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Numerical investigation on formation and stability of a hollow electron beam in the presence of a plasma wake field driven by an ultra-short electron bunch

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The model

- **Plasma**
  - Collisionless, unmagnetized
  - Overdense regime: $n_0 >> n_b$

    n_0 = unperturbed plasma density
    n_b = unperturbed beam density

  - The ions are supposed infinitely massive and constitute a background of positive charge with density n_0
- **Driving electron bunch**
  - Relativistic, travelling along z-direction with an initial velocity βc

Wake field generation

- Within fluid theory, the system is described by the **Lorentz-Maxwell system** in the relativistic regime
- The interaction is taken into account via the generation of plasma wake field in **electrostatic approximation** ($\xi = z\text{-}\beta ct$)
- The longitudinal sharpness of the bunch has been taken into account carefully compared to its high energy conditions (γ factor values)
- Small perturbations are introduced for all the physical quantities

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{k_{pe}^2}{\beta^2}\right)\left(\frac{1}{\gamma^2}\frac{\partial^2}{\partial\xi^2} + \nabla_\perp^2 - k_{pe}^2\right)\Omega = -4\pi e\left(\frac{1}{\gamma^2}\frac{\partial^2}{\partial\xi^2} - k_{pe}^2\right)\rho_b$$

$$\Omega = \beta A_{1z} - \phi$$

$\Downarrow$

Wake potential

- The equation for wake potential:
  - differs from the standard theoretical model of PWF theory [ref.]
  - contains second and fourth order derivatives with respect to the longitudinal coordinate

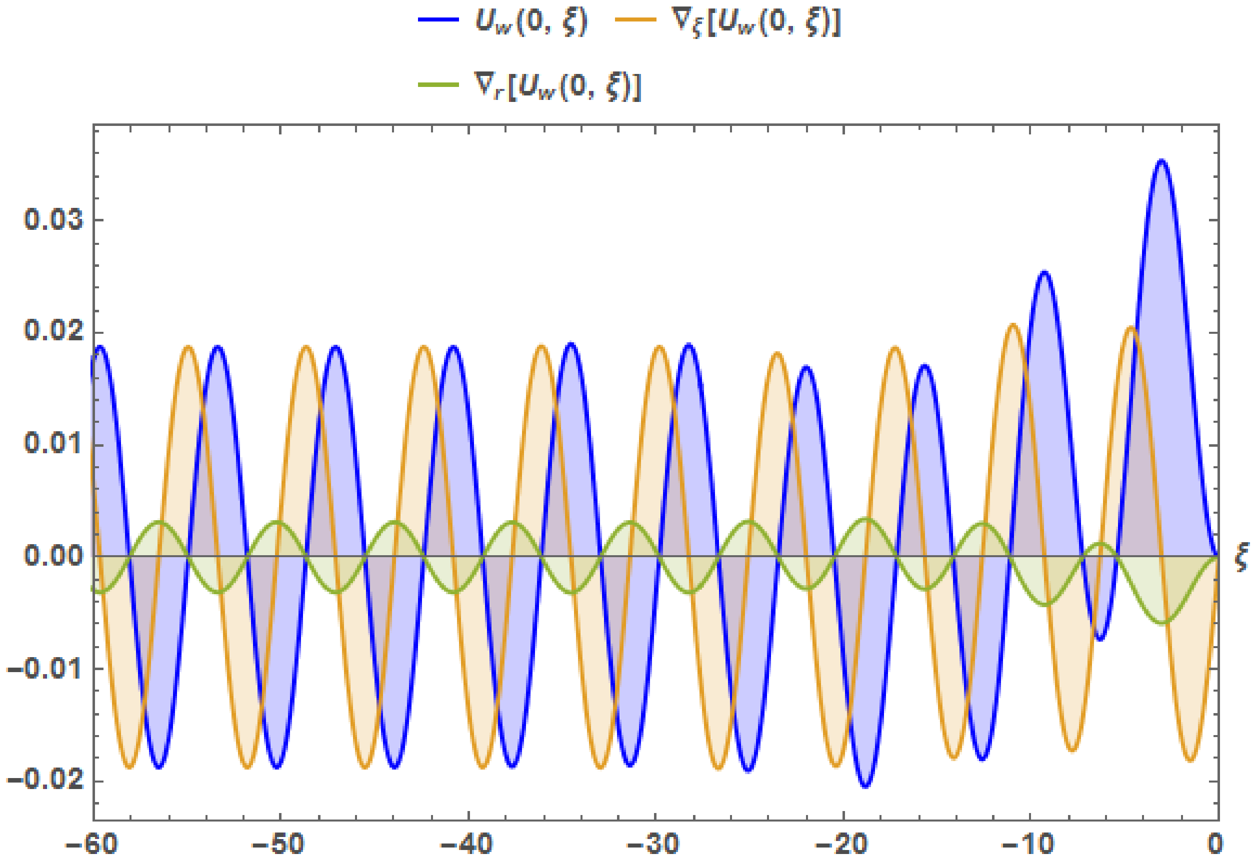
[ref.] P. Chen, J. M. Dawson, R.W. Hu, and T. Katsouleas, *Phys. Rev. Lett.* **54**, 693 (1985)

