JÜLICH FORSCHUNGSZENTRUM

Dispersive approach to the eta transition form factor

Andreas Wirzba (IAS-4 / IKP-3, Forschungszentrum Jülich)

in collaboration with

C. Hanhart, A. Kupść, U.-G. Meißner, F. Stollenwerk, G.Tukhashvili, T. Dato

MesonNet Meeting | Frascati | 29 September – 1 October, 2014

PROLOGUE

SM results of anomalous magnetic moments $a = \frac{1}{2}(g-2)$

1 **Electron:** $a_e = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{had.}) \simeq a_e(\text{QED})$

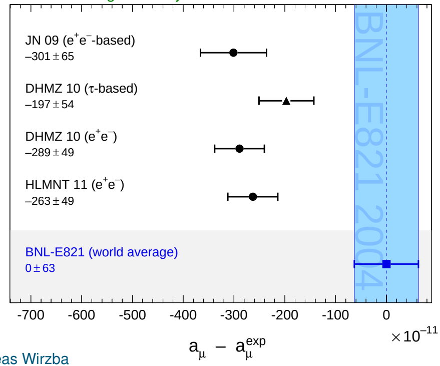
Experiment: 0.00115965218073(28) Hanneke et al., PRL **110** ('08),

Theory: 0.00115965218178(77) $\alpha/2\pi$ by Schwinger, PR **73** ('48),

11 digits! $\mathcal{O}(\alpha^5)$ by Aoyama et al., PRL **109** ('12).

2 **Muon:**

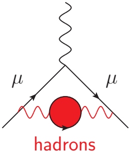
Fig. courtesy of PDG 2014



- visible deviation ($3.5\,\sigma$)
- " - " larger than $a_\mu(\text{weak})$
  $\longrightarrow$ New Physics ?
- $a_\mu^{\text{PDG}}(\text{had.}) = (6930 \pm 50) \cdot 10^{-11}$
  dominates uncertainty

Hadronic contribution to a_μ

1. Hadronic vacuum polarization:

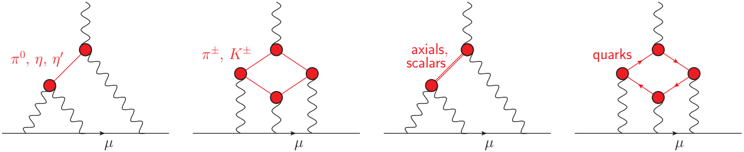


$a_\mu^{\text{PDG}}(\text{had. LO}) = (6923 \pm 42) \cdot 10^{-11}$

Goal: $< 0.3\%$ uncertainty

$\hookrightarrow$ to be fixed by experiment

2. Hadronic light-by-light:



$a_\mu(\text{had. light-by-light}) = (105 \pm 26) \cdot 10^{-11}$ (solely from models!)

J. Prades et al., arXiv:0901.0306

Model indep.: $\gamma^*\gamma^* \to \pi^0$ Hoferichter, Kubis, Leupold, Niecknig, Schneider;

$\gamma^*\gamma^* \to \pi\pi$ Colangelo, Hoferichter, Procura, Stoffer ('14);

$\gamma^*\gamma^* \to \eta$ this work.

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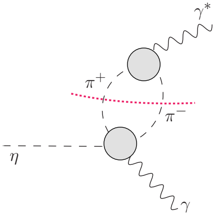
Ultimate goal: double off-shell form factor of $\eta \to \gamma^*\gamma^*$

1. Interim goal: the transition form factor of $\eta \to \gamma^*\gamma$



$\Longrightarrow$ $\eta \to \pi^+\pi^-\gamma$ transition amplitude

↪ FSI the same as in pion vector form factor $e^+e^- \to \pi^+\pi^-$

↪ Dispersion relation, universal and reaction-specific terms

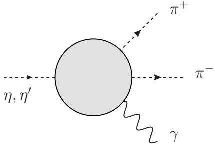
2. From the $\eta \to \pi^+\pi^-\gamma$ amplitude to the $\eta \to \gamma^*\gamma$ form factor
3. Results of slope and behavior of the transition form factor
4. Summary and outlook to $\eta \to \gamma^*\gamma^*$

