Methods in Amplitude Analysis: theory, models and implementation

Adam Szczepaniak
Indiana University/JLab

- Constraints and Examples

- 1-to-3 decay amplitude models

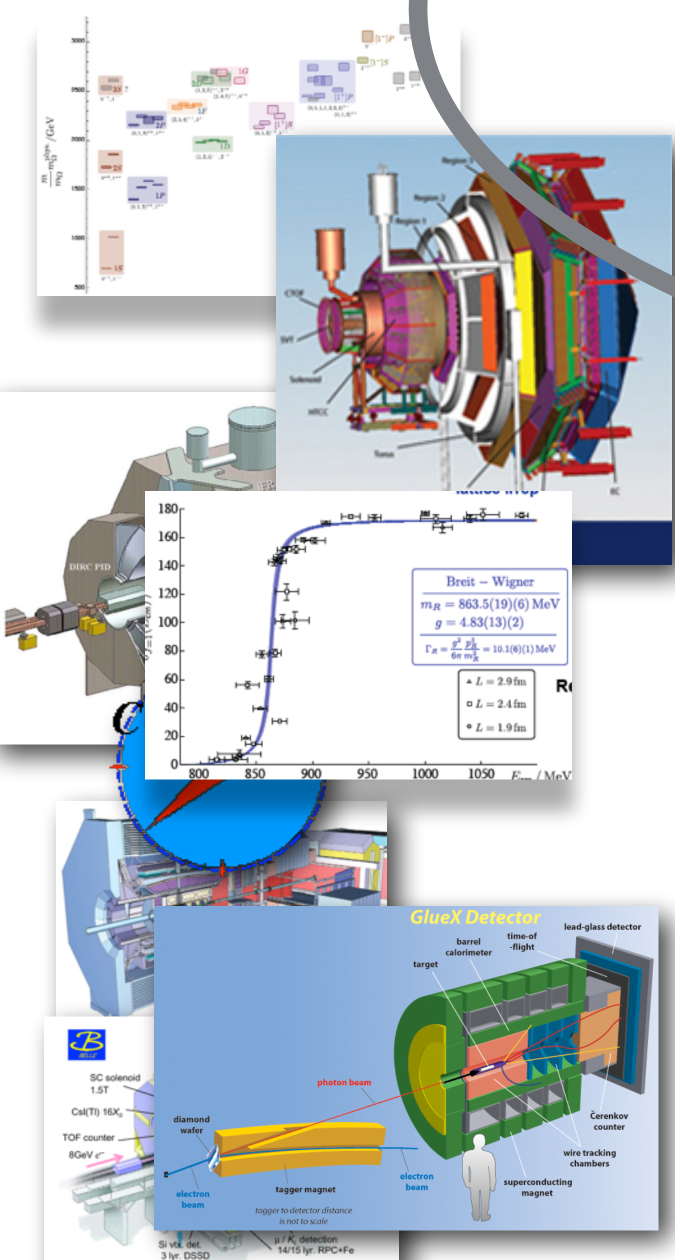
- KT Equations

- Collaboration strategy

Why Amplitude Analysis

Experimental Measurement

QCD Measurement



Physics quantities: form factors, resonance parameters masses, etc.

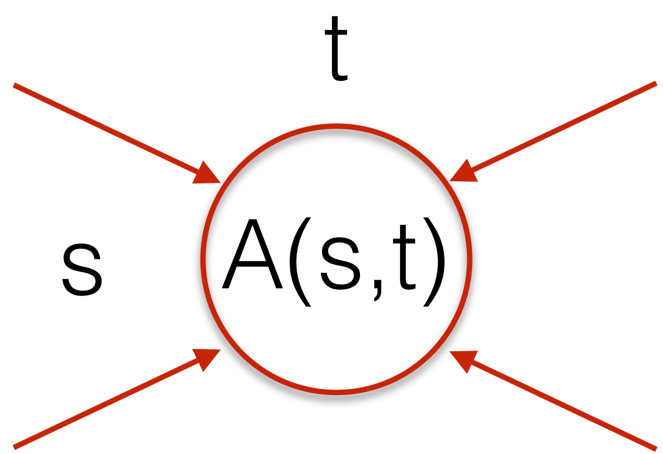
Reaction amplitudes

$d\sigma$Measured = Detector Acceptance $\otimes$ dPS $|A|^2$

Amplitude construction

Axiomatic S-matrix principles: (not the same as based on a a microscopic model/theory, e.g. unitary diagrams vs Feynman diagrams)

- Crossing relations:



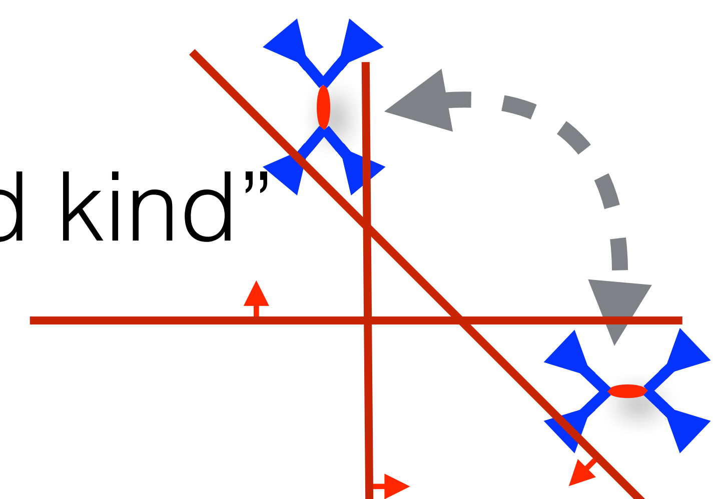
A(s,t) describes all processes related by line reversal

- Analyticity: Cuts determined by unitarity (i.e. in the physical region, continuation is complicated, Mandelstam representation known only for 4-point function)

  Asymptotic behavior ($A(s_i) < s_i\, O(\log s_j)$)

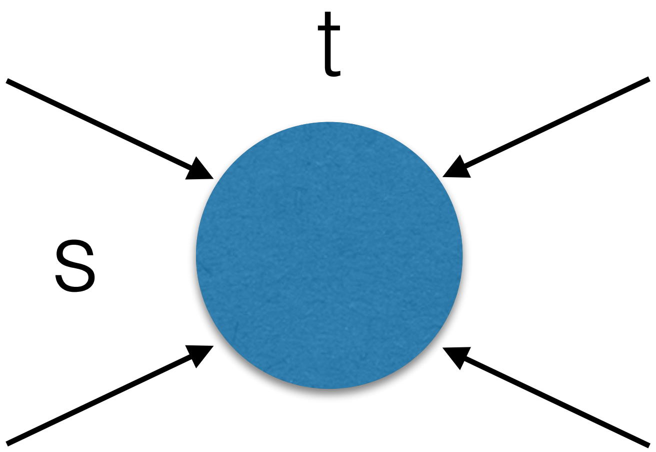
  Bound state poles

- Regge behavior: Analyticity of “the second kind”

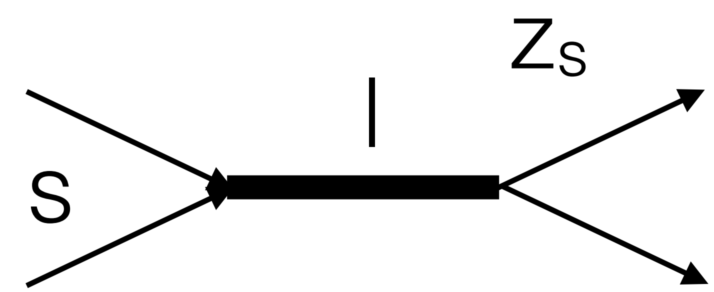


- Global symmetries: EM, chiral, …

- “All constraints are equal but some may be more equal then other”



if close to s-channel singularities and far from t/u channel singularities (left hand cut)



$$A(s,t) = \sum_{l}^{\infty} (2l+1) f_l(s) P_l(z_s) \rightarrow \sum_{l}^{\text{finite}} (2l+1) f_l(s) P_l(z_s)$$

