Dispersive Analysis of the π^0 Transition Form Factor

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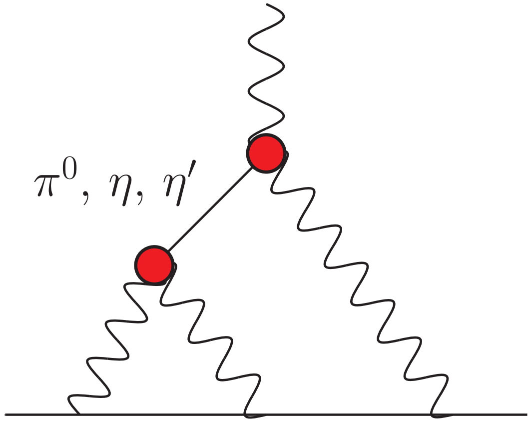
The π^0 transition form factor and $(g-2)_\mu$

- largest individual HLbL contribution: π^0 pole
  singly / doubly virtual form factors
  $F_{\pi^0\gamma\gamma^*}(q^2, 0)$ and $F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$



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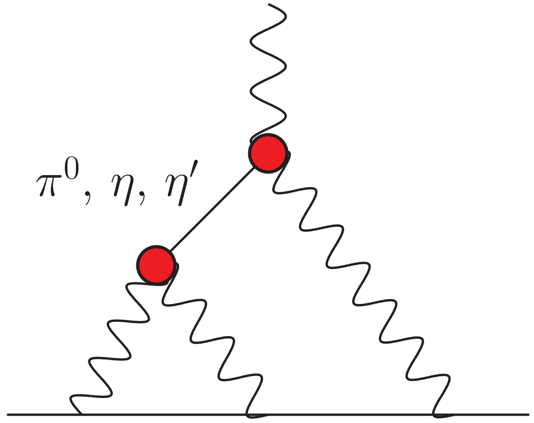

- normalisation fixed by Wess–Zumino–Witten anomaly:

$$F_{\pi^0\gamma\gamma}(0,0) = \frac{e^2}{4\pi^2 F_\pi}$$

F_π: pion decay constant $\longrightarrow$ measured at 1.5% level PrimEx 2011

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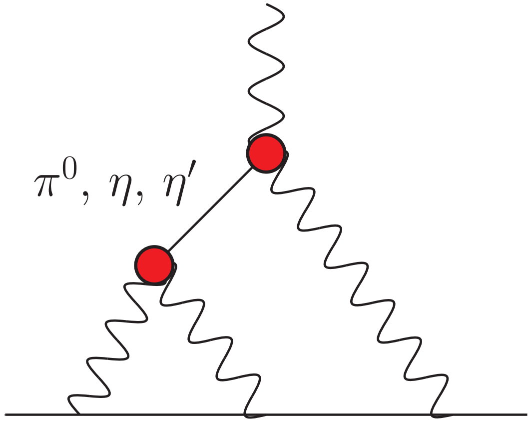

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- q_i^2-dependence: often analysed by vector-meson dominance
  $\longrightarrow$ what can we learn from analyticity and unitarity constraints?
  $\longrightarrow$ what experimental input sharpens these constraints?

Dispersive analysis of $\pi^0 \to \gamma^*\gamma^*$

- isospin decomposition:

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{vs}(q_1^2, q_2^2) + F_{vs}(q_2^2, q_1^2)$$

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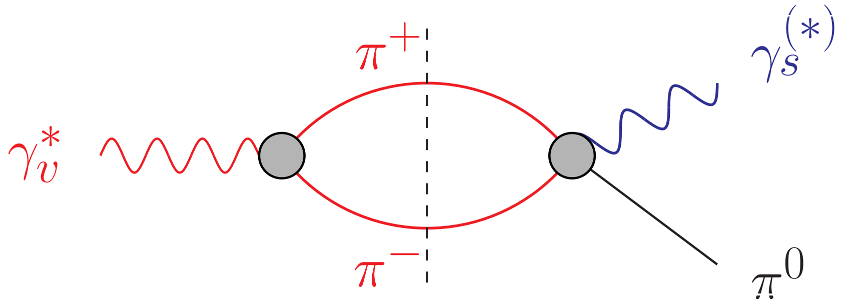
- analyze the leading hadronic intermediate states:

see also Gorchtein, Guo, Szczepaniak 2012



▷ isovector photon: 2 pions

$\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$

all determined in terms of pion–pion P-wave phase shift

+ Wess–Zumino–Witten anomaly for normalisation

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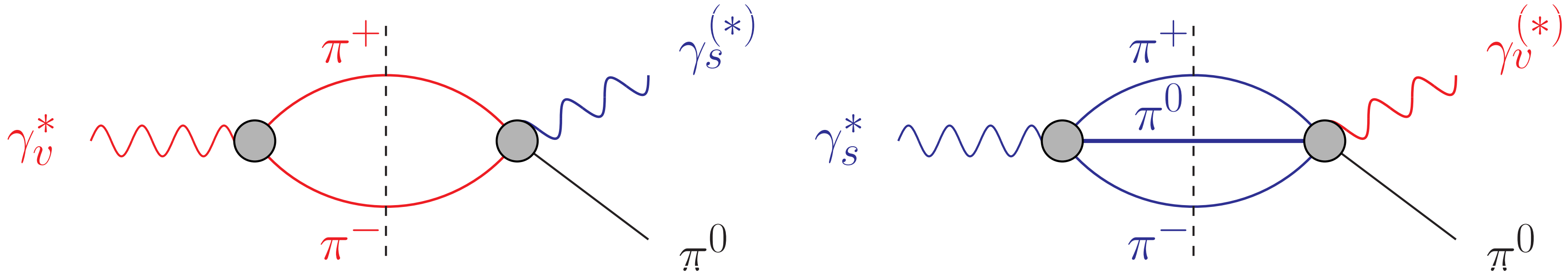

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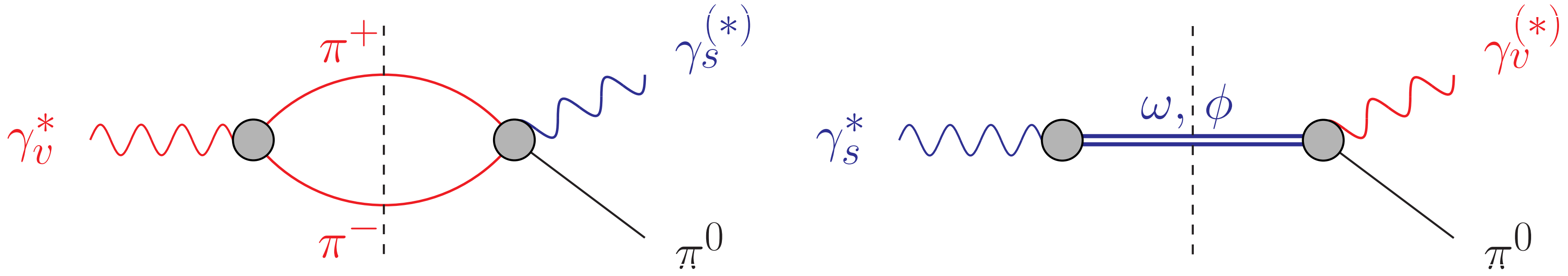

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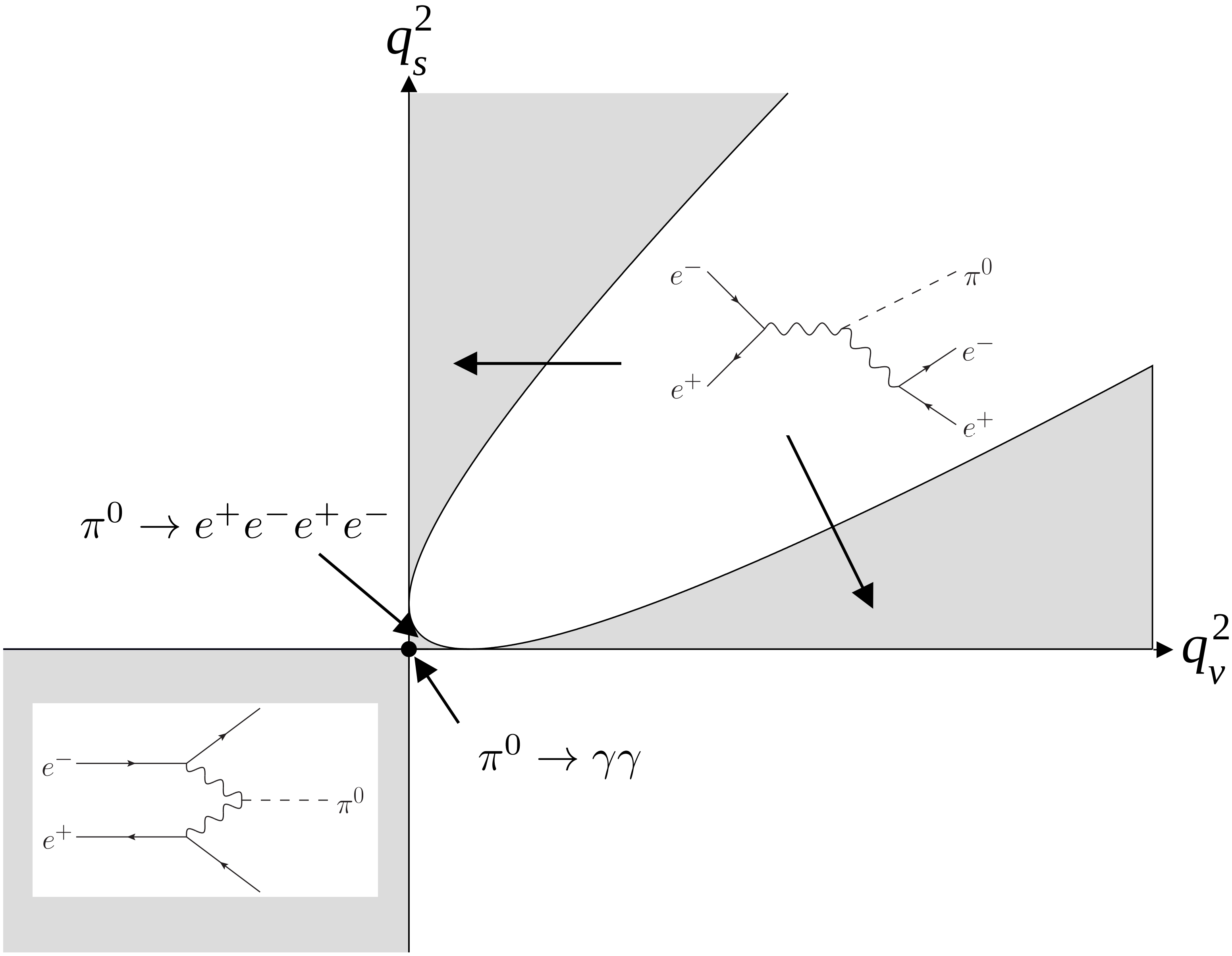
dominated by narrow resonances ω, ϕ