F. Stollenwerk, C. Hanhart, A. Kupść, U.-G. Meißner & A.W., PLB **707** (2012) 184 arXiv:1108.2419v3 [nucl-th]

C. Hanhart, A. Kupść, U.-G. Meißner, F. Stollenwerk & A.W., EPJC **73** (2013) 2668 arXiv:1307.5654v2 [nucl-th]

RADIATIVE TWO–PION DECAY OF ETA & FSI

Basics of $\eta, \eta' \to \pi^+\pi^-\gamma$



- driven by chiral anomaly
- branching ratios

  $\eta \to \pi^+\pi^-\gamma$: $(4.22 \pm 0.08)\%$ PDG

  $\eta' \to \pi^+\pi^-\gamma$: $(29.1 \pm 0.5)\%$ PDG

- $\boxed{\begin{array}{lcl} C_{\eta,\eta'} & = & +1 \\ C_\gamma & = & -1 \end{array}}$ $\longrightarrow$ $\boxed{C_{\pi^+\pi^-} = -1}$

$\downarrow$

$\mathrm{L}_{\pi^+\pi^-}$ odd

$\downarrow$

- **Assumption**

  partial waves
  $\mathrm{L}_{\pi^+\pi^-} \geq 3$
  negligible

$\longrightarrow$

Pion quantum numbers

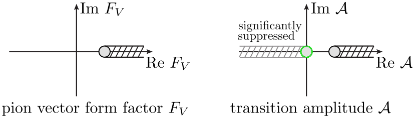
$$\mathrm{J}^\mathrm{P}(\pi^+\pi^-) = 1^- \ (\text{vector})$$

$$\mathrm{I}\ (\pi^+\pi^-) = 1 \ (\text{isovector})$$

Transition amplitude $\eta \to \pi\pi\gamma$

Same FSI & right-hand cut as pion vector FF F_V of $e^+e^- \to \pi^+\pi^-$

Dispersion rel. fixes F_V & $\mathcal{A}$ up to multiplicative analytic functions $\longrightarrow$



Left-hand cut of $\eta \to \pi^+\pi^-\gamma$

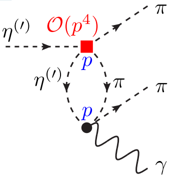
suppressed

chirally and

kinematically

because $\eta\pi$ in p-wave

Bernard et al. (1991); Kubis, Schneider (2009)



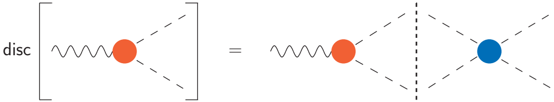
$\rightsquigarrow$ ansatz:

$$\mathcal{A}^{\eta}_{\pi\pi\gamma} = N\,P(s_{\pi\pi})\,F_V(s_{\pi\pi})$$

with $P(s_{\pi\pi})$ polynomial

Form factor constrained by analyticity and unitarity

for a single channel:



$$\mathrm{Im}\,F_{\mathrm{I}}(s) = \frac{1}{2i}\,\mathrm{disc}\,F_{\mathrm{I}}(s) = F_{\mathrm{I}}(s)\,\Theta(s-4m_\pi^2)\,\sin\delta_{\mathrm{I}}(s)\,e^{-i\delta_{\mathrm{I}}(s)} \quad \in \mathbb{R}\,(!)$$

$\hookrightarrow$ final-state theorem (Watson '54)

phase of $F_{\mathrm{I}}(s)$ is just $\delta_{\mathrm{I}}(s)$

$$\hookrightarrow\ \Omega_{\mathrm{I}}(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_{\mathrm{I}}(s')}{s'(s'-s-i\epsilon)}\right)$$

Omnès function (Omnès '58)

– fixed in elastic regime –

Data: B. Hyams et al., NPB **64** ('73) & **100** ('75)

