Lattice Technicolor
gauge theories in presence of an infrared fixed point

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The Standard Model

Bosonic sector

$$\begin{array}{ccccc} SU(3)_c & \times & SU(2)_L & \times & U(1)_Y \\ G_{\mu\nu} & & W_{\mu\nu} & & B_{\mu\nu} \end{array}$$

Fermionic sector

$$\begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \quad , \quad u^i_R \quad , \quad d^i_R$$

$$\begin{pmatrix} e^i_L \\ \nu^i_L \end{pmatrix} \quad , \quad e^i_R \quad , \quad \nu^i_R(?) \qquad i = 1, 2, 3$$

Higgs sector

Higgs field
– complex scalar field in the $\frac{1}{2}$ repr. of $SU(2)_L$
Mexican-hat potential
– $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ and Higgs mechanism
Yukawa coupling
– fermion masses

. . . and Beyond

The Higgs mass is expected to get corrections of the order of the natural cut-off (Planck scale); what does keep it of the order of a few hundred GeV?

$\hookrightarrow$ The Standard Model is an effective theory valid only at energy scales below the TeV!

. . . and Beyond

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$\hookrightarrow$ The Standard Model is an effective theory valid only at energy scales below the TeV!

An extension of the Standard Model must

1. give mass to the fermions and break the gauge symmetry while keeping the theory consistent
2. be compatible with electroweak precision measurement
3. solve the problems of the current formulation

. . . and Beyond

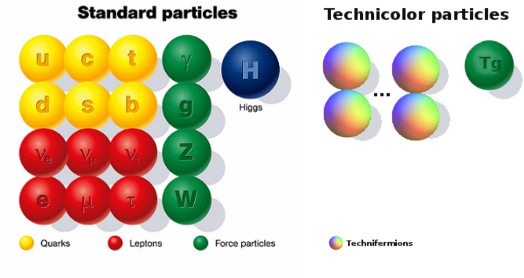
The Higgs mass is expected to get corrections of the order of the natural cut-off (Planck scale); what does keep it of the order of a few hundred GeV?

$\hookrightarrow$ The Standard Model is an effective theory valid only at energy scales below the TeV!

Some possible extensions

1. Supersymmetry
   A new symmetry that interchanges bosons with fermions valid for scales ≈ 1 TeV is conjectured; the Higgs is the lowest scalar state of this theory
2. (Compact) extra dimensions
   Fields are defined in 4+D dimensions, with the 4 dimensions detectable to us; field modes in the extra dimensions give rise to a tower of particles, among which could be the Higgs
3. Strongly interacting dynamics
   A new strongly-interacting sector exists whose phenomenology gives the Higgs sector at low energies

Some possible extensions



- Strongly interacting dynamics
  A new strongly-interacting sector exists whose phenomenology gives the Higgs sector at low energies.
  The Higgs particle is no longer elementary.

Color $\rightarrow$ Technicolor

As a result of chiral symmetry breaking, in QCD there is a quark condensate

$$\langle \bar{u}u + \bar{d}d \rangle \approx (200 \text{ MeV})^3$$

that is not invariant under $SU(2)_L \otimes U(1)_Y$
Not enough for accounting for the symmetry breaking of the Standard Model:

$$\langle \phi \rangle = 246 \text{ GeV}$$

Similarities

	EWSB	χSB
condesate	Higgs vev	$\bar{\psi}\psi$ chiral condensate
goldstone	eaten (gauged) by W,Z	π-mesons
radial excitations	Higgs particle	scalar meson

Color → Technicolor

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Technicolor

$$SU(N_{TC}) \text{ gauge theory}$$

Technicolor sector

$$\begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix} \quad , \quad U_R^i \quad , \quad D_R^i \qquad i = 1, \ldots, N_f$$

$$\langle \bar{U}_R U_L + \bar{D}_R D_L \rangle = \frac{\Sigma_{TC}}{2}$$

Technicolor extension

- In TC theories 4-fermion operators are effectively generated at low energy.

$$\Delta\mathcal{L}_{TC} = \frac{a}{\Lambda_{TC}^2}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\psi}\psi + \frac{b}{\Lambda_{TC}^2}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\Psi}\Psi + \frac{c}{\Lambda_{TC}^2}\bar{\psi}\psi\bar{\psi}\psi$$

A simple scaled-up version of QCD doesn't work:

Technicolor extension

- In TC theories 4-fermion operators are effectively generated at low energy.

$$\Delta\mathcal{L}_{TC} = \frac{a}{\Lambda_{TC}^2}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\psi}\psi + \frac{b}{\Lambda_{TC}^2}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\Psi}\Psi + \frac{c}{\Lambda_{TC}^2}\bar{\psi}\psi\bar{\psi}\psi$$

A simple scaled-up version of QCD doesn't work:

Technicolor is dead!