Numerical integration of the wake

- We consider a bunch profile with the Gaussian distribution of the form

$$\rho_b(r,\xi) = n_b\, exp\left[-\left(\frac{\xi^2}{2\sigma_z^2} + \frac{r^2}{\sigma_\perp^2}\right)\right]$$

- We numerically integrated the equation for the wake potential assuming this Gaussian profile



Dimensionless variables

$\xi' \rightarrow k_{pe}\xi, \qquad r' \rightarrow k_{pe}r$

$U_w \rightarrow \frac{n_0\gamma}{n_b}\frac{e\Omega}{m_0\gamma c^2}$

$\sigma_z' \rightarrow k_{pe}\sigma_z \simeq 0.002 \;\; (\sigma_z \simeq 0.1\,\mu m)$

$\sigma_\perp' \rightarrow k_{pe}\sigma_\perp \simeq 3 \qquad (\sigma_\perp \simeq 160\,\mu m)$

$\gamma = 10^3$

Driven beam

- A second electron beam of Gaussian profile is **externally injected** in phase locking with the plasma wake field
- This beam would experience the effect of the wake field generated by the driving bunch in a longer time scale
- The second beam is very flat radially i.e., **transverse dynamics is neglected**
- Quantum formalisms (quantum-like domain) provided by the *thermal wave model* (TWM) [refs.] have been used to describe the longitudinal dynamics of the externally injected beam

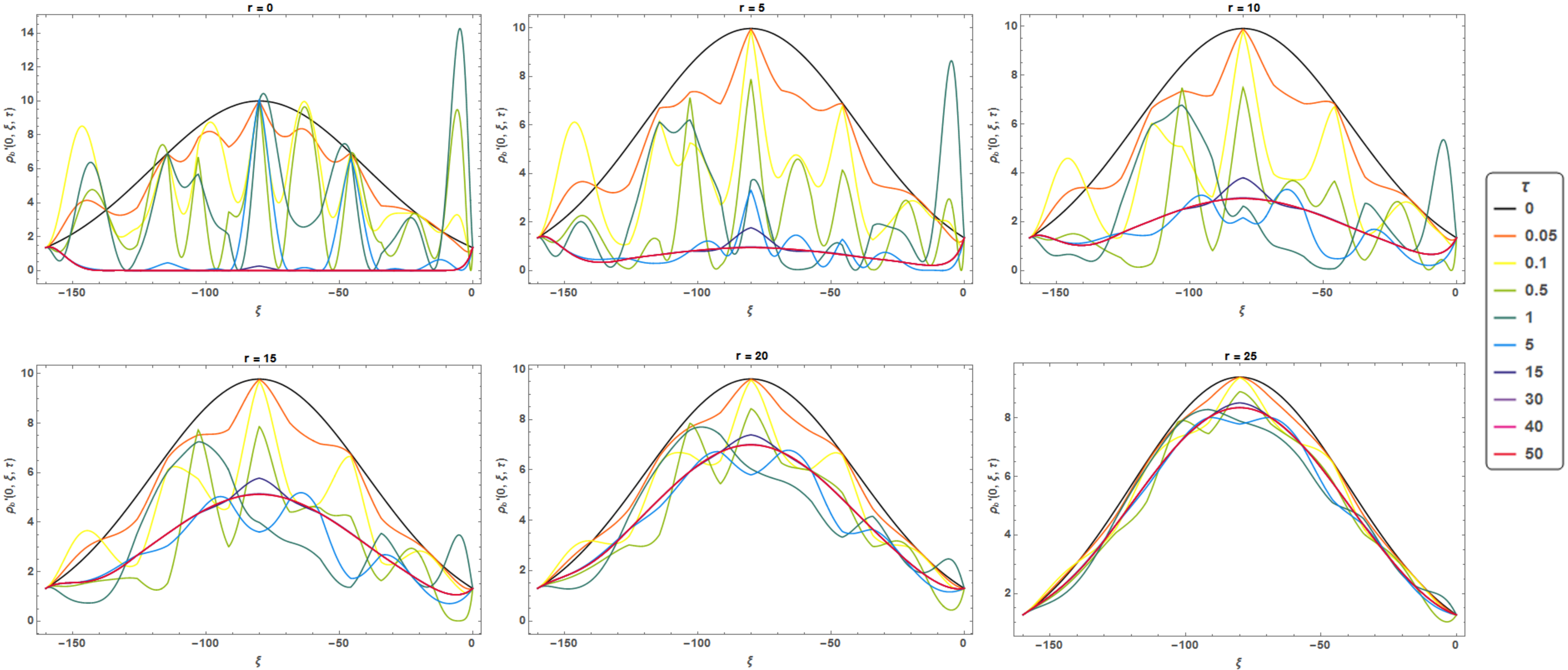
$$i\epsilon' \frac{\partial \psi}{\partial \tau'} = -\frac{\epsilon'^2}{2} \frac{\partial^2 \psi}{\partial \xi'^2} + U_w \psi$$

$\epsilon' \equiv k_{pe}\epsilon \Longrightarrow$ Thermal beam emittance

[refs.] R. Fedele, G. Miele, *Nuovo Cim. D* 13, 1527 (1991); R. Fedele, F. Tanjia, S. De Nicola, D. Jovanović, and P.K. Shukla, *Phys. Plasmas* 19, 102106 (2012); R. Fedele, F. Tanjia, D. Jovanović, S. De Nicola, C. Ronsivalle, *J. Plasma Phys.* 80, 145 (2014)

