Wakefield-induced ionization injection in beam-driven plasma accelerators and Self-Similar Staging

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An injection technique for the beam-driven plasma wakefield accelerator in the blowout regime that utilizes only the wakefields to induce ionization and trapping of high-quality electron bunches.
Wakefield-Induced Ionization Injection

A high-current $e^-$ beam excites a strong plasma wake that accelerates a witness bunch up to 3 GeV energy in 18 mm.

Driver beam (570 pC)
- High-current: 10 kA
- Length (resonant): 7 $\mu$m
- low-$\varepsilon_n$: 1.5 $\mu$m
- 1 GeV energy / $e^-$

Witness beam (6 pC)
- High-current: 5 kA
- Ultra-Short: 800 as
- low-$\varepsilon_n$: 300 nm
- 3 GeV energy / $e^-$

High Transformer ratio: 3

180 GV/m

Wakefield-Induced Ionization Injection

FLASHForward example

\[ n_0 = 12 \times 10^{17} \text{ cm}^{-3} \]
\[ k_p \sigma_z = 1.41 \]

dopant species: helium

\[ Q_b = 574 \text{ pC} \]
\[ I_b = 10 \text{ kA} \]
\[ E_b = 1 \text{ GeV} \]
\[ \sigma_z = 7 \mu m \]
\[ \sigma_r = 4 \mu m \]
\[ \varepsilon_n = 1 \mu m \]

- **Preionization laser** with an intensity capable to fully ionize a gas with a low ionization threshold (LIT), e.g. Hydrogen.
- **Micro-nozzle** fed by the same LIT gas doped with a high-ionization threshold (HIT) gas, e.g. Helium.
The Blowout regime \cite{1}

*High-current, tightly focused e\(^-\) beams blow out all the plasma electrons from their ions creating a clear ion cavity that propagates at the speed of light.*

High-current beams \( I_b \gtrsim 1 \text{ kA} \)

\[
\begin{align*}
    k_p \sigma_z &= \sqrt{2} \\
    k_p \sigma_x &= 0.8
\end{align*}
\]

OSIRIS 3D simulations \cite{0}


\cite{1} J. B. Rosenzweig et al., Phys. Rev. A 44, R6189 (1991)
The Blowout regime \[1\]

High-current, tightly focused $e^-$ beams blow out all the plasma electrons from their ions creating a clear ion cavity that propagates at the speed of light.

Blowout “size” \[2\]

\[
(k_plm)^2 \approx \sqrt{32\pi \Lambda_b^0 (k_p\sigma_z)} \quad \Rightarrow \quad k_plm \approx 3.77 \left(\Lambda_b^0\right)^{1/4}
\]

Blowout size scales with drivers current

\[3\]

\[
k_p\sigma_z = \sqrt{2}
\]

Normalized beam current

\[
\Lambda_b \equiv \left( \frac{n_b}{n_0} \right) (k_p\sigma_x)^2 = \frac{I_b}{I_0}
\]

\[
I_0 \equiv 2\pi e_0 \frac{mc^2}{e} = 8.52 \text{ kA}
\]

skindepth

\[
k_p^{-1} = \sqrt{\epsilon_0 mc^2 / n_0 e^2}
\]

Ion cavity (bubble)

resonant length

\[
k_p\sigma_z = \sqrt{2}
\]

longitudinal semi-axis

transverse radius

\[
l_m = \frac{\pi \rho}{4}
\]


distance from the plasma wake to the ion cavity center

The Blowout regime of beam-driven plasma wakes

Electron density

The Blowout regime

High-current, tightly focused $e^-$ beams blow out all the plasma electrons from their ions creating a clear ion cavity that propagates at the speed of light.
The Blowout regime of beam-driven plasma wakes

**Longitudinal Electric field**

\[ E_0 = \left( \frac{mc^2}{e} \right) k_p \]

wavebreaking field

From linear theory [4]

\[ \frac{E_{z \max}}{E_0} \approx 1.3 \Lambda_b \ln \sqrt{\frac{10}{\Lambda_b}} \]

Max. acc. field

\[ \frac{E_{z \max}}{E_0} \approx \frac{k_p l_m}{2} \]

