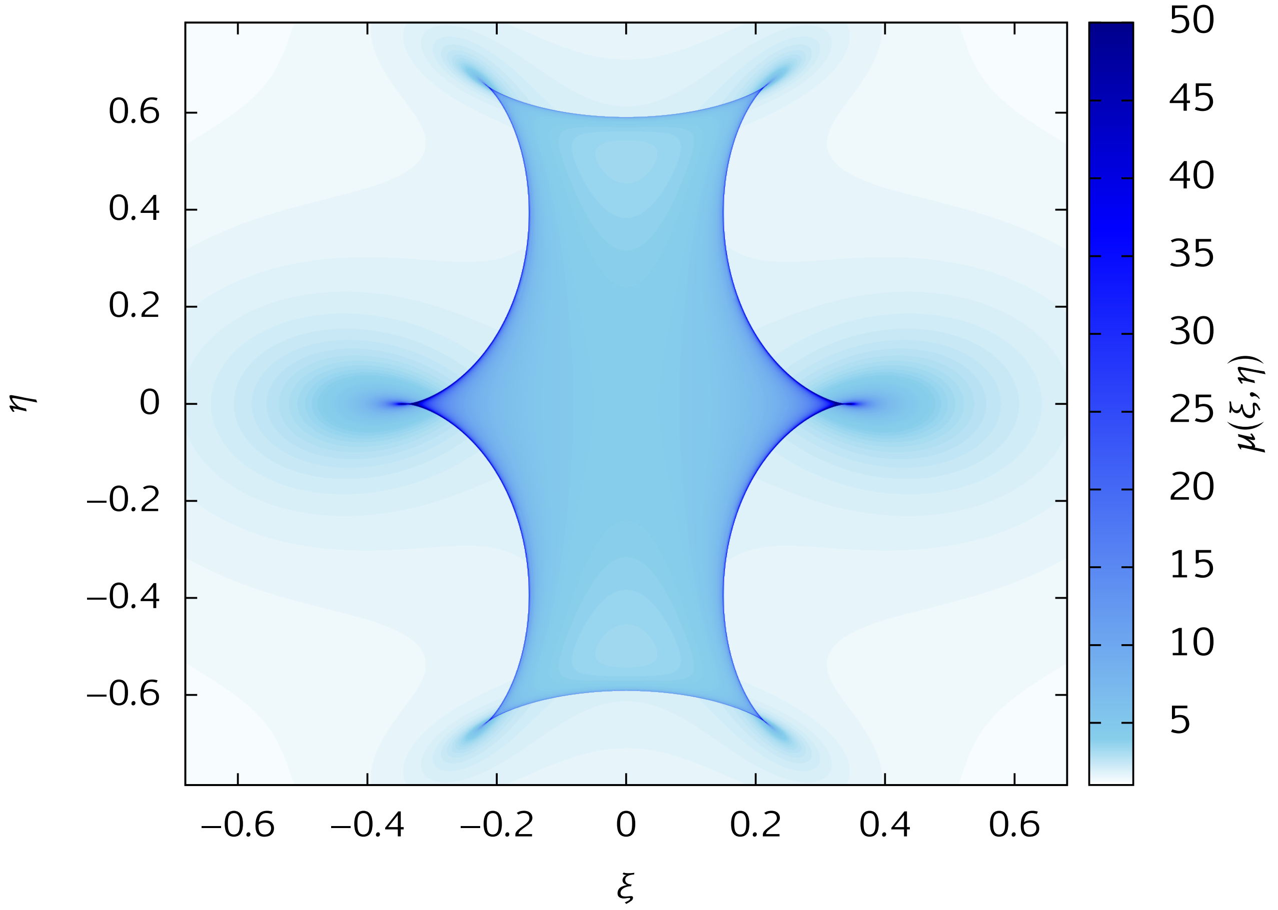
Amplification

$$\mu(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

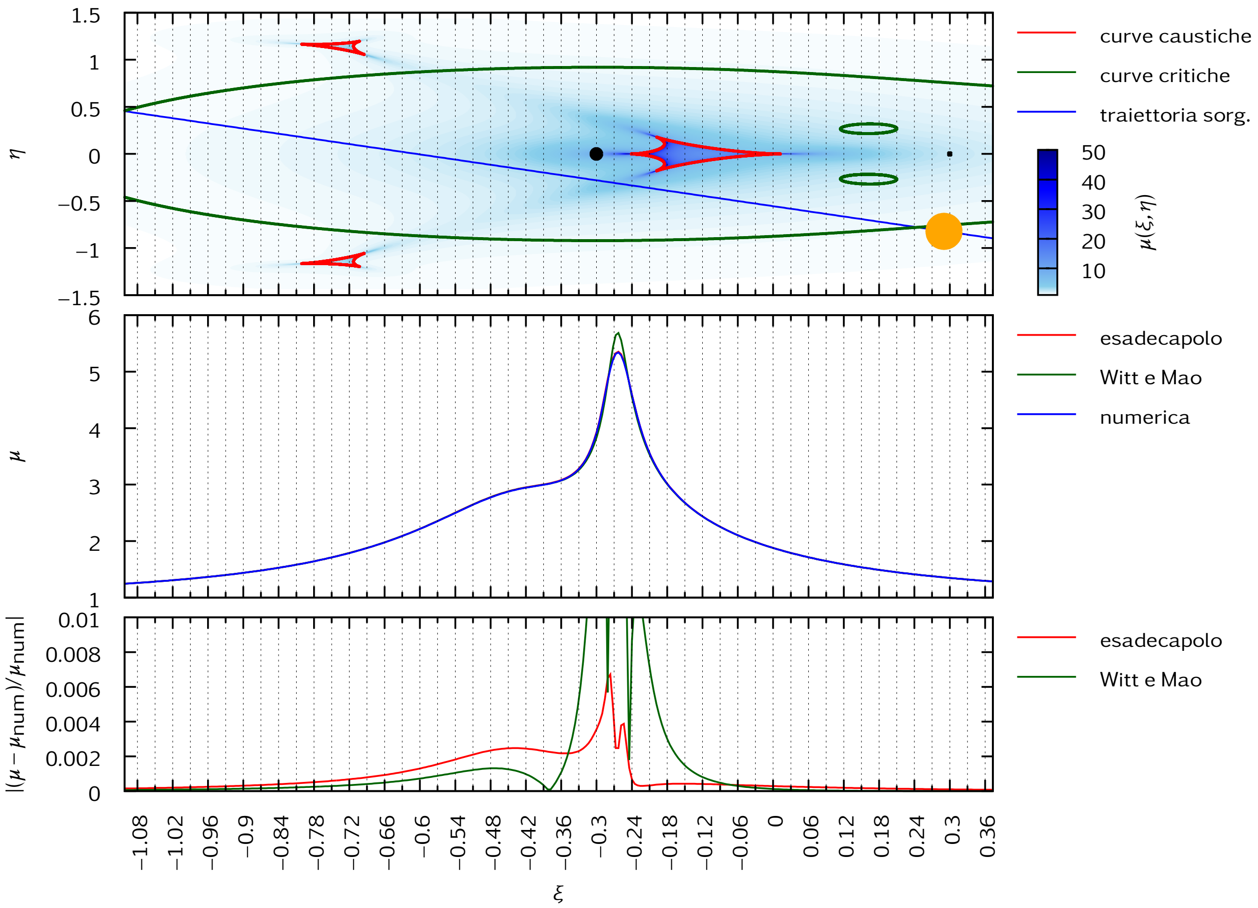
with $u = \sqrt{\xi^2 + \eta^2}$ distance between lens and source projected onto the sky plane, at a fixed time



Amplification Map



Static Binary Lens



Binary Lens with Orbital Motion

Parameters to be determined using a fit in microlensing events by binary lens with orbital motion

- base parameters: $t_0 \quad u_0 \quad t_E \quad \theta$
- finite source effects: $\rho_\star$
- binary lens: $b \quad q$
- binary lens with orbital motion: $a \quad e \quad i \quad \varphi$

In addition, with small mass ratios q there is the close-wide degeneracy $b \longleftrightarrow b^{-1}$

What if we knew the orbital period of the lenses

$$P = 2\pi\sqrt{\frac{a^3}{GM}}$$

independently?