Density oscillation- 1D

- Initial longitudinally off-axis Gaussian beam: $\psi(r', \xi', 0) = n'_b\, exp\left[-\left(\frac{(\xi'+\bar{\xi})^2}{2\sigma_z'^2} + \frac{r'^2}{\sigma_\perp'^2}\right)\right]$
- We follow the spatio temporal evolution of the density of the driver $\rho'_b(r', \xi', \tau) = N|\psi(r', \xi', \tau)|^2$



Dimensionless variables

$\tau' \rightarrow \omega_{pe}\tau \rightarrow \omega_{pe}ct'$,
$\xi' \rightarrow k_{pe}\xi$,
$r' \rightarrow k_{pe}r$
$\epsilon' \rightarrow k_{pe}\epsilon \simeq 10^{-3}$
$\sigma_z' \rightarrow k_{pe}\sigma_z \simeq 40$,
$\sigma_\perp' \rightarrow k_{pe}\sigma_\perp \simeq 100$

Density oscillation- 1D

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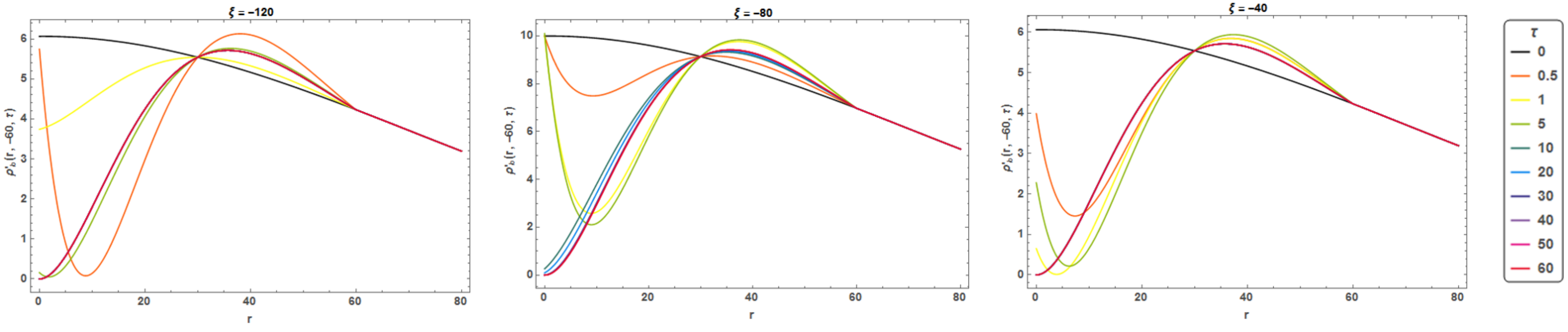

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Density oscillation- 2D

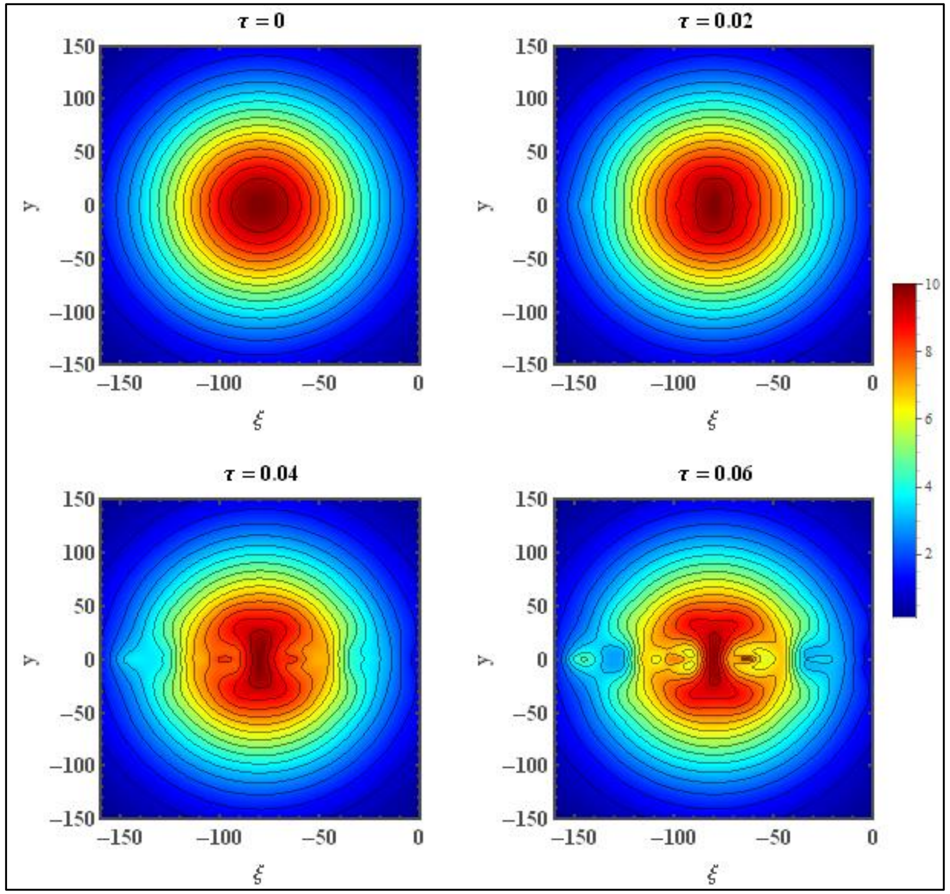
- The spatio temporal evolution in 2D of the driven $\rho'_b(x, y, \xi', \tau')$ is followed
- Several interesting phenomena we observed while τ' incresease
  - **Formation of filaments and voids** $(\tau' = 0 - 0.5)$
  - **Coalescence of voids and channeling** $(\tau = 0.75 - 5\ )$
  - **Hollow beam formation** $(\tau = 7.5 - 20)$