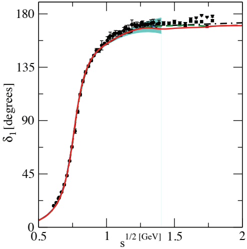
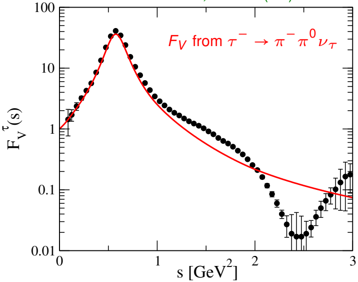


Fig. courtesy of C. Hanhart, arXiv:1203.6839

such that for negligible left-hand cuts: $\boxed{F_{\mathrm{I}}(s) = P_{\mathrm{I}}(s)\Omega_{\mathrm{I}}(s)}$

Universality of FSI (in pion vector FF and $\eta, \eta' \to \pi^+\pi^-\gamma$ spectra)

Data from Belle Coll., PRD **78** ('08) 072006



— $F_V(s) \leftarrow$ $\Omega(s)$ *p*-wave Omnès

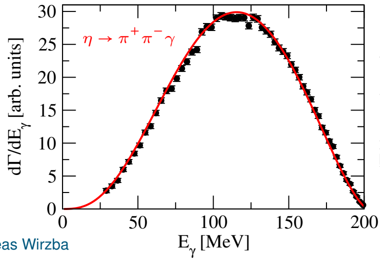
or $d\Gamma/dE_\gamma \propto |\Omega(s)|^2 \times$ kin. factors

$\hookrightarrow$ $\Omega(s)$ is universal with

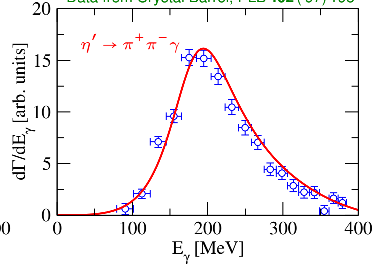
visible deviations above elastic reg.:

- higher thresholds
- inelastic resonances (ρ', ρ'')

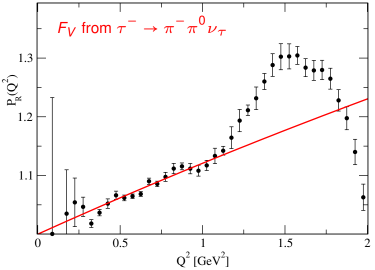
Data from KLOE/KLOE-2, PLB **718** ('13) 910



Data from Crystal Barrel, PLB **402** ('97) 195



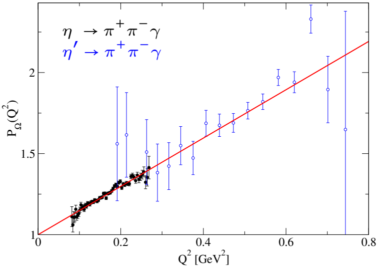
The residual P remaining after division by the Omnès function



$$P_R(Q^2) \equiv F_V(Q^2)/\Omega(Q^2)$$

- $P_R(Q^2)$ linear for $Q^2 < 1\,\mathrm{GeV}^2$:
  $P_R(Q^2) = 1 + \alpha_R Q^2$
- remaining deviations from ρ' & ρ''

C. Hanhart et al., EPJC **73** ('13) 2668



$$P_\Omega(Q^2) \equiv |\mathcal{A}_{\eta,\eta'}(Q^2)|/N_{\eta^{(\prime)}}\Omega(Q^2)$$

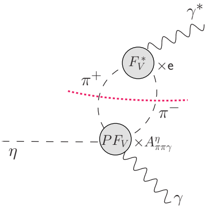
- $P_\Omega(Q^2)$ linear in both cases,
  at least for $Q^2 \lesssim (m_{\eta,\eta'} - 2m_\pi)^2$:
  $P_\Omega(Q^2) = (1 + \alpha_{\eta,\eta'} Q^2)$
- α_η, $\alpha_{\eta'}$ and normalizations N_η, $N_{\eta'}$
  reaction specific
- $\alpha_{\eta'} \simeq \alpha_\eta$ (1-loop ChPT & large N_c)

