Dipartimento di Matematica e Fisica
"Ennio De Giorgi"

INFN

Consiglio di sezione INFN

Presentazione delle attività relative all'iniziativa specifica QFT-HEP a Lecce

Luigi Delle Rose – Lecce, 21/07/2014

Generalities

- ✓ People:
  C. Corianò, M. Serino, A. Costantini, C.Marzo, L. D. R.

- ✓ Collaborations:
  E. Mottola (Los Alamos)
  L. Trentadue (Parma and INFN Milano)
  E. Gabrielli (CERN and INFN Trieste)
  P. Colangelo (INFN Bari)

Publications 2013-2014

- ✓ C. Corianò, A. Costantini, L. D. R., M. Serino
  "Superconformal sum rules and the spectral density flow of the composite dilaton (ADD) multiplet in N=1 theories"
  Published in JHEP 1406 (2014) 136, arXiv:1402.6369 [hep-th]

- ✓ C. Corianò, L. D. R., E. Gabrielli, L. Trentadue
  "Fermion Scattering in a Gravitational Background: Electroweak Corrections and Flavour Transitions"
  Published in JHEP 1403 (2014), arXiv:1312.7657 [hep-ph]

- ✓ C. Corianò, L. D. R., C. Marzo, M. Serino
  "The dilaton Wess-Zumino action in six dimensions from Weyl gauging: local anomalies and trace relations"
  Published in Class.Quant.Grav. 31 (2014) 105009, arXiv:1311.1804 [hep-th]

- ✓ C. Corianò, L. D. R., C. Marzo, M. Serino
  "Conformal Trace Relations from the Dilaton Wess-Zumino Action"
  Published in Phys.Lett. B726 (2013) 4-5. 896-905, arXiv:1306.4248 [hep-th]

Publications 2013-2014

- ✓ C. Corianò, L. D. R., E. Mottola, M. Serino
  "Solving the Conformal Constraints for Scalar Operator in Momentum Spaces and the Evaluation of Feynman's Master Integrals"
  Published in JHEP 1307 (2013) 011, arXiv:1304.6944 [hep-th]

- ✓ C. Corianò, L. D. R., E. Gabrielli, L. Trentadue
  "Mass Corrections to Flavor-Changing Fermion-Graviton Vertices in the Standard Model"
  Published in Phys. Rev. D88 (2013) 085008, arXiv:1303.1305 [hep-th]

- ✓ C. Corianò, L. D. R., E. Gabrielli, L. Trentadue
  "One loop Standard Model corrections to flavor diagonal fermion-graviton vertices"
  Published in Phys. Rev. D87 (2013) 054020, arXiv:1212.5029 [hep-ph]

Generalities

- ✓ Organization of QCD@Work 2014 --- 16-19 June 2014, Giovinazzo
- ✓ Thesis
  - A. Quintavalle – "Dilaton Interactions and the Anomalous Breaking of Scale Invariance of the Standard Model" – Dottorato, Luglio 2013
  - L.D.R. - "The Standard Model in a Weak Gravitational Background: Dilatons, Scale Anomalies and Conformal Methods" – Dottorato, Luglio 2013
  - A. Politano – "L'equazione di Dirac in campi esterni" – Triennale, 2013
  - M. Maglio – "Scattering di un fotone in un background gravitazionale" – Triennale, 2014
  - M. Serino – "Conformal anomaly actions and dilaton interactions" – Dottorato, Luglio 2014

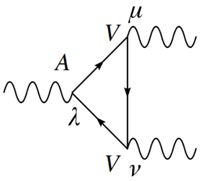
Outline

- ✓ *Superconformal sum rules and the spectral density flow*
- ✓ *Fermion interactions in a gravitational background*
- ✓ *Vacuum stability in abelian extensions of the Standard Model*
- ✓ *Dilaton Wess-Zumino action from the conformal anomaly*

Superconformal sum rules and the spectral density flow

Review: The *AVV* diagram and the pion

The anomalous *AVV* diagram with an axial-vector current (A) and two vector currents (V) is characterized by a massless singularity



$$\Delta^{\lambda\mu\nu} = a_n \, \frac{k^\lambda}{k^2} \, \epsilon^{\mu\nu\alpha\beta} \, k_{1\alpha} \, k_{2\beta}$$

Dolgov, Zakharov. Nucl. Phys. B27, 525 (1971)

The axial anomaly pole appears in the AVV diagram

The pole structure clearly describes the pseudoscalar pion π

the anomalous spectral density in the AVV possesses a sum rule

$$\frac{1}{\pi}\int_0^\infty \rho(s, m^2) ds = f$$

Horejsi. Phys. Rev. D32, 1029 (1985)

this is a general feature of anomalies (chiral, conformal, superconformal)

Sum rules

The existence of a sum rule implies a particular behaviour of the spectral density