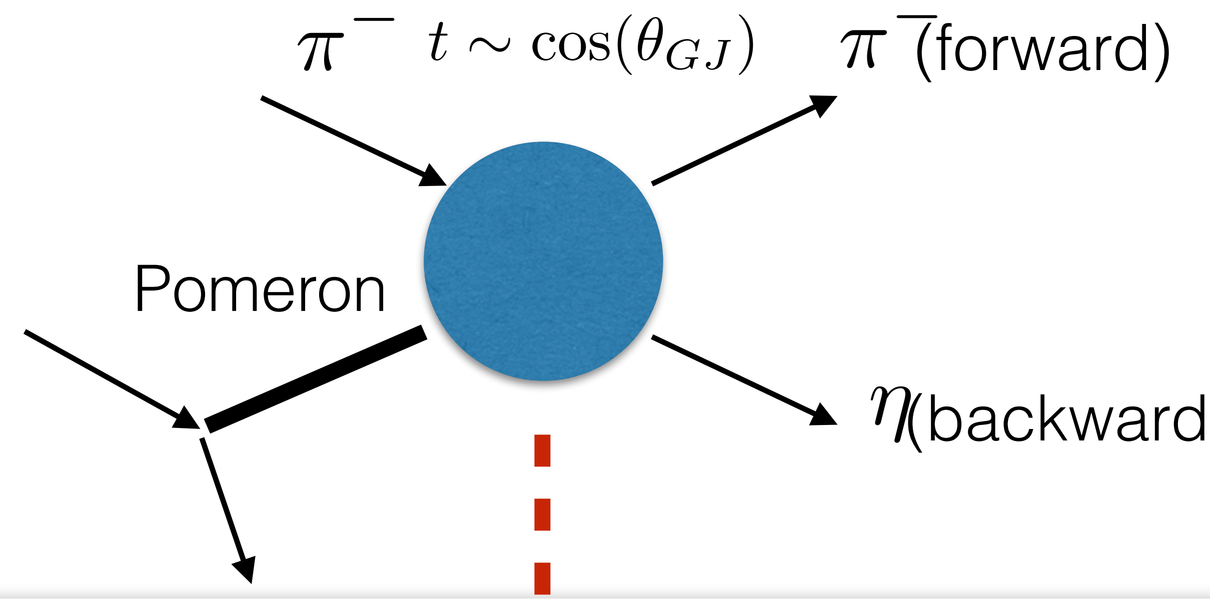
(s-channel) Isobar model

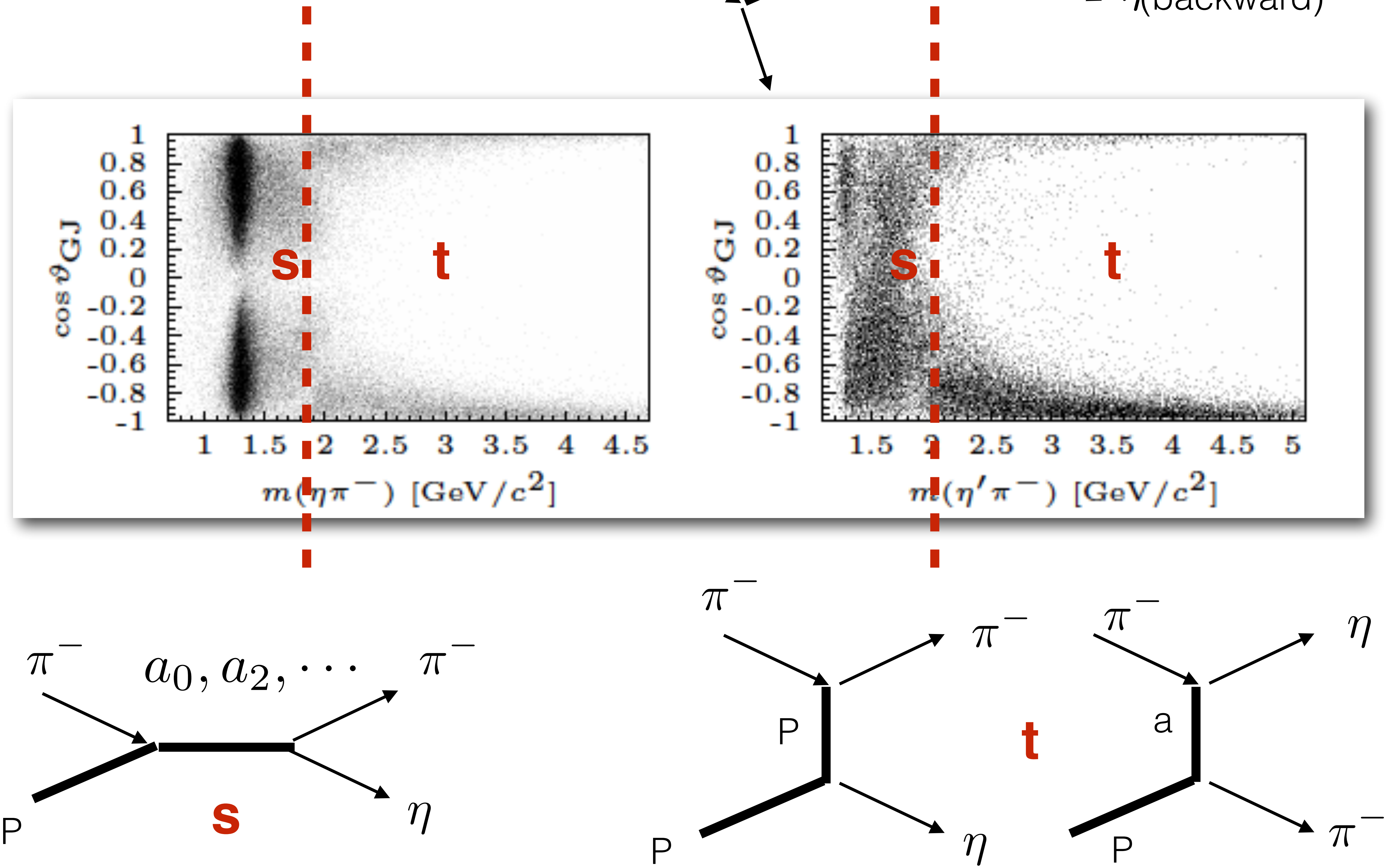
$$(s-4)R^2 \sim \frac{s-4}{m_e^2} << 1$$

alternatively use t/u channel isobars

COMPASS

$$\pi^-(190\text{GeV}^2)p \to \eta(')\pi^- p$$





- When cross-channel channel singularities are all nearby, there are no known amplitudes that satisfy all S-matrix constraints

(except perturbatively, e.g. chiral p.t.)

$$M(\eta, \omega, \phi, \cdots) \to m + m + m \; (\pi^+, \pi^-, \pi^0, \cdots)$$

$$M >\sim 3m$$

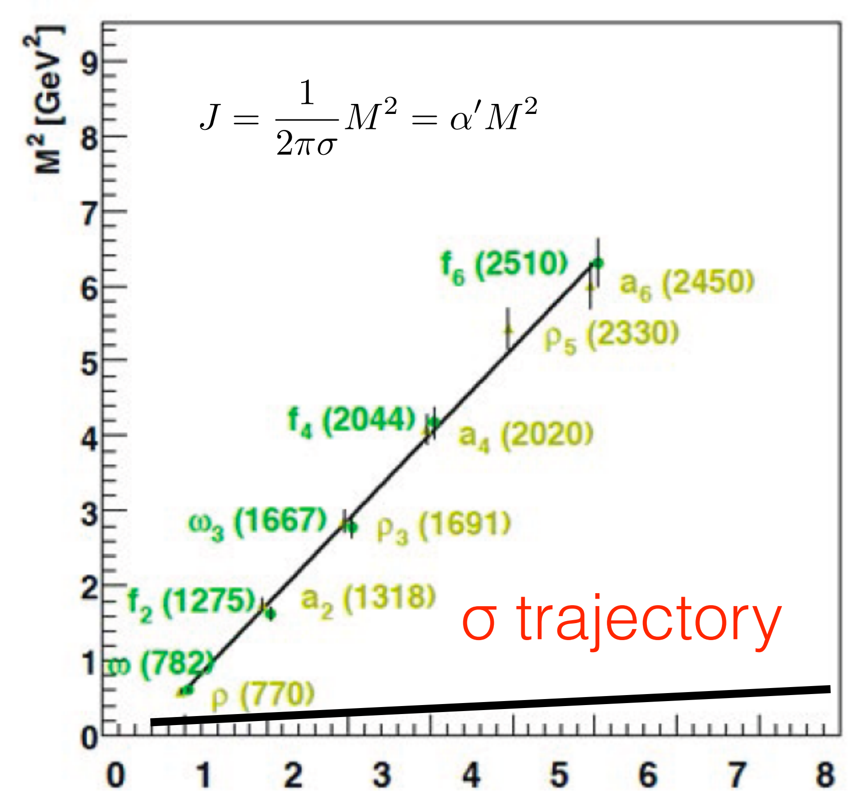
Two general class of models

- Two-body unitarization, of low partial waves

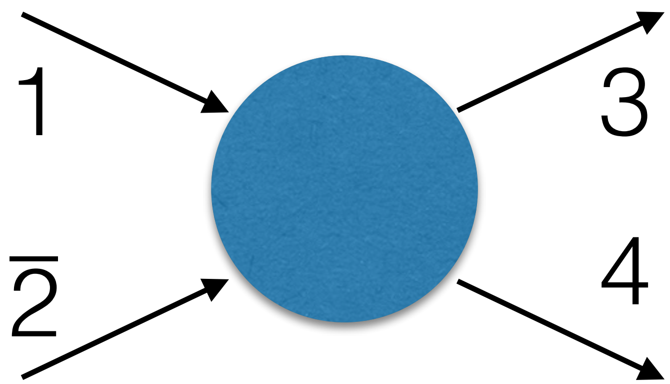
violate analyticity of the 2nd kind

- Resonance/Regge Duality

violate analyticity of individual partial waves



- General properties:



s-channel: $M(1) + \bar{m}(\bar{2}) \to m(3) + m(4)$

$$s = (p_3 + p_4)$$
$$t = (p_4 - p_{\bar{2}})^2 = (p_4 + p_2)^2$$
$$u = (p_3 - p_{\bar{2}})^2 = (p_3 + p_2)^2$$

in decay-channel, s,t,u become the Dalitz variables

- Partial waves:

$$A(s,t) = \sum_{l}^{\infty} (2l+1) A_l(s) P_l(z_s)$$
$$A_l(s) = A_l^R(s) + A_l^L(s)$$

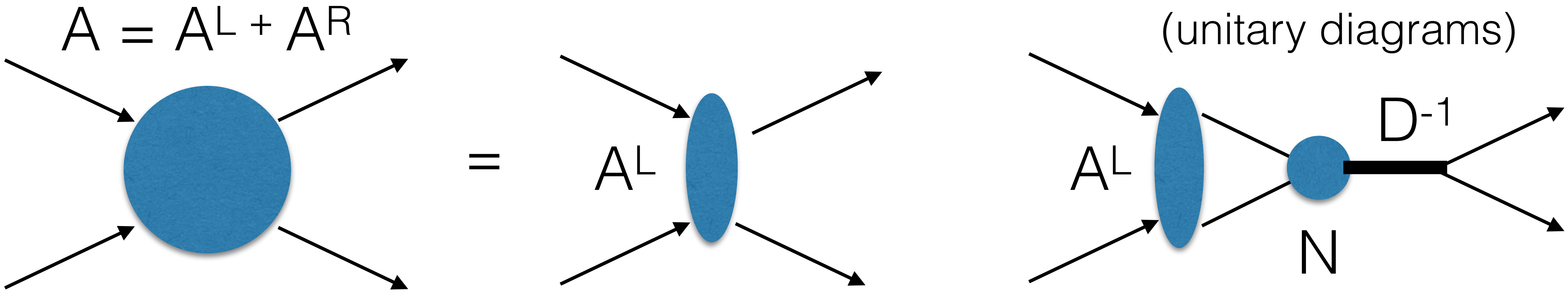
- Two-body unitarity:

$$\Delta A_l^R(s) = \rho(s) \frac{N_l(s)}{D_l^*(s)} A_l(s)$$

$\frac{N_l(s)}{D_l(s)}$ = m + m → m + m l-wave p.w

- Solution: (Frazer-Fulco/Omnes/Mandelstam)

$$A_l(s) = A_l^L(s) + \frac{1}{D(s)} \int_{4m^2} \frac{ds'}{\pi} \frac{\rho(s') N_l(s') A_l^L(s')}{s' - s}$$