- We have followed the evolution $\rho'_b(x, y, \xi', \tau')$ for different depths of x

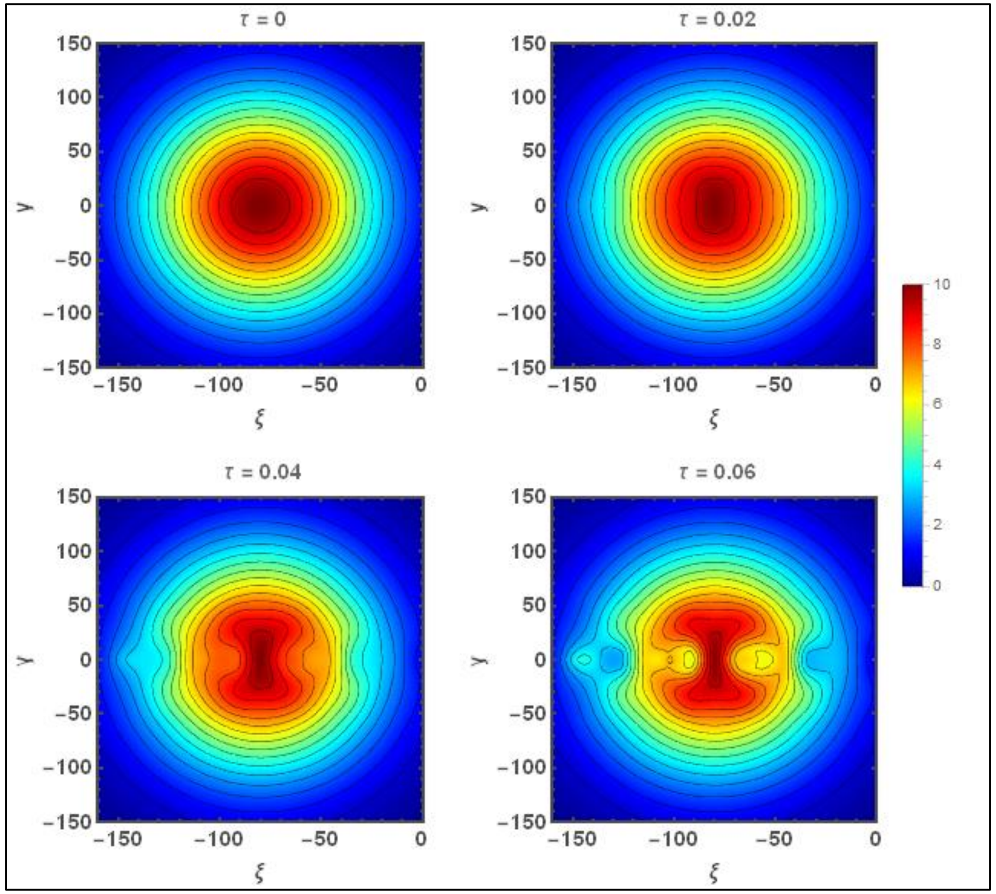
Next slides to follow!

Deformation and formation of filaments and voids

$\rho_b'(x, y, \xi, \tau\ )$ at the depth $x = 0$



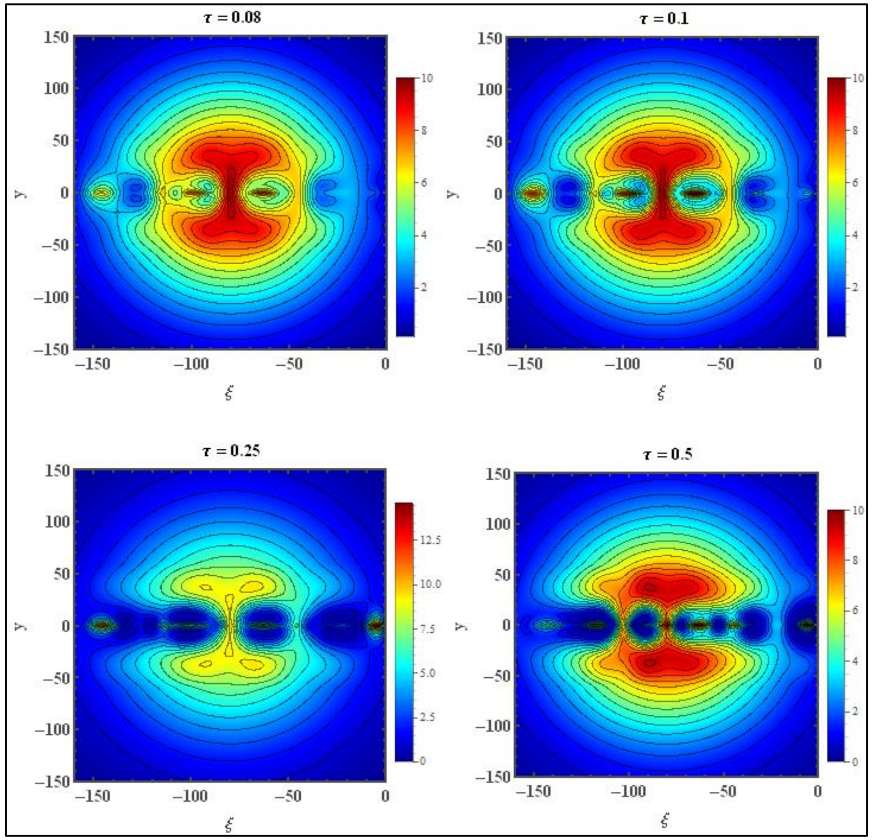
$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 5$