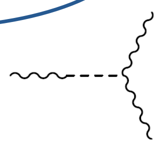
we consider a sum rule with a mass deformation parameter m to control the spectral density away from the chiral/conformal point

$$\frac{1}{\pi}\int_0^\infty \rho(s,m^2)ds = f$$ with f ≠ 0 and mass independent

- the spectral density is integrable
- its scale dimension is fixed
- $\rho(s, m^2)$ flows towards a $\delta(s)$ as m goes to zero
- a pole-like behaviour appears also in the UV

this corresponds to a massless state propagating in the theory

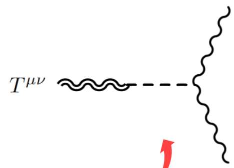
anomalies are IR/UV phenomena $\lim_{k^2\to\infty} k^2 F(k^2,m^2) = f$



The conformal anomaly case

- Conformally anomalous correlators possess spectral densities with a convergent sum rule
- As the AVV case, it has been shown that the TVV correlator, (T is the energy-momentum tensor EMT) shares a similar behaviour dictated by the conformal anomaly

  this has been proved in QED, QCD and in EW sector of the SM



this can be interpreted as an effective scalar, the ***dilaton***, coupling to the trace of the EMT, in full analogy with the pion

exchange of a conformal anomaly pole

In the following the superconformal anomaly case is considered, in which chiral, conformal and superconformal symmetries are treated in a unified way

Anomalies in *N*=1 Super Yang-Mills

Consider a *N*=1 Super Yang-Mills (for instance SuperQCD)

vector (gauge) supermultiplet $V = \left(A^a_\mu, \lambda^a, D^a\right)$

chiral (matter) supermultiplet $\Phi = (\phi_i, \chi_i, F_i)$

we can introduce the Ferrara-Zumino (FZ) hypercurrent

$$\mathcal{J}_{A\dot{A}} = \mathrm{Tr}\left[\bar{W}_{\dot{A}} e^V W_A e^{-V}\right] - \frac{1}{3}\bar{\Phi}\left[\overleftarrow{\bar{\nabla}}_{\dot{A}}\, e^V \nabla_A - e^V \bar{D}_{\dot{A}} \nabla_A + \overleftarrow{\bar{\nabla}}_{\dot{A}} \overleftarrow{D}_A\, e^V\right]\Phi$$

$$W_A = 2g W^a_A T^a = -\frac{1}{4}\bar{D}^2 e^{-V} D_A\, e^V$$

which describes a chiral supermultiplet containing the ***EMT***, the ***R-current*** and the ***supersymmetric current***

$$\mathcal{J} = (R^\mu, T^{\mu\nu}, S^\mu)$$

- *in a classical superconformal theory the FZ hypercurrent is conserved* $\bar{D}^{\dot{A}} \mathcal{J}_{A\dot{A}} = 0$

the chiral superpotential $\mathcal{W}$ must be cubic or vanishing

Anomalies in *N*=1 Super Yang-Mills

in the quantum mechanical framework, the FZ develops a superconformal anomaly

$$\bar{D}^{\dot{A}}\mathcal{J}_{A\dot{A}} = a_n D_A W^2 \qquad\qquad a_n = -\frac{2}{3}g^2\frac{3T_G - \sum_f T(R_f)}{16\pi^2}$$

The components of the FZ supermultiplet are

$$\begin{aligned}
R^\mu &= \bar{\lambda}^a\bar{\sigma}^\mu\lambda^a + \frac{1}{3}\left(-\bar{\chi}_i\bar{\sigma}^\mu\chi_i + 2i\phi_i^\dagger\mathcal{D}^\mu_{ij}\phi_j - 2i(\mathcal{D}^\mu_{ij}\phi_j)^\dagger\phi_i\right), \\
S^\mu_A &= i(\sigma^{\nu\rho}\sigma^\mu\bar{\lambda}^a)_A F^a_{\nu\rho} - \sqrt{2}(\sigma_\nu\bar{\sigma}^\mu\chi_i)_A(\mathcal{D}^\nu_{ij}\phi_j)^\dagger - i\sqrt{2}(\sigma^\mu\bar{\chi}_i)\mathcal{W}_i^\dagger(\phi^\dagger) \\
&- ig(\phi_i^\dagger T^a_{ij}\phi_j)(\sigma^\mu\bar{\lambda}^a)_A + S^\mu_{I\,A}, \\
T^{\mu\nu} &= -F^{a\,\mu\rho}F^{a\,\nu}{}_\rho + \frac{i}{4}\left[\bar{\lambda}^a\bar{\sigma}^\mu(\delta^{ac}\overrightarrow{\partial^\nu} - g\,t^{abc}A^{b\,\nu})\lambda^c + \bar{\lambda}^a\bar{\sigma}^\mu(-\delta^{ac}\overleftarrow{\partial^\nu} - g\,t^{abc}A^{b\,\nu})\lambda^c + (\mu\leftrightarrow\nu)\right] \\
&+ (\mathcal{D}^\mu_{ij}\phi_j)^\dagger(\mathcal{D}^\nu_{ik}\phi_k) + (\mathcal{D}^\nu_{ij}\phi_j)^\dagger(\mathcal{D}^\mu_{ik}\phi_k) + \frac{i}{4}\Big[\bar{\chi}_i\bar{\sigma}^\mu(\delta_{ij}\overrightarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j \\
&+ \bar{\chi}_i\bar{\sigma}^\mu(-\delta_{ij}\overleftarrow{\partial^\nu} + igT^a_{ij}A^{a\,\nu})\chi_j + (\mu\leftrightarrow\nu)\Big] - \eta^{\mu\nu}\mathcal{L} + T^{\mu\nu}_I,
\end{aligned}$$