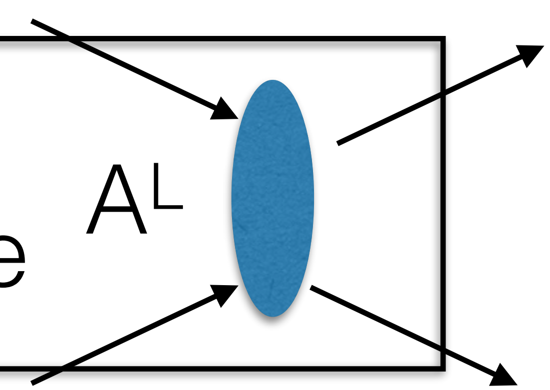
$$A_l(s) = A_l^L(s) + \frac{1}{D(s)} \int_{4m^2} \frac{ds'}{\pi} \frac{\rho(s') N_l(s') A_l^L(s')}{s' - s}$$

can also be represented in the standard form

$$A_l(s) = \frac{1}{D_l(s)} \left(G_l^L(s) = \int_{-\infty} \frac{ds'}{\pi} \frac{D_l(s') A_l^L(s')}{s' - s} \right)$$

left hand cut

The solution depends on
and generalized to inelastic case



exact representation:

$$A_l(s) = \frac{1}{D_l^{el}(s)} \left(\sum_n a_n^l \omega_L^n(s) + \sum_m b_m^l \omega_{in}^m(s) \right)$$

Model examples (implementation of crossing defines the l.h.c)

- "product form"

$$A(s,t,u) = \frac{f(s,t,u)\,\text{(analytical)}}{D_0(s)D_0(t)D_0(u)}$$

$$G_0^L(s) = f(s,t,u)\int_{-1} 1 \frac{dz_s}{D_0(t)D_0(u)}$$

- "sum form" = Khuri-Treiman (equation)

$$A(s,t,u) = \frac{f_0(s)}{D_0(s)} + \frac{f_0(t)}{D_0(t)} + \frac{f_0(u)}{D_0(u)}$$

$$A_l(s) = \frac{1}{D_l(s)}\int_{4m^2} ds' \frac{\rho(s')N_l(s')A_l^L(s')}{s'-s} + A_l^L(s)$$

General remarks

- Various forms satisfy: 2-body unitarity and crossing symmetry
- Can be analytically continued to the decay region

minor complications from anomalous thresholds
(Mandelstam/Kacser/Aitchison/Brehm)

- Can be extended to higher partial waves
- Analytical continuation of unitarity from 2-to-2 to 1-to-3 is not the same as imposing unitarity on 3-to-3 amplitude at the M-particle pole

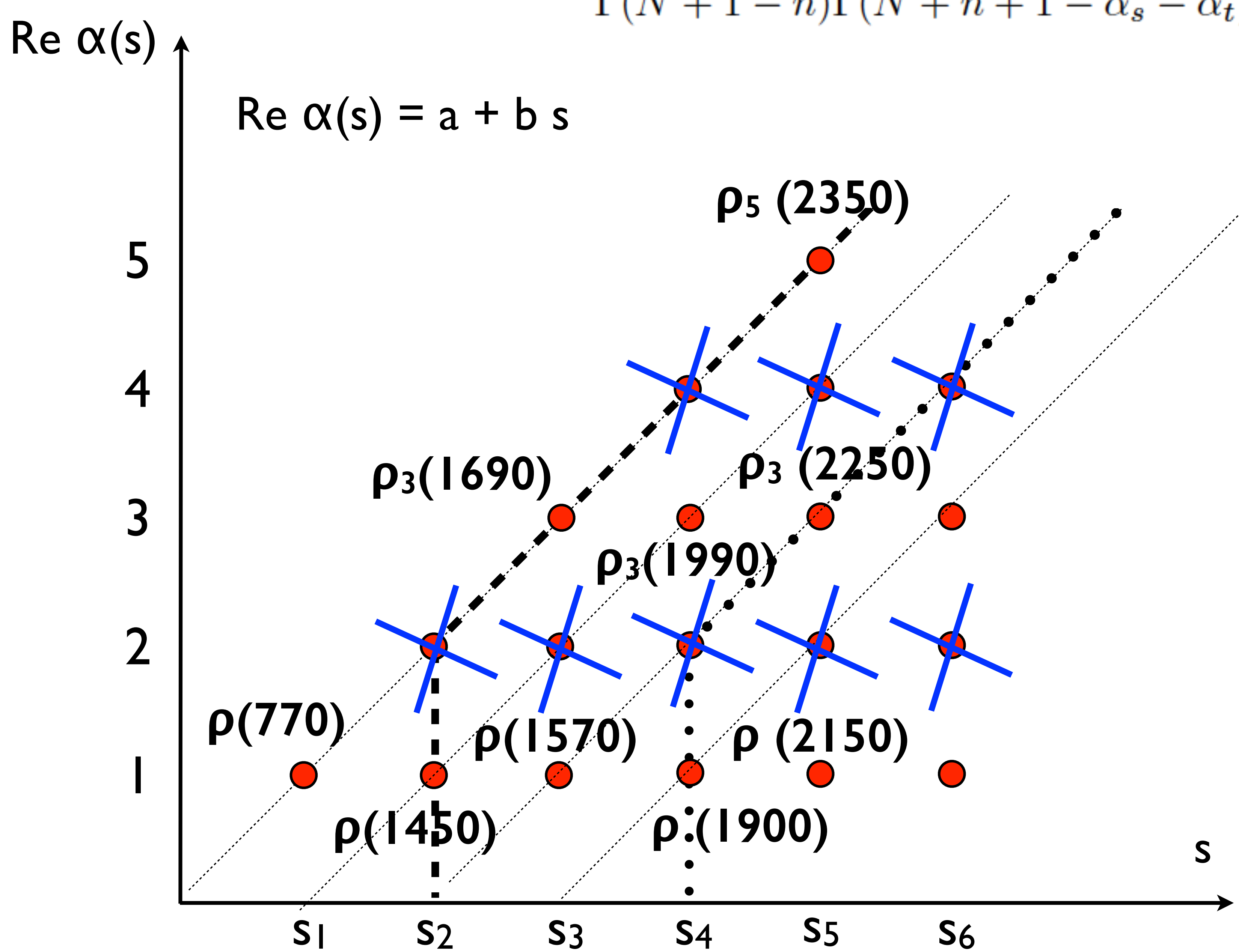
Any model which truncate partial waves will

- Violate Mandelstam analyticity
- Have incorrect asymptotic behavior
- Violate analyticity in the complex-l plane

- Dual models (Veneziano) $A(s,t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)+\alpha(t))}$

$$A(s,t) = \sum_k \frac{\beta_k(t)}{k-\alpha(s)} = \sum_k \frac{\beta_k(s)}{k-\alpha(t)}$$

$$\mathcal{A}_n(s,t;N) = \frac{2n-\alpha_s-\alpha_t}{(n-\alpha_s)(n-\alpha_t)} \sum_{i=1}^{n} a_{n,i}(-\alpha_s-\alpha_t)^{i-1} \times \frac{\Gamma(N+1-\alpha_s)\Gamma(N+1-\alpha_t)}{\Gamma(N+1-n)\Gamma(N+n+1-\alpha_s-\alpha_t)}.$$