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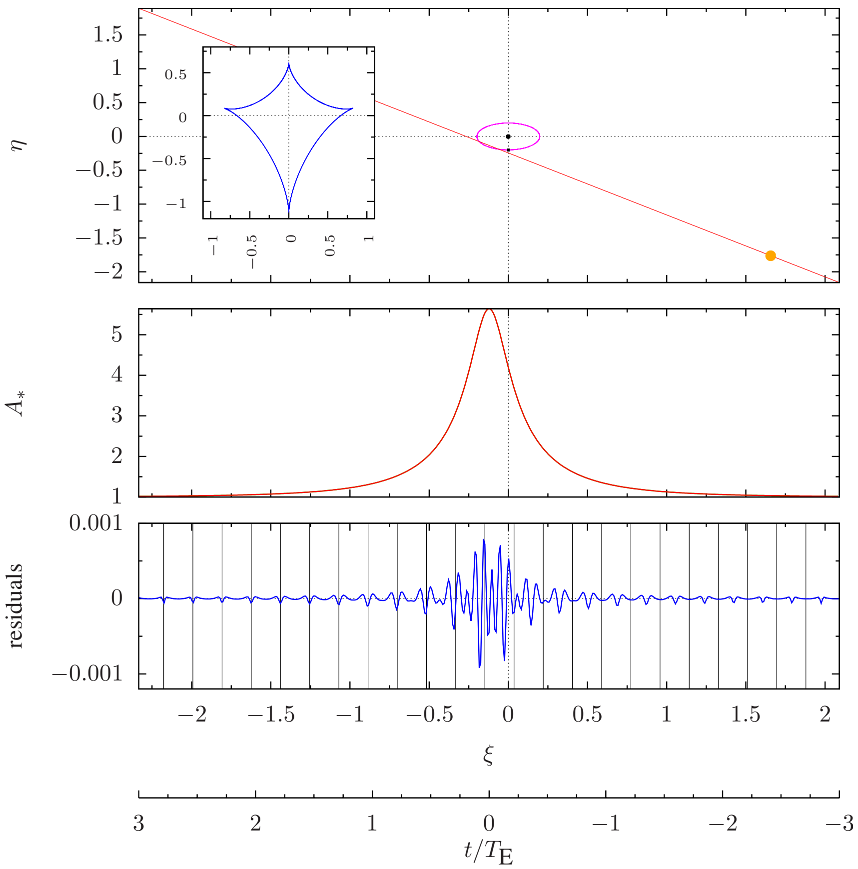
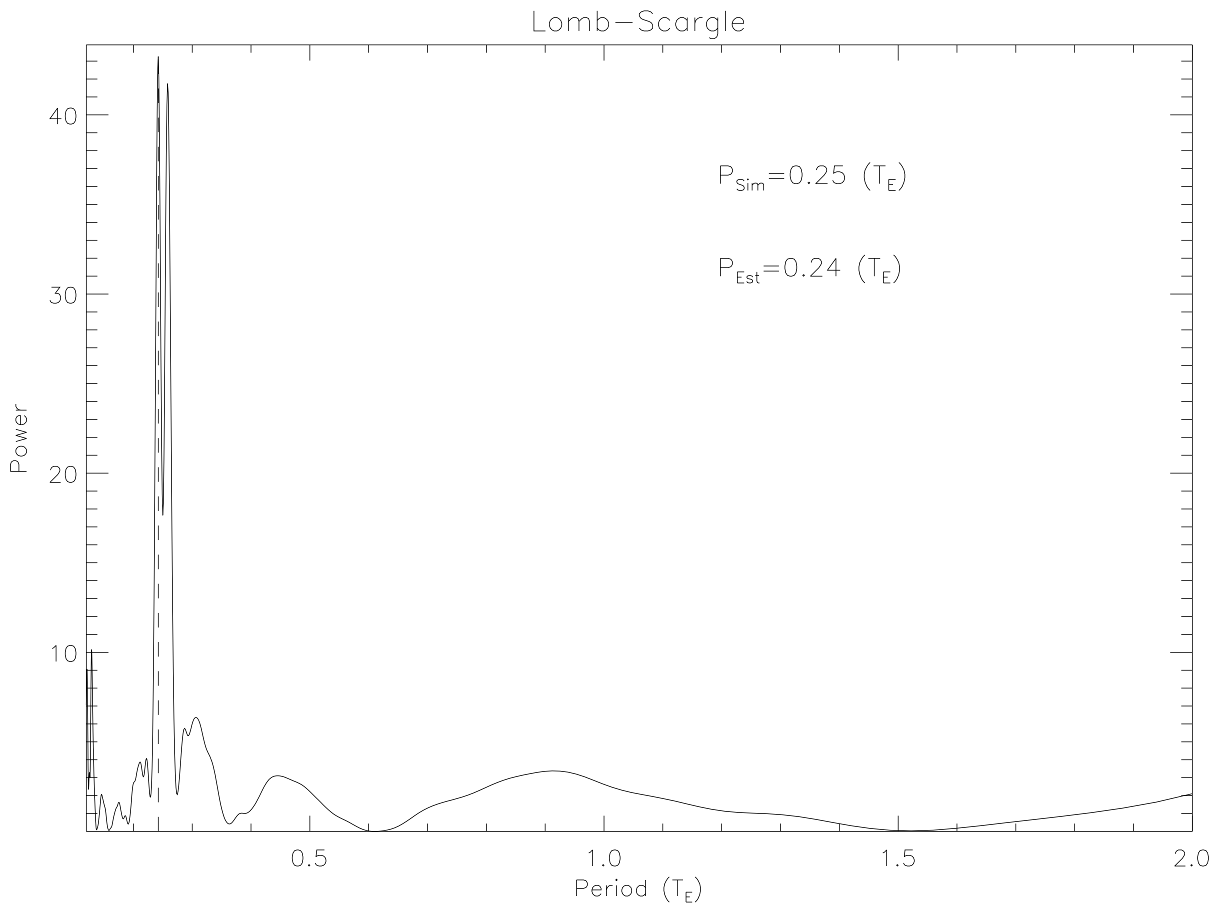


Figure: $q = 10^{-3}$, $a = 0.2$, $e = 0$, $i = \varphi = 0°$, $P = t_E/4$

Simulation 1 (periodogram)