$\pi^0 \to \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor



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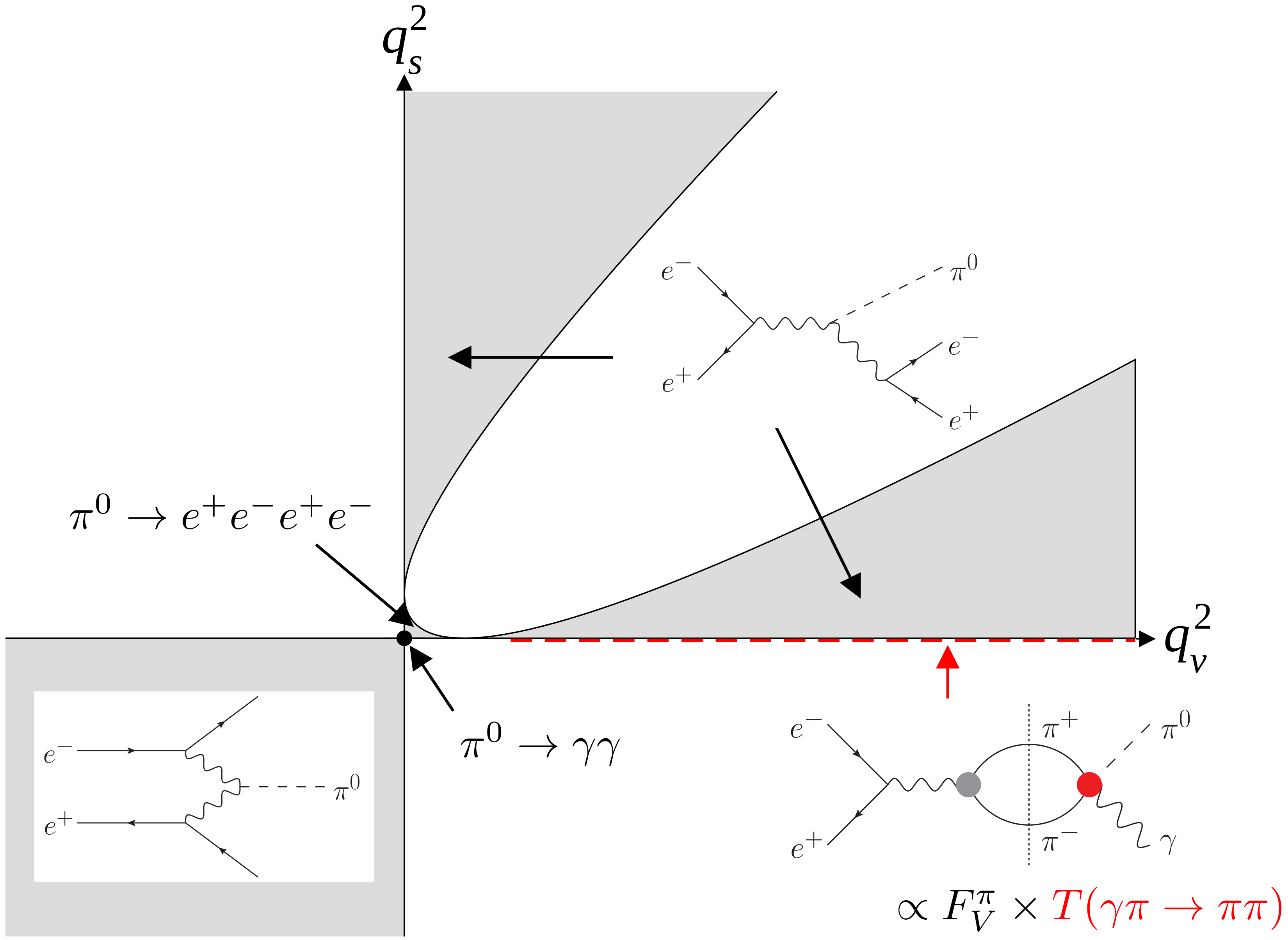

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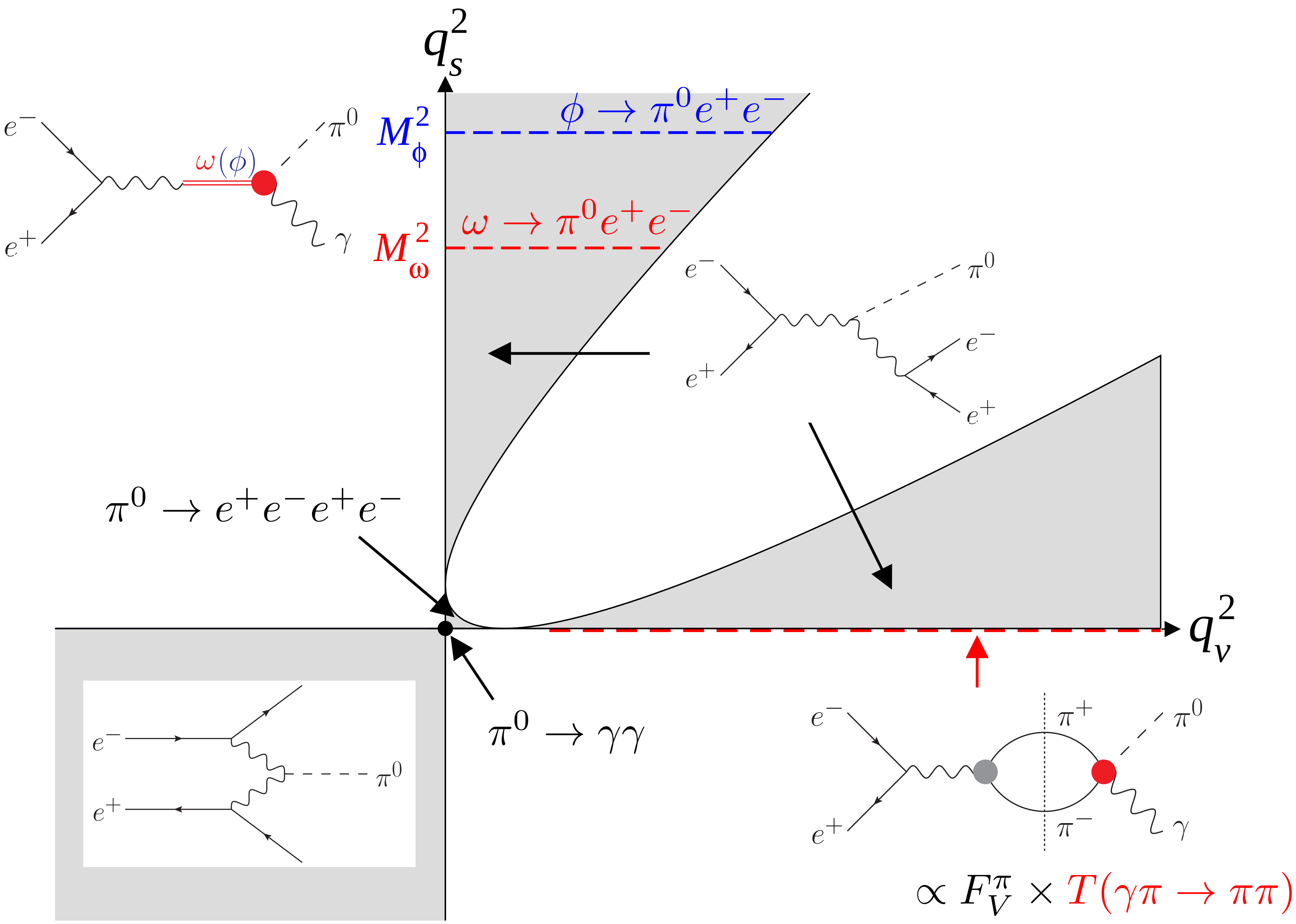
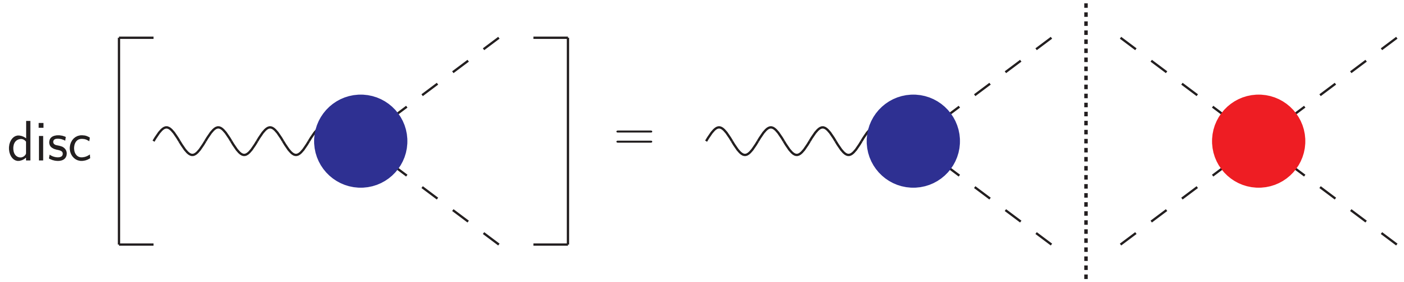


Warm-up: charged pion form factor



$$\frac{1}{2i}\,\text{disc}\,F_\pi^V(s) = \text{Im}\,F_\pi^V(s) \;=\; F_\pi^V(s)\times\theta(s-4\,M_\pi^2)\times\sin\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

$\longrightarrow$ final-state theorem: phase of $F_\pi^V(s)$ is just $\delta_1^1(s)$ Watson 1954

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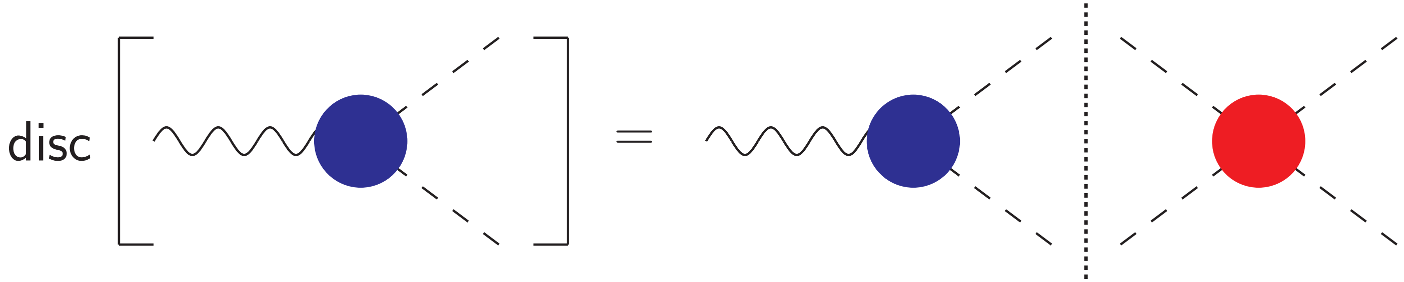

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- solution:

$$F_\pi^V(s) = P(s)\Omega(s)\;, \quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$P(s)$ polynomial, $\Omega(s)$ Omnès function — Omnès 1958

  - ▷ $\pi\pi$ phase shifts from Roy equations
    Ananthanarayan et al. 2001, García-Martín et al. 2011
  - ▷ $P(0)=1$ from symmetries (gauge invariance)

- inclusion of inelastic effects $\longrightarrow$ high-precision description of $e^+e^-\to\pi^+\pi^-$ and $\tau^-\to\nu_\tau\pi^-\pi^0$ — e.g. Hanhart 2012

Anomalous process $\gamma\pi \to \pi\pi$

- $\gamma\pi \to \pi\pi$ relatively simple system: odd partial waves
  $\longrightarrow$ P-wave phase shifts only (neglecting F- and higher)
- amplitude decomposed into single-variable functions

$$\mathcal{M}(s,t,u) = i\epsilon_{\mu\nu\alpha\beta} n^\mu p^\nu_{\pi^+} p^\alpha_{\pi^-} p^\beta_{\pi^0} \, \mathcal{F}(s,t,u)$$

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

representation exact up to three-loop corrections ($\mathcal{O}(p^{10})$)

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  representation exact up to three-loop corrections ($\mathcal{O}(p^{10})$)
- low-energy theorem / Wess–Zumino–Witten anomaly:

$$\mathcal{F}(0,0,0) = F_{3\pi} = \frac{e}{4\pi^2 F_\pi^3}$$

  $\longrightarrow$ $F_{3\pi}$ only verified experimentally to 10% accuracy

  Serpukhov 1987, Ametller et al. 2001

Dispersion relations for $\gamma\pi \to \pi\pi$

Unitarity relation for $\mathcal{F}(s)$:

$$\text{disc}\,\mathcal{F}(s) = 2i\{\underbrace{\mathcal{F}(s)}_{\text{right-hand cut}} + \underbrace{\hat{\mathcal{F}}(s)}_{\text{left-hand cut}}\} \times \theta(s - 4\,M_\pi^2) \times \sin\,\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