F. Stollenwerk et al., PLB **707** ('12) 184

TRANSITION FORM FACTOR

From $\eta \to \pi^+\pi^-\gamma$ to $\eta \to \gamma^*\gamma$

Parameter-free prediction of the isovector part of the transition FF



$$F_{\eta\gamma^*\gamma}(Q^2,0) \equiv 1 + \Delta F^{(I=1)}_{\eta\gamma^*\gamma}(Q^2,0) + \Delta F^{(I=0)}_{\eta\gamma^*\gamma}(Q^2,0)$$

$$\Delta F^{(I=1)}_{\eta\gamma^*\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^{\infty} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

$\Delta F^{(I=0)}_{\eta\gamma^*\gamma}$ (isoscalar part) needs to be modeled

Thus

- $\eta \to \gamma^*\gamma$ and $\eta \to \pi^+\pi^-\gamma$ are closely connected:
- FSI $\left(\Omega(Q^2)\text{ part of } F_V(Q^2)\right)$ universal, driven by ρ pole and residue
- $P(Q^2)$ and κ_η are reaction-dependent and modify $\eta \to \gamma^*\gamma$

C. Hanhart et al., EPJC **73** ('13) 2668, **arXiv:1307.5654v2**

The isoscalar contribution of the transition form factor

Modeled à la VMD (okay because of narrow ω and ϕ resonances)

$$\Delta F^{(I=0)}_{\eta\gamma^*\gamma}(Q^2,0) = \frac{w^{\eta}_{\omega\gamma}\, Q^2}{m_\omega^2 - Q^2 - i m_\omega \Gamma_\omega} + \frac{w^{\eta}_{\phi\gamma}\, Q^2}{m_\phi^2 - Q^2 - i m_\phi \Gamma_\phi}$$

from 'data' (isoscalar contribution isolated in exp. analysis) via

$$\sigma(e^+e^- \to \eta\gamma)\Big|^{\text{isoscalar}}_{s=m_V^2} = \frac{12\pi \mathrm{BR}(V\to\eta\gamma)\,\mathrm{BR}(V\to e^+e^-)}{m_V^2}, \quad V=\omega,\phi$$

with the weight factors (residua)

$w^{\eta}_{\omega\gamma} \approx (0.78 \pm 0.04) \times 1/8$ and $w^{\eta}_{\phi\gamma} \approx (0.75 \pm 0.03) \times (-2/8)$

where $1/8$ and $-2/8$ are the standard VMD values

see, e.g., L.G. Landsberg, Phys. Rep. **128** ('85) 301

$\hookrightarrow$ The isoscalar contribution is small for the η ($\sim 2\%$)!

Input for $\Delta F^{(I=1)}_{\eta\gamma^*\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^{\infty} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$

1. $P(s') = (1 + \alpha s')$ with $\alpha = (1.32 \pm 0.13)\,\mathrm{GeV}^{-2}$ from KLOE/-2
   D. Babusci et al., PLB **718** ('13) 910
2. $\kappa_\eta \equiv e f_\pi^2 A^\eta_{\pi\pi\gamma}/A^\eta_{\gamma\gamma}$ from data (tabulated branching ratios)
   Review of Particle Properties (**PDG**)
3. $F_V(s')$ (indeed only its modulus) from recent parametrization
   C. Hanhart, PLB **715** ('12) 170

(4) the upper cutoff Λ^2 of the s' integration (see next slide)

\+ the ca. 2% correction from $\Delta F^{(I=0)}_{\eta\gamma^*\gamma}(Q^2, 0)$ with

(5) the poles m_ρ, m_ϕ and the widths Γ_ρ, Γ_ϕ directly from **PDG** and

(6) the residua $w^\eta_{\omega\gamma}$ and $w^\eta_{\phi\gamma}$ of the modified VMD expression from the tabulated $\mathrm{BR}(\omega, \phi \to \eta\gamma)$ and $\mathrm{BR}(\omega, \phi \to e^+e^-)$.