Simulation 2

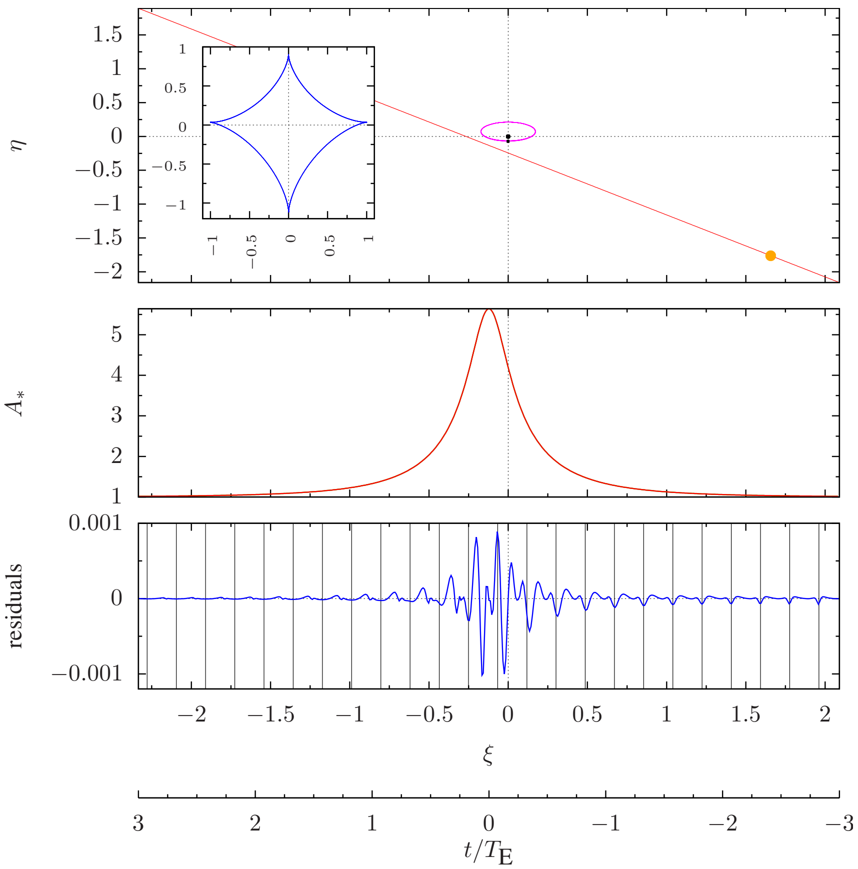
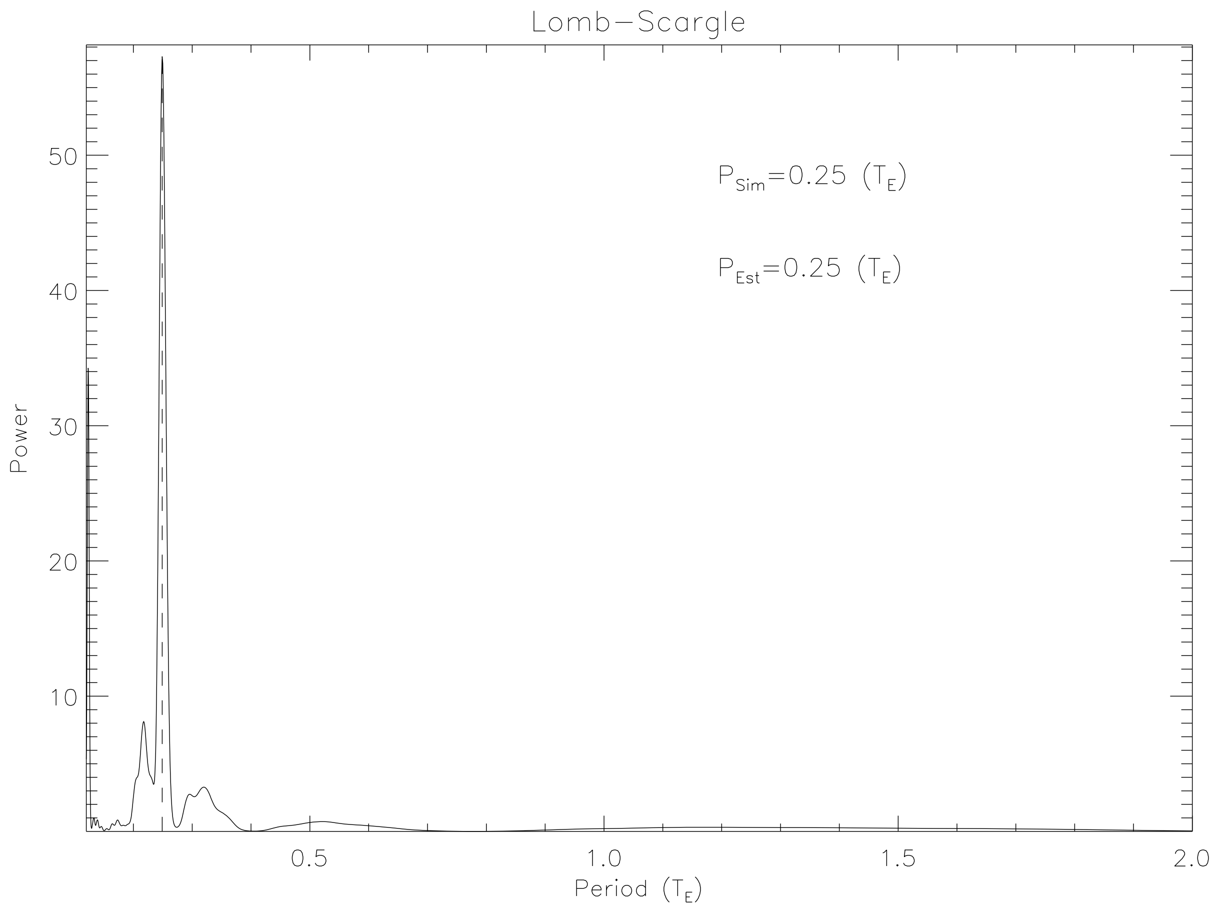


Figure: $q = 10^{-3}$, $a = 0.2$, $e = 0.5$, $i = 45°$, $\varphi = 0°$, $P = t_E/4$

Simulation 2 (periodogram)