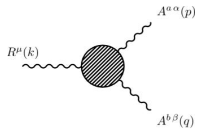
and satisfy the anomaly equations

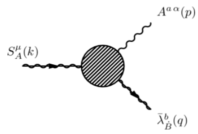
$$\left\{\begin{aligned}
\partial_\mu R^\mu &= -\frac{a_n}{2}F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \\
T^\mu{}_\mu &= \frac{3}{4}a_n F^a_{\mu\nu}F^{a\,\mu\nu} \\
\bar{\sigma}_\mu S^\mu &= 3ia_n\bar{\lambda}^a\bar{\sigma}^{\mu\nu}F^a_{\mu\nu}
\end{aligned}\right.$$

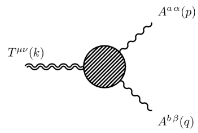
Perturbative computation

We look at the three correlation functions responsible for the appearance of the superconformal anomaly

$$
\begin{aligned}
\delta^{ab}\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q) &\equiv \langle R^{\mu}(k)\,A^{a\,\alpha}(p)\,A^{b\,\beta}(q)\rangle \qquad \langle RVV\rangle\,,\\
\delta^{ab}\,\Gamma^{\mu\alpha}_{(S)\,A\dot{B}}(p,q) &\equiv \langle S^{\mu}_{A}(k)\,A^{a\,\alpha}(p)\,\bar{\lambda}^{b}_{\dot{B}}(q)\rangle \qquad \langle SVF\rangle\,,\\
\delta^{ab}\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &\equiv \langle T^{\mu\nu}(k)\,A^{a\,\alpha}(p)\,A^{b\,\beta}(q)\rangle \qquad \langle TVV\rangle
\end{aligned}
$$







they are constrained by the vector current conservation

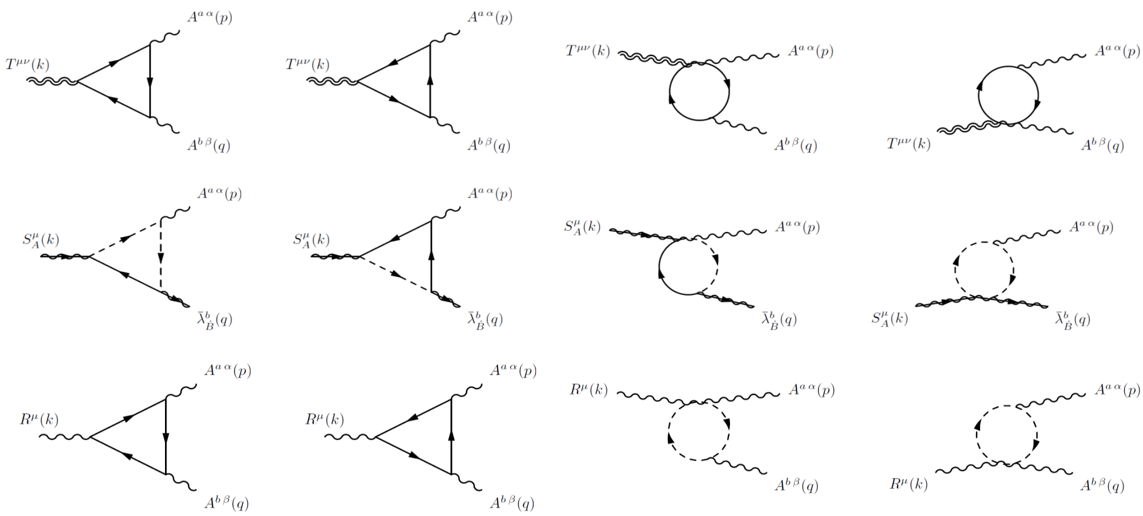
$$
\begin{aligned}
&p_{\alpha}\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q)=0\,, \quad q_{\beta}\,\Gamma^{\mu\alpha\beta}_{(R)}(p,q)=0\,,\\
&p_{\alpha}\,\Gamma^{\mu\alpha}_{(S)}(p,q)=0\,,\\
&p_{\alpha}\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q)=0\,, \quad q_{\beta}\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q)=0
\end{aligned}
$$

and by the conservation of the supercurrent and of the EMT

$$
\begin{aligned}
i\,k_{\mu}\,\Gamma^{\mu\alpha}_{(S)}(p,q) &= -2p_{\mu}\,\sigma^{\mu\alpha}\hat{\Gamma}_{(\lambda\bar{\lambda})}(q) - i\sigma_{\mu}\hat{\Gamma}^{\mu\alpha}_{(AA)}(p)\,,\\
i\,k_{\mu}\,\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &= q_{\mu}\hat{\Gamma}^{\alpha\mu}_{(AA)}(p)\eta^{\beta\nu} + p_{\mu}\hat{\Gamma}^{\beta\mu}_{(AA)}(q)\eta^{\alpha\nu} - q^{\nu}\hat{\Gamma}^{\alpha\beta}_{(AA)}(p) - p^{\nu}\hat{\Gamma}^{\alpha\beta}_{(AA)}(q)
\end{aligned}
$$