Technicolor extension

- TC is embedded in an ETC and the interaction is mediated by the ETC particles.

$$\Delta\mathcal{L}_{ETC} = \frac{a}{\Lambda^2_{ETC}}\langle\bar{\Psi}\Psi\rangle_{ETC}\bar{\psi}\psi + \frac{b}{\Lambda^2_{ETC}}\langle\bar{\Psi}\Psi\rangle_{ETC}\bar{\Psi}\Psi + \frac{c}{\Lambda^2_{ETC}}\bar{\psi}\psi\bar{\psi}\psi$$

- Vicinity to an IR-fixed point.

$$\langle\bar{\Psi}\Psi\rangle_{ETC} = \langle\bar{\Psi}\Psi\rangle_{TC}\exp\int_{\Lambda_{TC}}^{\Lambda_{ETC}}\gamma(\mu)\frac{d\mu}{\mu} \simeq \langle\bar{\Psi}\Psi\rangle_{TC}\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma*}$$

$$\Delta\mathcal{L}_{ETC} = \frac{a}{\Lambda^{1-\gamma*}_{ETC}\Lambda^{\gamma*}_{TC}}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\psi}\psi + \frac{b}{\Lambda^{1-\gamma*}_{ETC}\Lambda^{\gamma*}_{TC}}\langle\bar{\Psi}\Psi\rangle_{TC}\bar{\Psi}\Psi + \frac{c}{\Lambda^2_{ETC}}\bar{\psi}\psi\bar{\psi}\psi$$

Technicolor → Walking

- $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- Λ_{TC} is tuned to give the right mass to the $W^{\pm}$, Z bosons
- 4-operator coupling $\bar{Q}Q\bar{q}q$ to give mass to the SM fermions; effectively generated by some more fundamental theory (*extended technicolor*, ETC) at higher energy Λ_{ETC}
- in general too many technipions exists
- ETC generates also masses for the extra technipions (good!) and flavor changing neutral currents (FCNC, bad!)
- we can require Λ_{ETC} to be high enough in order to suppress FCNC, but then we need an enhancement mechanism to get reasonable masses for the SM fermions and high masses for the extra technipions...
- The problems of the technicolor models can be traced back to the logarithmic running of the coupling in QCD ⇒ QCD-like dynamics is unviable
- Ultimately, QCD-like dynamics will dominate in the infrared (confinement) and in the ultraviolet (asymptotic freedom) ⇒ there is still the possibility that in the intermediate region the running is different from standard QCD

Running coupling

Confinement and χSB.
• Conformal anomaly + asymptotic freedom.
• The RG flow has an UV gaussian fixed point.
• Λ separates the asymptotically free and non-perturbative regions.

$$g(\mu) = \begin{cases} \dfrac{1}{2b_0 \log\left(\frac{\mu}{\Lambda}\right)} & \mu \to \infty \\ +\infty & \mu \to 0 \end{cases}$$

IR conformality.
• Conformal anomaly + asymptotic freedom.
• The RG flow has an UV gaussian and an IR fixed point.
The theory flows from the UV to the IR fixed point.
• Λ separates the asymptotically free and scale-invariant regions.

$$g(\mu) = \begin{cases} \dfrac{1}{2b_0 \log\left(\frac{\mu}{\Lambda}\right)} & \mu \to \infty \\ g_* & \mu \to 0 \end{cases}$$

Conformality.
• Conformal symmetry.
• The RG flow has an UV gaussian and an IR fixed point.
The theory sits in the IR fixed point.

$$g(\mu) = g_*$$

From QCD to Walking

The running of the coupling in QCD is determined by the β-function

$$\beta(\mu) = \mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \dots \ ,$$

with

$$b_0 = \frac{1}{(4\pi)^2} \left(\frac{11}{3} N - \frac{4}{3} T_R N_f \right)$$

$$b_1 = \frac{1}{(4\pi)^4} \left(\frac{34}{3} N^2 - \frac{20}{3} N T_R N_f - 4 \frac{N^2 - 1}{d_R} T_R^2 N_f \right)$$

For conformal field theories the coupling is constant and the β-function is zero

At points for which $\beta(g) = 0$ the coupling is constant (infrared fixed point)

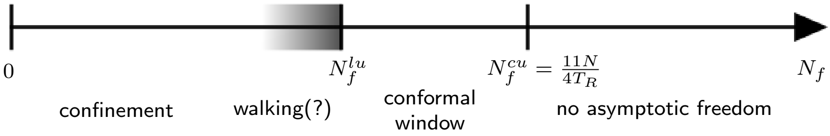
Near zeros of the β-function the coupling walks

Walking and Conformal window

$$\beta(\mu) = \mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \dots$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N - \frac{4}{3}T_R N_f\right)$$

$$b_1 = \frac{1}{(4\pi)^4}\left(\frac{34}{3}N^2 - \frac{20}{3}N T_R N_f - 4\frac{N^2-1}{d_R}T_R^2 N_f\right)$$

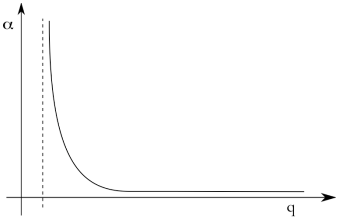


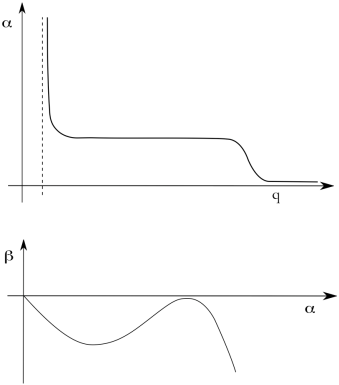
Banks-Zaks (perturbative) fixed point:

$$g_*^2 \simeq -\frac{b_0}{b_1} << 1$$

Walking and β-function

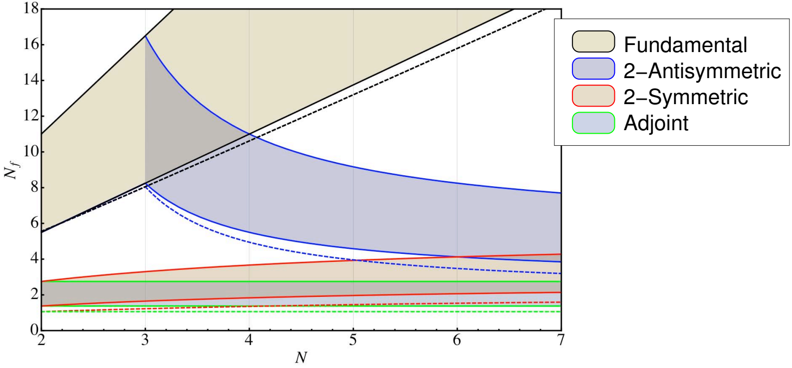
Walking needs two separate scales Λ_{ETC} and Λ_{TC}





If the anomalous dimension is large the difficulties of technicolor disappear

Higher Representations Fermions



Dietrich and Sannino: Phys. Rev. D **75** (2007) 085018 [arXiv:hep-ph/0611341].

Fermions in higher representations have a lower contribution to the S-parameter, the minimal being given by SU(2) with two flavours of adjoint (symmetric) fermions (Minimal Walking Technicolor)

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \ \langle \Phi_R(t,\mathbf{x})\Phi_R(0)\rangle(g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\,[d_\Phi - \gamma_\Phi(g)] \right\} C(t,g,m,\mu) = 0$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$
$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \; \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

Weinberg-Callan-Symanzik equation. Close to the fixed point...

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\left[d_\Phi - \gamma_\Phi(g)\right] \right\} C(t,g,m,\mu) = 0 + \text{corrections}$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$

$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \ \langle \Phi_R(t,\mathbf{x})\Phi_R(0)\rangle(g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} - [1+\gamma]\, m\frac{\partial}{\partial m} + 2\,[d_\Phi - \gamma_\Phi] \right\} C(t,g,m,\mu) = 0$$

Solution of the Weinberg-Callan-Symanzik equation.

$$C(t,g,m,\mu) \simeq b^{2(d_\Phi-\gamma_\Phi)} C(bt, g_*, b^{-(1+\gamma)}m, \mu) =$$
$$\simeq \mu^{2d_\Phi} \left(\frac{m}{\mu}\right)^{2\frac{d_\Phi-\gamma_\Phi}{1+\gamma}} F\left(tm^{\frac{1}{1+\gamma}}, \mu\right)$$

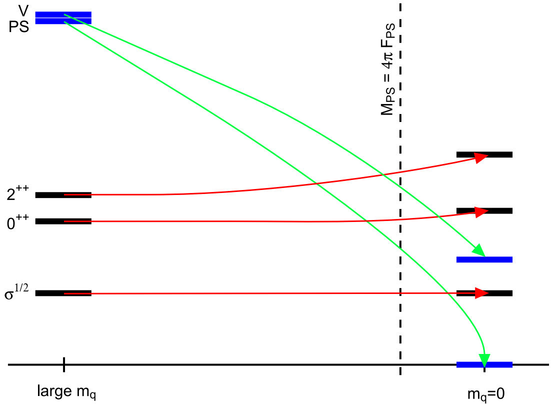
The mass term breaks the asymptotic scale invariance. A mass gap is expected to be generated.

$$C(t,g,m,\mu) \simeq A \exp\left(-M_\Phi t\right)$$

$$M_\Phi = a_\Phi \mu \left(\frac{m}{\mu}\right)^{\frac{1}{1+\gamma}} \qquad m \to 0$$

Some cartoons

Confinement and χSB



large fermion mass
(pure YM + NR fermions)

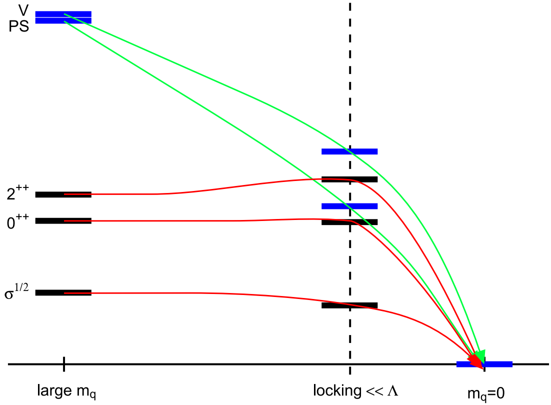
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V/M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$

chiral limit (χPT)

$$M_{PS}^2 \simeq -\frac{\langle\bar{\Psi}\Psi\rangle}{F_{PS}^2} m_q$$
$$M_V \simeq M_V^{(0)}$$
$$M_G \simeq M_G^{(0)}$$