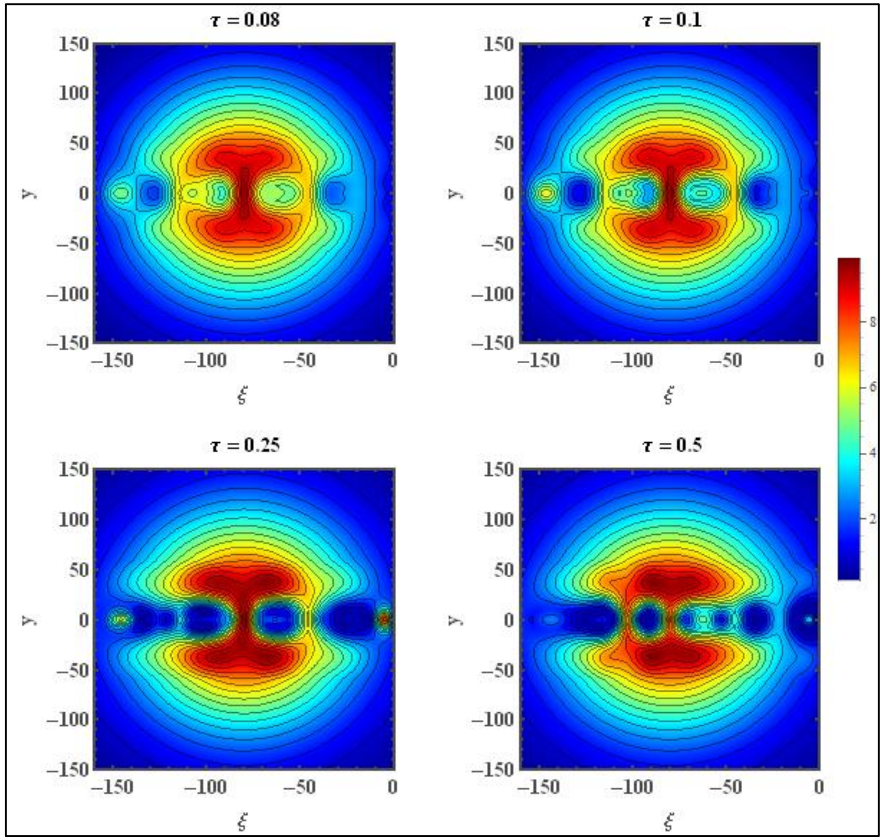
- ❖ Density evolves very fast
- ❖ Deformation of the core of the initial profile
- ❖ Core evolves experiencing a contraction along ξ

Deformation and formation of filaments and voids

$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 0$



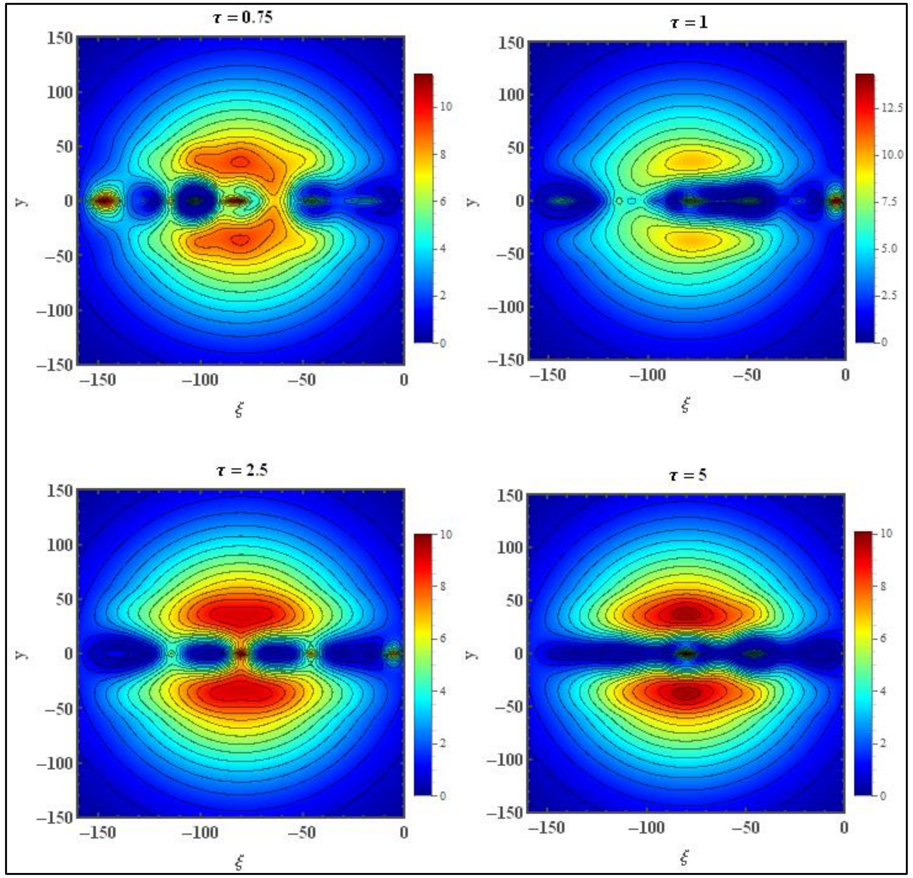
$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 5$