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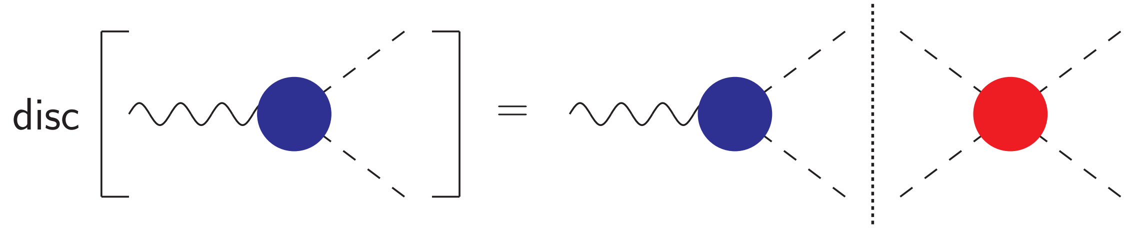
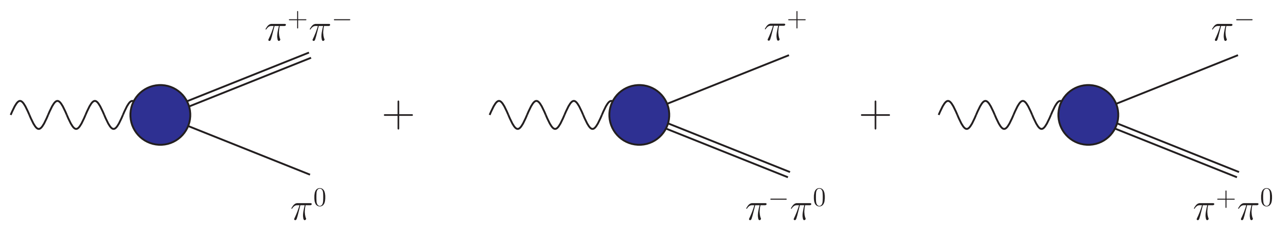


- right-hand cut only $\longrightarrow$ Omnès problem

$$\mathcal{F}(s) = P(s)\,\Omega(s)\;, \qquad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_1^1(s')}{s'-s}\right\}$$

$\longrightarrow$ amplitude given in terms of pion vector form factor

$\mathcal{F}(s,t,u) =$



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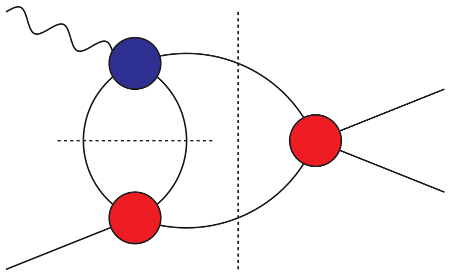
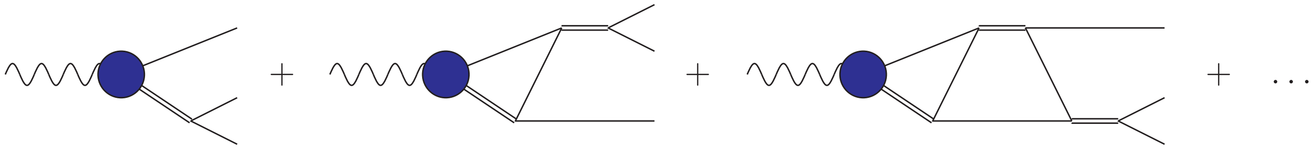


- inhomogeneities $\hat{\mathcal{F}}(s)$: angular averages over the $\mathcal{F}(t)$, $\mathcal{F}(u)$

$$\mathcal{F}(s) = \Omega(s)\left\{\frac{C_1}{3} + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$$\hat{\mathcal{F}}(s) = \frac{3}{2}\int_{-1}^{1} dz\,(1-z^2)\mathcal{F}\big(t(s,z)\big)$$

$\mathcal{F}(s) =$ 

$\gamma\pi \longrightarrow \pi\pi$: potential improvements

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- $\pi\pi$ P-wave phase shift as input
- $F_{3\pi} \simeq C_1 + \mathcal{O}(M_\pi^2)$, corrections controlled in ChPT $\longrightarrow \simeq 1\%$
- effect of inelasticities below $\sqrt{s} \approx 1\,$GeV?

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- high-accuracy data from Primakoff spectrum

  COMPASS

- twice-subtracted dispersive representation

  $\longrightarrow$ fit to data, extract

  $$F_{3\pi} \simeq C_2 = C_2^{(1)} + C_2^{(2)} M_\pi^2$$

  Hoferichter, BK, Sakkas 2012

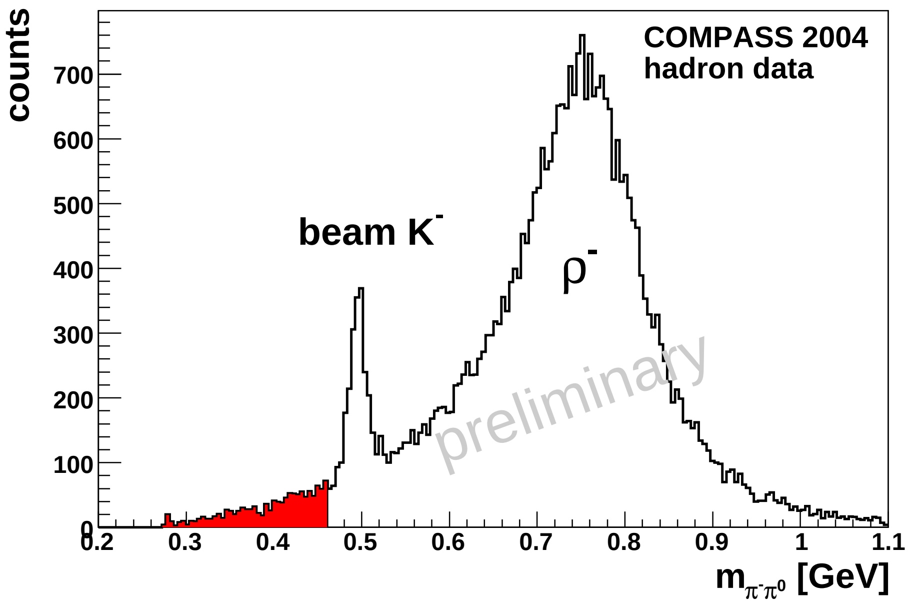


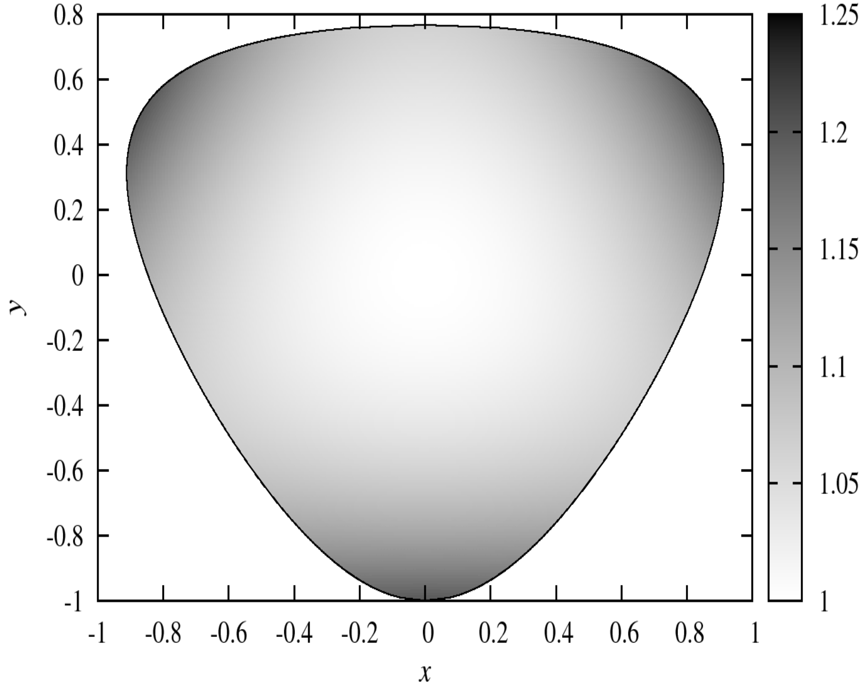
figure courtesy of T. Nagel 2009

Extension to decays: $\omega/\phi \to 3\pi$

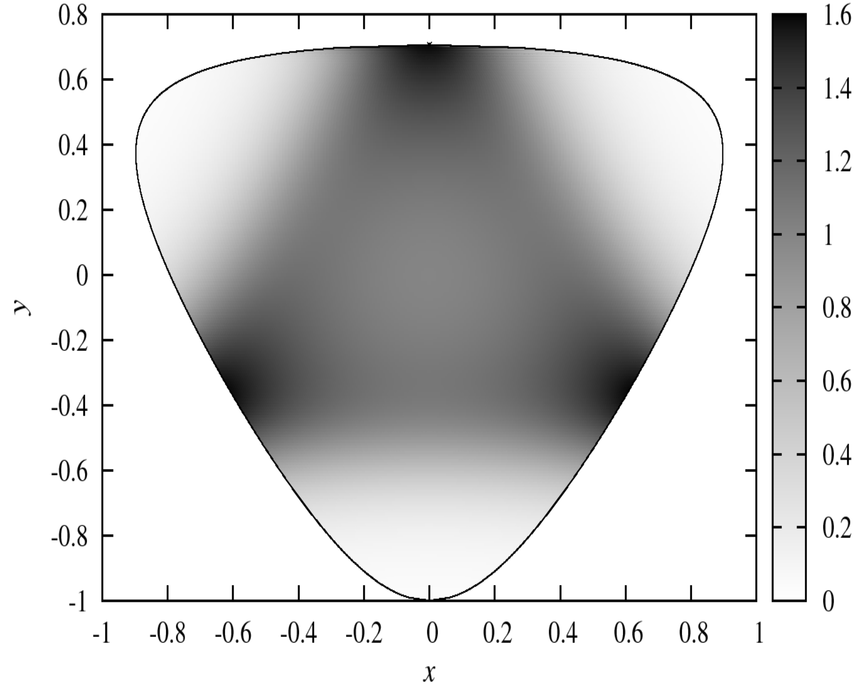
- identical quantum numbers to $\gamma\pi \to \pi\pi$
- fix subtraction constants $a_{\omega/\phi}$ to partial width(s) $\omega/\phi \to 3\pi$

  $\longrightarrow$ normalised Dalitz plot a prediction Niecknig, BK, Schneider 2012

$\omega \to 3\pi$:



$\phi \to 3\pi$:



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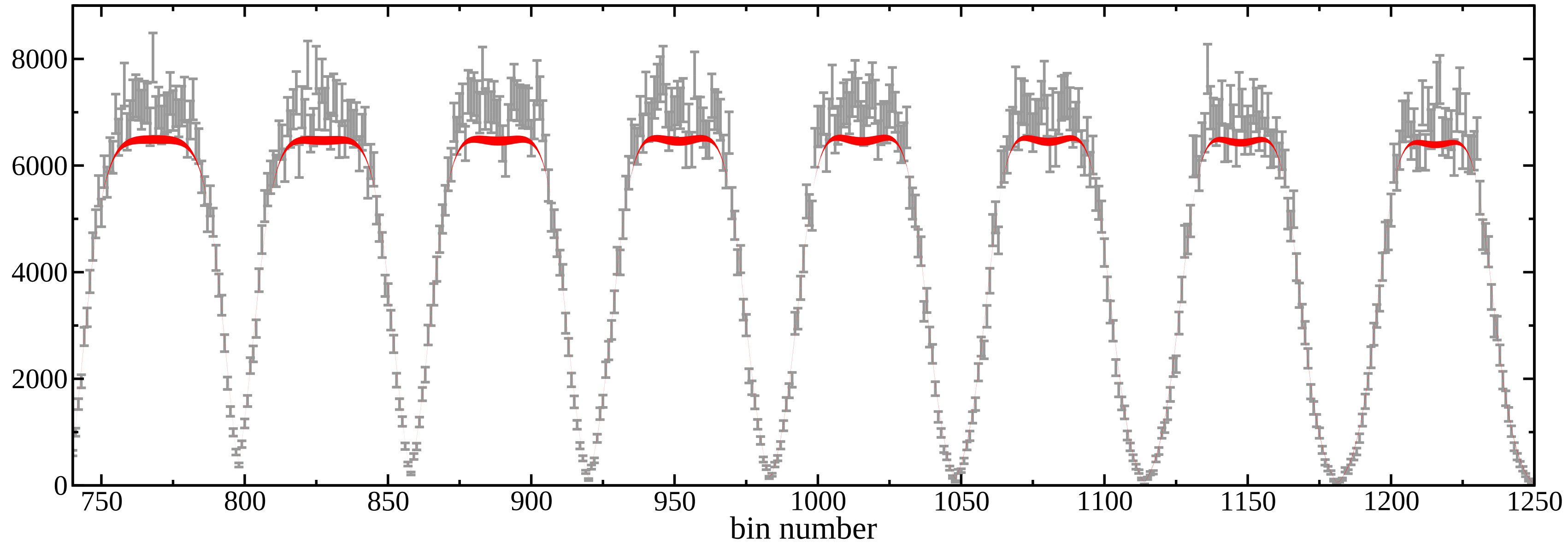
- test accuracy on KLOE Dalitz plot: $2 \cdot 10^6$ events, 1834 bins



	$\hat{\mathcal{F}} = 0$
χ^2/ndof	$1.71 \ldots 2.06$

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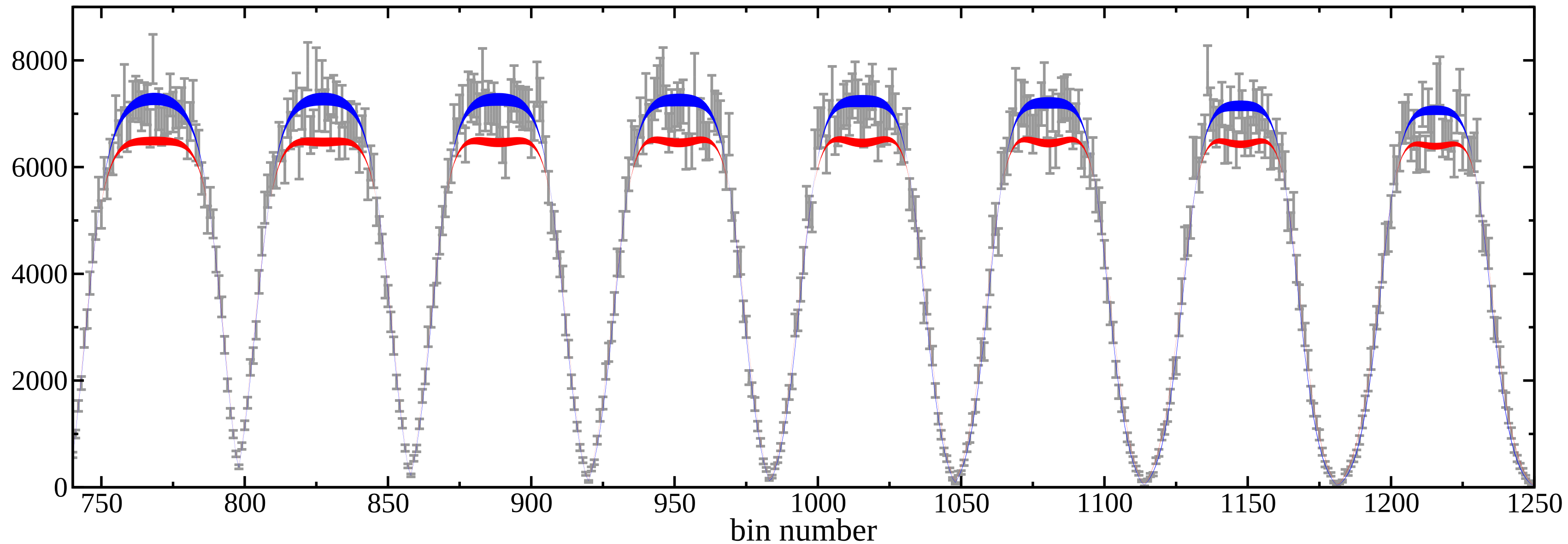

	$\hat{\mathcal{F}} = 0$	once-subtracted
χ^2/ndof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$

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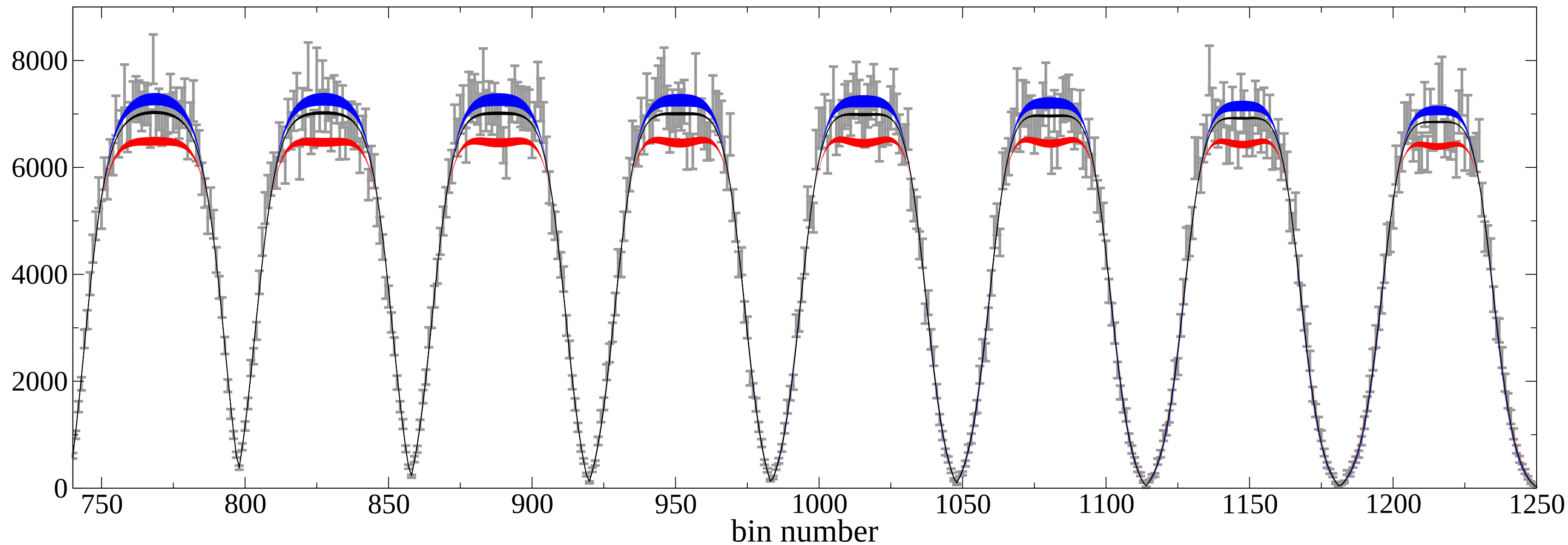

	$\hat{\mathcal{F}} = 0$	once-subtracted	twice-subtracted
χ^2/ndof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$	$1.02 \ldots 1.03$

Extension to decays: $\omega/\phi \to 3\pi$

- identical quantum numbers to $\gamma\pi \to \pi\pi$
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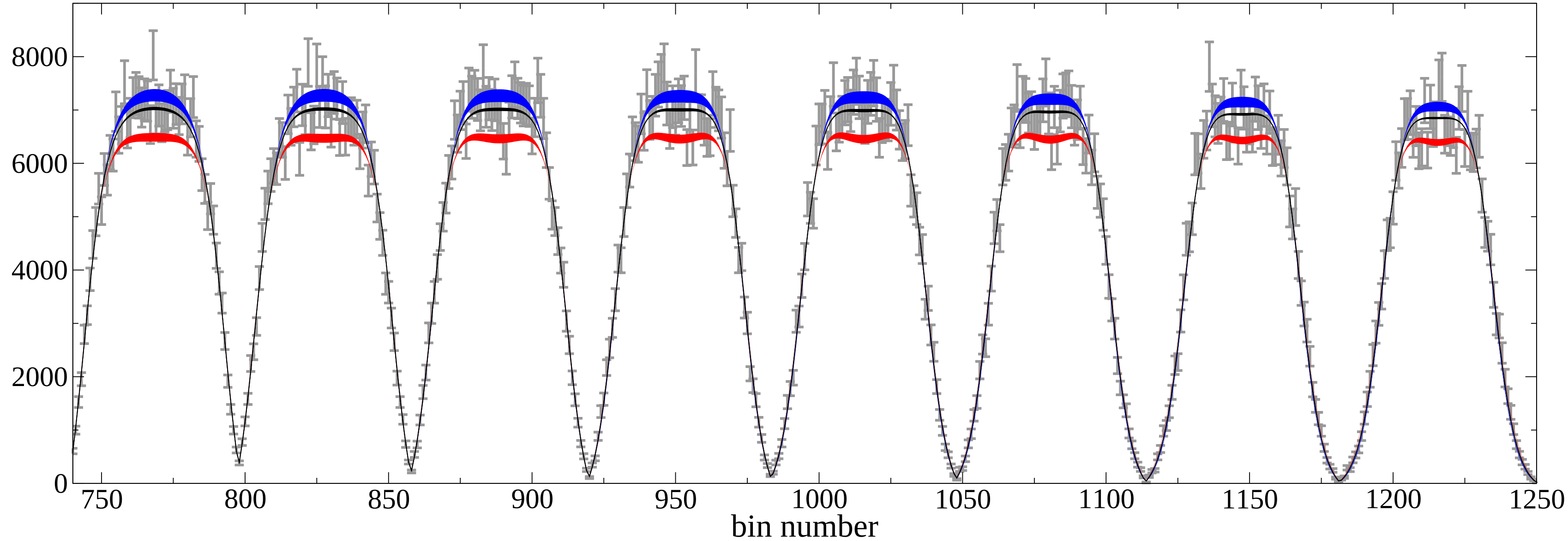
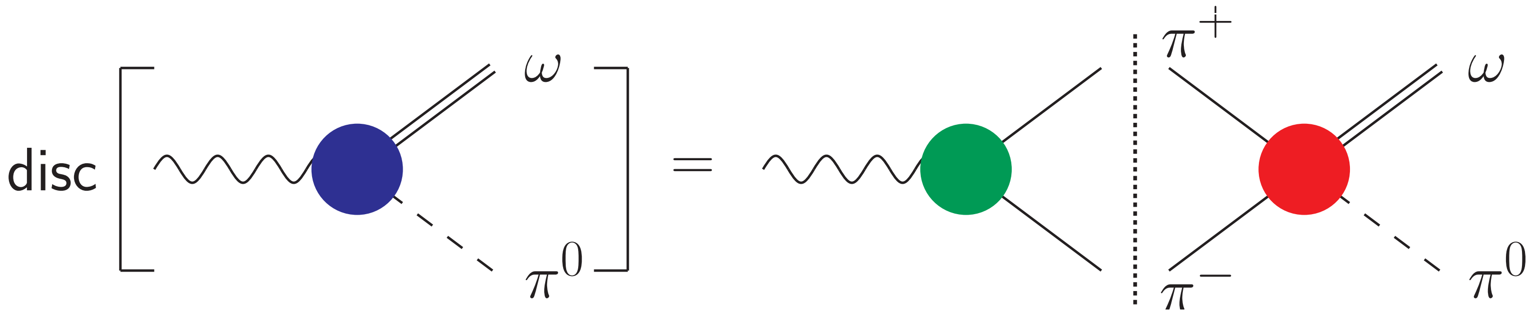


- second subtraction improves accuracy — Niecknig, BK, Schneider 2012
- $\omega \to 3\pi$ Dalitz plot? — KLOE, WASA-at-COSY, CLAS?