Perturbative computation

Samples of the one-loop perturbative expansion of the three anomalous correlators



Explicit results

- We show the chiral matter contribution in the on-shell gauge currents kinetimatic region
- A massive chiral supermultiplet is employed to stay away from the conformal point and to study the flow of the spectral densities

$$\Gamma^{\mu\alpha\beta}_{(R)}(p,q) = i\frac{g^2\,T(R)}{12\pi^2}\,\Phi_1(k^2,m^2)\,\frac{k^\mu}{k^2}\varepsilon[p,q,\alpha,\beta]\,,$$

$$\Gamma^{\mu\alpha}_{(S)}(p,q) = i\frac{g^2 T(R)}{6\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,s_1^{\mu\alpha} + i\frac{g^2 T(R)}{64\pi^2}\,\Phi_2(k^2,m^2)\,s_2^{\mu\alpha}\,,$$

$$\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) = \frac{g^2\,T(R)}{24\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,t_{1S}^{\mu\nu\alpha\beta}(p,q) + \frac{g^2\,T(R)}{16\pi^2}\,\Phi_2(k^2,m^2)\,t_{2S}^{\mu\nu\alpha\beta}(p,q)$$

Explicit results

- We show the chiral matter contribution in the on-shell gauge currents kinetimatic region
- A massive chiral supermultiplet is employed to stay away from the conformal point and to study the flow of the spectral densities

superconformal anomaly poles

$$
\begin{aligned}
\Gamma^{\mu\alpha\beta}_{(R)}(p,q) &= i\frac{g^2\,T(R)}{12\pi^2}\,\Phi_1(k^2,m^2)\,\frac{k^\mu}{k^2}\,\varepsilon[p,q,\alpha,\beta]\,,\\
\Gamma^{\mu\alpha}_{(S)}(p,q) &= i\frac{g^2 T(R)}{6\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,s_1^{\mu\alpha} + i\frac{g^2 T(R)}{64\pi^2}\,\Phi_2(k^2,m^2)\,s_2^{\mu\alpha}\,,\\
\Gamma^{\mu\nu\alpha\beta}_{(T)}(p,q) &= \frac{g^2\,T(R)}{24\pi^2\,k^2}\,\Phi_1(k^2,m^2)\,t_{1S}^{\mu\nu\alpha\beta}(p,q) + \frac{g^2\,T(R)}{16\pi^2}\,\Phi_2(k^2,m^2)\,t_{2S}^{\mu\nu\alpha\beta}(p,q)
\end{aligned}
$$

- The first form factor is the anomaly contribution

this is interpreted as the residue of the pole

$$
\begin{aligned}
\Phi_1(k^2,m^2) &= -1 - 2\,m^2\,\mathcal{C}_0(k^2,m^2)\,,\\
\Phi_2(k^2,m^2) &= 1 - \mathcal{B}_0(0,m^2) + \mathcal{B}_0(k^2,m^2) + 2m^2\mathcal{C}_0(k^2,m^2)
\end{aligned}
$$

Anomalous spectral density

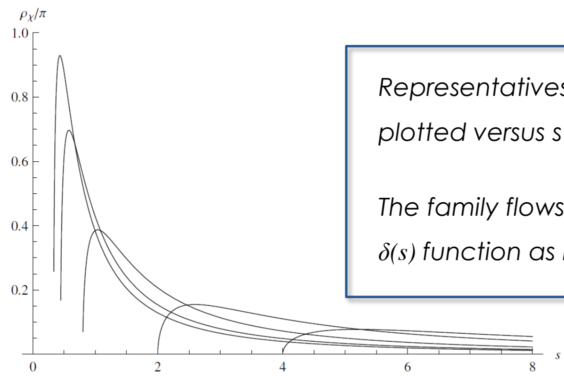
Spectral densities can be computed using cutting rules or exploiting directly the analytic continuations of the two- and three-point scalar integrals

The spectral density of the anomalous form factor

$$\chi(k^2, m^2) \equiv \Phi_1(k^2, m^2)/k^2$$

$$\rho_\chi(s, m^2) = \frac{1}{2i}\text{Disc}\,\chi(s, m^2) = \frac{2\pi m^2}{s^2}\log\left(\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right)\theta(s-4m^2)$$

$$\tau(k^2, m^2) = \sqrt{1-4m^2/k^2}$$



Representatives of the family of spectral densities plotted versus s in units of m^2.