Simulation 3

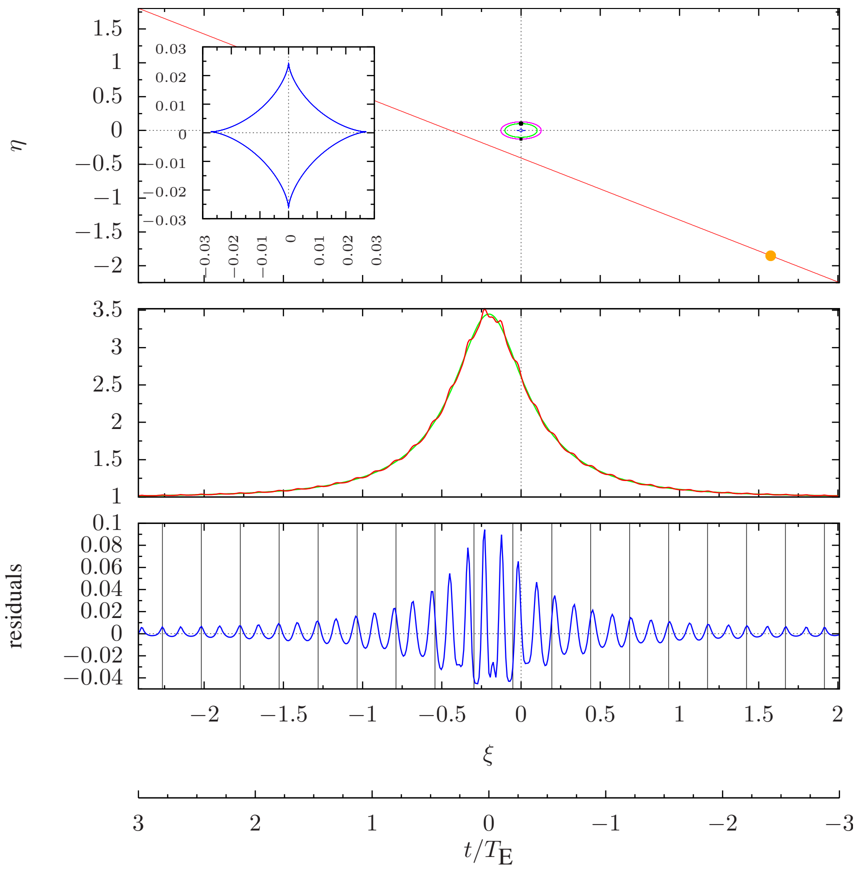
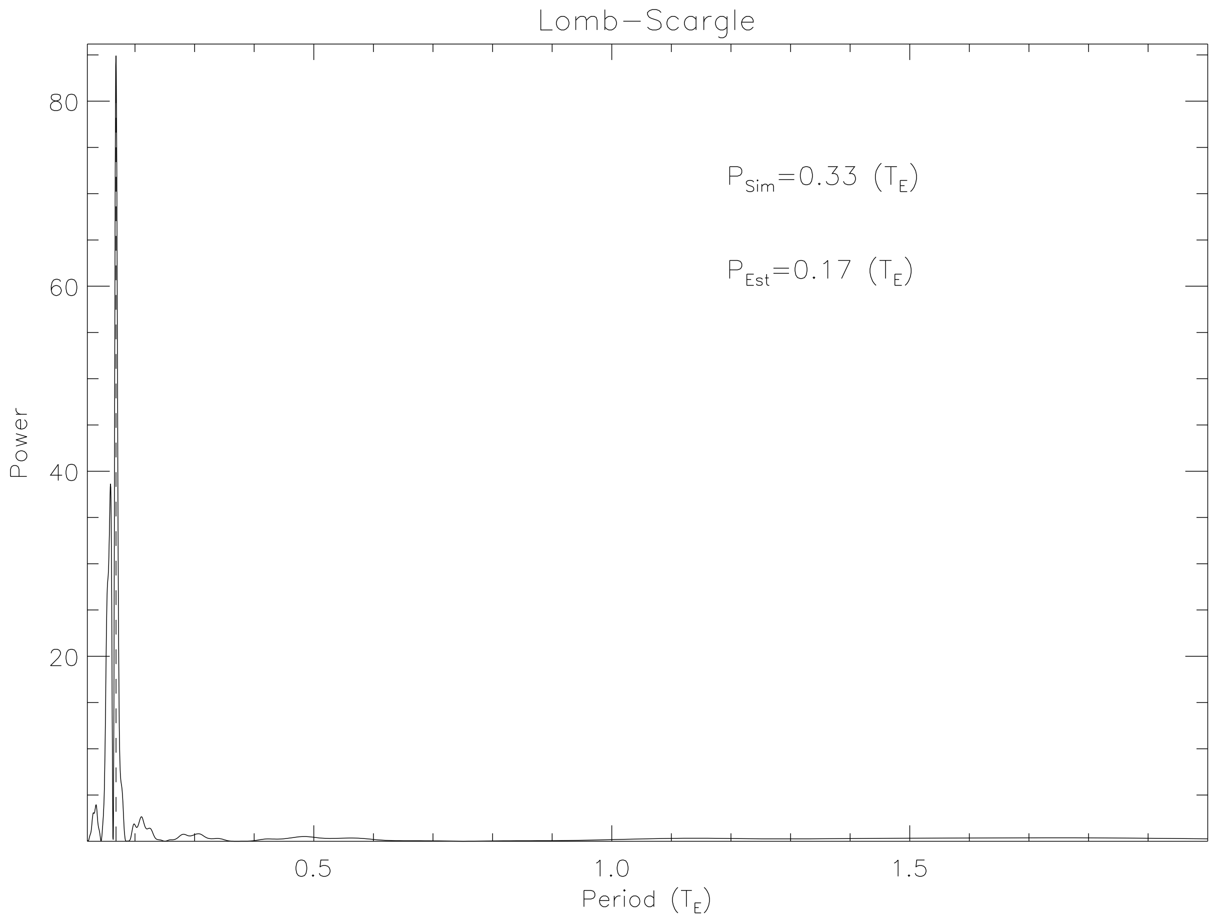


Figure: $q = 0.8$, $a = 0.23$, $e = 0$, $i = \varphi = 0^\circ$, $P = t_E/3$

Simulation 3 (periodogram)



Simulation 4

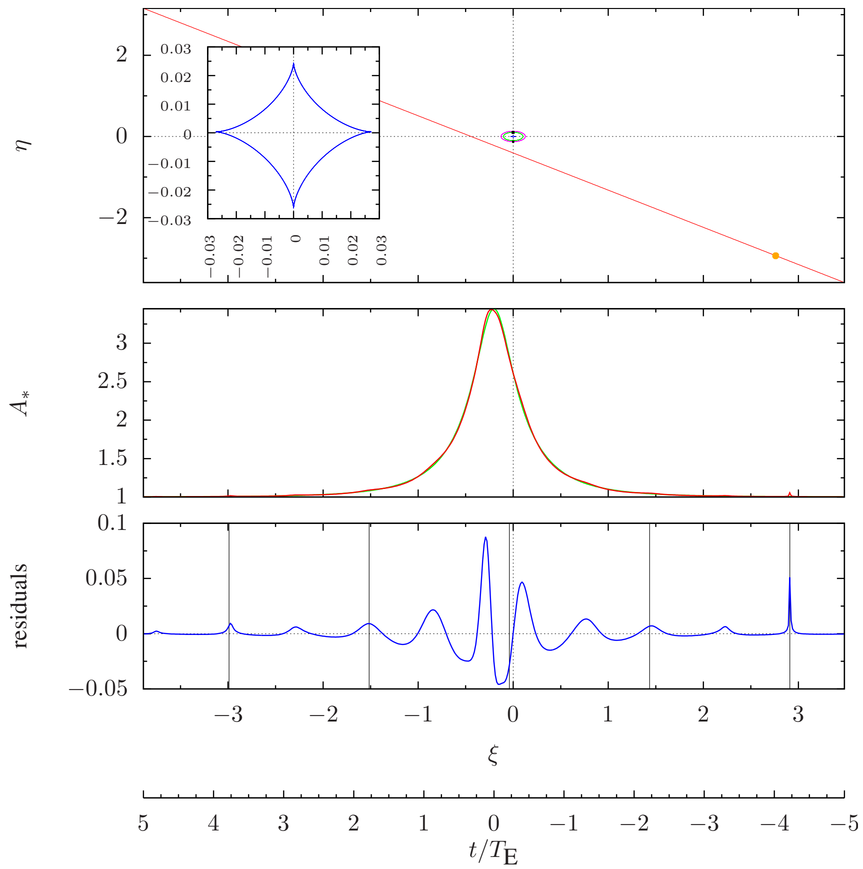
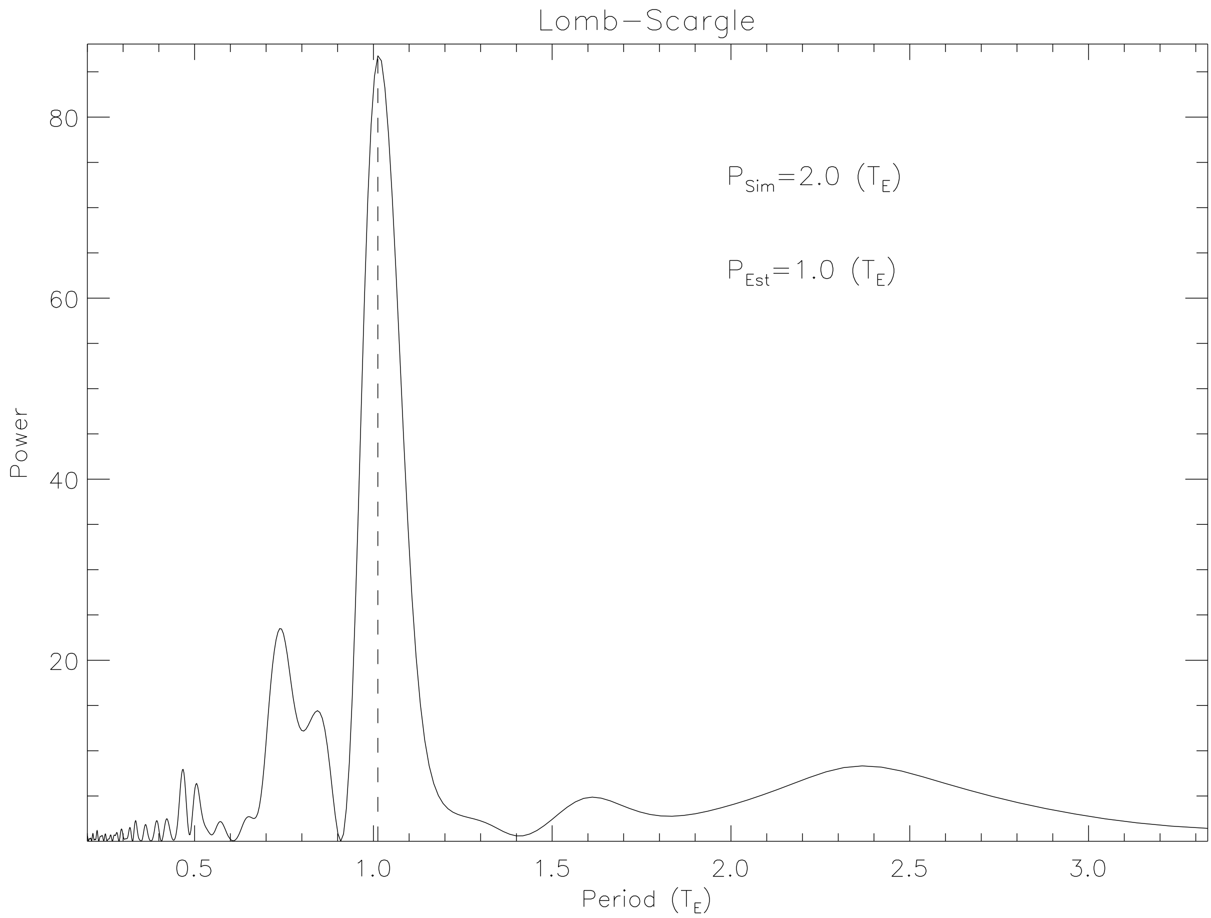