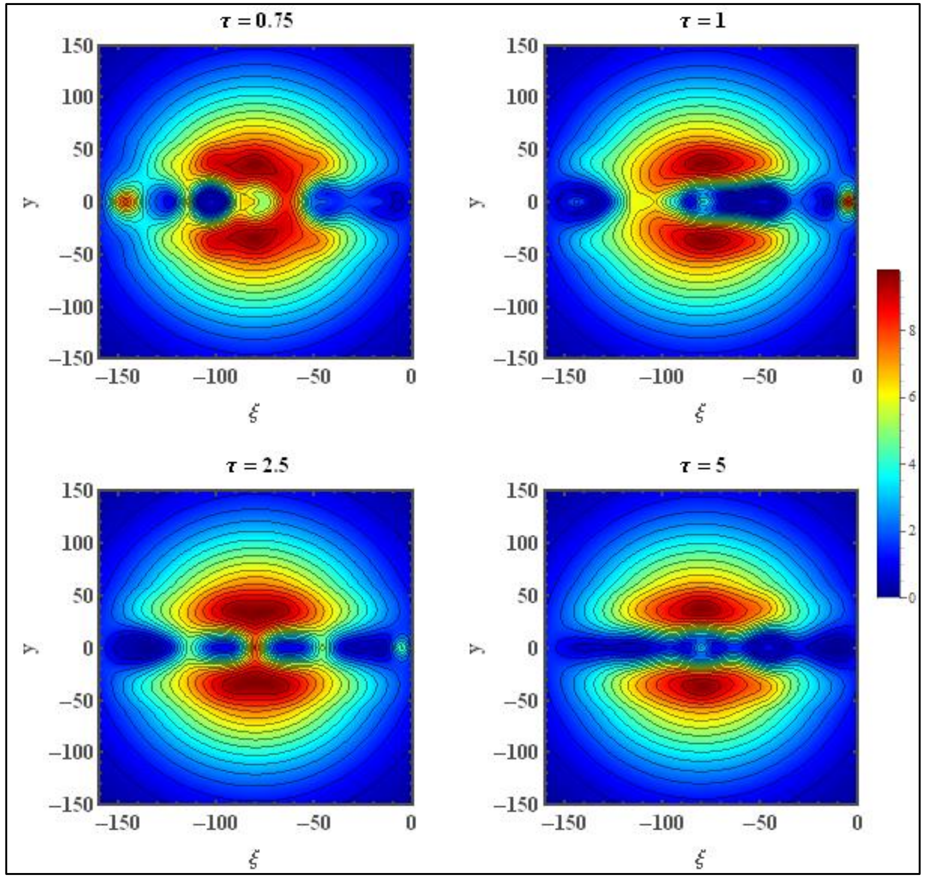
- ❖ Deformation of the core of the initial profile
- ❖ Cigar shaped bone-like structures, filaments and voids
- ❖ The distribution of particles are different in different planes of x