Uncertainties of $\Delta F^{(I=1)}_{\eta\gamma^*\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right)\int_{4m_\pi^2}^{\Lambda^2} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$

1. Uncertainties in the input: especially for α in $P(s') = 1 + \alpha s'$
2. isoscalar piece: here we needed the VMD model (especially for the phases of the ω and ϕ contribution)
3. Large s' behavior of $P(s')$ and $F_V(s')$:

   FSI in $e^+e^- \to \pi^+\pi^-$ and $\gamma\eta \to \pi^+\pi^-$ different in inelastic regime (e.g. different couplings of ρ' and ρ'' in both reactions):

   $\hookrightarrow$ $\boxed{\Lambda = m_{\eta'} \ldots 2\,\text{GeV}}$

   cross check à la

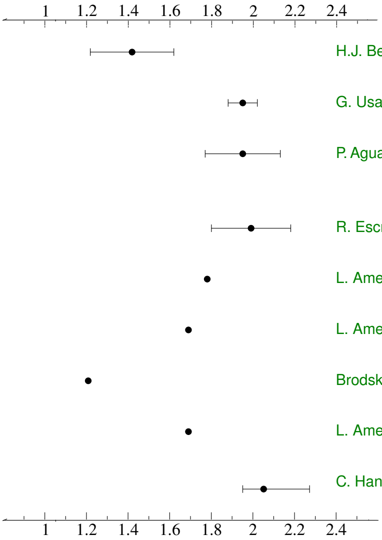
   S.P. Schneider et al., PRD **86** ('12) 054013; M. Hoferichter et al., PRD **86** ('12) 116009:

$$A^{\eta\,(I=1)}_{\gamma\gamma} = \frac{eA^\eta_{\pi\pi\gamma}}{96\pi^2}\int_{4m_\pi^2}^{\Lambda^2} ds' \sigma_\pi(s')^3 P(s') |F_V(s')|^2$$

   overestimates the experimental value by $(7 \pm 5)\%$ for $\Lambda = 1\,\text{GeV}$.

RESULTS

Results: slope parameter $b_\eta \equiv \frac{d}{dQ^2} F_{\eta\gamma^*\gamma}(Q^2, 0)\Big|_{Q^2=0}$



Recent experiments

$e^+e^- \to e^+e^-\eta$ (CELLO) — H.J. Behrend et al., ZPC **49** ('91) 401

$\eta \to \mu^+\mu^-\gamma$ (NA60) — G. Usai et al., NPA **855** ('11) 189

$\eta \to e^+e^-\gamma$ (A2) — P. Aguar-Bartolome et al., PRC**89**('14)044608

Theory

Padé approx. fit to data — R. Escribano et al., PRD **89** ('14) 034014

VMD — L. Ametller et al., PRD **45** ('92) 986

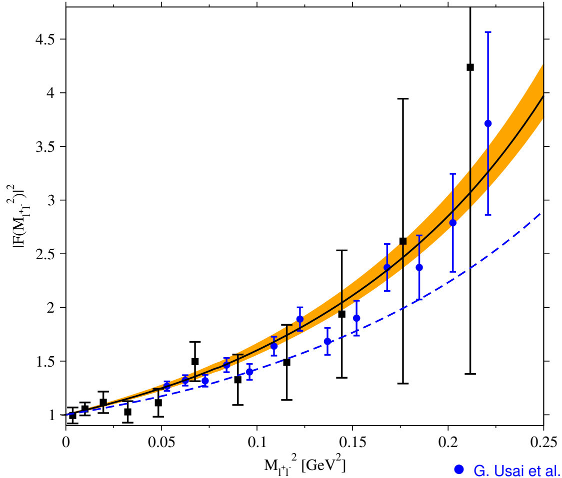
Quark loop — L. Ametller et al., PRD **45** ('92) 986

