Numerical investigation on formation and stability of a hollow electron beam in the presence of a plasma wake field driven by an ultra-short electron bunch

F. Tanjia\textsuperscript{a,b}, R. Fedele\textsuperscript{a,b}, S. De Nicola\textsuperscript{c,b}, T. Akhter\textsuperscript{a,b}, and D. Jovanović\textsuperscript{d}

\textsuperscript{a} Dipartimento di Fisica, Università di Napoli Federico II, Napoli, Italy
\textsuperscript{b} INFN Sezione di Napoli, Italy
\textsuperscript{c} CNR-SPIN, Sezione di Napoli, Napoli, Italy
\textsuperscript{d} Institute of Physics Belgrade, Serbia
The model

• Plasma
  - Collisionless, unmagnetized
  - Overdense regime: $n_0 \gg n_b$
    
    \[ n_0 = \text{unperturbed plasma density} \]
    \[ n_b = \text{unperturbed beam density} \]

    The ions are supposed infinitely massive and constitute a background of positive charge with density $n_0$

• Driving electron bunch
  - Relativistic, travelling along $z$-direction with an initial velocity $\beta c$
Wake field generation

- Within fluid theory, the system is described by the Lorentz-Maxwell system in the relativistic regime.
- The interaction is taken into account via the generation of plasma wake field in electrostatic approximation (\( \xi = z - \beta ct \)).
- The longitudinal sharpness of the bunch has been taken into account carefully compared to its high energy conditions (\( \gamma \) factor values).
- Small perturbations are introduced for all the physical quantities.

\[
\Omega = -4\pi e \left( \frac{1}{\gamma^2} \frac{\partial^2}{\partial \xi^2} - k_{pe}^2 \right) \rho_b
\]

- The equation for wake potential:
  - differs from the standard theoretical model of PWF theory [ref.]
  - contains second and fourth order derivatives with respect to the longitudinal coordinate.

Numerical integration of the wake

- We consider a bunch profile with the Gaussian distribution of the form

\[ \rho_b(r, \xi) = n_b \exp \left[-\left(\frac{\xi^2}{2\sigma_z^2} + \frac{r^2}{\sigma_\perp^2}\right)\right] \]

- We numerically integrated the equation for the wake potential assuming this Gaussian profile

**Dimensionless variables**

\[ \begin{align*}
\xi' & \rightarrow k_p e \xi, \quad r' \rightarrow k_p r \\
U_w & \rightarrow n_0 \gamma \frac{e\Omega}{n_b m_0 \gamma c^2} \\
\sigma_z' & \rightarrow k_p \sigma_z \approx 0.002 \ (\sigma_z \approx 0.1 \mu m) \\
\sigma_\perp' & \rightarrow k_p \sigma_\perp \approx 3 \ (\sigma_\perp \approx 160 \mu m) \\
\gamma & = 10^3
\end{align*} \]
Driven beam

- A second electron beam of Gaussian profile is externally injected in phase locking with the plasma wake field.
- This beam would experience the effect of the wake field generated by the driving bunch in a longer time scale.
- The second beam is very flat radially i.e., transverse dynamics is neglected.
- Quantum formalisms (quantum-like domain) provided by the thermal wave model (TWM) [refs.] have been used to describe the longitudinal dynamics of the externally injected beam.

\[ i\epsilon' \frac{\partial \psi}{\partial \tau'} = -\frac{\epsilon'^2}{2} \frac{\partial^2 \psi}{\partial \xi'^2} + U_w \psi \]

\[ \epsilon' \equiv k_{pe}\epsilon \Rightarrow \text{Thermal beam emittance} \]

Density oscillation- 1D

- Initial longitudinally off-axis Gaussian beam: \( \psi(r', \xi', 0) = n'_b \exp \left[ - \left( \frac{(\xi' + \bar{\xi})^2}{2\sigma_{z'}^2} + \frac{r'^2}{\sigma_{\perp'}^2} \right) \right] \)

- We follow the spatio temporal evolution of the density of the driver \( \rho'_b(r', \xi', \tau) = N|\psi(r', \xi', \tau)|^2 \)

**Dimensionless variables**

- \( r' \rightarrow \omega_{pe}t \rightarrow \omega_{pe}ct' \)
- \( \xi' \rightarrow k_{pe} \xi \)
- \( r' \rightarrow k_{pe}r \)
- \( \epsilon' \rightarrow k_{pe}\epsilon \approx 10^{-3} \)
- \( \sigma_{z'} \rightarrow k_{pe}\sigma_z \approx 40 \)
- \( \sigma_{\perp'} \rightarrow k_{pe}\sigma_{\perp} \approx 100 \)
Density oscillation- 1D