Figure: $q = 0.8$, $a = 0.23$, $e = 0$, $i = \varphi = 0°$, $P = 2t_E$

Simulation 4 (periodogram)



Simulation 5

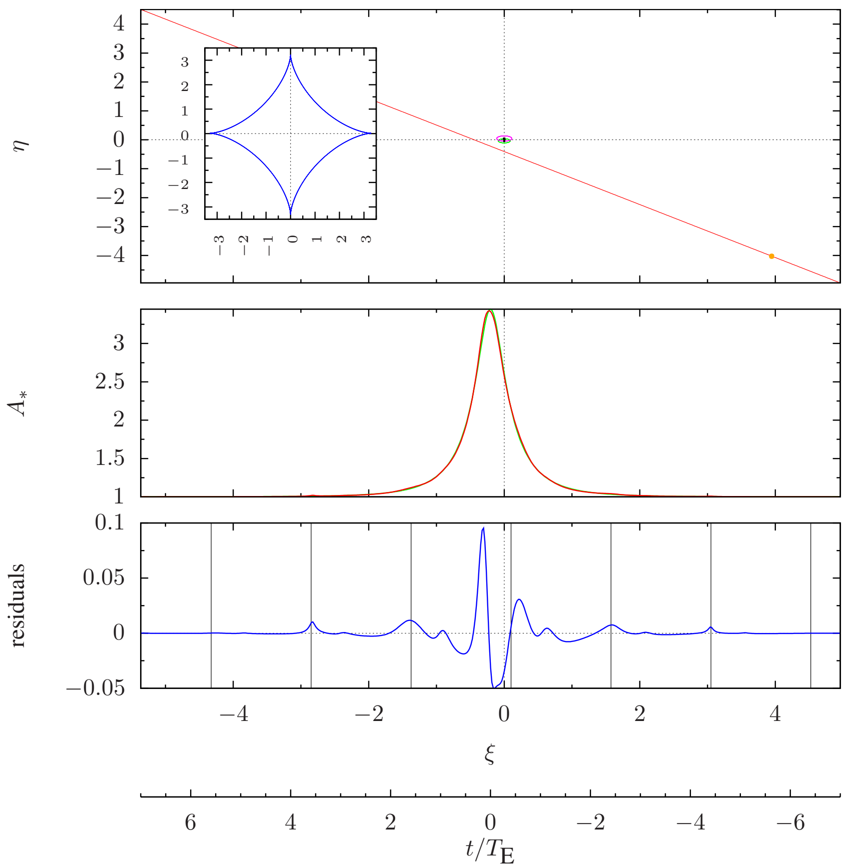
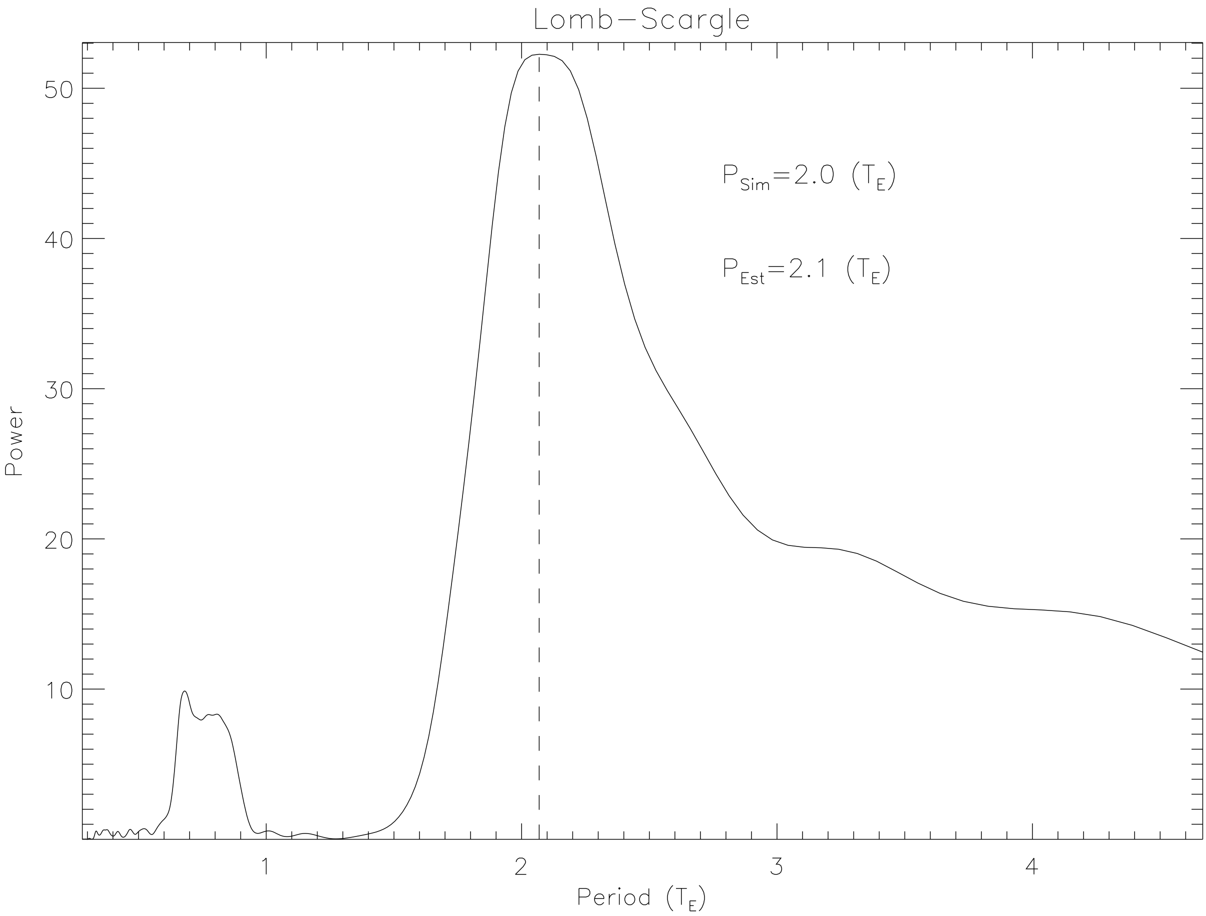