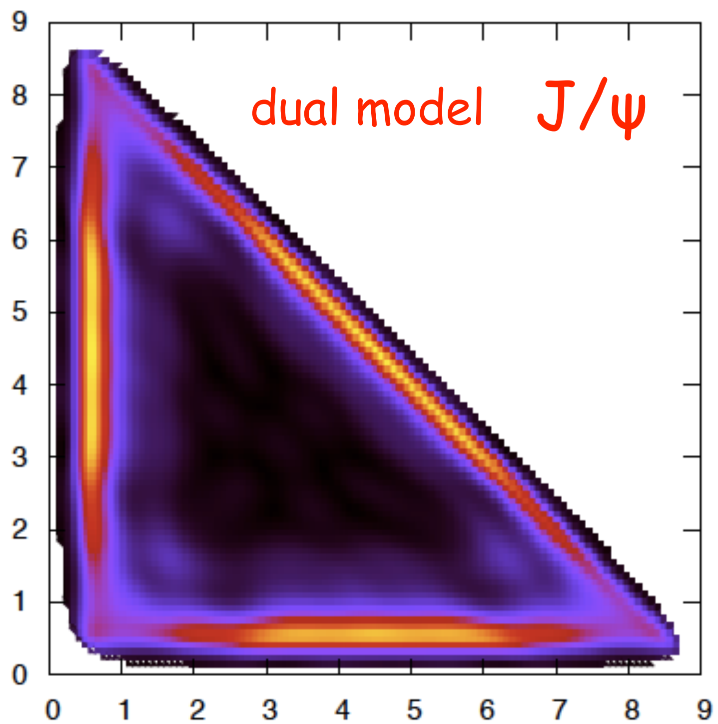


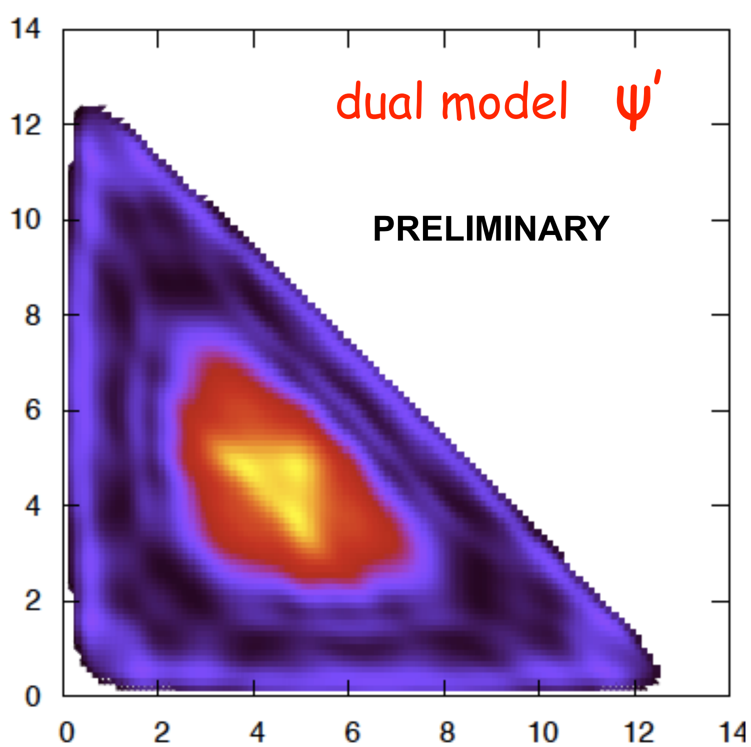
Regge/Resonance duality

Can be generalized to any number of external particles

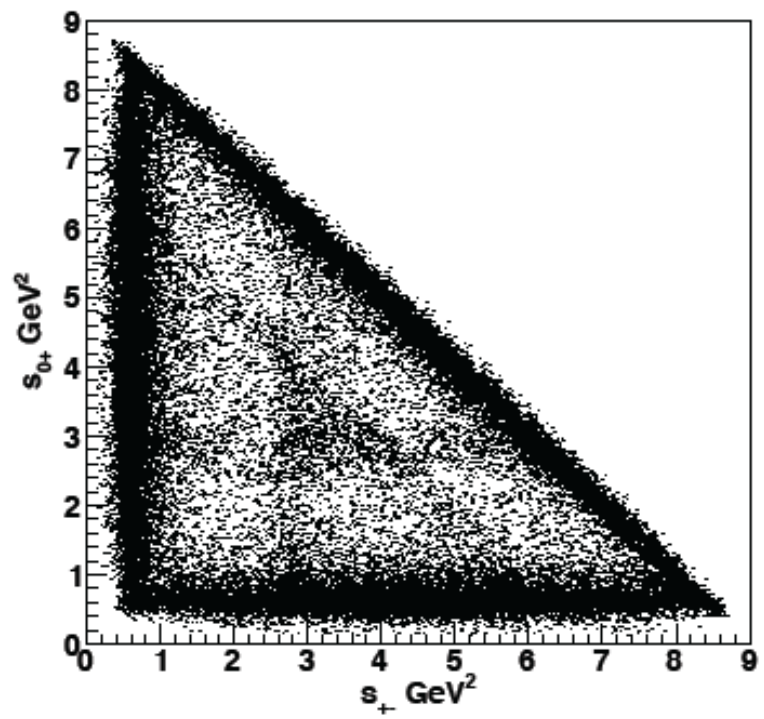
Can be extend to satisfy Mandelstam duality, but not known extensions to several trajectories

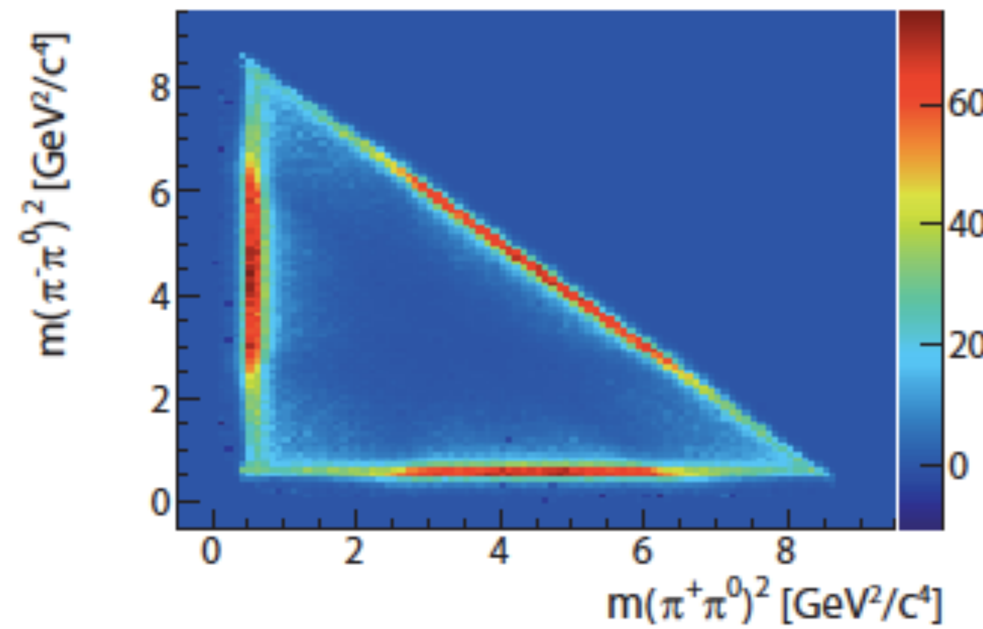
Regge

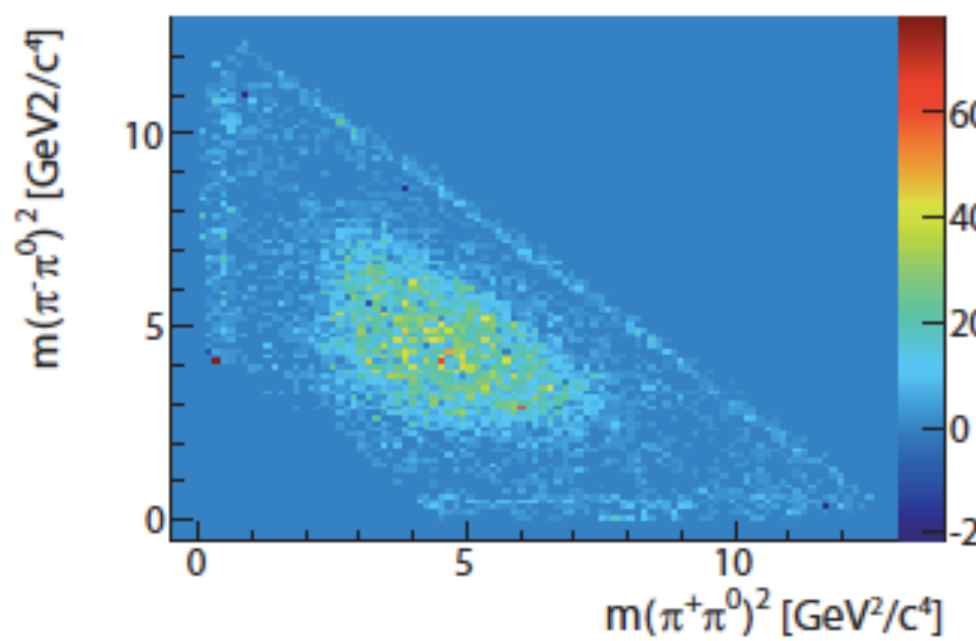




standard isobar

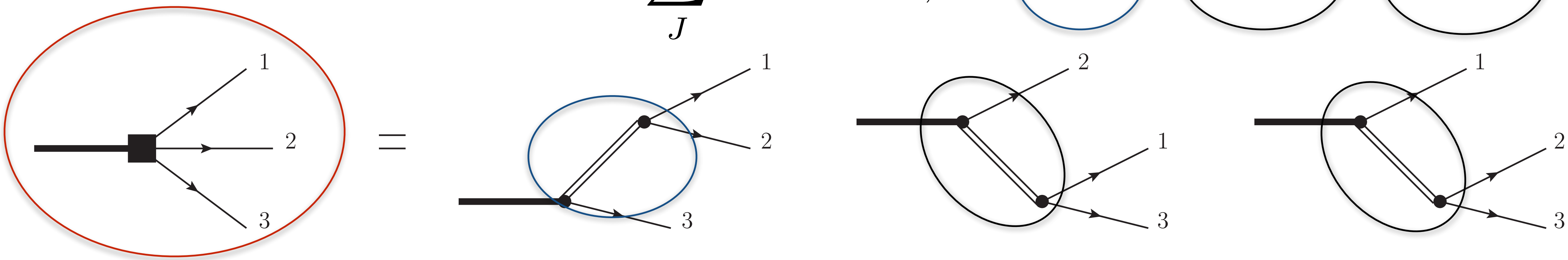






Dispersive analysis of ω/ϕ→3π

$$A(s,t) = \sum_J^{J_{max}} (2J+1) d^J_{1,0}(z_s) a^R_J(s) + (s \to t) + (s \to u)$$



$$A^L(s) = 3 \int_{-1}^{1} dz_s \frac{1-z_s^2}{2} a^R(t(s,z_s))$$

$$A(s,t) = \sum_J^{J_{max}} (2J+1) d^J_{1,0}(z_s) A_J(s)$$

Solution: (e.g. P-waves only)

$$a^R(s) = \frac{1}{D(s)} \int_{4m^2} \frac{ds'}{\pi} \frac{\rho(s')N(s')A^L(s')}{s'-s}$$