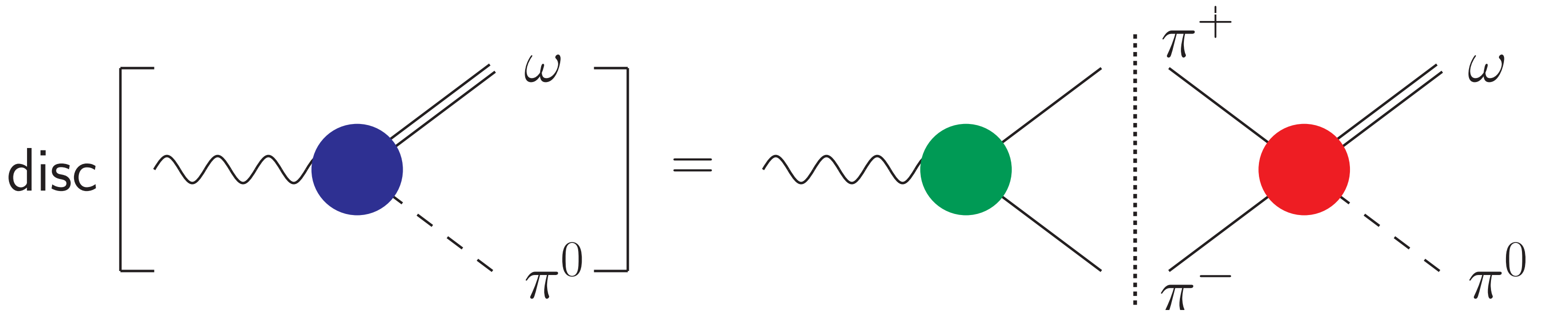
Transition form factor $\omega(\phi) \to \pi^0 \ell^+ \ell^-$



- ω transition form factor related to

  pion vector form factor $\times$ $\omega \to 3\pi$ decay amplitude

Transition form factor $\omega(\phi) \to \pi^0\ell^+\ell^-$



$$f_{\omega\pi^0}(s) = f_{\omega\pi^0}(0) + \frac{s}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s') F_\pi^{V*}(s') f_1(s')}{s'^{3/2}(s'-s)}$$ Köpp 1974

- $f_1(s) = f_1^{\omega\to 3\pi}(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$ P-wave projection of $\mathcal{F}(s,t,u)$

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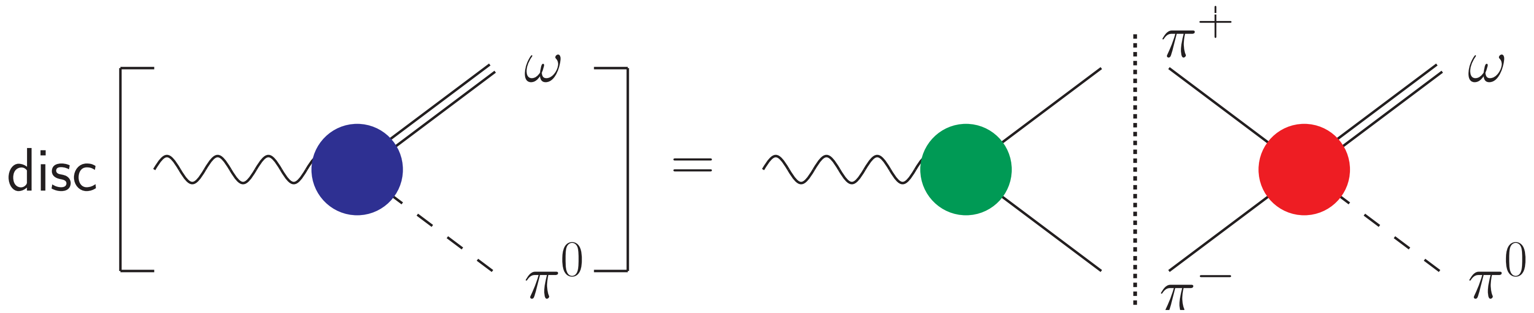

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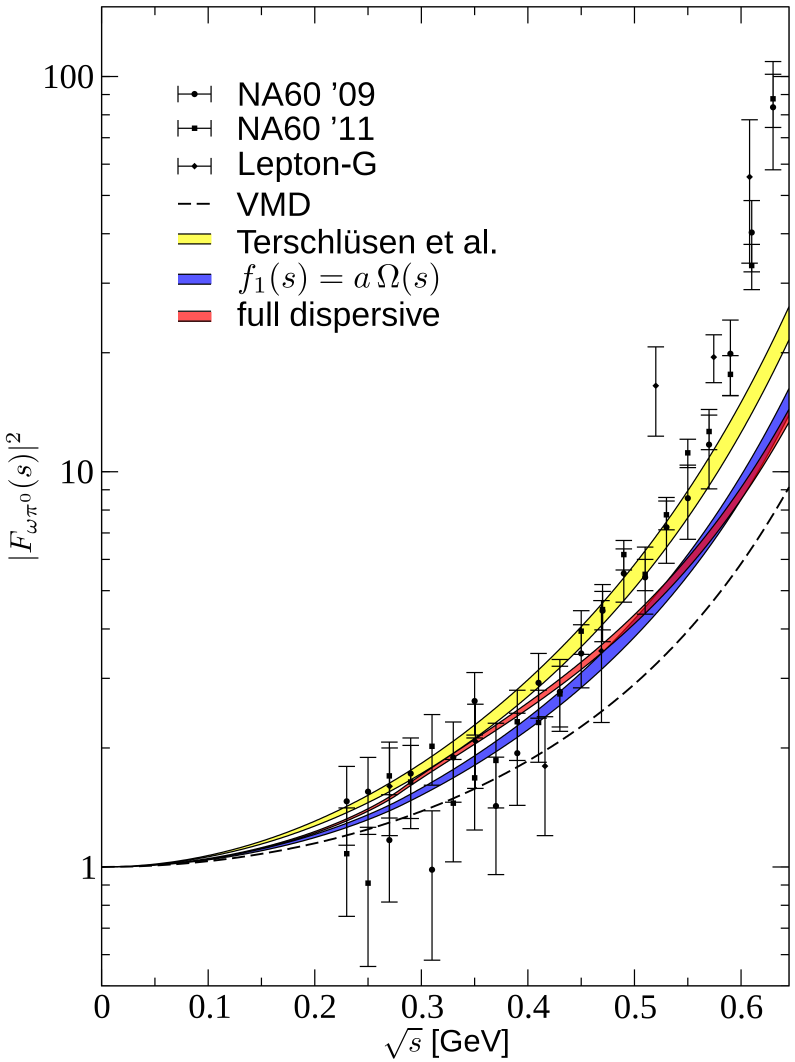
- $f_1(s) = f_1^{\omega\to 3\pi}(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$ P-wave projection of $\mathcal{F}(s,t,u)$
- sum rule for $\omega \to \pi^0\gamma \longrightarrow$ saturated at 90–95%

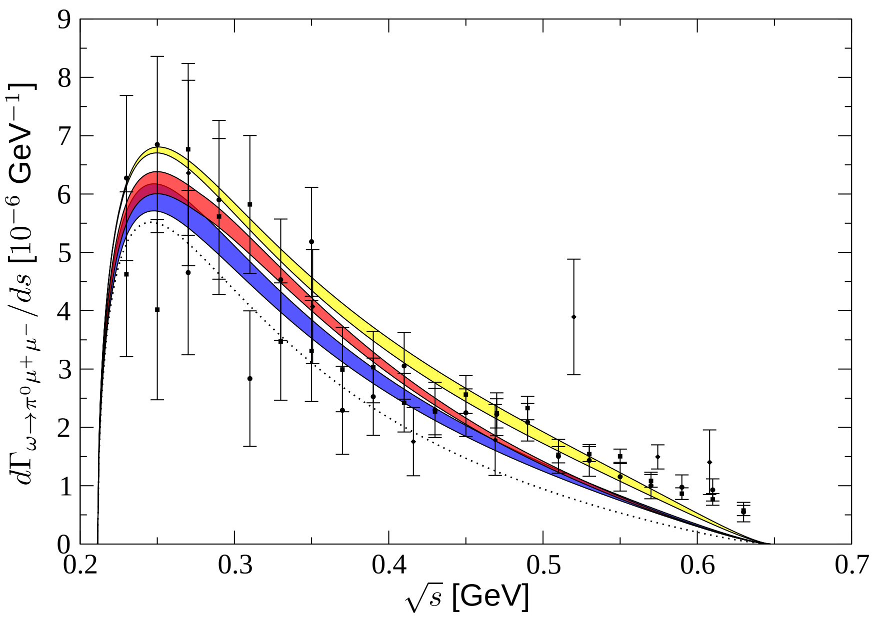
$$f_{\omega\pi^0}(0) = \frac{1}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')}{s'^{3/2}} F_\pi^{V*}(s') f_1(s') \,, \quad \Gamma_{\omega\to\pi^0\gamma} \propto |f_{V\pi^0}(0)|^2$$

$\longrightarrow$ expect better convergence for $\omega \to \pi^0\gamma^*$ transition form factor

Schneider, BK, Niecknig 2012

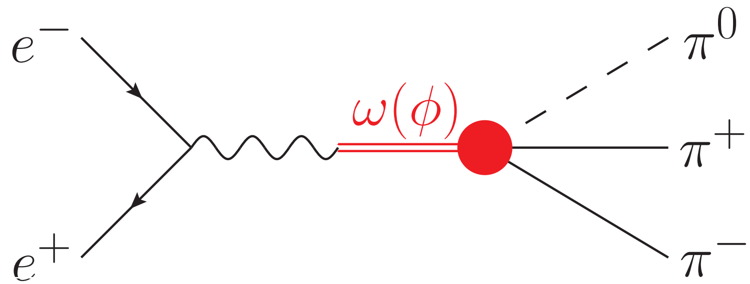
Numerical results: $\omega \to \pi^0 \mu^+ \mu^-$





- clear enhancement vs. naive vector-meson dominance
- incompatible with data (from heavy-ion coll.) NA60 2009, 2011
- more "exclusive" data?! CLAS?