Some cartoons

IR conformality



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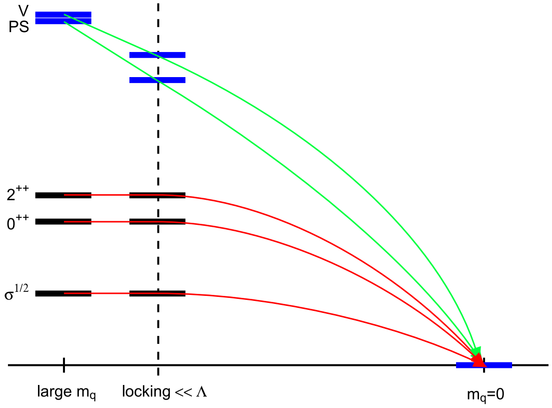
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V / M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$
$$\sigma \simeq \sigma^{(YM)}$$

chiral limit (scaling region)

$$M_X \simeq a_X \mu^{1-\alpha} m_q^{\alpha}$$
$$\sigma^{1/2} \simeq a_\sigma \mu^{1-\alpha} m_q^{\alpha}$$
$$M_X / M_Y \simeq a_X / a_Y$$
$$M_X / \sigma^{1/2} \simeq a_X / a_\sigma$$

Some cartoons

IR conformality



at all scales

$M_V/M_{PS} \simeq 1+\epsilon$

$M_{PS} \gg \sigma^{1/2}$

$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$

Miransky's scenario for Banks-Zaks fixed point

V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

- Banks-Zaks fixed point $g_*^2 = -\frac{b_0}{b_1} \ll 1$.
- The fermionic mass destroys the IR fixed point. Define the fermionic pole mass M_q.

$$M_q = a_q \mu \left(\frac{m_q}{\mu}\right)^{\frac{1}{1+\gamma}} \qquad M_q \ll \Lambda$$

- In the limit $M_q \gg \Lambda$, the fermions decouple and theory is pure YM. Consider the regime $M_q \ll \Lambda$.
- The running coupling constant:

$$g(m,\mu) = \begin{cases} g(0,\mu) & \mu > M_q \\ g_* & \mu = M_q \\ \frac{1}{2b_0^{YM} \ln \frac{\mu}{\Lambda_{YM}}} & \mu \ll M_q \end{cases}$$

- Λ_{YM} is not a new scale in the theory, but is computed by requiring continuity about the energy scale $\mu \simeq M_q$.

$$\Lambda_{YM} = M_q \; e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q$$

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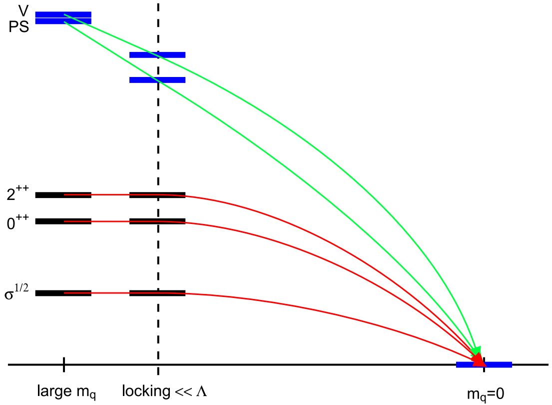
$$\Lambda_{YM} = M_q \ e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q \ll \Lambda$$

- At energies much lower than M_q, the original theory is effectively described by a pure Yang-Mills theory with scale Λ_{YM}.
- Glueballs are lighter than mesons.
- A deconfinement transition occurs at a temperature $T_c \simeq \Lambda_{YM}$.
- Mesons are effectively quenched. The mesons are bound states of the quark-antiquark pair interacting via the YM static potential, the bound energy is small with respect to the mass of the constituents, and the correction to the potential due to quark-antiquark pair creation are negligible.
- As the mass M_q is reduced, the IR physics is always the same, provided that all the masses are rescaled with M_q.

Some cartoons again

IR conformality

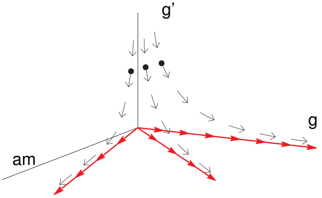


$$M_V/M_{PS} \simeq 1 + \epsilon$$

$$M_{PS} \gg \sigma^{1/2}$$

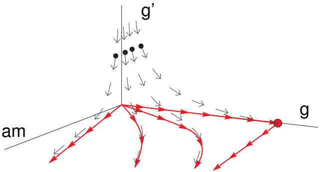
$$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$$

And on the lattice?



- the theory has two UV-relevant parameters g, m
- renormalized trajectories lie in the (g, m) plane
- simulations are performed away from this plane
- am small in order to have small discretization effects

- scale invariance is broken by m AND 1/L
- large physical volume + light masses!
- deviations from QCD spectrum



General strategy: go chiral!

- In order to distinguish between confinement and IR-conformality, the study of the chiral limit is essential.
- The IR-conformality is characterised by the presence of a scaling region (all the masses go to zero with the same power law).
- In principle one could investigate the existence of a power-law behaviour, but unfortunately the quality of the fits is poor. A stabler numerical strategy is to look for plateaux in ratios of masses.
- In the case of a perturbative-like IR fixed point, the quasi-degeneracy of PS and V mesons is a common feature to the high-mass and chiral regimes. Instead the glueballs and the string tension have drammatically different behaviours in the two opposite regimes.