Max. dec. field [2]

\[ \frac{E_z}{E_0} \approx \frac{1}{2} k_p (\zeta - \zeta_0) \]

Max. dec. field

\[ \frac{E_z}{E_0} \approx \sqrt{\frac{\Lambda_b}{2}} \]

High transformer ratio

"Most efficient energy transfer“

Wakefield potential

\[ \psi \equiv - \int \left( \frac{E_z}{E_0} \right) k_p \, d\zeta \]

\[ \Delta \psi = 1 \]

Last contour in focusing region

\[ \Delta \psi_{\text{max}} \approx \frac{(k_p l_m)^2}{4} = \sqrt{4\pi \Lambda_0^2} \]

Trapping from ionization requires high current drivers

\[ I_b^0 \gtrsim 5 \, \text{kA} \]

[Necessary (but not sufficient) trapping condition]

\[ \psi_i - \psi_f = 1 \]

The Blowout regime of beam-driven plasma wakes

**Blowout model scalings vs 3D PIC (OSIRIS)**

\[ k_p \sigma_z = \sqrt{2} \quad k_p \sigma_x = 0.1 \]

- **Driver-beam**
  - Resonant length
  - Narrow beam
  - Matched emittance

\[ k_p \epsilon_n = (k_p \sigma_x)^2 \sqrt{\gamma/2} \]

- **Trapping from Ionization**
  - \( I_b^0 \gtrsim 5 \text{ kA} \)

- **WII injection**
  - \( I_b^0 \gtrsim 8.5 \text{ kA} \)
The Blowout regime of beam-driven plasma wakes

**Electric field magnitude**

- Beam current: \( I_b = 10 \, \text{kA} \)
- Resonant length: \( k_p \sigma_z = \sqrt{2} \)
- Spot size: \( k_p \sigma_x = 0.8 \)

**The total electric field can induce ionization inside the trapping zone**

- Max. trap. e-field: \( E_{\text{wake}}/E_0 \approx 1.25 \)
- Max. beam e-field: \( E_{\text{beam}}/E_0 \approx 0.5 \)

**Appropriate ionization potential**

- \( E_{\text{ion}} \approx 0.87 \, E_0 \)

- \( E_{\text{wake}} > E_{\text{ion}} > E_{\text{beam}} \)

**Field magnitude proportional to plasma density**

- \( E_0 \propto \sqrt{n_0} \)
Match the beam to the plasma density
\[ k_p \sigma_z \approx \sqrt{2} \]

Select the appropriate dopant species
\[ E_0 \propto \sqrt{n_0} \]

Ionization probability rate \([7]\)
\[
W_{ADK}[\text{fs}^{-1}] \approx 1.52 \frac{4^{n^*} \xi_i [\text{eV}]}{n^* \Gamma (2n^*)} \left( \frac{20.5 \xi_i^{3/2} [\text{eV}]}{E [\text{GV/m}]} \right)^{2n^*-1} \exp \left( -6.83 \frac{\xi_i^{3/2} [\text{eV}]}{E [\text{GV/m}]} \right).
\]

Select the appropriate dopant species
\[ n^* \approx 3.69 \frac{Z}{\xi_i^{1/2} [\text{eV}]} \]

ADK ionization probability rate \([6]\)

$n_0 = 1.2 \times 10^{18} \text{ cm}^{-3}$

Beam current

$I_b = 10 \text{ kA}$

resonant length

$\sigma_z = 7 \mu\text{m}$

spot size

$\sigma_x = 4 \mu\text{m}$

The total electric field can induce ionization inside the trapping zone

Appropriate ionization potential

$E_{\text{ion}} \approx 91 \text{ GV/m}$

$E_{\text{wake}} > E_{\text{ion}} > E_{\text{beam}}$

Max. trap. e-field

$E_{\text{wake}} \approx 130 \text{ GV/m}$

Max. beam e-field

$E_{\text{beam}} \approx 50 \text{ GV/m}$

$E_0 = 105 \text{ GV/m}$

Hydrogen

Helium (1\textsuperscript{st})

Helium (2\textsuperscript{nd})

Neon (1\textsuperscript{st})

Neon (2\textsuperscript{nd})

Beam region (low field)