Figure: $q = 0.8$, $a = 0.23$, $e = 0.5$, $i = 45°$, $\varphi = 0°$, $P = 2t_E$

Simulation 5 (periodogram)



Fit to Real Data

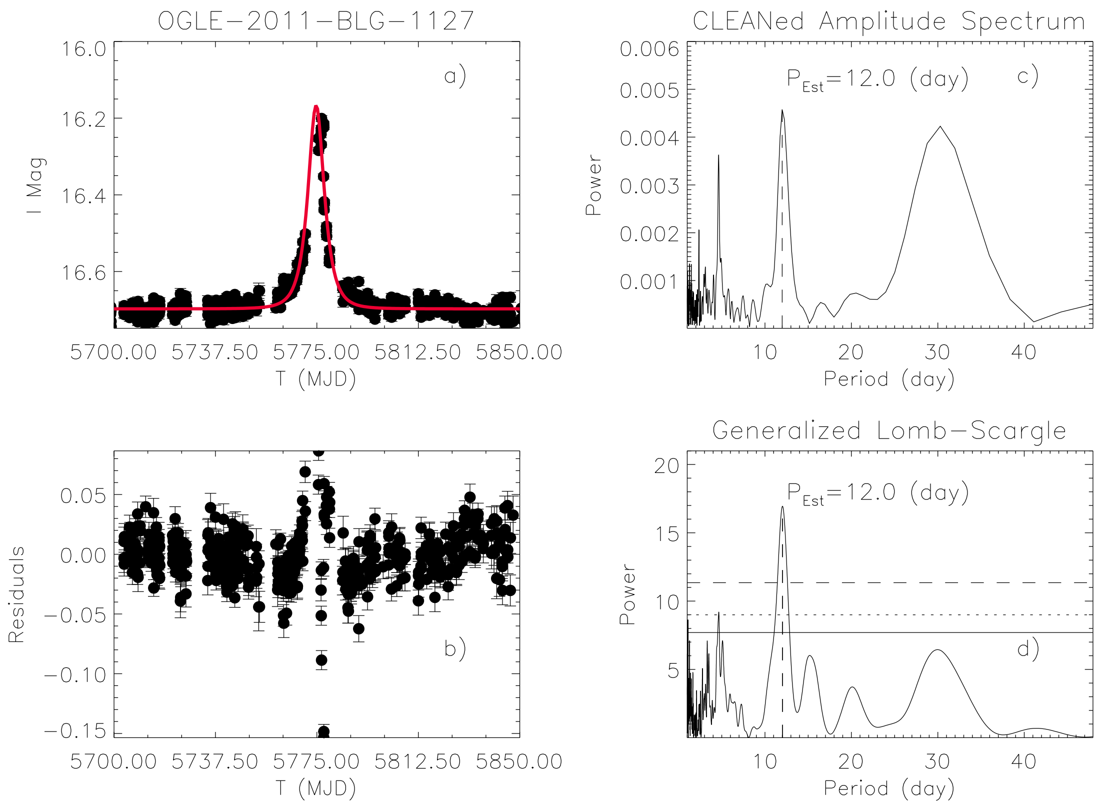


Figure: Event OGLE-2011-BLG-1127/MOA-2011-BLG-322

Discussions

- ✓ Orbital period of the lenses should be shorter than the Einstein time or we must have long observational window
- ✓ We fit the observed amplification curve to a simple Paczyński curve, with four easily-guessable free parameters
- ✓ We need to remove a very small region around the maximum amplification peak from the residuals curve to perform the periodogram
- ✗ Periodic feature with the same period far from the peak $\implies$ source periodicity (binary system, intrinsic variable, etc. . .)

Exoplanet?

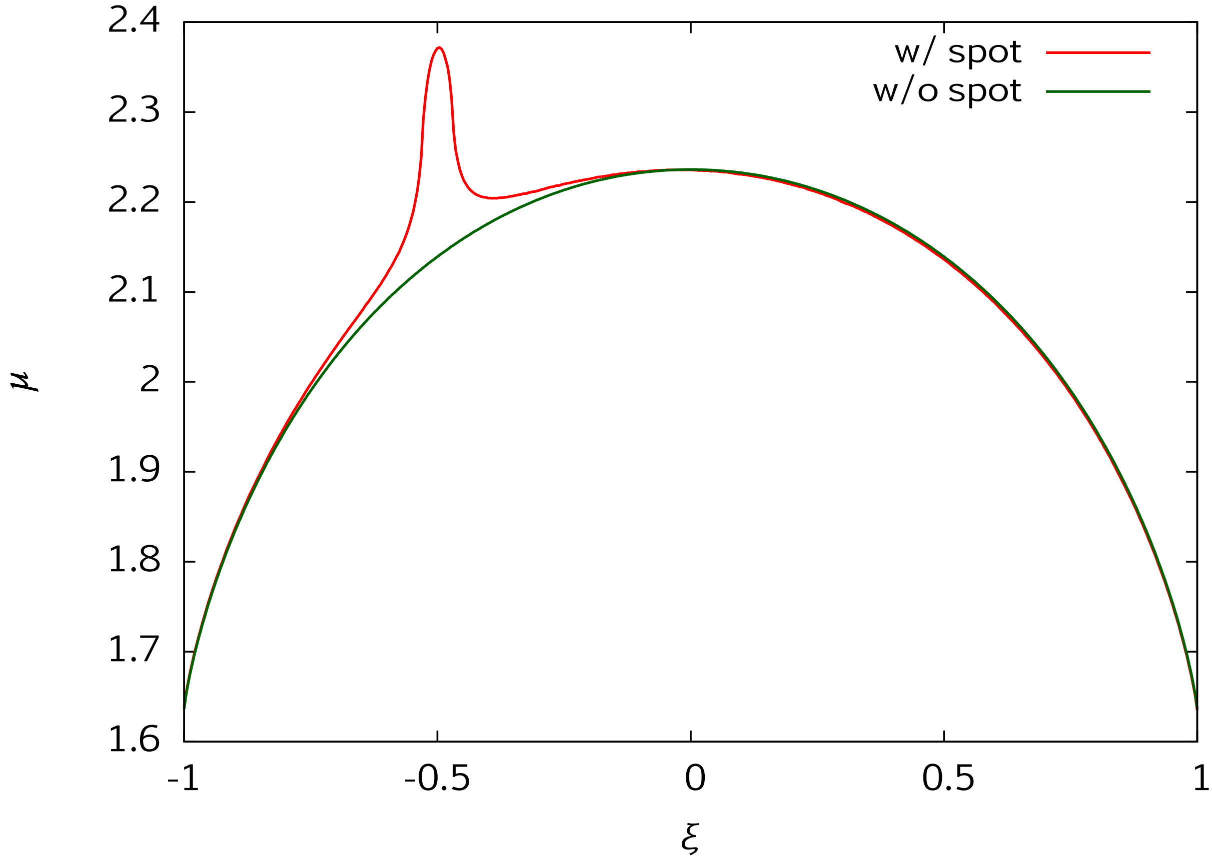


Figure: $L_{\text{spot}}/L_{\text{star}} = 5$, $r_{\text{spot}}/r_{\text{star}} = 0.03$