Coalescence of voids and channelling

$\rho_b'(x, y, \xi, \tau\ )$ at the depth $x = 0$



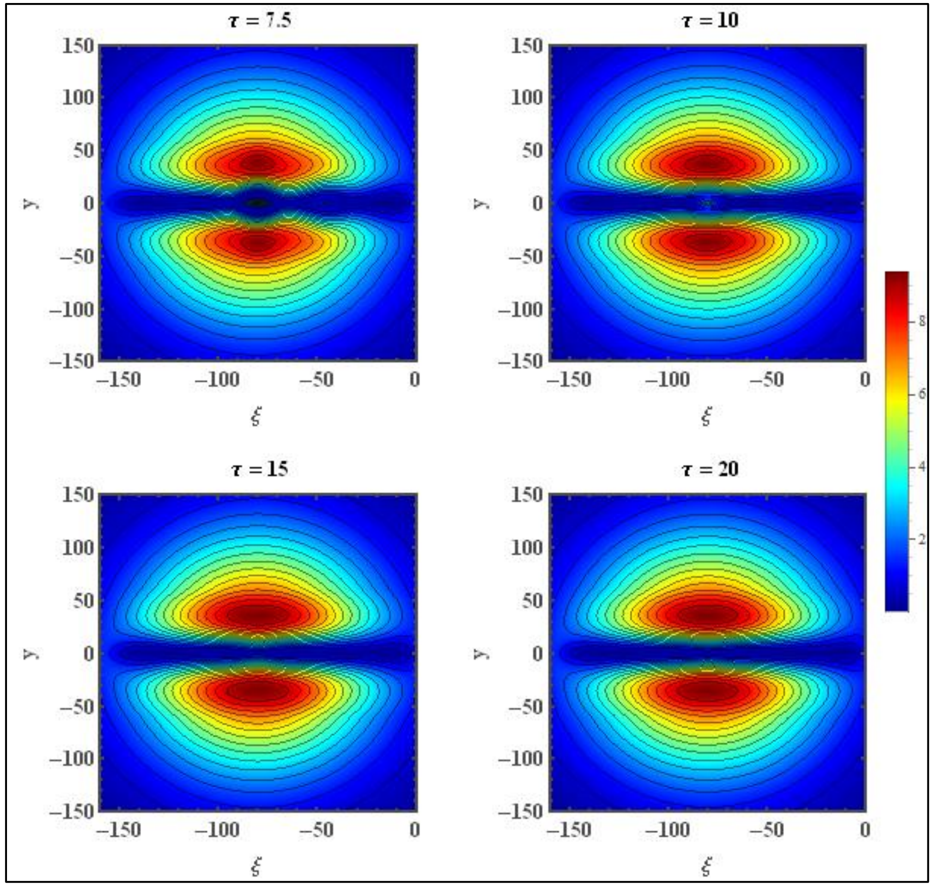
$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 5$



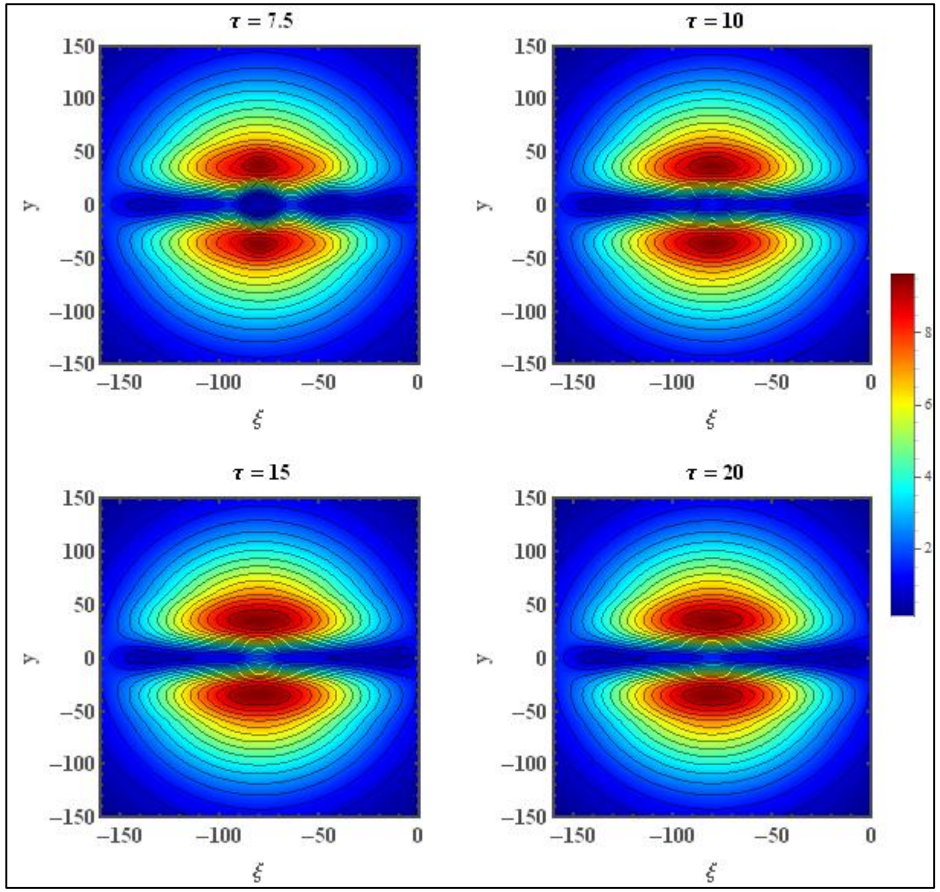
- ❖ System further evolves exhibiting the channeling formation
- ❖ In this stage the filaments progressively disappear
- ❖ Voids progressively coalesce