Back cavity (high field)
Ionization probability rate for He

Well-defined injection volume

Narrow disc centered on-axis:

$$k_p\epsilon_n \approx \frac{(k_p r_{\text{max}})^2}{12}$$

Emittance is defined by the transverse extension of the volume of injection

$$k_p\epsilon_n \approx k_p^2 \langle x_0^2 \rangle / 4$$ \[8\]

WIII bunches are short and low emittance by construction

$$k_p\sigma_z \sim k_p\epsilon_n \sim 0.1$$

Wakefield-Induced Ionization Injection

Injection Volume

The accelerating wakefields are so intense that they can ionize and trap electrons from a high-Z species (e.g. helium).
Betatron wavelength \[ \lambda_\beta = \sqrt{2\gamma} \lambda_p \]

\[ \frac{E_x^{\text{max}}}{E_0} \approx 0.45 \frac{\Lambda_b}{k_p \sigma_x} \]

Ionization by the radial electric fields of the beam strongly depends on the beam’s radius, which undergoes betatron oscillations.

The dopant high-Z species is confined in a narrow jet at the beginning of the plasma cell.

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Wakefield-Induced Ionization Injection

Injected bunch properties

Driver and witness bunch after 14.6 mm

Witness properties after 14.6 mm

Transformer ratio

\[ R \equiv \left| \frac{E_z^{\text{witness}}}{E_z^{\text{driver}}} \right| \approx 3 \]

Up to 3 GeV/e\(^{-}\) in 2 cm

\[ \langle p_z \rangle = 2.41 \text{ GeV/c} \]

\[ z = 14.6 \text{ mm} \]

\[ Q = -6.43 \text{ pC} \]

\[ \Delta \zeta = 0.24 \text{ \(\mu\)m} \]

\[ \Delta \gamma / \langle \gamma \rangle = 2.1 \% \]

\[ \epsilon_n = 2.60 \times 100 \text{ nm} \]

\[ \langle p_z \rangle = 2.41 \text{ GeV/c} \]

\[ z = 14.6 \text{ mm} \]

\[ Q = -6.43 \text{ pC} \]

\[ \Delta \zeta = 0.24 \text{ \(\mu\)m} \]

\[ \Delta \gamma / \langle \gamma \rangle = 2.1 \% \]

\[ \epsilon_n = 2.60 \times 100 \text{ nm} \]

High-current (tunable): 5 kA

Ultra-short bunches: 770 as

Low emittance: 300 nm

Low uncorrelated energy spread \(\sim 1\%\)
Wakefield-Induced Ionization Injection

FACET example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_b$</td>
<td>2.7 nC</td>
</tr>
<tr>
<td>$I_b$</td>
<td>22 kA</td>
</tr>
<tr>
<td>$E_b$</td>
<td>23 GeV</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>14 µm</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>10 µm</td>
</tr>
<tr>
<td>$\epsilon_x$</td>
<td>50 µm</td>
</tr>
<tr>
<td>$\epsilon_y$</td>
<td>5 µm</td>
</tr>
</tbody>
</table>

**Wakefield Injection**

- **plasma density**
- $n_0 = 5 \times 10^{17}$ cm$^{-3}$
- $k_p \sigma_z = 1.86$

**Dopant species: helium**

- $Q_b = 2.7$ nC
- $I_b = 22$ kA
- $E_b = 23$ GeV
- $\sigma_z = 14$ µm
- $\sigma_x = 10$ µm
- $\epsilon_x = 50$ µm
- $\epsilon_y = 5$ µm

Wakefield-Induced Ionization Injection

Injected bunch properties

Driver and witness bunch after 20.8 mm

\[ n_0 = 0.50 \times 10^{18} \text{ cm}^{-3} \]
\[ z = 20802.9 \text{ \( \mu \)m} \]

\[ E_z = 133 \text{ GV/m} \]

\[ n_p / n_0 \approx 10^2 \]
\[ n_p / n_0 \approx 10^3 \]

\[ \| \gamma - 1 \| = 0.80 \]
\[ \Delta \gamma / \langle \gamma \rangle = 2.7\% \]
\[ \text{width} = 2.7 \text{ \( \mu \)m} \]
\[ \langle p_z \rangle = 2.70 \text{ GeV/c} \]