Brodsky & Lepage — Brodsky & Lepage, PRD **34** ('81) 1808

1-loop ChPT — L. Ametller et al., PRD **45** ('92) 986

Dispersion integral — C. Hanhart et al., EPJC **73** ('13) 2668

Results: $|\text{transition FF}_{\eta\to\gamma^*\gamma}|^2$ versus $M^2_{l^+l^-}$



● G. Usai et al. (NA60), NPA **855** ('11) 189

■ H. Berghäuser et al. (CB/TAPS), PLB **701** ('11) 562

—— $\alpha = (1.32 \pm 0.13)\,\text{GeV}^{-2}$, – – – $\alpha := 0$

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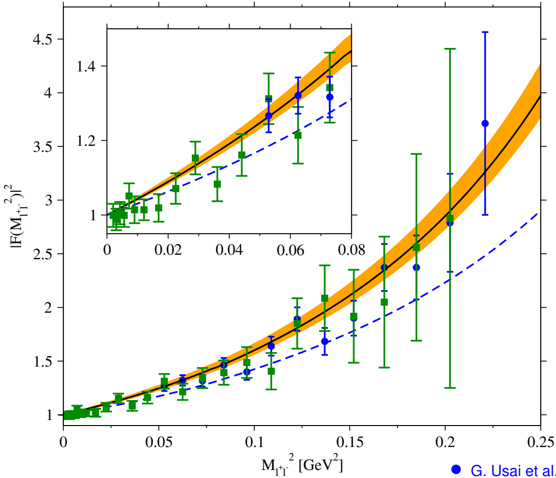

● G. Usai et al. (NA60), NPA **855** ('11) 189

■ P. Aguar-Bartolome et al. (A2), PRC **89** ('14) 044608

—— $\alpha = (1.32\ \pm 0.13\ )\ \text{GeV}^{-2}$, - - - $\alpha := 0$

Summary

- $\eta \to \pi^+\pi^-\gamma$ and $\eta \to \gamma^*\gamma$ are characterized by two scales:
  - a universal one ($\sim m_\rho$) from the $\pi\pi$ final-state interaction,
  - a reaction-specific one from the vertex and the left-hand cuts.

    The latter are related for the two reactions, but not equal.

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    The latter are related for the two reactions, but not equal.

Outlook

- Modify ρ' and ρ'' couplings such that simultaneously
  - $F^{\eta}_{\pi\pi\gamma}(s) = P(s)F_V(s)$ for $s < 1\,\text{GeV}^2$ and
  - $F^{\eta}_{\pi\pi\gamma}(s) \propto 1/s$ for $s \to \infty$.

  $\hookrightarrow$ unsubtracted dispersion integral plus non-2-pion channels.

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  - $F^{\eta}_{\pi\pi\gamma}(s) \propto 1/s$ for $s \to \infty$.

  $\hookrightarrow$ unsubtracted dispersion integral plus non-2-pion channels.
- Proposal to measure $e^+e^- \to \eta\pi^+\pi^-$ both as function of s and $s_{\pi\pi}$

  $\hookrightarrow$ allows extraction of the isovector piece of $\eta \to \gamma^*\gamma^*$

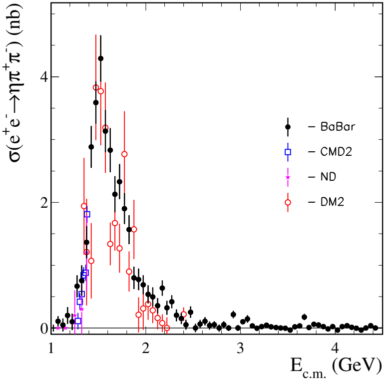
first step: 'energy-averaged' BaBar data from 2007 $\hookrightarrow$

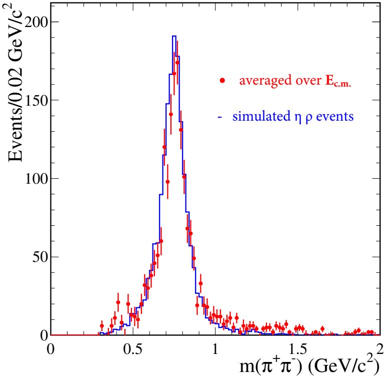
EPILOGUE

Outlook: $e^+e^- \to \eta\pi^+\pi^-$

B. Aubert et al. (**BaBar**), PRD **76** ('07) 092005

Figs. courtesy of B. Aubert et al. (BaBar), arXiv:0708.2461v2 [hep-ex]