The family flows towards the s=0 region becoming a $\delta(s)$ *function as m*^{2} *goes to zero.*

Anomalous spectral density

- The spectral density is integrable and satisfies a convergent sum rule

$$\frac{1}{\pi}\int_{4m^2}^{\infty} ds\,\rho_\chi(s,m^2) = 1$$

the anomaly coefficient has been factorized for convenience

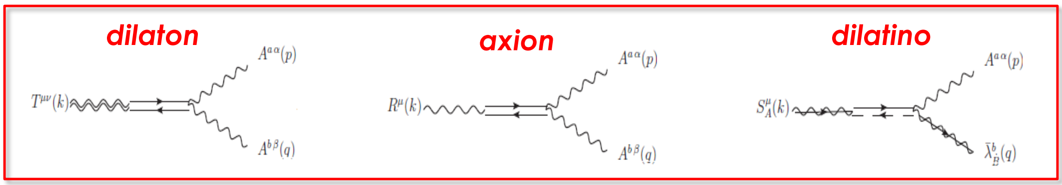
- The discontinuity of the anomalous form factor is characterized by a cut for $k^2 > 4\,m^2$.

  There is any resonant state of the pole at k^2=0 in the massive case (*decoupling*)

- The spectral density flows towards a $\delta(s)$ as m goes to zero $\lim_{m\to 0}\frac{1}{\pi}\rho_\chi(s,m^2) = \delta(s)$

anomaly accounts for the appearance of massless states in the spectrum,

one for each component of the superconformal hypercurrent

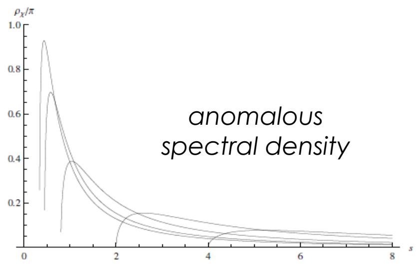


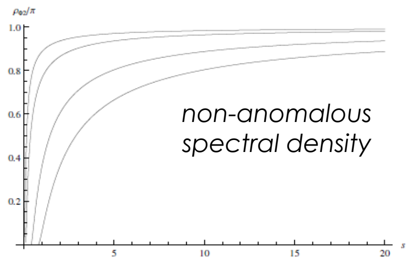
Non-anomalous spectral density

The non-anomalous form factor Φ_2

- needs a subtraction for its integrability (is affected by renormalization)
- does not possess a sum rule
- tends to a uniform distribution as m goes to zero $\frac{1}{\pi} \lim_{m \to 0} \rho_{\Phi_2}(k^2, m^2) = 1$

there is a sort of ***duality*** between the two spectral densities for m->0





Conformal anomaly in QCD

The anomalous correlators (TVV) expands onto ***three*** form factors

$$\Gamma^{\mu\nu\alpha\beta}(p,q) = \Gamma_q^{\mu\nu\alpha\beta}(p,q) + \Gamma_g^{\mu\nu\alpha\beta}(p,q) \qquad \Gamma_{q/g}^{\mu\nu\alpha\beta}(p,q) = \sum_{i=1}^{3} \Phi_{i\,q/g}(k^2,m^2)\,\phi_i^{\mu\nu\alpha\beta}(p,q)$$

The explicit results in quark sector are

$$\Phi_{1\,q}(k^2,m^2) = \frac{g^2}{6\pi^2 k^2}\left\{-\frac{1}{6}+\frac{m^2}{k^2}-m^2\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}-\frac{2m^2}{k^2}\right]\right\},$$

anomalous

$$\Phi_{2\,q}(k^2,m^2) = -\frac{g^2}{4\pi^2 k^2}\left\{\frac{1}{72}+\frac{m^2}{6k^2}+\frac{m^2}{2k^2}\mathcal{D}(k^2,m^2)+\frac{m^2}{3}\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}+\frac{m^2}{k^2}\right]\right\},$$

$$\Phi_{3\,q}(k^2,m^2) = \frac{g^2}{4\pi^2}\left\{\frac{11}{72}+\frac{m^2}{2k^2}+m^2\mathcal{C}_0(k^2,m^2)\left[\frac{1}{2}+\frac{m^2}{k^2}\right]+\frac{5\,m^2}{6k^2}\mathcal{D}(k^2,m^2)+\frac{1}{6}\mathcal{B}_0^{\overline{MS}}(k^2,m^2)\right\}$$

non anomalous

$$\rho_{2q}(s,m^2) = -\frac{g^2}{24\pi}\frac{m^2}{s^2}\left[3\sqrt{\tau(s,m^2)}-\left(1+\frac{2m^2}{s}\right)\log\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right]\theta(s-4m^2)$$

This spectral density is not anomalous,
but is integrable and with a convergent sum rule

another massless state
in the spectrum?