One step further: $e^+e^- \to 3\pi$, $e^+e^- \to \pi^0\gamma^*$



- decay amplitude for $\omega/\phi \to 3\pi$: $\mathcal{M}_{\omega/\phi} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s) = a_{\omega/\phi}\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

$a_{\omega/\phi}$ adjusted to reproduce total width $\omega/\phi \to 3\pi$

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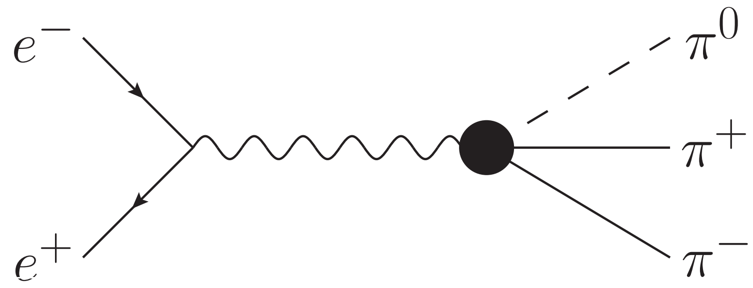

- decay amplitude for $e^+e^- \to 3\pi$: $\mathcal{M}_{e^+e^-} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s,q^2) = a_{e^+e^-}(q^2)\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s',q^2)}{|\Omega(s')|(s'-s)}\right\}$$

$a_{e^+e^-}(q^2)$ adjusted to reproduce spectrum $e^+e^- \to 3\pi$

contains 3π resonances $\longrightarrow$ no dispersive prediction

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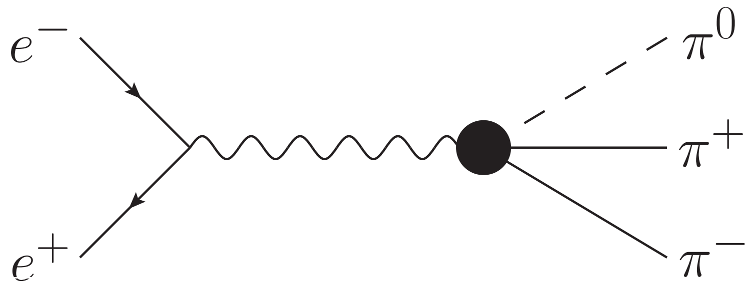

- decay amplitude for $e^+e^- \to 3\pi$: $\mathcal{M}_{e^+e^-} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s,q^2) = a_{e^+e^-}(q^2)\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s',q^2)}{|\Omega(s')|(s'-s)}\right\}$$

$a_{e^+e^-}(q^2)$ adjusted to reproduce spectrum $e^+e^- \to 3\pi$

- parameterisation:

$$a_{e^+e^-}(q^2) = \frac{F_{3\pi}}{3} + \beta\, q^2 + \frac{q^4}{\pi}\int_{\rm thr}^{\infty} ds'\,\frac{{\rm Im}BW(s')}{s'^2(s'-q^2)}$$

$$BW(q^2) = \sum_{V=\omega,\phi}\frac{c_V}{M_V^2 - q^2 - i\sqrt{q^2}\,\Gamma_V(q^2)}$$

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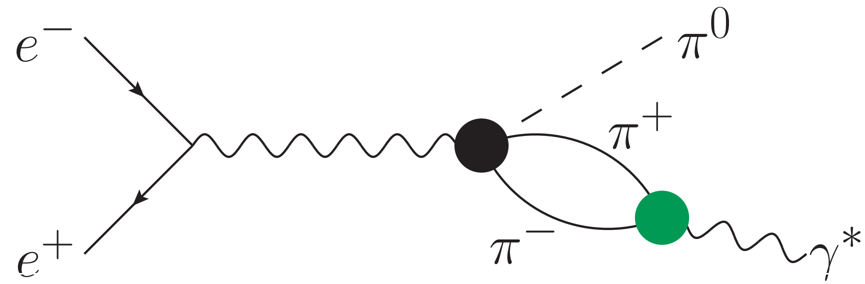

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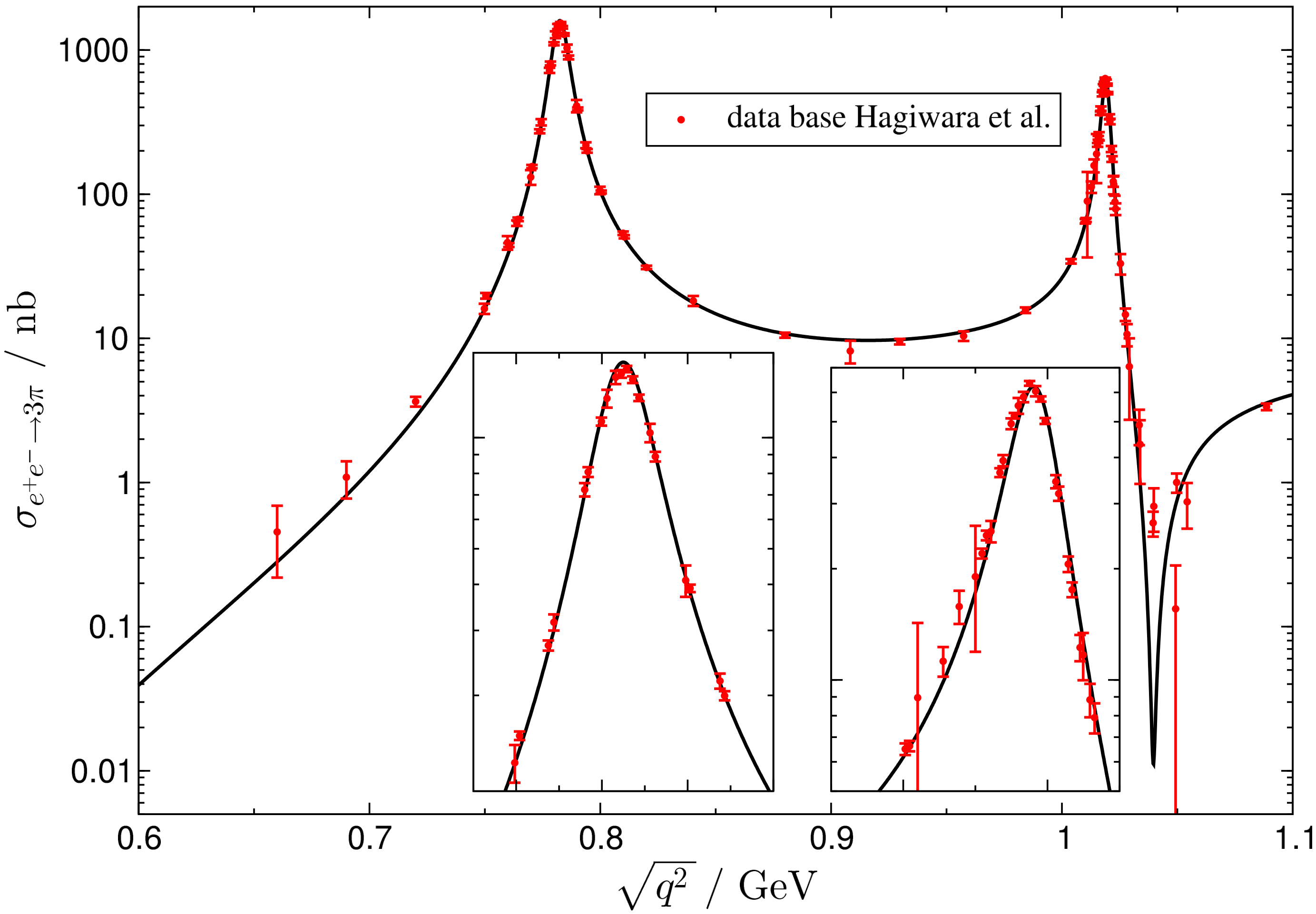
- fit to $e^+e^- \to 3\pi$ data $\longrightarrow$ prediction for $e^+e^- \to \pi^0\gamma^*$

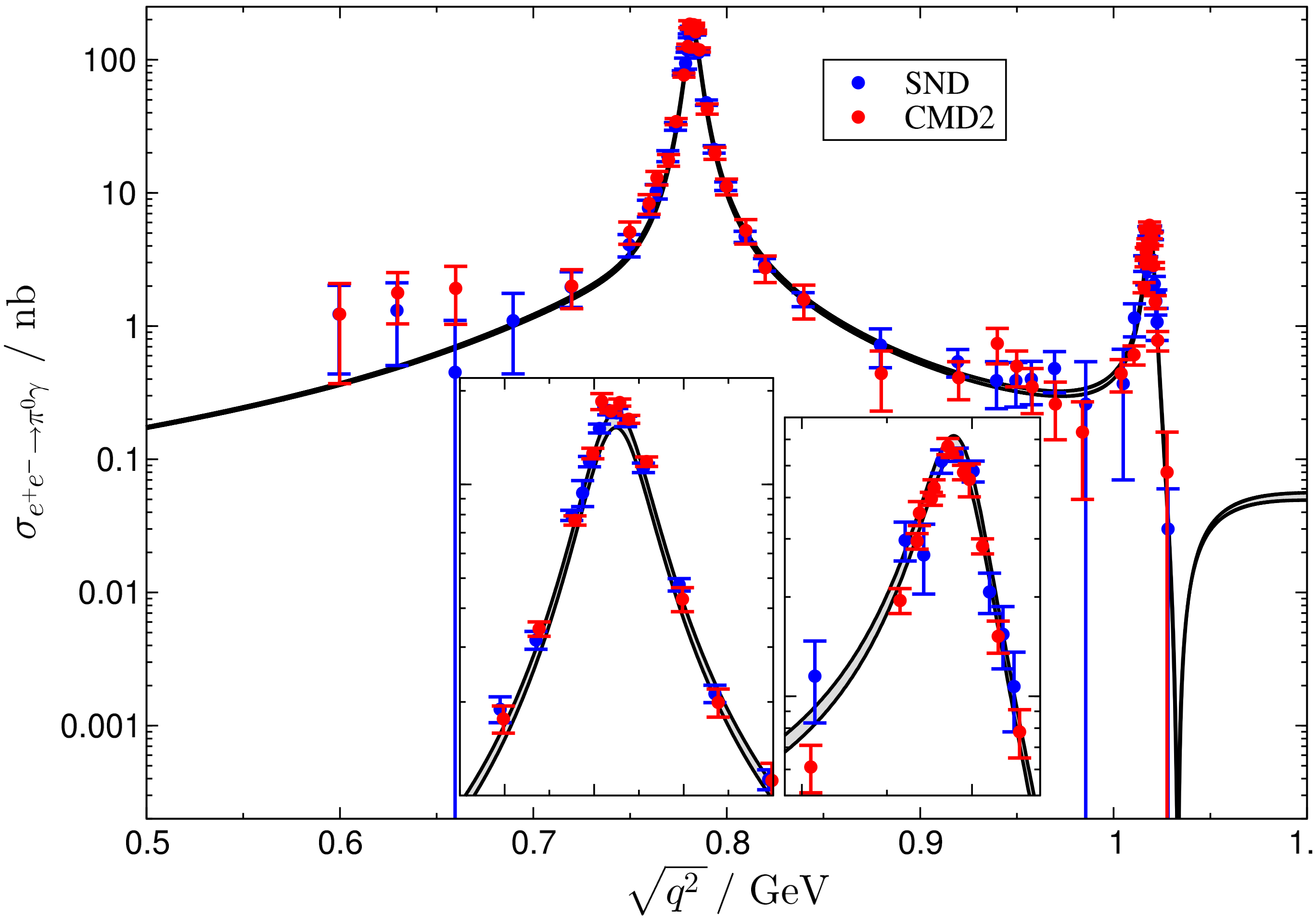
Fit to $e^+e^- \to 3\pi$ data



Hoferichter, BK, Leupold, Niecknig, Schneider, *in preparation*

- one subtraction/normalisation at $q^2 = 0$ fixed by $\gamma \to 3\pi$
- fitted: ω, ϕ residues, linear subtraction β

Comparison to $e^+e^- \to \pi^0\gamma$ data



Hoferichter, BK, Leupold, Niecknig, Schneider, *in preparation*

- "prediction"—no further parameters adjusted
- data well reproduced

Extension to spacelike region; slope

- continuation to spacelike region: use another dispersion relation

$$F_{\pi^0\gamma^*\gamma}(q^2,0) = F_{\pi\gamma\gamma} + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\operatorname{Im} F_{\pi^0\gamma^*\gamma}(s',0)}{s'(s'-q^2)}$$

$\longrightarrow$ work in progress; high-energy completion of the integral? convergence/uncertainties?