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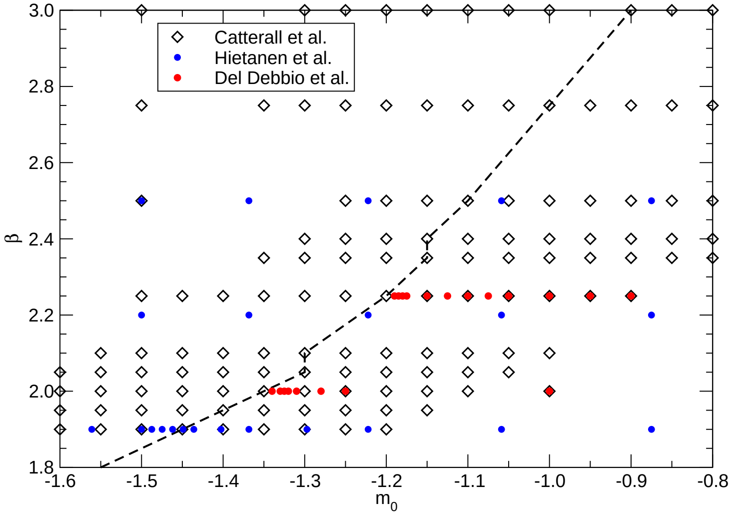
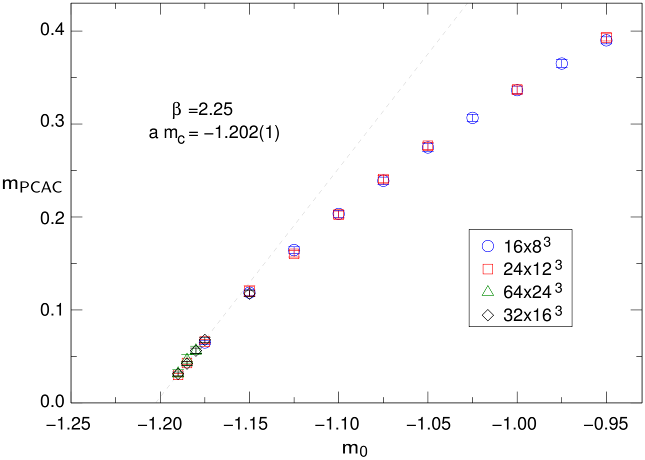
Many of these behaviors can be masked in numerical simulations
by finite-volume effects and discretization artifacts.

What has been simulated so far



Chiral Limit

The quark mass from the axial Ward identity (PCAC mass) is used.



Old results: the spectrum

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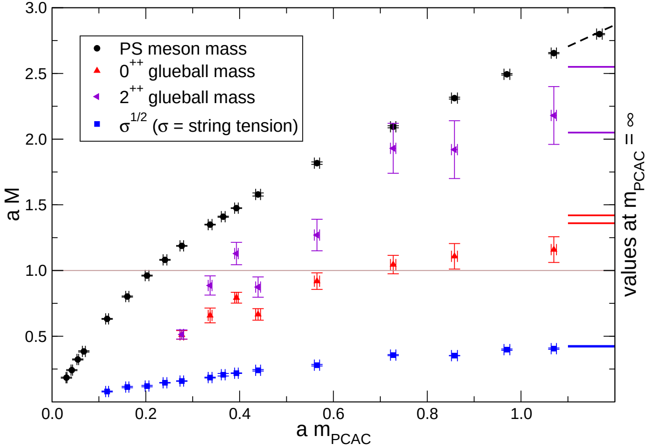
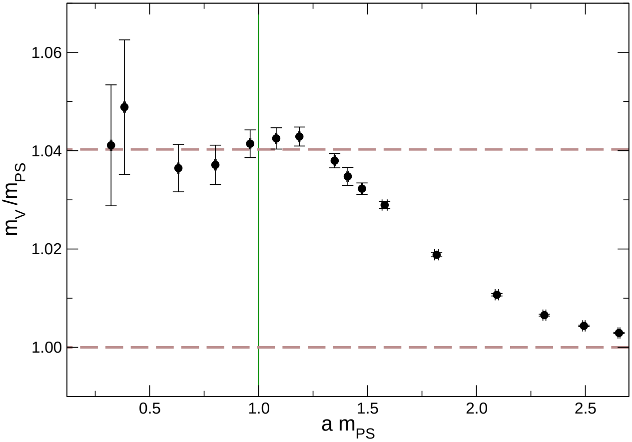


Old results: ratios

All the masses must scale with the same exponent, ratios have to be constant.



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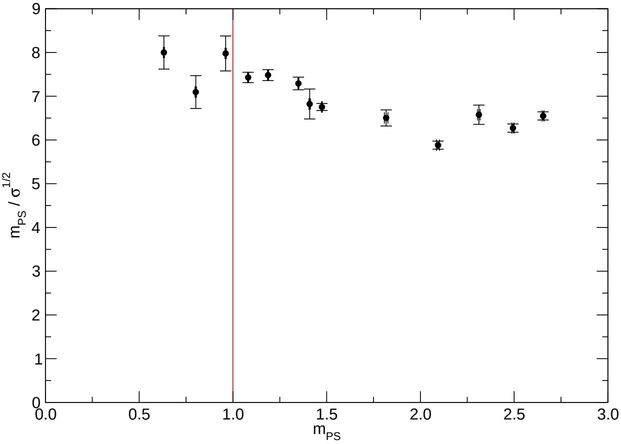
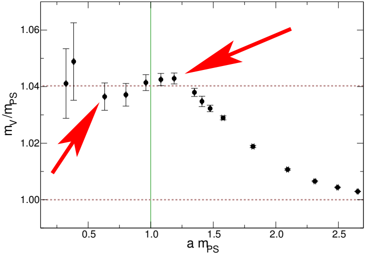