Hollow formation

$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 0$

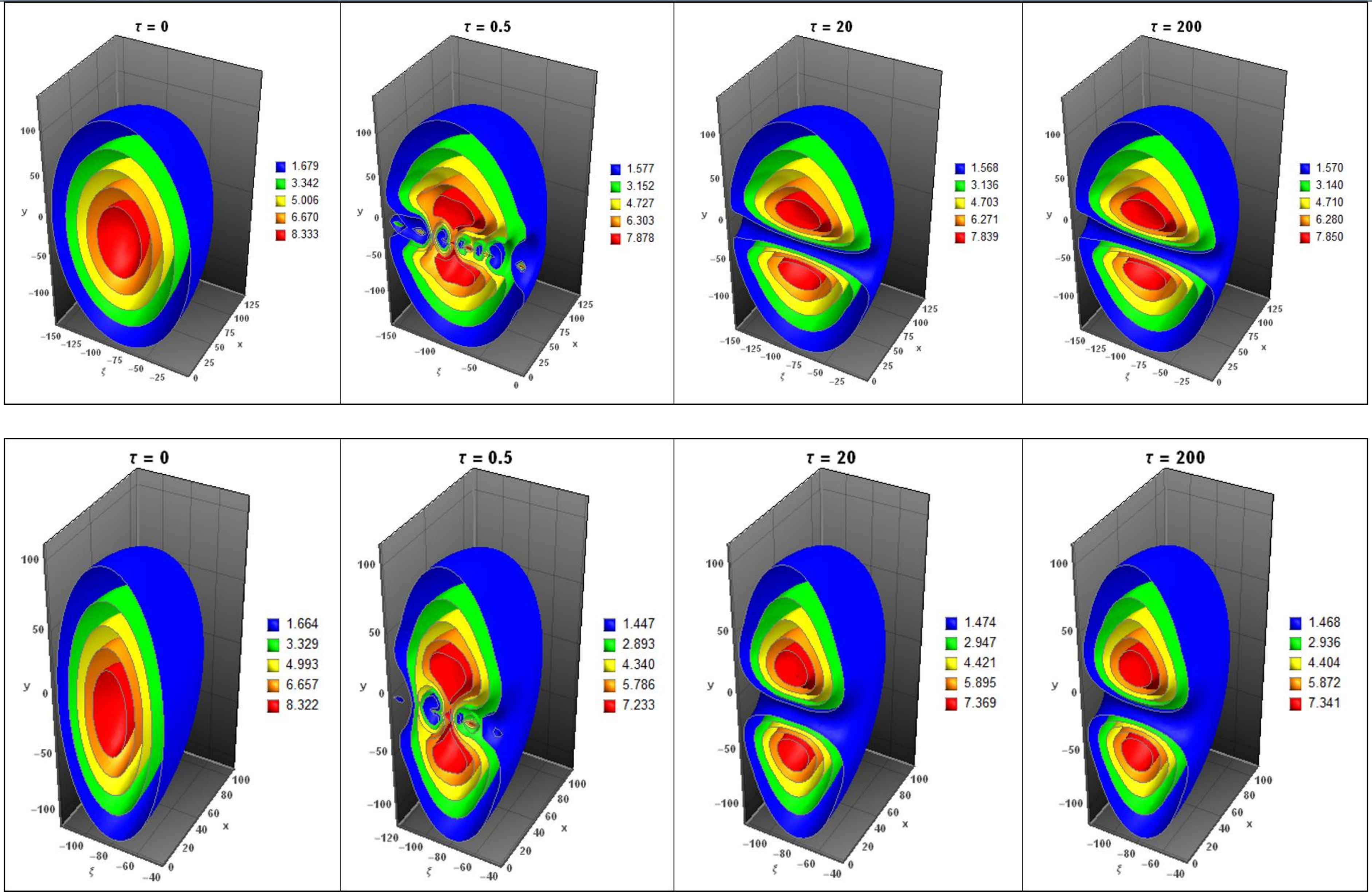


$\rho_b'(x, y, \xi, \tau)$ at the depth $x = 5$



- ❖ Evolution becomes slower
- ❖ Through the process of channeling, hollow structure is created
- ❖ the evolution preserves this hollow formation

3D structures



- ❖ Remains stable
- ❖ The number of particles are conserved

Summary- I

- A theoretical model of PWF theory in the overdense regime has been presented introducing the effective careful analysis of the longitudinal sharpness of the bunch compared to its high energy conditions
- The PWF has a periodic spatial structure and both longitudinal and radial component amplitudes are compatible with the physical conditions of the overdense regime $(n_b \ll n_b)$.
- The longitudinal beam dynamics of an externally injected second beam has been analysed within the context of quantum formalisms (quantum-like domain) provided by TWM.
- The driven beam experiences the effects of the PWF and its length is comparable to the wavelength of the PWF
- The TWM evolution equation has been numerically integrated by taking into account typical values for the beam and plasma parameters. We have found that

Summary- II

- The TWM evolution equation has been numerically integrated by taking into account typical values for the beam and plasma parameters. We have found that-
  - ➢ The number of particles are pushed forward or backword in such a way that they are longitudinally squeezed in specific regions, thus **modulating *the longitudinal beam profile*** (the effect resembles the bucket formation due to a spatially periodic electric field structure)
  - ➢ In the transverse direction, the effect seems to be mainly due to the radial dependence of the wake potential that leads to the formation of ***hollow beam***
  - ➢ After some certain time interval the beam profile becomes stable both longitudinally and radially thus ***preserving it from being collapsed***
- Remarkably-
  - the density structures and processes, (i.e., filaments and voids, coalescence of voids, channeling, and bone-like structures, etc) are all related to the longitudinal and radial density oscillations as result of the PWF action.
  - These oscillations are coupled due to the conservation of the partilcle number.
- The analysis that is under way-
  - Involves the collective treatment of the behaviour of charged-particles in terms of the superposition of Floquet-like states while the beam spreading, due to the thermal emittance, takes place. In particular, the Floquet-like states account for the particle dynamics in a spatially periodic potential.

Thanks for your attention!

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