- sum rule for slope $\quad F_{\pi^0\gamma^*\gamma}(q^2,0) = F_{\pi\gamma\gamma}\left\{1 + a_\pi \frac{q^2}{M_{\pi^0}^2} + \mathcal{O}(q^4)\right\}$

$$a_\pi = \frac{M_{\pi^0}^2}{F_{\pi\gamma\gamma}} \times \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2} \operatorname{Im} F_{\pi^0\gamma^*\gamma}(s',0)$$

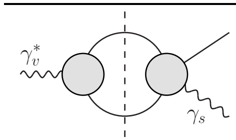
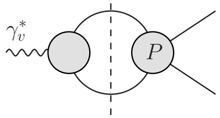
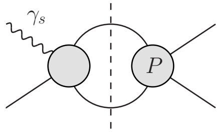
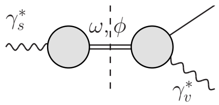
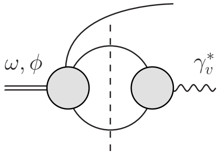
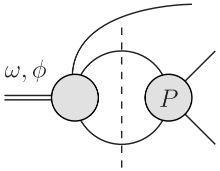
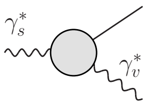
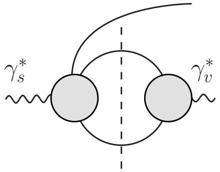
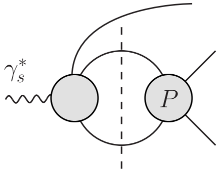
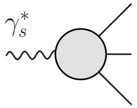
$$= (30.6 \pm 0.4) \times 10^{-3}$$

preliminary!

compare: $\quad a_\pi = (32 \pm 4) \times 10^{-3}$ PDG 2014

- theory error estimate: $\pi\pi$ phases and cutoff effects (in $\gamma^* \to 3\pi$ partial waves and $[\gamma^* \to 3\pi] \longrightarrow [\gamma^* \to \pi^0\gamma]$) only!

Summary: processes and unitarity relations for $\pi^0 \to \gamma^*\gamma^*$

process	unitarity relations	SC 1	SC 2
			$F_{\pi^0\gamma\gamma}$
		$F_{3\pi}$	$\sigma(\gamma\pi \to \pi\pi)$
			$\Gamma_{\pi^0\gamma}$
		$\Gamma_{3\pi}$	$\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
			$\sigma(e^+e^- \to \pi^0\gamma)$
		$\sigma(e^+e^- \to 3\pi)$	$\sigma(\gamma\pi \to \pi\pi)$ $\frac{d^2\Gamma}{ds\,dt}(\omega,\phi \to 3\pi)$
		$F_{3\pi}$	$\sigma(e^+e^- \to 3\pi)$

Colangelo, Hoferichter, BK, Procura, Stoffer 2014

$\gamma\pi \to \pi\pi$

$\omega \to 3\pi$, $\phi \to 3\pi$

$\gamma^* \to 3\pi$

common theme:
resum $\pi\pi$ rescattering

Outlook / future improvement

3-pion amplitudes $V \to 3\pi$

- normalisation fixed by anomaly $F_{3\pi}$ and widths $\Gamma(\omega, \phi \to 3\pi)$
- improved partial waves (2nd subtraction):

$$\gamma\pi \to \pi\pi \qquad \omega \to 3\pi \qquad \phi \to 3\pi$$

$\longrightarrow$ improved $e^+e^- \to 3\pi$ amplitudes via interpolation

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Vector meson transition form factors

- only realistic way (?) to test doubly virtual $F_{\pi^0\gamma^*\gamma^*}$ with precision
  - ▷ large deviations from data and VMD in $\omega \to \pi^0\gamma^*$
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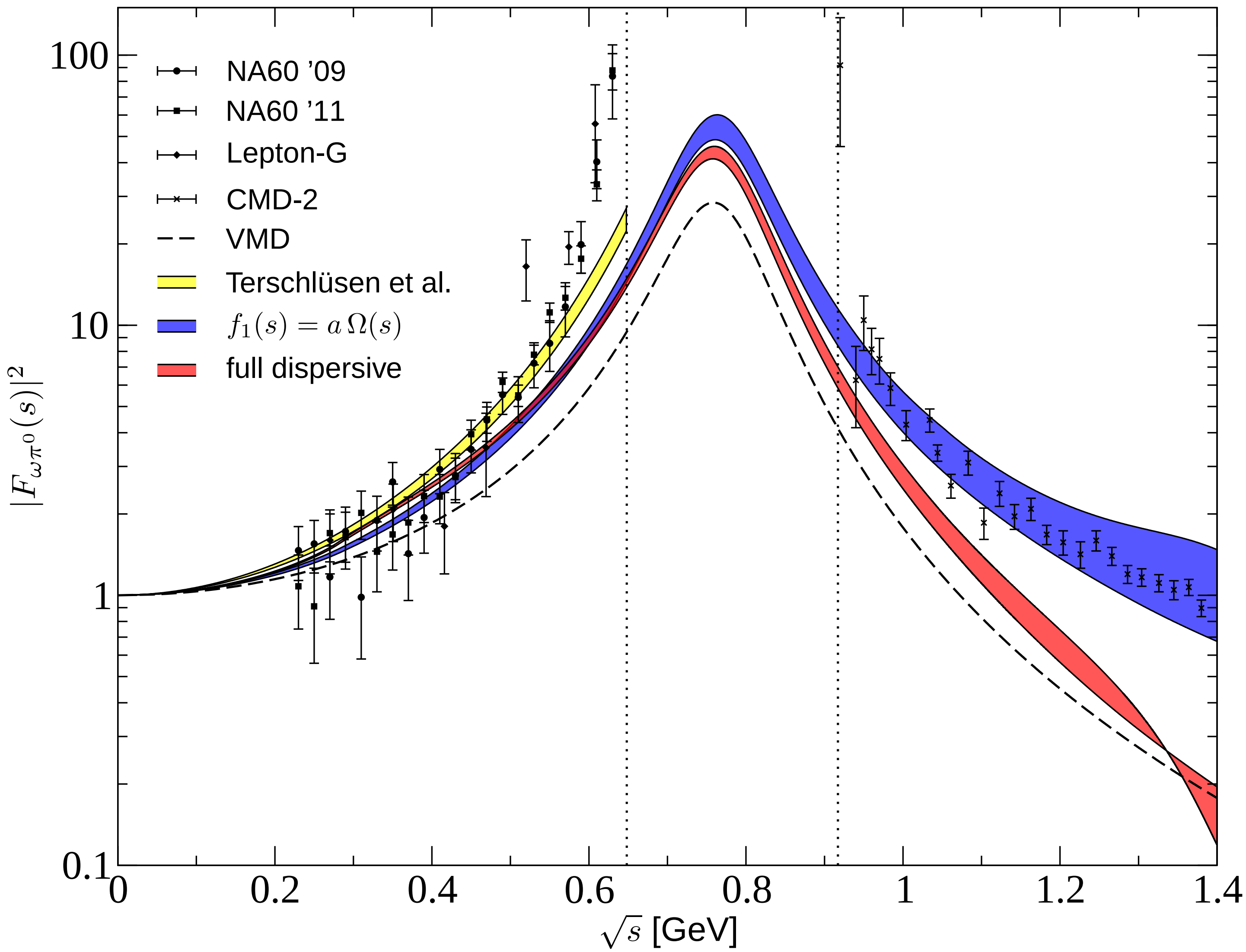
π^0 transition form factor

- successful description of $e^+e^- \to \pi^0\gamma$
  on the level of (unsubtracted) $\omega, \phi \to \pi^0\gamma$ sum rules
- doubly-virtual: use $e^+e^- \to \pi^0\gamma$ as subtraction function
  subtracted prediction for $e^+e^- \to \pi^0\gamma^*$ “safe”