Digging into data

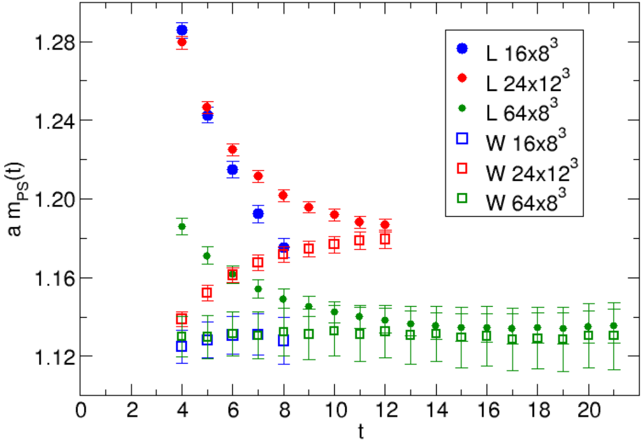
Fermionic observables

How can I classify the finite volume effects affecting my data?

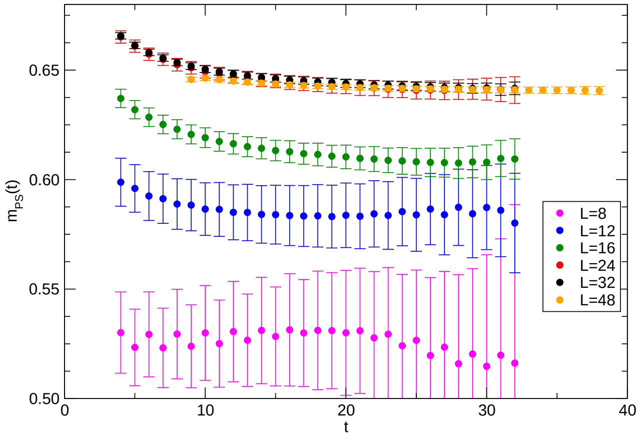
- Change the volume in a controlled way:
  - Increase the temporal direction
  - Increase the spatial direction
- Change boundary conditions (...)



Temporal finite size effects



Spatial finite size effects



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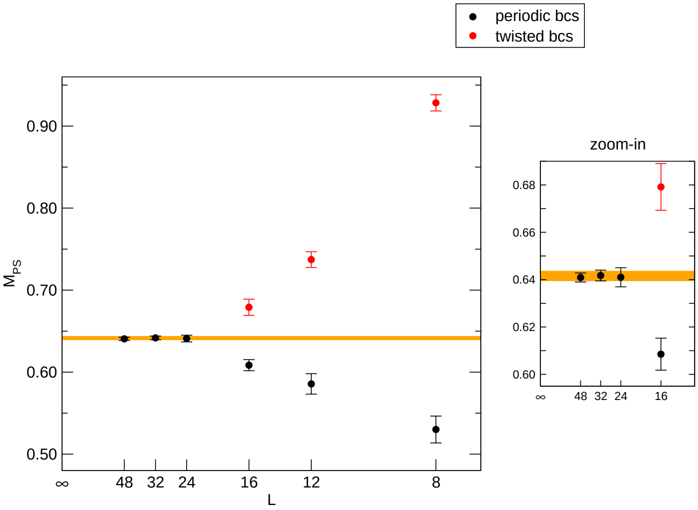

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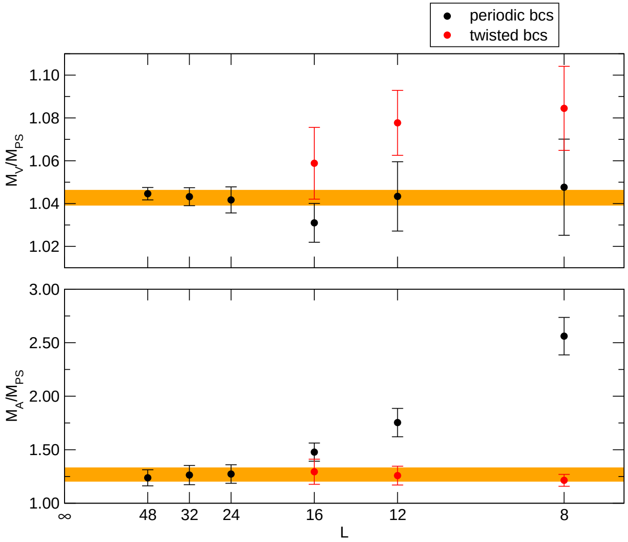
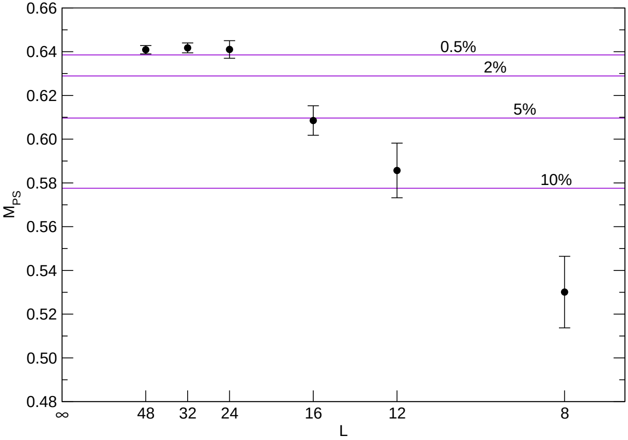
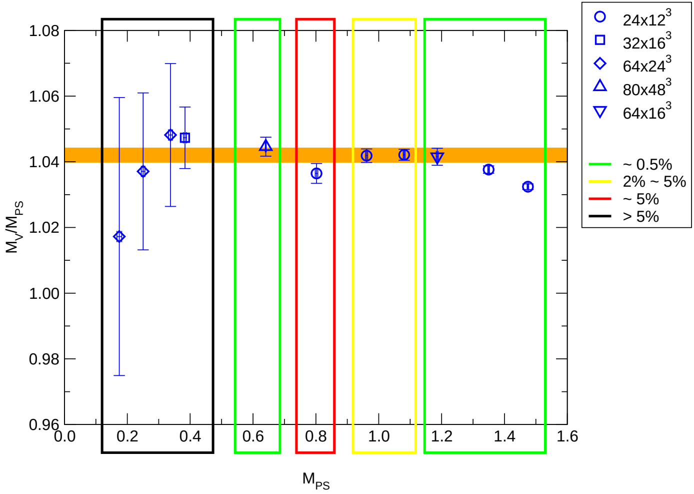