Extra pole cancellation in susy theories

Supersymmetric theories force the extra pole to cancel

In a general Yang-Mills theory the contribution to the non-anomalous form factor with the pole-like behaviour is

$$f_2(k^2) = \frac{N_f}{2} f_2^{(f)}(k^2) + N_s\, f_2^{(s)}(k^2) + N_A\, f_2^{(A)}(k^2)$$
$$= \frac{g^2}{144\pi^2\, k^2}\left[-\frac{N_f}{2}T(R_f) + N_s\,\frac{T(R_s)}{2} + N_A\,\frac{T(A)}{2}\right]$$

Bosonic and fermionic contributions have opposite signs

$f_2^{(f)}$ for a Weyl fermion, $f_2^{(s)}$ for a complex scalar, $f_2^{(A)}$ for a gauge vector

- chiral supermultiplet $N_f = 1, N_s = 1, N_A = 0$ $\quad T(R_f) = T(R_s)$
- vector supermultiplet $N_f = 1, N_s = 0, N_A = 1$ $\quad T(R_f) = T(A)$

there is no ambiguity in a susy theory

Superconformal anomalies ⟷ ***Anomaly poles (massless states)***

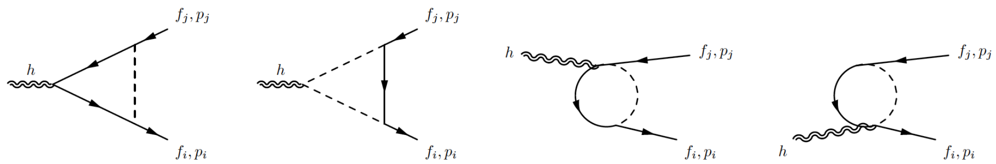
Remarks

- ✓ Analysis of chiral, conformal and superconformal anomalies
  and their sum rules in perturbation theory
- ✓ Anomalous spectral densities flow towards massless states
- ✓ Axion-dilaton-dilatino supermultiplet naturally emerges from the quantum breaking of the superconformal symmetry
- ✓ Unambiguous interpretation in a supersymmetric context:
  one-to-one correspondece between anomalies and anomaly poles

Fermion interactions in a gravitational background

Fermion interactions in a gravitational background

- One-loop Standard Model (QCD & EW) corrections to $\langle T f_i f_j \rangle$ in the *flavor diagonal* and *off-diagonal* sector
  - classification of the operators
  - computation of the form factors (including the full mass dependece)
  - computation of the deflection angle of a fermion in a gravitational background (*gravitational lensing of neutrino*)



Dashed lines can be gluons, photons, Higgs, Z and W bosons

Fermion interactions in a gravitational background

- Compute the SM effective action at one-loop order in a dilaton and a gravitational background
- Inspection of the electroweak corrections to the Newton's potential
- Scattering of a fermion in an external gravitational field generated by heavy sources (star, black hole, galaxy)
- Analysis of the gravitational interactions of neutrinos (differences between Dirac and Majorana neutrinos?)

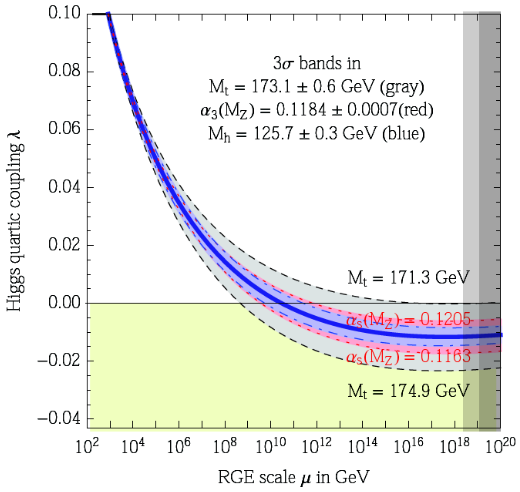
Vacuum stability in abelian extensions of the Standard Model

(in preparation)

Review: the vacuum stability of the SM

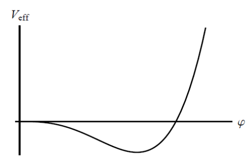
β function of the Higgs quartic coupling $\quad V_\varphi = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$

$$\beta_\lambda = \frac{1}{(4\pi)^2}\left[24\lambda^2 - 6y_t^4 + \frac{3}{8}\left(2g^4 + \left(g^2 + g'^2\right)^2\right) + \left(-9g^2 - 3g'^2 + 12y_t^2\right)\lambda\right]$$

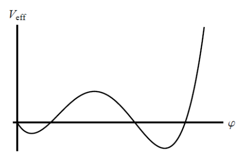


Degrassi et al.
JHEP 1208 (2012) 098

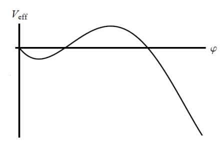
Effective potential:



stable



metastable



unstable

Vacuum stability in abelian extensions of SM

- *The requirement of vacuum stability of a SM extension, up to a given scale (GUT or Planck), can be used to constrain its parameter space*

- We are analyzing the stability problem in one of the simplest extensions of the SM $SU(3)_c \times SU(2)_w \times U(1)_Y \times U(1)'$

- The spectrum of the SM is augmented by new states:
  - ✓ A gauge boson Z'
  - ✓ A complex scalar φ to achieve U(1)' SSB (vev ~ TeV) → *new heavy scalar*
  - ✓ SM singlet fermions to cancel U(1)' anomalies → *explains small neutrino masses through seesaw*