Easily generalized to inelastic case

$$a^R(s) = \frac{1}{D^{el}(s)} \left(\int_{4m^2}^{s_i} \frac{ds'}{\pi} \frac{\rho(s')N(s)(s')A^L(s')}{s'-s} + A^{in}(s) \right)$$

el = only elastic cut   in = only inelastic cut

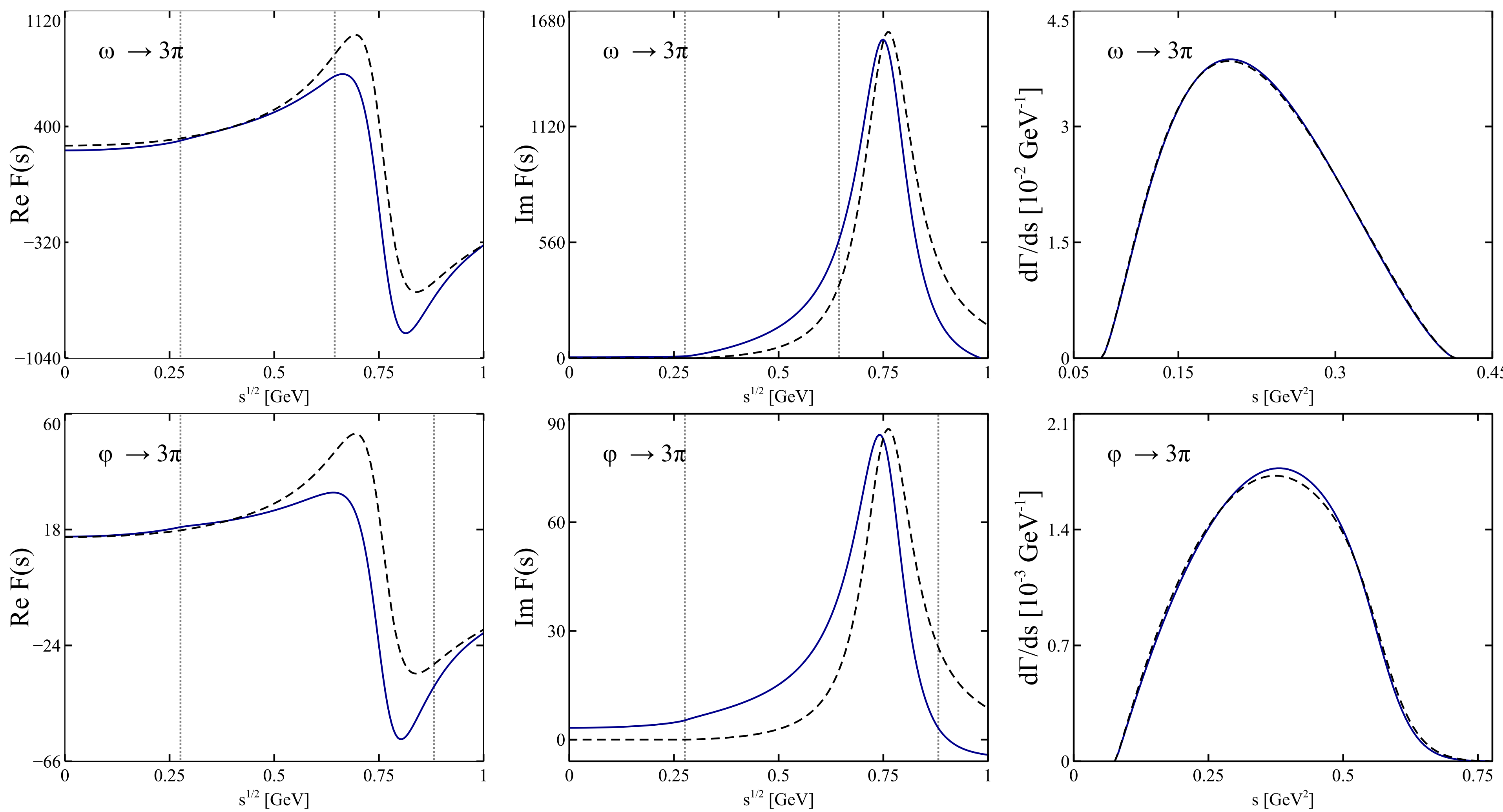
Dispersive analysis of ω/ϕ→3π

Integral equation

$$a^R(s) = \frac{1}{D^{el}(s)}\left(\int_{4m^2}^{s_i}\frac{ds'}{\pi}\,\frac{\rho(s')N(s)(s')A^L(s')}{s'-s} + \sum_{i=0}^{N} a_i\omega^i(s)\right)$$

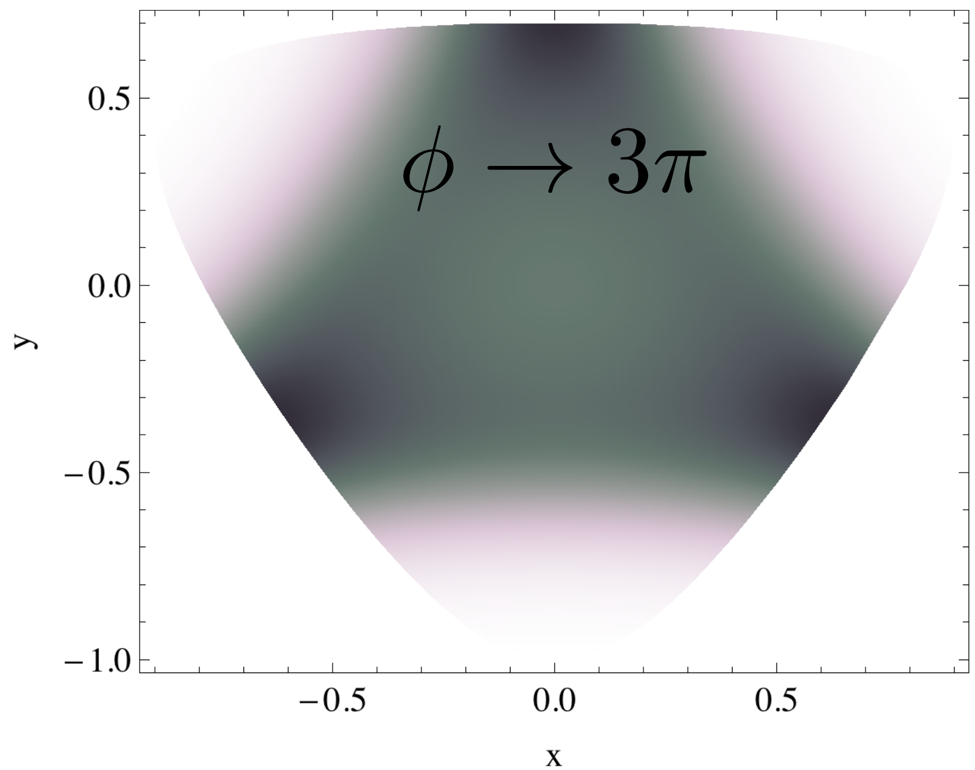
- w(s) is the conformal map of inelastic contributions:
  Coefficients a_i play the role of improved subtraction constants

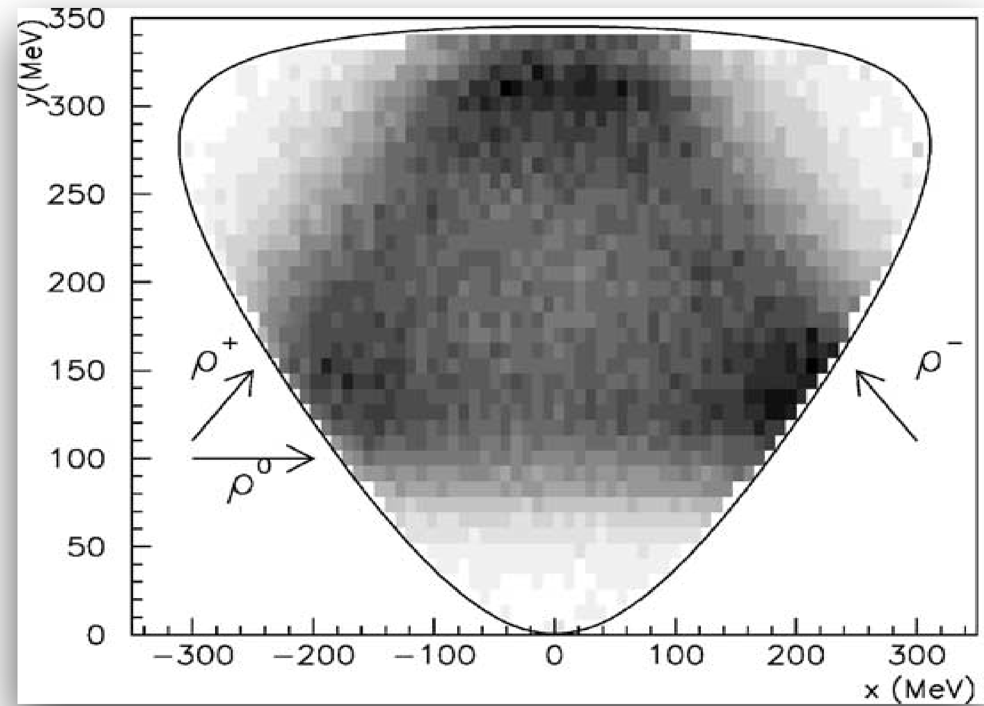
different from
Niecknig et. al. 2012
Anisovich et. al. 1998