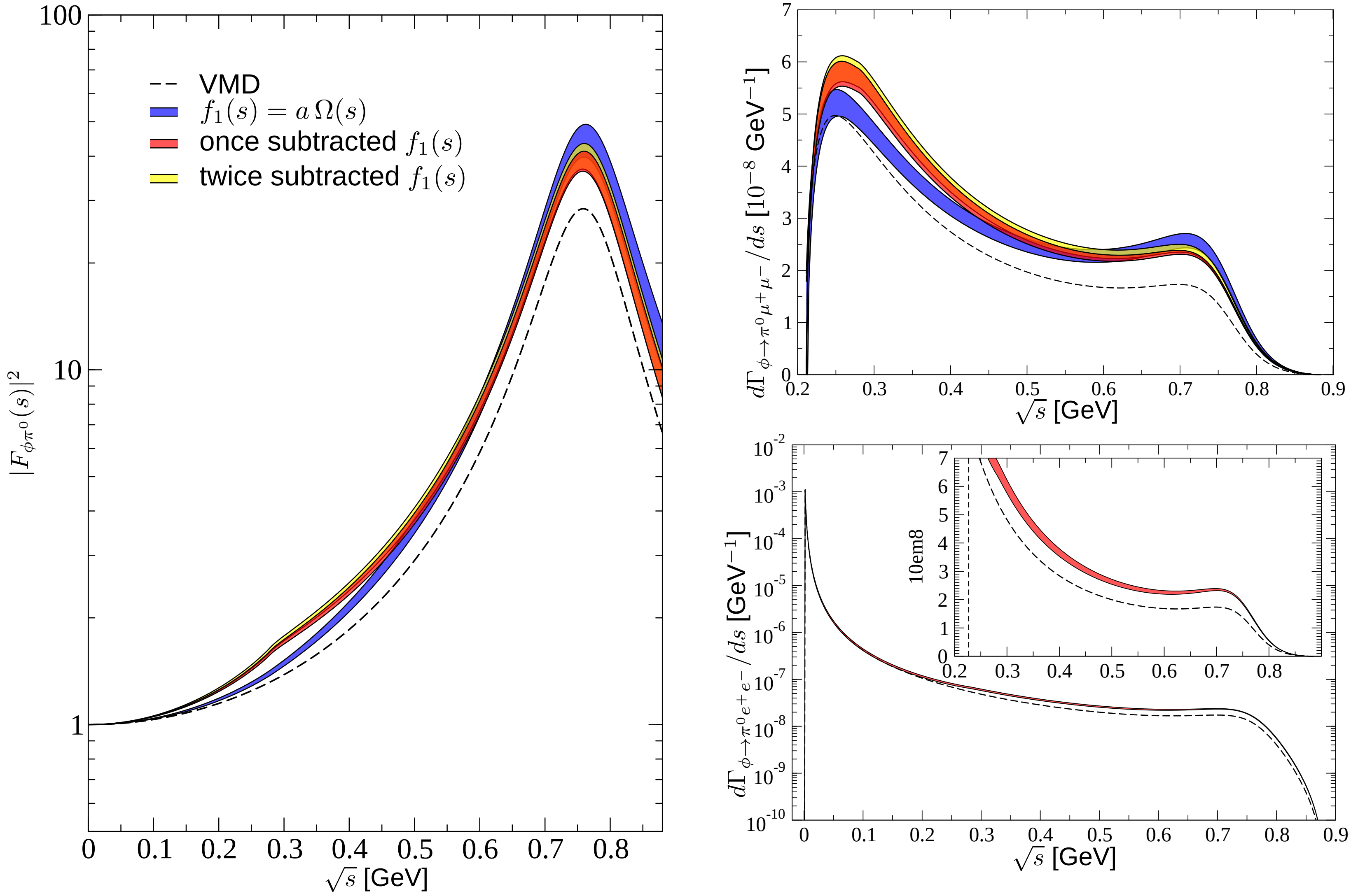
Spares

Naive extension to $e^+e^- \to \pi^0\omega$



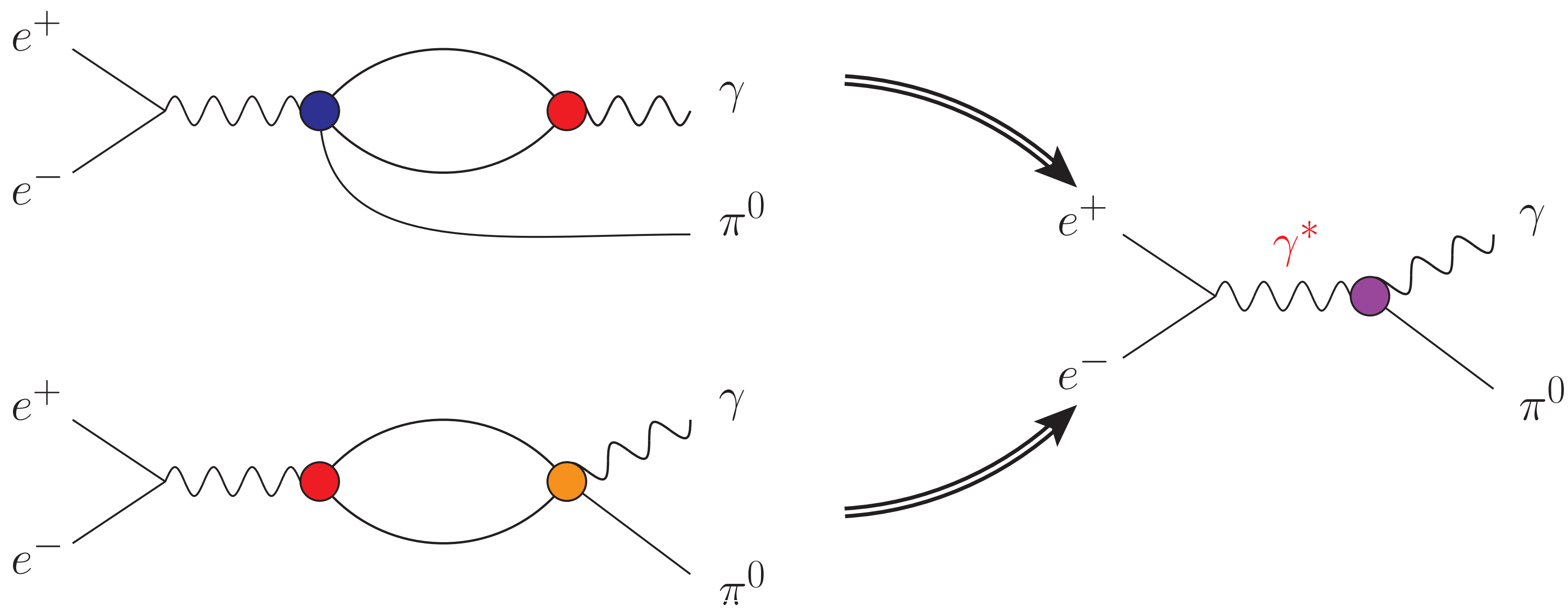
- data on $\omega \to \pi^0\mu^+\mu^-$ and $e^+e^- \to \pi^0\omega$ compatible???

Numerical results: $\phi \to \pi^0 \ell^+ \ell^-$



- measurement would be extremely helpful: ρ in physical region!
- partial-wave amplitude backed up by experiment

Towards a dispersive analysis of $e^+e^- \to \pi^0\gamma$



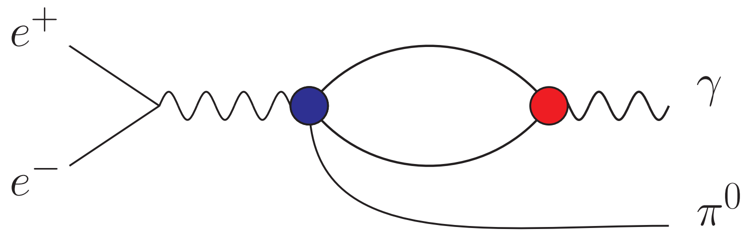
- combine isoscalar and isovector contribution to $e^+e^- \to \pi^0\gamma$

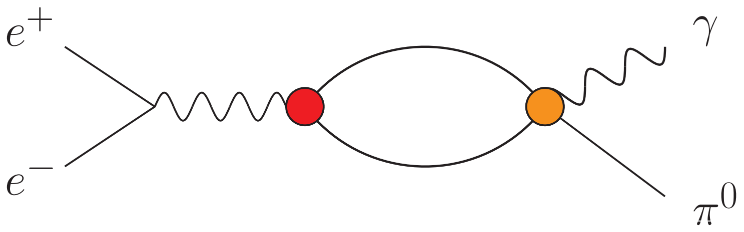
$$F_{\pi\gamma^*\gamma}(q^2,0) = F_{vs}(0,q^2) + F_{vs}(q^2,0)$$

$$= \frac{1}{12\pi^2}\int_{4M_\pi^2}^{\infty} ds' \frac{q_\pi^3(s')}{\sqrt{s'}} \left\{ \frac{f_1^{\gamma^*\to 3\pi}(q^2,s')}{s'} + \frac{f_1^{\gamma\pi\to\pi\pi}(s')}{s'-q^2} \right\} F_\pi^{V*}(s')$$

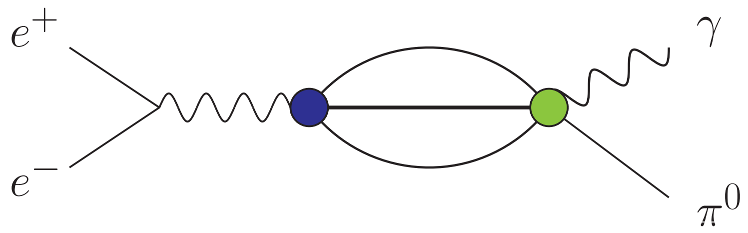
On the approximation for the 3-pion cut

Compare:





$\longrightarrow$ isoscalar contribution looks simplistic; why not instead



$\longrightarrow$ contains amplitude $3\pi \to \gamma\pi$

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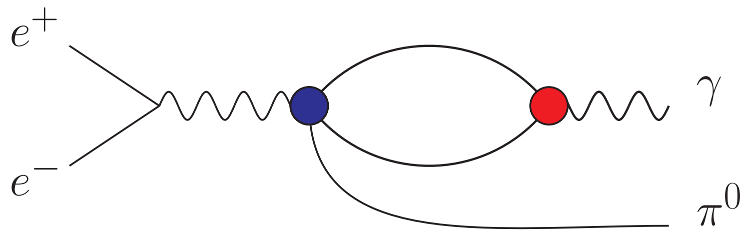

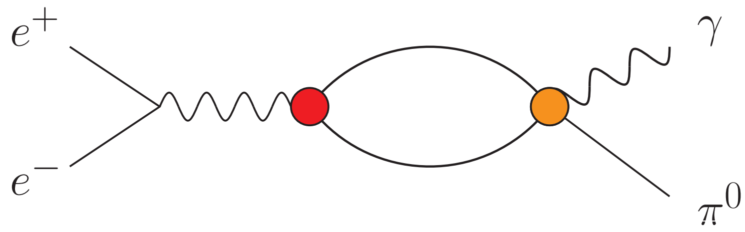

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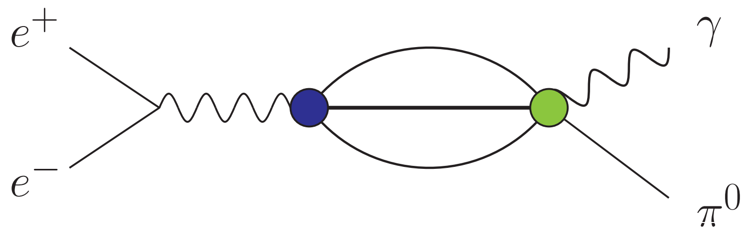

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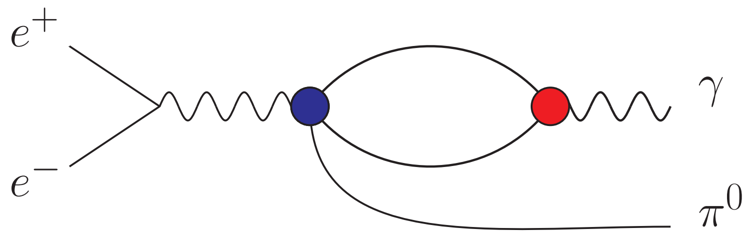
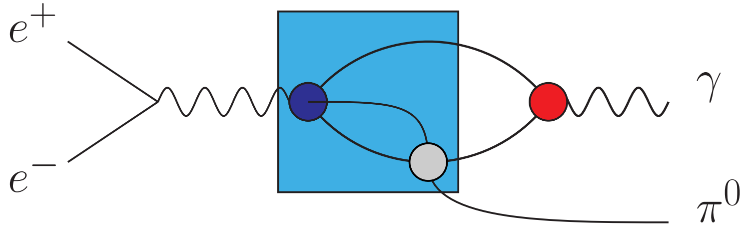
Our approximation:



includes



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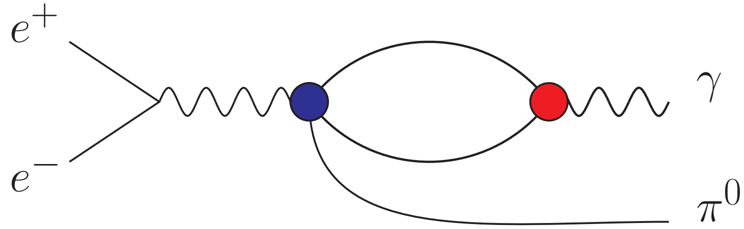

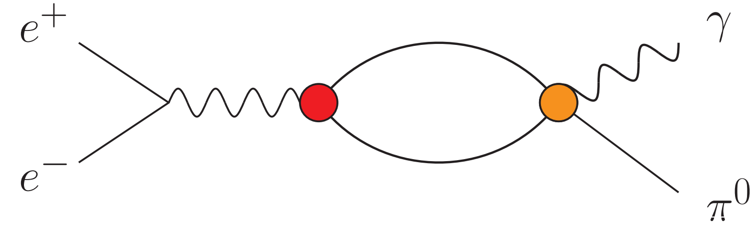

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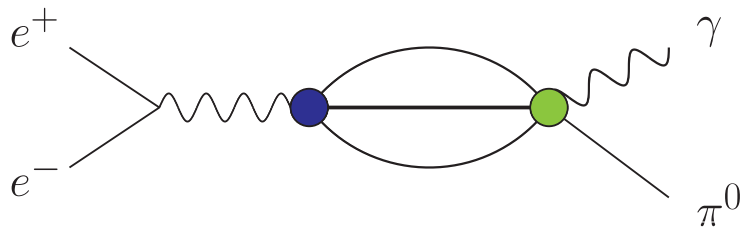

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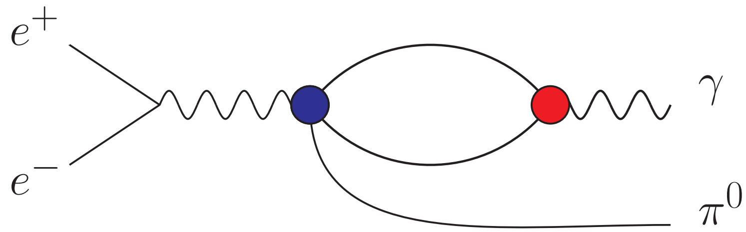

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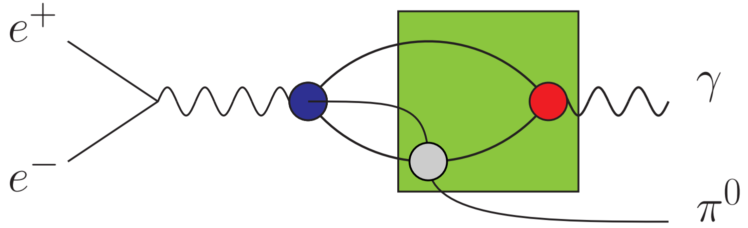

$\longrightarrow$ simplifies left-hand-cut structure in $3\pi \to \gamma\pi$ to pion pole terms