Vacuum stability in abelian extensions of SM

- Consider for instance the B-L *(Baryon – Lepton numbers)* extension

  ✓ The type-I seesaw is implemented in the Yukawa sector

$$\mathcal{L}_{yuk} = \mathcal{L}_{SM\ yuk} - Y_\nu\, L \cdot H \nu_R^c - Y_N\, \phi^\dagger\, \nu_R^c\, \nu_R^c$$

  ✓ The scalar potential is given by

$$V(H,\phi) = m_1^2 H^\dagger H + m_2^2 \phi^\dagger \phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (H^\dagger H)(\phi^\dagger \phi)$$

  ✓ The vacuum stability conditions are

$$\lambda_1 > 0\,, \quad \lambda_2 > 0\,, \quad 4\lambda_1\lambda_2 - \lambda_3^2 > 0$$

RGEs are employed to study vacuum stability up to the Planck scale

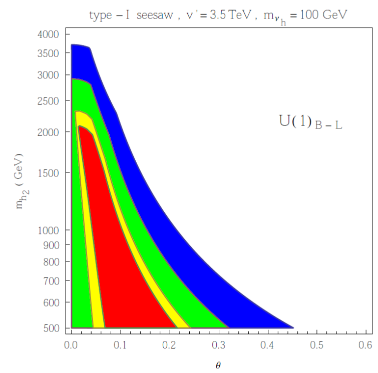
RGE@NLO

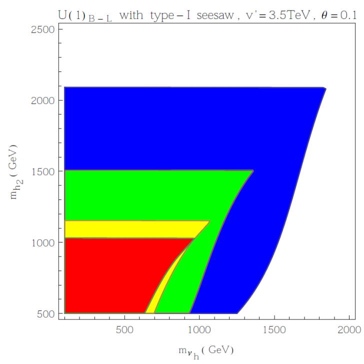
Mathematica package for the automatic computation and the numerical solution of the two-loop RGE in a general QFT

- **Input**: *Gauge group (SU, SO, Sp, G2, F4, E6, E7, E8),*
  Field (Superfield) content, (Irreps may be specified by Dynkin labels)
  Potential (Superpotential)

- **Output**: *β functions and anomalous dimensions at two-loop order*
  results are saved in Mathematica and LaTeX files
  a code for the numerical analysis is generated

Some preliminary results





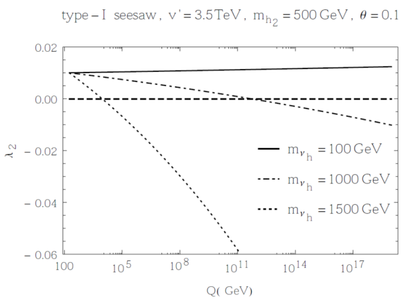
Regions in which the stability conditions are preserved up to

10^5 GeV (blue)

10^9 GeV (green)

10^{15} GeV (yellow)

10^{19} GeV (red)



m_{h2} - mass of the heaviest scalar

θ - mixing angle in the scalar sector

$m_{\nu h}$ - mass of the heavy neutrinos

Remarks

- ✓ The requirement of vacuum stability may constrain the parameter space of a model
- ✓ Different seesaw mechanisms can be analyzed in the context of vacuum stability and RG evolution
- ✓ The complete two-loop analysis requires the computation of effective potential, matching conditions at the EW scale and possible threshold corrections

Backup slides

The N=1 supersymmetric Lagrangian in the component formalism is

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + i\lambda^a\sigma^\mu\mathcal{D}^{ab}_\mu\bar\lambda^b + (\mathcal{D}^\mu_{ij}\phi_j)^\dagger(\mathcal{D}_{ik\,\mu}\phi_k) + i\chi_j\sigma_\mu\mathcal{D}^{\mu\,\dagger}_{ij}\bar\chi_i$$
$$-\sqrt{2}g\left(\bar\lambda^a\bar\chi_i T^a_{ij}\phi_j + \phi^\dagger_i T^a_{ij}\lambda^a\chi_j\right) - V(\phi,\phi^\dagger) - \frac{1}{2}\left(\chi_i\chi_j\mathcal{W}_{ij}(\phi) + h.c.\right)$$

The component expansion of the superfields are

$$\Phi_i = \phi_i + \sqrt{2}\theta\chi_i + \theta^2 F_i$$
$$W^a_A = \lambda^a_A + \theta_A D^a - (\sigma^{\mu\nu}\theta)_A F^a_{\mu\nu} + i\theta^2\,\sigma^\mu_{A\dot B}\mathcal{D}_\mu\bar\lambda^{a\,\dot B},$$
$$V^a = \theta\sigma^\mu\bar\theta A^a_\mu + \theta^2\bar\theta\bar\lambda^a + \bar\theta^2\theta\lambda^a + \frac{1}{2}\theta^2\bar\theta^2\left(D^a + i\partial_\mu A^{a\,\mu}\right)$$