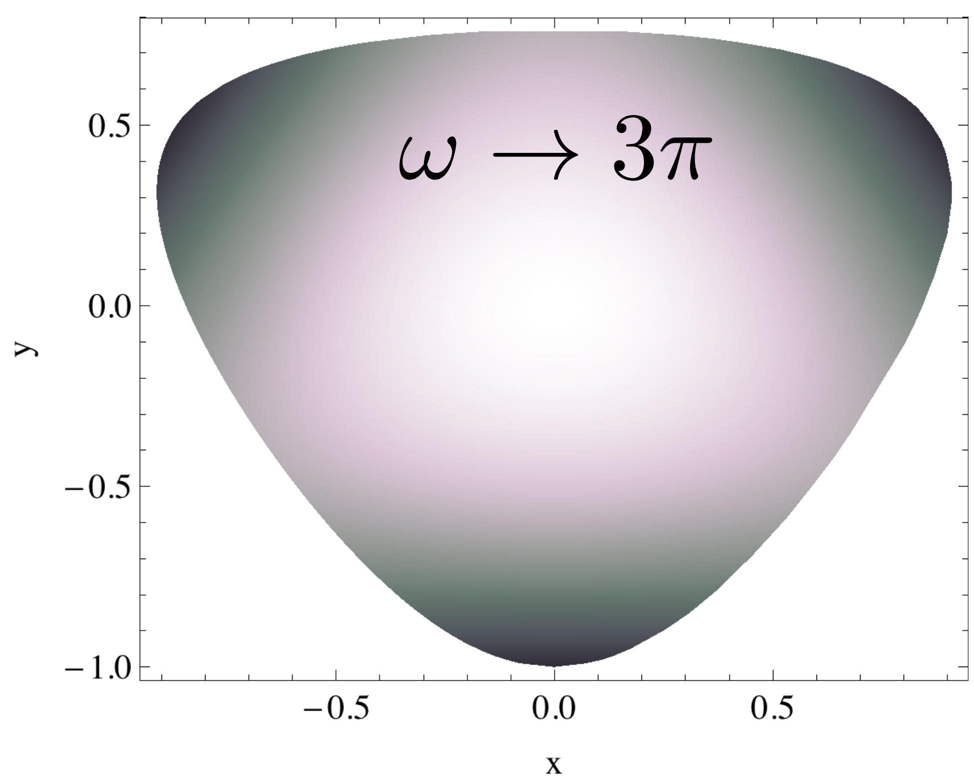
all details in: I. Danilkin et al., arXiv1076363

Dalitz plots





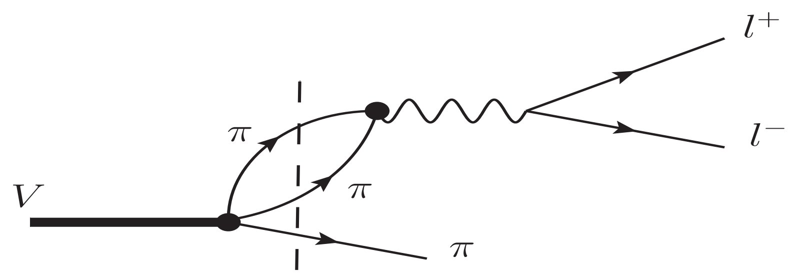
KLOE (2003)



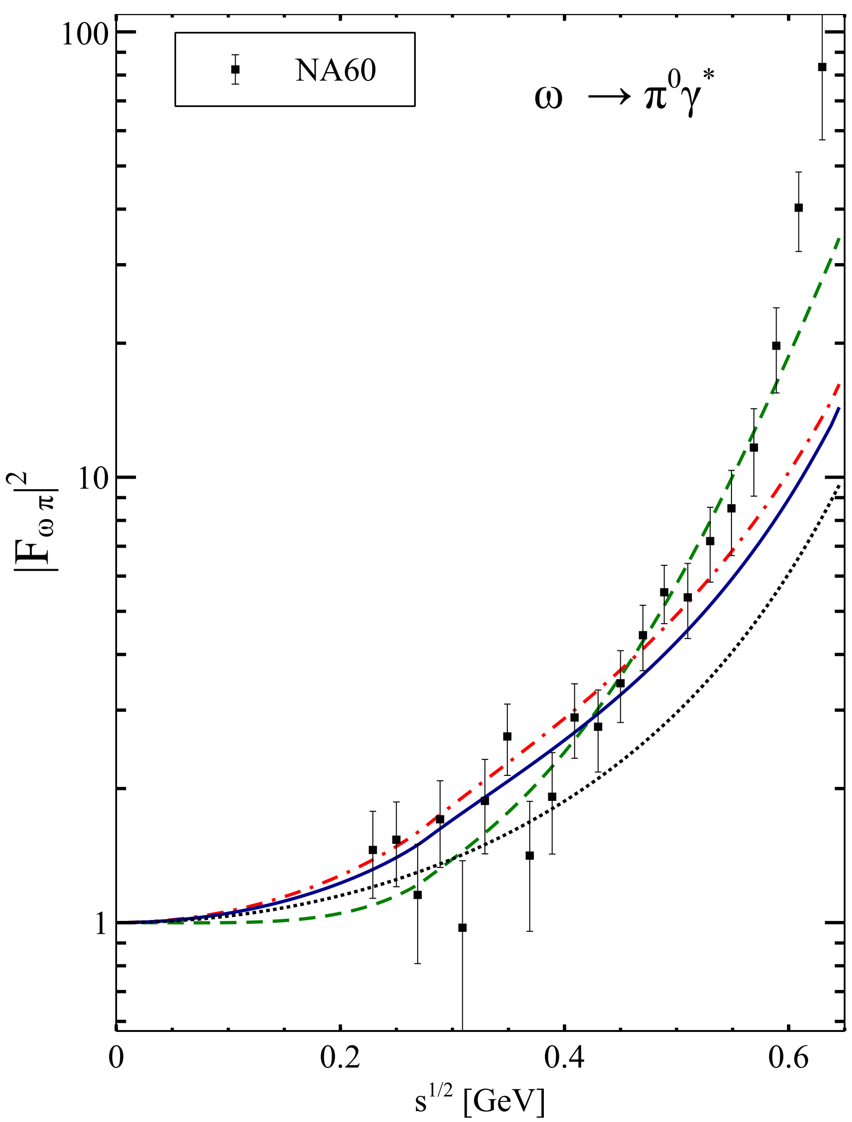
- Only one parameter (overall normalization) → fixed from $\Gamma_{exp}(\omega/\phi\to3\pi)$
- **φ→3π**: distribution clearly shows ρ-meson resonances
- **ω→3π**: distribution is relatively flat;
- upcoming high-statistic data from CLAS, KLOE, WASA, etc.

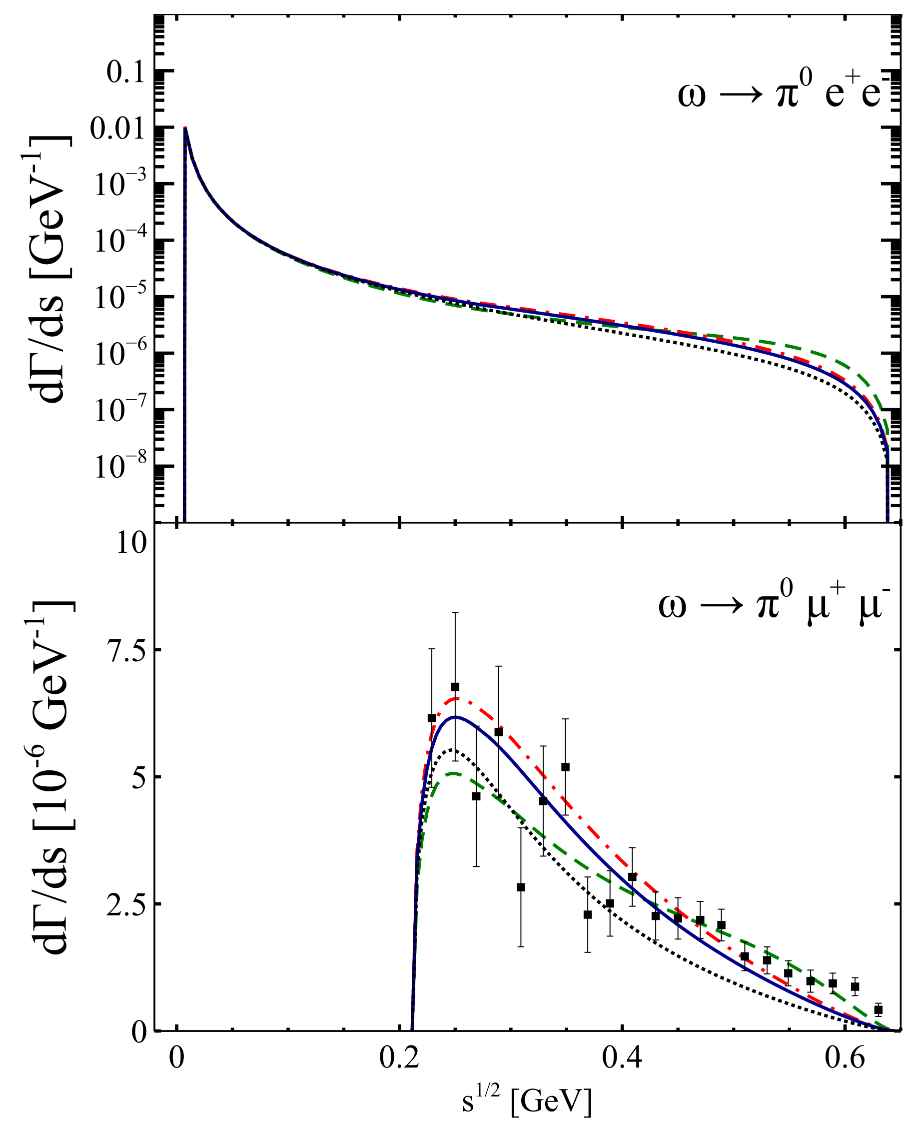
I. Danilkin et al.