$\hookrightarrow$ ρ' (and ρ'' ?) dominance in $e^+e^- \to \gamma^* \to \rho'$

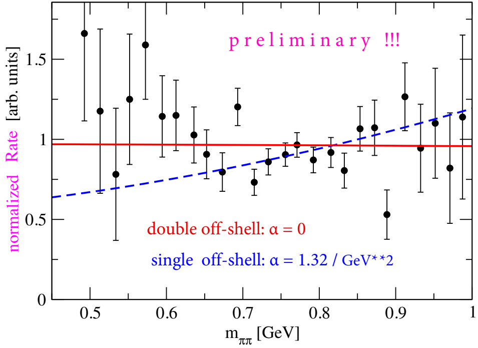
$\hookrightarrow$ ρ dominance in FSI of $\eta\rho \to \eta\,\pi^+\pi^-$

Outlook: $e^+e^- \to \eta\pi^+\pi^-$

unfolding the BaBar cross section – on the route to the double-off-shell TFF

$$\text{Rate} \equiv \int ds\, \frac{d\sigma}{dm_{\pi\pi}} = \mathcal{N}^{\frac{3}{2}}(s_{\pi\pi})\, \boxed{P(s_{\pi\pi})^2}\, \left|F_V(s_{\pi\pi})\right|^2 \int_{(\sqrt{s_{\pi\pi}}+m_\eta)^2}^{(4.5\,\text{GeV})^2} ds\, s^{-3/2} |\mathbf{p}^*_\eta|^3 |G^{\text{BW}}_{\rho'}(s)|^2$$

cross section data: B. Aubert et al. (BaBar), PRD 76 ('07) 092005



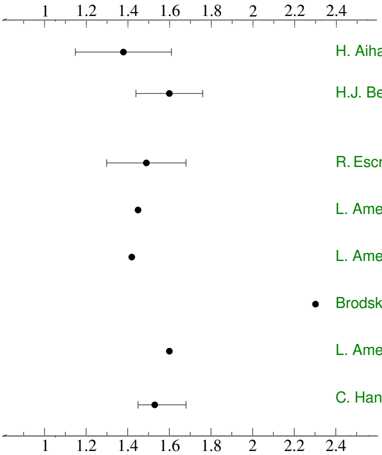
T. Dato, C. Hanhart, A. Kupść, U.-G. Meißner, A.W., in preparation.

$\mathcal{RE'SUME'}$

- First indication of $\alpha(s)$ dependence in $P = (1 + \alpha(s) s_{\pi\pi})$
- First indication for non-factorization, already below 2 GeV, of the two legs of the $\eta \to \gamma^* \gamma^*$ (double-off-shell) form factor
- New $e^+ e^- \to \eta \pi^+ \pi^-$ data with *both* s and $s_{\pi\pi}$ dependence urgently needed

BACKUP SLIDES

Results: slope parameter $b_{\eta'} \equiv \frac{d}{dQ^2} F_{\eta'\gamma^*\gamma}(Q^2, 0)\Big|_{Q^2=0}$



Recent experiments	
$e^+e^- \to e^+e^-\eta$ (TCP/2γ)	H. Aihara et al., PRL **64** ('90) 172
$e^+e^- \to e^+e^-\eta$ (CELLO)	H.J. Behrend et al., ZPC **49** ('91) 401
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1-loop ChPT	L. Ametller et al., PRD **45** ('92) 986
Dispersion integral	C. Hanhart et al., EPJC **73** ('13) 2668

Cross check: unsubtracted dispersion integral with $\Lambda = 1$ GeV underestimates the experimental value of $A^{\eta'}_{\gamma\gamma}$ by $(-2 \pm 7)\%$