\[ z = 20.8 \text{ mm} \]
\[ Q = -8.77 \text{ pC} \]
\[ \Delta \mu = 1.58 \text{ \( \mu \)m} \]

\[ n_\epsilon \]
\[ Z = 20.8 \text{ mm} \]

\[ R \approx 2 \]

\[ \text{Transformer ratio} \]

\[ E_z = 133 \text{ GV/m} \]

\[ \langle p_z \rangle = 2.70 \text{ GeV/c} \]

\[ \text{Charge [pC]} \]

\[ \text{Energy spread [%]} \]

\[ \text{Emittance [\( \mu \)m]} \]

\[ \text{Current [kA]} \]

\[ \text{Low uncorrelated energy spread} \sim 0.3\% \]

\[ \text{High-current (tunable):} \sim 2 \text{ kA} \]

\[ \text{Ultra-short bunches:} 2.7 \text{ fs} \]

\[ \text{Low emittance:} 1.5 \text{ \( \mu \)m} \]

Driver and witness bunch after 20.8 mm

Witness properties after 20.8 mm

\[ \text{He at 0.2\%} \]

\[ \langle p_z \rangle = 2.70 \text{ GeV/c} \]

\[ \text{Charge [pC]} \]

\[ \text{Energy spread [%]} \]

\[ \text{Emittance [\( \mu \)m]} \]

\[ \text{Current [kA]} \]

\[ \text{Low uncorrelated energy spread} \sim 0.3\% \]

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\[ \text{Ultra-short bunches:} 2.7 \text{ fs} \]

\[ \text{Low emittance:} 1.5 \text{ \( \mu \)m} \]
Wakefield-Induced Ionization Injection

Beam loading for energy spread reduction

Improved energy spread + higher current

Short (~μm), low-current (~2kA) low-emittance (~μm) and linearly chirped GeV-electron beams
Beam loading for energy spread reduction

Improved energy spread + higher current

Short (~μm), high-current (~20kA) low-emittance (~μm) and quasi-mono-energetic GeV-electron beams

Wakefield-Induced Ionization Injection

Charge [pC]

Energy spread [%]

Emittance [μm]

Current [10kA]

Energy spread [GeV/c]

Beam loading for energy spread reduction

Improved energy spread + higher current

Short (~μm), high-current (~20kA) low-emittance (~μm) and quasi-mono-energetic GeV-electron beams
Wakefield-Induced Ionization Injection

Beam loading for energy spread reduction

Witness bunch profile for perfect beam loading

\[ \Lambda_w = \sqrt{\left( \frac{E_w}{E_0} \right)^4 + \left( \frac{k_p r_m}{2} \right)^4} - \frac{E_w}{E_0} k_p (\zeta - \zeta_w). \]

Witness/driver current ratio

\[ \frac{\Lambda_w}{\Lambda_d} \approx \sqrt{\left( \frac{R_w}{\sqrt{2}} \right)^4 + 1} \]

for reduced energy spread

High-current (~20kA)

low-emittance (~μm)

quasi-mono-energetic

GeV-electron beams

High-brightness for applications

\[ B \propto \frac{I_b}{\epsilon^2 n} \]

Features and requirements

**Requirements**

- **High-current drivers** ($I_0^b > 8.5$ kA) to operate a strong blowout regime.
  
  \[
  k_p \sigma_z \approx \sqrt{2} \\
  k_p \sigma_x \approx 1 \\
  k_p \epsilon_n < (k_p \sigma_x)^2 \frac{\sqrt{\gamma}}{2}
  \]
  
  - Resonant length
  - Moderate spot size
  - Low emittance

- A jet with dopant species with **appropriate ionization threshold**.
  
  \[
  E_{\text{wake}} > E_{\text{ion}} > E_{\text{beam}}
  \]

- Plasma cell technology for the experimental setup.

**Features**

- **Ultra-short, high-quality bunches.**
  
  Injected beams are short and low-emittance

  \[
  k_p \sigma_z \approx k_p \sigma_r \approx k_p \epsilon_n \approx 0.1
  \]

- **High-current, low-energy spread** by controlled beam loading

- **Energy doubling/tripling:**
  
  Transformer ratio $\geq 2$

- **WII bunches can do WII injection in higher plasma densities !!**
Experimental setup

The witness e\textsuperscript{-} bunch is now the driver

\[ \frac{n_1}{n_0} \approx \left( \frac{\sigma_{dri}}{\sigma_{wit}} \right)^2 \approx 100 \]

WII bunches can do WII injection in $10^2$ times higher plasma densities !!