Results and Future Outlook

Results

- implementation of the hexadecapole method
- development of a method to estimate the orbital period of a binary system through fluctuations on the Paczyński curve

Future outlook:

- extend the program to triple lenses, to include a second planet or a moon
- fast fits to observed light curves, also with orbiting lenses
- study of microlensing of extended source, with accretion disks and cold/hot spots

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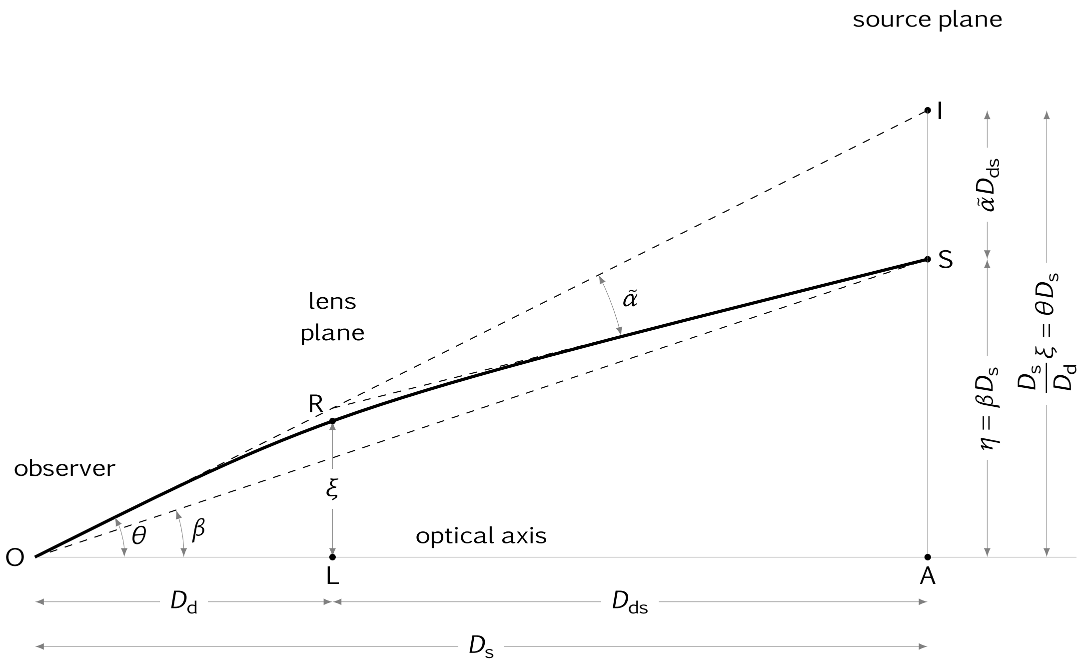
Publication list

A. Nucita, M. Giordano, F. De Paolis, and G. Ingrosso. "Signatures of rotating binaries in microlensing experiments". In: *Monthly Notices of the Royal Astronomical Society* 438 (Mar. 2014), pp. 2466–2473. DOI: 10.1093/mnras/stt2363. arXiv: 1401.6288.

Lens Equation



Lens Equation

$$\vec{\beta} = \vec{\theta} - \vec{\tilde{\alpha}} \frac{D_{ds}}{D_s} \iff \vec{\eta} = \vec{\xi} \frac{D_s}{D_d} - \vec{\tilde{\alpha}} D_{ds} \iff \vec{y} = \vec{x} - \vec{\alpha}$$

Critic and Caustic Curves

Amplification Matrix

$$\mathcal{J}_{ij} = \frac{\partial y_i}{\partial x_j}$$

Amplification

$$\mu = \frac{1}{\det \mathcal{J}}$$

Critic Curves

Locus of the points in the lens plane in which $\mu \to \infty \iff \det \mathcal{J} \to 0$

Caustic Curves

Locus of the points in the source plane in which $\mu \to \infty \iff \det \mathcal{J} \to 0$

Dimensionless Quantities

Einstein Radius

$$R_{\mathrm{E}} = \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{ds}}D_{\mathrm{d}}}{D_{\mathrm{s}}}}$$

Einstein Angle

$$\theta_{\mathrm{E}} = \frac{R_{\mathrm{E}}}{D_{\mathrm{d}}} = \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}D_{\mathrm{d}}}}$$

Critical Superficial Mass Density

$$\Sigma_{\mathrm{cr}} = \frac{c^2 D_{\mathrm{s}}}{4\pi G D_{\mathrm{d}} D_{\mathrm{ds}}}$$

Complex Formalism

Complex Coordinates:
Source Plane: $z = x + \mathrm{i}\, y$
Lens Plane: $\zeta = \xi + \mathrm{i}\, \eta$
Mass Distribution

$$\Sigma(z) = \sum_{j=1}^{N} m_j \delta^2 (z - z_j)$$

Lens Equation

$$\zeta = (1 - \kappa) z + \gamma \bar{z} - \sum_{j=1}^{N} \frac{\varepsilon_j}{\bar{z} - \bar{z}_j}$$

Critic Curves Parametrization

$$\sum_{j=1}^{N} \frac{\varepsilon_j}{(\bar{z} - \bar{z}_j)^2} = (1 - \kappa)\, \mathrm{e}^{\mathrm{i}\, \varphi} - \gamma$$

Witt & Mao Method

Binary-Lens Equation

$$\zeta = z + \frac{\varepsilon_1}{\overline{z}_1 - \overline{z}} + \frac{\varepsilon_2}{\overline{z}_2 - \overline{z}}$$

Put the lenses on points $z_1 = -z_2$ along the real axis $(z_j = \overline{z}_j)$