Transition form factors: ω/ϕ→πγ



$$f_{V\pi}(s) = \int_{4m^2}^{s_i} \frac{ds'}{\pi} \frac{\Delta f_{V\pi}(s')}{s'-s} + \sum_{i=0}^{N} C_i \omega^i(s)$$

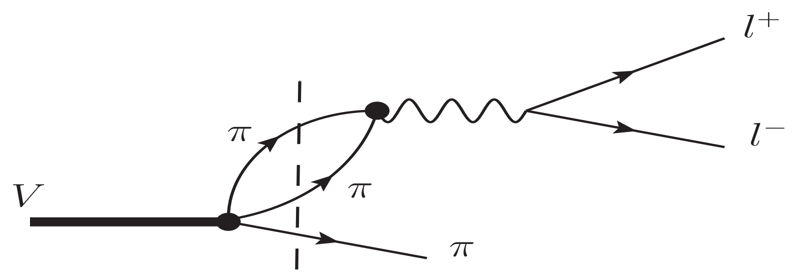




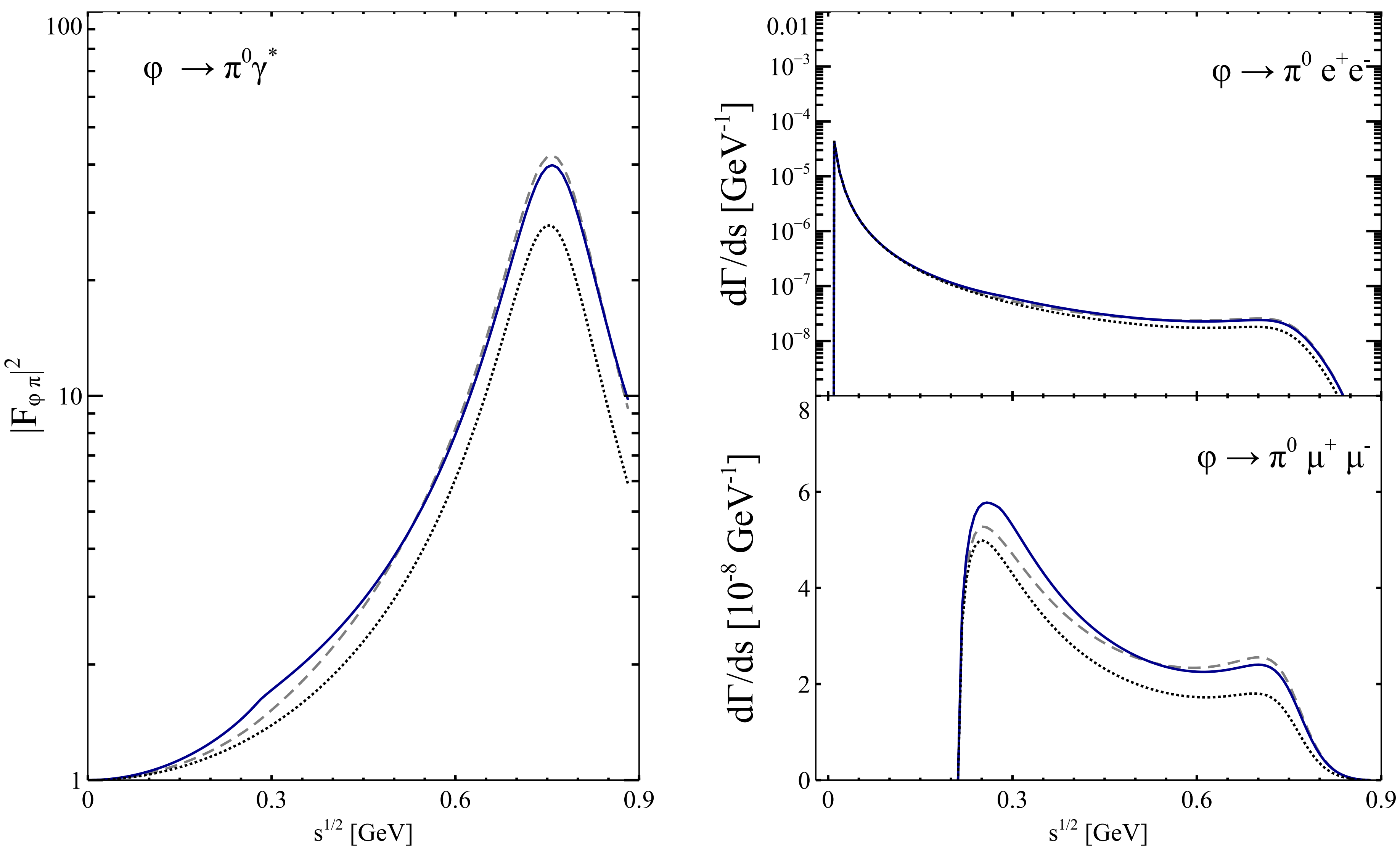
- **Black**: standard VMD (fails to describe the data)
- **Blue**: N=0 (C_0 from Γ_{exp}(V→πγ)) **Red**: N=1 **Green**: N=2 (fit to the data)
- Nature of the steep rise? Exp. analysis of ϕ→πγ is very important

I. Danilkin et al.

Transition form factors: ω/ϕ→πγ

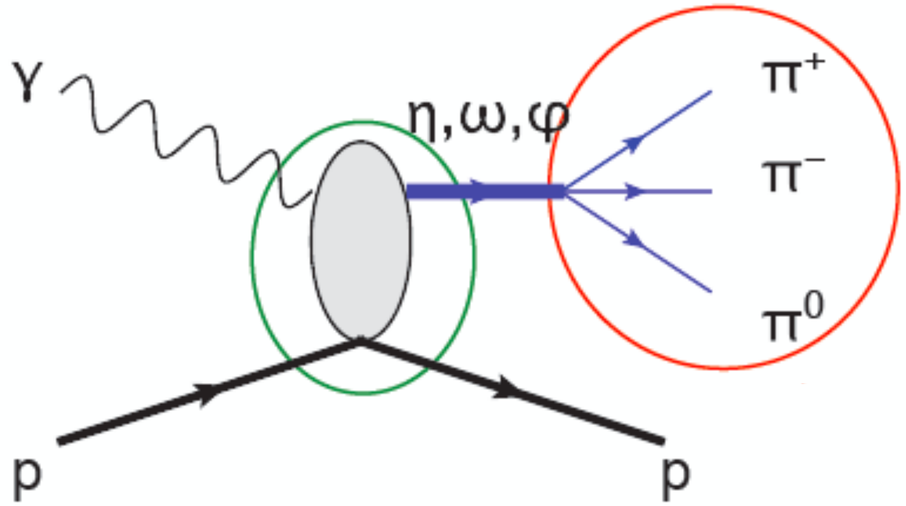


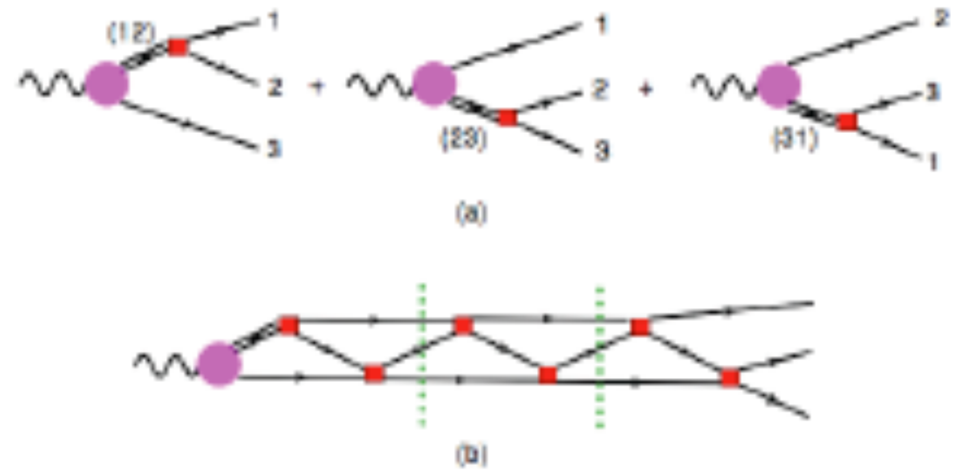
$$f_{V\pi}(s) = \int_{4m^2}^{s_i} \frac{ds'}{\pi} \frac{\Delta f_{V\pi}(s')}{s'-s} + \sum_{i=0}^{N} C_i \omega^i(s)$$