Witness in the second PWFA stage

$>>$ 10 times shorter and lower emittance.
$>>$ Higher current.
$>>$ 10$^2$ times brighter!
$>>$ “Redoubles” the energy per electron.
Self-Similar Staging: First simulations

\[ n_0 = 4 \times 10^{19} \text{ cm}^{-3} \]

Ionization threshold 90 eV
\( \text{N}^{5+} \) (IP 98 eV)

The witness bunch is now the driver

\[ E_{\text{wit}}^{\text{m}} \approx 1 \text{ TV/m} \]

Up to 92 GeV in ~9 cm

Properties

\( \rightarrow \) High-current 15 kA.
\( \rightarrow \) Ultra-short 140 as.
\( \rightarrow \) Ultra-low emitt. 60 nm.
\( \rightarrow \) Transformer ratio ~2.

\[ \langle p_x \rangle = 0.68 \text{ GeV/c} \]

\[ \langle p_y \rangle = 0.40 \text{ GeV/c} \]
Experimental setup

Electrons beams from LWFA can do WII injection!

if they have enough current ~10kA

Boost the energy and the quality of electron beams produced in LWFA accelerators to the next level

Witness in the PWFA stage

- 10 times shorter and lower emittance.
- Higher current.
- $10^2$ times brighter!
- “Doubles” the energy per electron.
Wakefield-Induced Ionization Injection
Staging from a LWFA accelerator

Laser driver

- \( \lambda_0 = 800 \, \text{nm} \)
- \( P_0 = 24.4 \, \text{TW} \)
- \( \tau = 25 \, \text{fs} \) (FWHM on intensity)

Energy in the pulse: 640 mJ

Matched conditions

- \( k_p R_b \simeq k_p w_0 = 2 \sqrt{a_0} \)

- \( n_0 = 5 \times 10^{18} \, \text{cm}^{-3} \quad \rightarrow \quad a_0 = 3.1 \)

High current witness from LWFA (experiments)

Injection-jet: H doped with N

- \( L_{\text{jet}} = 100 \, \mu\text{m} \)
- \( n_N/n_H = 1\% \)
Wakefield-Induced Ionization Injection
Staging from a LWFA accelerator

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High current witness from LWFA (experiments)

Injection-jet: H doped with N
\[ L_{\text{jet}} = 100 \mu\text{m} \quad n_{\text{N}}/n_{\text{H}} = 1\% \]
Wakefield-Induced Ionization Injection

**Needs**

- High-current electron drivers ($I_b > 8$ kA) to operate a strong blowout regime.
- A dopant species $E_{\text{wake}} > E_{\text{ion}} > E_{\text{beam}}$ with appropriate ionization threshold.
- Plasma cell technology for the experimental setup.

**Features**

- Controlled ionization-based injection.
  - Injects from a narrow and well-defined region
- Simple experimental setup.
  - No need for extra devices
- Stable operation.
  - Not very sensitive to driver fluctuations

**Produces**

- Ultra-short, high-quality bunches. Naturally, Injected beams are short and low-emittance $k_p \sigma_z \sim k_p \epsilon_n \sim 0.1$
- High-current, quasi-mono-energetic electron-beams by controlled beam loading
- Energy doubling: Transformer ratio $\geq 2$

**Self-Similar Staging**

- The produced bunches could do WII injection at $10^2$ times higher plasma density.
- Short and high-current electron beams from LWFA could do it as well.
- To produce 10 times reduction in emittance and length with double the energy per electron.
- $10^2$ times brightness enhancement.
Wakefield-Induced Ionization Injection

Summary slide

- Preionization laser with an intensity capable to fully ionize a gas with a low ionization threshold (LIT), e.g. Hydrogen.
- Micro-nozzle fed by the same LIT gas doped with a high-ionization threshold (HIT) gas, e.g. Helium.

WII bunches can do WII injection in $10^2$ times higher plasma densities for the production of $10^2$ times higher brightness $e^-$ beams.