$$p_5(z) = \sum_{i=0}^{5} c_i z^i = 0$$

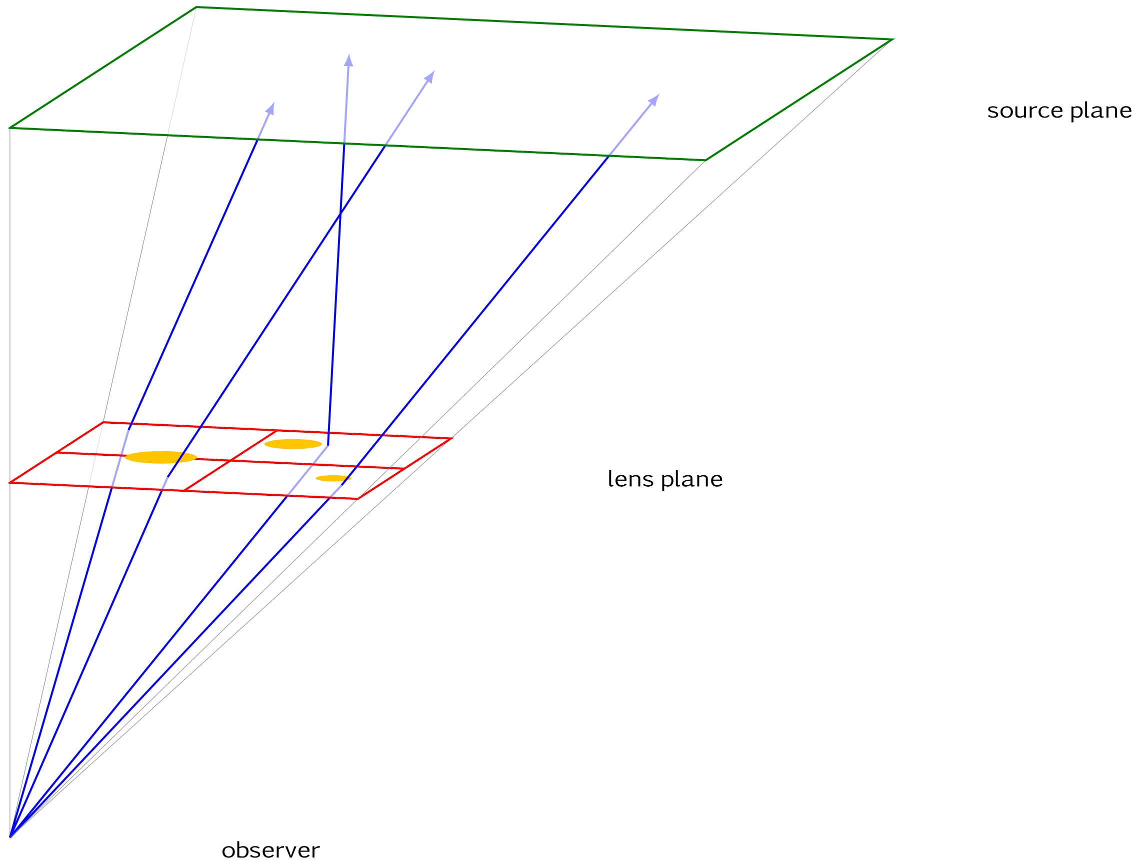
Amplification

$$\mu(\zeta) = \sum_{i=1}^{N} |\mu_i| = \sum_{i=1}^{N} \frac{\pi_i}{\det \mathcal{J}}\bigg|_{z=z_i}$$

Pros and cons

- ✓ fast
- ✗ only point-like source
- ✓ any lens configuration
- ✗ doesn't work near caustics

Inverse Ray Shooting



Inverse Ray Shooting (cont.)

Solve the lens equation "backwards"

$$\zeta = z - \sum_{i=1}^{N} \frac{\varepsilon_i (z - z_i)}{\|z - z_i\|^2}$$

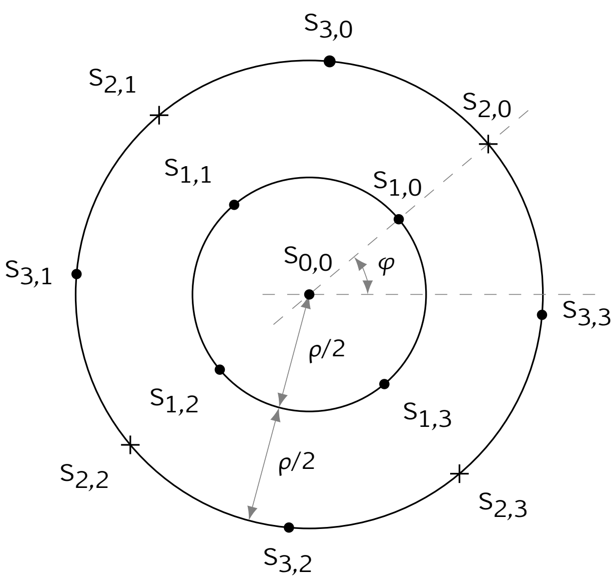
Conditions

- source area subdivided in at least 10^3 pixels
- each pixel on the source plane matches at least 100 pixels on the lens plane

Pros and cons

- ✓ precise, also on caustics
- ✗ very slow, high number of photons to be "shot"
- ✓ any lens configuration
- ✗ only point-like source

Hexadecapole Approximation



Approximation of the amplification function with a Taylor series up to the fourth order

$$\mu_{\text{finite}}(\rho) = \frac{2\pi}{F} \sum_{n=0}^{\infty} \mu_{2n} \int_0^{\rho} S(w) w^{2n+1}\, \mathrm{d}w$$
$$= \mu_0 + \frac{\mu_2 \rho^2}{2}\left(1 - \frac{\Gamma}{5}\right) + \frac{\mu_4 \rho^4}{3}\left(1 - \frac{11\Gamma}{35}\right) + \cdots$$

Pros and cons

- ✓ fast (no amplification map required)
- ✓ extended source
- ✓ any lens configuration and any radial luminosity profile of the source
- ✗ far enough from the caustics

Hexadecapole Approximation (cont.)

Far from the caustics, amplification can be expanded in Taylor series

$$\mu(\xi,\eta) = \sum_{n=0}^{\infty}\sum_{i=0}^{n} \mu_{n,i}(\xi-\xi_0)^i(\eta-\eta_0)^{n-i}$$

Amplification of an extended source

$$\mu_{\text{finite}}(\rho;\xi_0,\eta_0) = \frac{\int_0^\rho wS(w)\,\mathrm{d}w \int_0^{2\pi} \mu(\xi_0 + w\cos\theta, \eta_0 + w\sin\theta)\,\mathrm{d}\theta}{\int_0^\rho wS(w)\,\mathrm{d}w \int_0^{2\pi} \mathrm{d}\theta}$$

$$= \frac{2\pi}{F}\sum_{n=0}^{\infty} \mu_{2n} \int_0^\rho S(w) w^{2n+1}\,\mathrm{d}w$$

With linear limb-darkening ($S(w) = (1 - \Gamma(1-(3/2)\sqrt{1-w^2/\rho^2}))F/\pi\rho^2$)

$$\mu_{\text{finite}}(\rho;\xi_0,\eta_0) = \mu_0 + \frac{\mu_2\rho^2}{2}\left(1-\frac{\Gamma}{5}\right) + \frac{\mu_4\rho^4}{3}\left(1-\frac{11\Gamma}{35}\right) + \cdots$$

Hexadecapole Approximation (cont.)

$$
\begin{aligned}
M_{w,+} &= \frac{1}{4}\sum_{j=0}^{3}\mu(\xi_0 + w\cos(\varphi + j\pi/2), \eta_0 + w\sin(\varphi + w\sin(\varphi + j\pi/2))) - \mu_0 \\
&\approx \frac{1}{4}\sum_{j=0}^{3}\sum_{n=0}^{4}\sum_{i=0}^{n}\mu_{n,i}w^n(\cos(\varphi + j\pi/2))^i(\sin(\varphi + j\pi/2))^{n-i} - \mu_0 \\
&= \frac{(\mu_{4,0}+\mu_{4,4})(3+\cos(4\varphi)) + (\mu_{4,3}+\mu_{4,1})\sin(4\varphi) + \mu_{4,2}(1-\cos(4\varphi))}{8} \\
&\quad + \mu_2 w^2 \\
M_{w,\times} &= \frac{1}{4}\sum_{j=0}^{3}\mu(\xi_0 + w\cos(\varphi + (2j+1)\pi/4), \eta_0 + w\sin(\varphi + w\sin(\varphi + (2j+1)\pi/4))) \\
&\quad - \mu_0 \\
&\approx \frac{(\mu_{4,0}+\mu_{4,4})(3-\cos(4\varphi)) - (\mu_{4,3}+\mu_{4,1})\sin(4\varphi) + \mu_{4,2}(1+\cos(4\varphi))}{8} w^4 \\
&\quad + \mu_2 w^2
\end{aligned}
$$

Hexadecapole Approximation (cont.)

Recipe:

- determine amplification on the thirteen points
- use these amplifications to calculate $M_{\rho,+}$, $M_{\rho,\times}$, and $M_{\rho/2,+}$
- calculate $\mu_2\rho^2$ and $\mu_4\rho^4$ with relations

$$\mu_2\rho^2 = \frac{16M_{\rho/2,+} - M_{\rho,+}}{3}$$
$$\mu_4\rho^4 = \frac{M_{\rho,+} + M_{\rho,\times}}{2} - \mu_2\rho^2$$

- insert $\mu_2\rho^2$, $\mu_4\rho^4$, and amplification μ_0 of the central monopole inside equation

$$\mu_{\text{finite}}(\rho;\xi_0,\eta_0) = \mu_0 + \frac{\mu_2\rho^2}{2}\left(1 - \frac{\Gamma}{5}\right) + \frac{\mu_4\rho^4}{3}\left(1 - \frac{11\Gamma}{35}\right) + \cdots$$

to get the amplification of a finite source