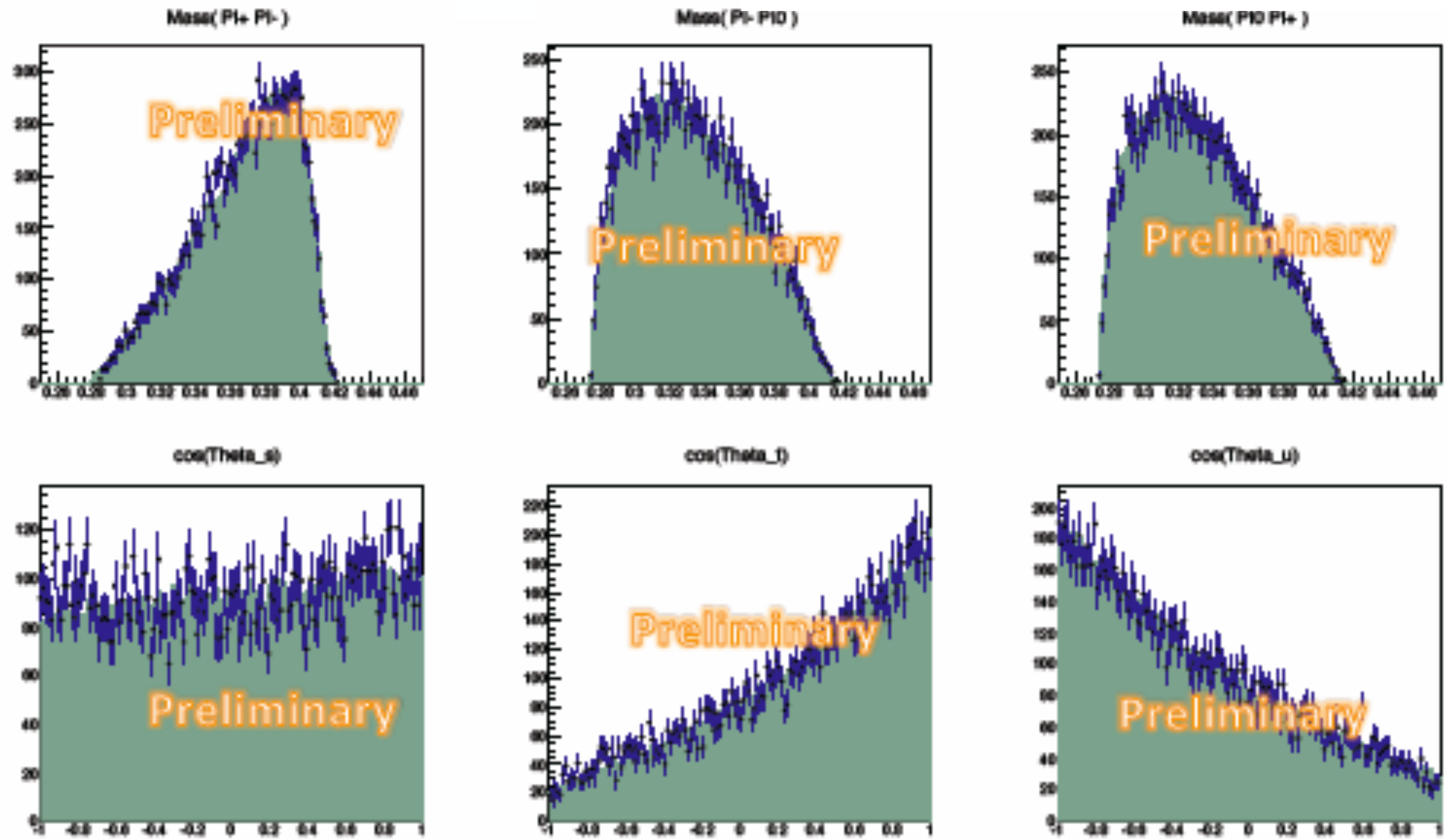


- **Black:** standard VMD (fails to describe the data)
- **Blue:** N=0 (C_0 from $\Gamma_{exp}(V\to\pi\gamma)$)
  Grey: N=0 (no 3b effects)

I. Danilkin et al.





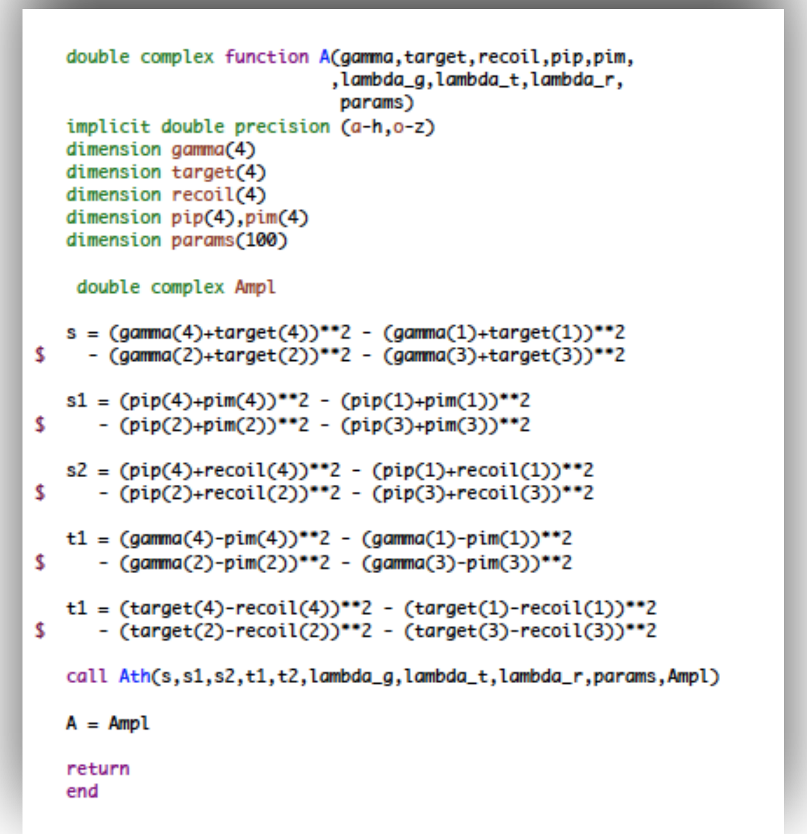


P.Guo, D.Schott, et al.

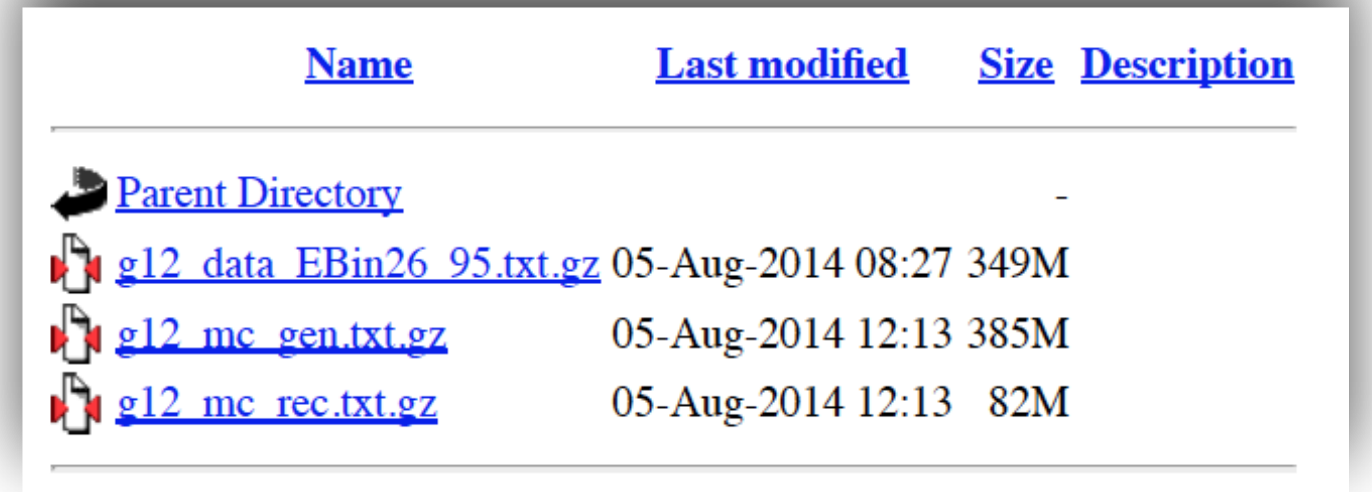
Joint Physics Analysis Center (JPAC)

theory

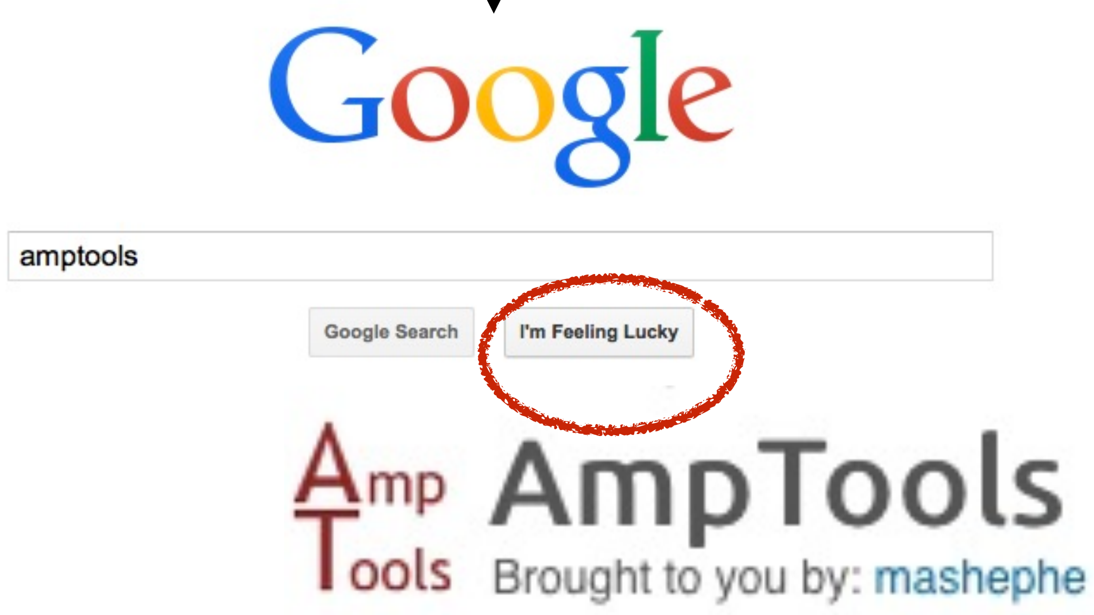
experiment



```
double complex function A(gamma,target,recoil,pip,pim,
                          ,lambda_g,lambda_t,lambda_r,
                          params)
implicit double precision (a-h,o-z)
dimension gamma(4)
dimension target(4)
dimension recoil(4)
dimension pip(4),pim(4)
dimension params(100)

 double complex Ampl

 s = (gamma(4)+target(4))**2 - (gamma(1)+target(1))**2
$   - (gamma(2)+target(2))**2 - (gamma(3)+target(3))**2

 s1 = (pip(4)+pim(4))**2 - (pip(1)+pim(1))**2
$    - (pip(2)+pim(2))**2 - (pip(3)+pim(3))**2

 s2 = (pip(4)+recoil(4))**2 - (pip(1)+recoil(1))**2
$    - (pip(2)+recoil(2))**2 - (pip(3)+recoil(3))**2

 t1 = (gamma(4)-pim(4))**2 - (gamma(1)-pim(1))**2
$    - (gamma(2)-pim(2))**2 - (gamma(3)-pim(3))**2

 t1 = (target(4)-recoil(4))**2 - (target(1)-recoil(1))**2
$    - (target(2)-recoil(2))**2 - (target(3)-recoil(3))**2

 call Ath(s,s1,s2,t1,t2,lambda_g,lambda_t,lambda_r,params,Ampl)

 A = Ampl

 return
 end
```



Name	Last modified	Size	Description
Parent Directory		-	
g12_data_EBin26_95.txt.gz	05-Aug-2014 08:27	349M	
g12_mc_gen.txt.gz	05-Aug-2014 12:13	385M	
g12_mc_rec.txt.gz	05-Aug-2014 12:13	82M	



working on CLAS data now

η and ω codes will be available online

"generic" parametrizations will be available online