Spatial finite size effects

Assume scaling and characterise my uncertainties



In the end: not too bad

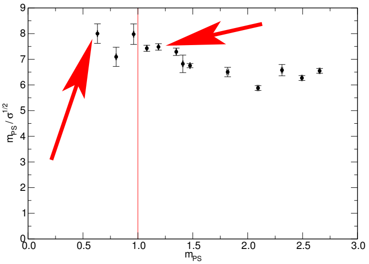


Digging into data

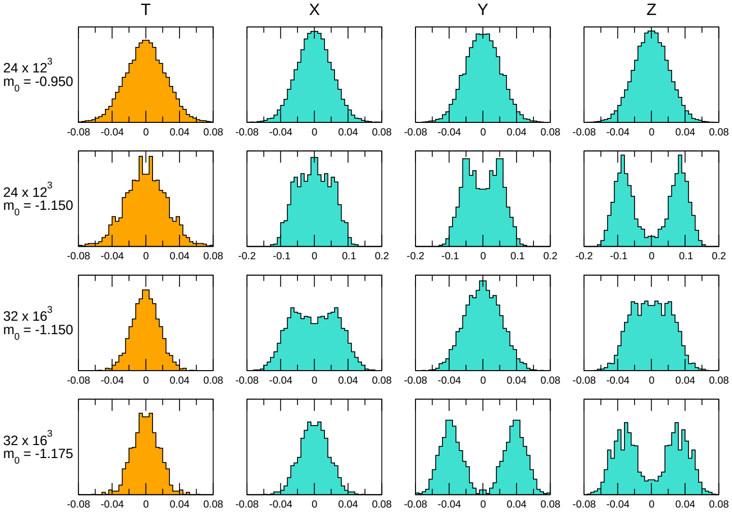
Gluonic observables

How can I classify the finite volume effects affecting my data?

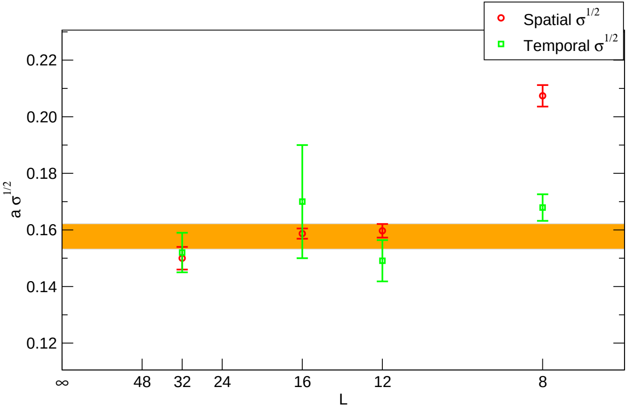
- Control center symmetry.
- Coherence of spatial and string tension.
- Change the volume in a controlled way:
  - Increase the spatial direction.