The terms of improvement, necessary only for a scalar field, are

$$S^\mu_{I\,A} = \frac{4\sqrt{2}}{3}i\left[\sigma^{\mu\nu}\partial_\nu(\chi_i\phi^\dagger_i)\right]_A$$
$$T^{\mu\nu}_I = \frac{1}{3}\left(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\phi^\dagger_i\phi_i$$

These ensure the vanishing of the classical trace of the EMT and of the gamma-trace of the supercurrent for a classical superconformal theory

The two-point functions appearing in the Ward identities are

$$\Gamma^{(AA)}_{\mu\nu}(p) = -i\delta^{ab}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Sigma^{(AA)}(p^2)$$

$$\Gamma^{(\lambda\bar{\lambda})}_{A\dot{B}}(p) = i\delta^{ab}\, p_\mu \sigma^\mu_{A\dot{B}}\, \Sigma^{(\lambda\bar{\lambda})}(p^2)\,,$$

with

$$\Sigma^{(AA)}(p^2) = \frac{g^2}{16\pi^2}p^2\left\{T(R)\,\mathcal{B}_0(p^2,m^2) - T(A)\,\mathcal{B}_0(p^2,0)\right\}$$

$$\Sigma^{(\lambda\bar{\lambda})}(p^2) = \frac{g^2}{16\pi^2}\left\{T(R)\,\mathcal{B}_0(p^2,m^2) + T(A)\,\mathcal{B}_0(p^2,0)\right\}\,.$$

Two- and three-point scalar integrals

$$\mathcal{B}_0(p_1^2, m_0^2, m_1^2) = \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l+p_1)^2 - m_1^2)}\,,$$

$$\mathcal{C}_0((p+q)^2, p^2, q^2, m_0^2, m_1^2, m_2^2) = \frac{1}{i\pi^2}\int d^n l \frac{1}{(l^2 - m_0^2)((l-p)^2 - m_1^2)((l-p-q)^2 - m_2^2)}\,.$$

$$\mathcal{B}_0(p_1^2, m^2) \equiv \mathcal{B}_0(p_1^2, m^2, m^2) \qquad \mathcal{C}_0((p+q)^2, m^2) \equiv \mathcal{C}_0((p+q)^2, 0, 0, m^2, m^2, m^2)$$

tensor structures in the susy computation

$$
\begin{aligned}
s_1^{\mu\alpha} &= \sigma^{\mu\nu} k_\nu \, \sigma^\rho k_\rho \, \bar{\sigma}^{\alpha\beta} p_\beta \\
s_2^{\mu\alpha} &= 2 p_\beta \, \sigma^{\alpha\beta} \sigma^\mu \, .
\end{aligned}
$$

$$u^{\alpha\beta}(p,q) = \eta^{\alpha\beta} p \cdot q - p^\beta q^\alpha$$

$$
\begin{aligned}
t_{1S}^{\mu\nu\alpha\beta}(p,q) &\equiv \phi_1^{\mu\nu\alpha\beta}(p,q) = (\eta^{\mu\nu} k^2 - k^\mu k^\nu) u^{\alpha\beta}(p,q) \, , \\
t_{2S}^{\mu\nu\alpha\beta}(p,q) &\equiv \phi_3^{\mu\nu\alpha\beta}(p,q) = (p^\mu q^\nu + p^\nu q^\mu)\eta^{\alpha\beta} + p \cdot q(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) - \eta^{\mu\nu} u^{\alpha\beta}(p,q) \\
&- (\eta^{\beta\nu} p^\mu + \eta^{\beta\mu} p^\nu) q^\alpha - (\eta^{\alpha\nu} q^\mu + \eta^{\alpha\mu} q^\nu) p^\beta \, ,
\end{aligned}
$$

tensor structures in the qcd computation

$$
\begin{aligned}
\phi_1^{\mu\nu\alpha\beta}(p,q) &\equiv t_1^{\mu\nu\alpha\beta}(p,q) = (k^2 \eta^{\mu\nu} - k^\mu k^\nu) u^{\alpha\beta}(p,q) \, , \\
\phi_2^{\mu\nu\alpha\beta}(p,q) &\equiv t_3^{\mu\nu\alpha\beta}(p,q) + t_5^{\mu\nu\alpha\beta}(p,q) - 4 t_7^{\mu\nu\alpha\beta}(p,q) = -2 u^{\alpha\beta}(p,q)[k^2 \eta^{\mu\nu} + 2(p^\mu p^\nu + q^\mu q^\nu) \\
&- 4(p^\mu q^\nu + q^\mu p^\nu)] \, , \\
\phi_3^{\mu\nu\alpha\beta}(p,q) &\equiv t_{13}^{\mu\nu\alpha\beta}(p,q) = (p^\mu q^\nu + p^\nu q^\mu)\eta^{\alpha\beta} + p \cdot q(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) - \eta^{\mu\nu} u^{\alpha\beta}(p,q) \\
&- (\eta^{\beta\nu} p^\mu + \eta^{\beta\mu} p^\nu) q^\alpha - (\eta^{\alpha\nu} q^\mu + \eta^{\alpha\mu} q^\nu) p^\beta ,
\end{aligned}
$$