• Initial longitudinally off-axis Gaussian beam: 
  \[ \psi(r', \xi', 0) = n'_b \exp \left[ -\left( \frac{(\xi' + \bar{\xi})^2}{2\sigma'^2_z} + \frac{r'^2}{\sigma'^2_\perp} \right) \right] \]

• We follow the spatio temporal evolution of the density of the driver 
  \[ \rho'_b(r', \xi', \tau) = N|\psi(r', \xi', \tau)|^2 \]

Dimensionless variables

- \( r' \rightarrow \omega_{pe}\tau \rightarrow \omega_{pe}ct' \)
- \( \xi' \rightarrow k_{pe}\xi \)
- \( r' \rightarrow k_{pe}\tau \)
- \( \epsilon' \rightarrow k_{pe}\epsilon \approx 10^{-3} \)
- \( \sigma'_z \rightarrow k_{pe}\sigma_z \approx 40 \)
- \( \sigma'_\perp \rightarrow k_{pe}\sigma_\perp \approx 100 \)
Density oscillation- 2D

• The spatio temporal evolution in 2D of the driven $\rho'_b(x, y, \xi', \tau')$ is followed

• Several interesting phenomena we observed while $\tau'$ increase
  • **Formation of filaments and voids** ($\tau' = 0 - 0.5$)
  • **Coalescence of voids and channeling** ($\tau = 0.75 - 5$)
  • **Hollow beam formation** ($\tau = 7.5 - 20$)

• We have followed the evolution $\rho'_b(x, y, \xi', \tau')$ for different depths of $x$

Next slides to follow!
Deformation and formation of filaments and voids

$\rho'_b(x, y, \xi, \tau)$ at the depth $x = 0$

$\rho'_b(x, y, \xi, \tau)$ at the depth $x = 5$

- Density evolves very fast
- Deformation of the core of the initial profile
- Core evolves experiencing a contraction along $\xi$
Deformation and formation of filaments and voids

\[ \rho_b'(x, y, \xi, \tau) \text{ at the depth } x = 0 \]

\[ \rho_b'(x, y, \xi, \tau) \text{ at the depth } x = 5 \]

- Deformation of the core of the initial profile
- Cigar shaped bone-like structures, filaments and voids
- The distribution of particles are different in different planes of \( x \)
Coalescence of voids and channelling

\[ \rho_b'(x, y, \xi, \tau) \] at the depth \( x = 0 \)

\[ \rho_b'(x, y, \xi, \tau) \] at the depth \( x = 5 \)

- System further evolves exhibiting the channeling formation
- In this stage the filaments progressively disappear
- Voids progressively coalesce
Hollow formation

\[ \rho_b'(x, y, \xi, \tau) \] at the depth \( x = 0 \)

\[ \rho_b'(x, y, \xi, \tau) \] at the depth \( x = 5 \)

- Evolution becomes slower
- Through the process of channeling, hollow structure is created
- the evolution preserves this hollow formation
3D structures

- Remains stable
- The number of particles are conserved
• A theoretical model of PWF theory in the overdense regime has been presented introducing the effective careful analysis of the longitudinal sharpness of the bunch compared to its high energy conditions

• The PWF has a periodic spatial structure and both longitudinal and radial component amplitudes are compatible with the physical conditions of the overdense regime ($n_b \ll n_b$).

• The longitudinal beam dynamics of an externally injected second beam has been analysed within the context of quantum formalisms (quantum-like domain) provided by TWM.

• The driven beam experiences the effects of the PWF and its length is comparable to the wavelength of the PWF

• The TWM evolution equation has been numerically integrated by taking into account typical values for the beam and plasma parameters. We have found that
The TWM evolution equation has been numerically integrated by taking into account typical values for the beam and plasma parameters. We have found that:

- The number of particles are pushed forward or backward in such a way that they are longitudinally squeezed in specific regions, thus modulating the longitudinal beam profile (the effect resembles the bucket formation due to a spatially periodic electric field structure).
- In the transverse direction, the effect seems to be mainly due to the radial dependence of the wake potential that leads to the formation of hollow beam.
- After some certain time interval the beam profile becomes stable both longitudinally and radially thus preserving it from being collapsed.

Remarkably:

- the density structures and processes, (i.e., filaments and voids, coalescence of voids, channeling, and bone-like structures, etc) are all related to the longitudinal and radial density oscillations as result of the PWF action.
- These oscillations are coupled due to the conservation of the particle number.

The analysis that is under way:

- Involves the collective treatment of the behaviour of charged-particles in terms of the superposition of Floquet-like states while the beam spreading, due to the thermal emittance, takes place. In particular, the Floquet-like states account for the particle dynamics in a spatially periodic potential.
Thanks for your attention!

- Università di Napoli Federico II
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