Polyakov distribution

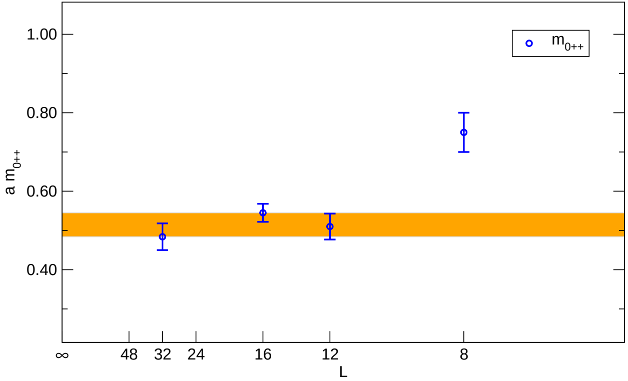


Finite size effects: $\sqrt{\sigma}$



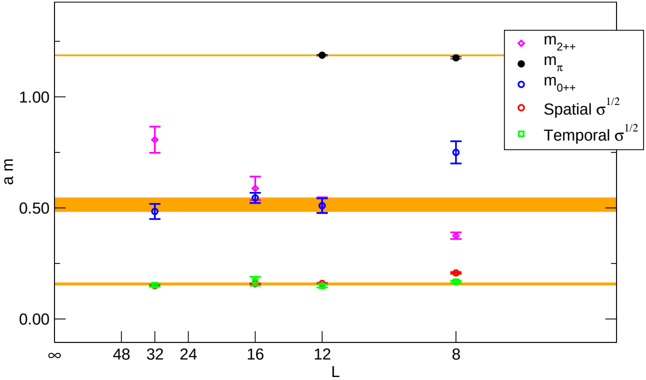
Remember: $M_\pi = 1.187(2)$

Finite size effects: M_{0++}



Remember: $M_\pi = 1.187(2)$

Summary



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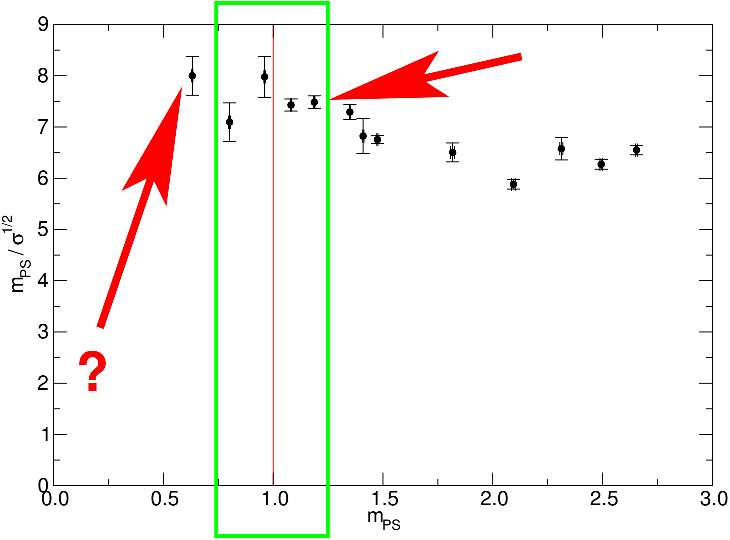
Summary



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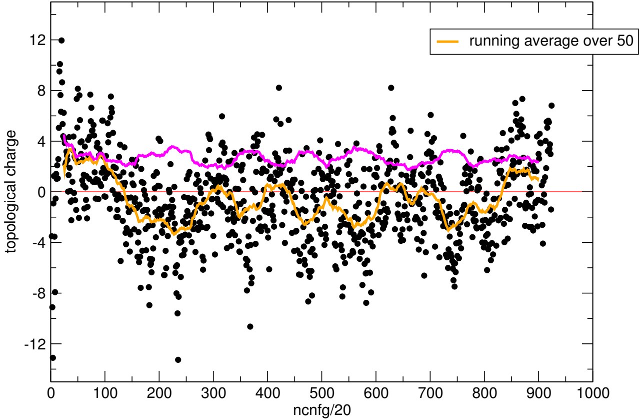
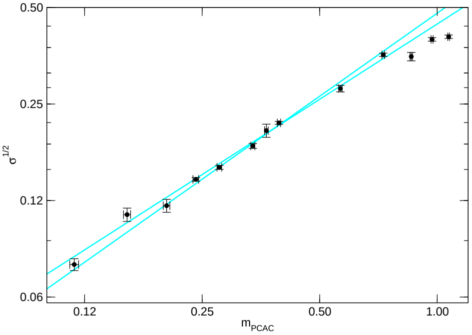


Scaling region



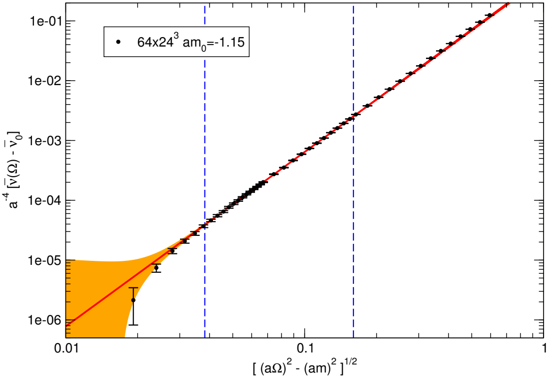
$$a\sigma^{1/2} = A_\sigma (am_q)^{\frac{1}{1+\gamma}} \qquad \gamma \simeq 0.16 \ \ldots \ 0.28$$

Not efficient!

A smart idea!

Study the scaling of the integral of the spectral density of the Dirac operator.

- The mode number $\bar{\nu}(\Omega) = 2\int_0^{\sqrt{\Omega^2 - m^2}} \rho(\omega) d\omega$
- It can be shown $\bar{\nu}(\Omega) \simeq \bar{\nu}_0 + A[\Omega^2 - m^2]^{\frac{2}{1+\gamma_*}}$



Conclusions

- **Control the Volume finite size effects.**
  In every channel interesting for you theory.
- **Use smart measurement.**
  An interesting method to evaluate the anomalous dimensions.