Cooling of relativistic electron beams in intense laser pulses: chirps and radiation spectra

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Outline

1. Introduction to radiation reaction
2. Importance and inclusion of quantum effects
3. Electron beam cooling in a collision with an intense laser pulse
4. Chirped laser pulses
5. Stochastic single-photon-emission model
6. Conclusions and future work.
Classical radiation reaction

- The motion of a charged particle in an external electromagnetic field is governed by the Lorentz force,

\[ \ddot{x}^a = -\frac{q}{m} F^a_b \dot{x}^b, \quad \text{or} \quad \frac{dp}{dt} = q(E + v \times B). \]  \hspace{1cm} (1)

- However, an accelerating charge radiates energy (and momentum) — *How does this emission affect the dynamics of the particle?*

- The *radiation reaction* force responsible for the particle’s recoil is typically very small compared to the applied force, and so neglected.

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However, an accelerating charge radiates energy (and momentum) — How does this emission affect the dynamics of the particle?

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But... not always:
As the field becomes strong, the charge radiates more and radiation reaction may become important.
- Future high-intensity laser facilities (such as ELI).

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Classical descriptions of radiation reaction

- **Lorentz–Abraham–Dirac equation:**

  \[ \ddot{x}^a = -\frac{q}{m} F^a_b \dot{x}^b + \tau \Delta^a_b \ddot{x}^b, \]  

  where \( \tau = q^2/6\pi m \simeq 6 \times 10^{-24} \text{ s} \) is the characteristic time of the electron, and \( \Delta^a_b = \delta^a_b + \dot{x}^a \dot{x}_b \) preserves the mass-shell condition.

- **Jerk** \( \dddot{x} \) leads to unphysical *runaway solutions* and *preacceleration*.

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▶ Jerk \( \ddot{x} \) leads to unphysical runaway solutions and preacceleration.

▶ Landau–Lifshitz: Treat radiation reaction as a small perturbation:

\[
\ddot{x}^a = -\frac{q}{m} F^a_b \dot{x}^b - \tau \frac{q}{m} \left[ \dot{x}^c \partial_c F^a_b \dot{x}^b - \frac{q}{m} \Delta^a_b F^b_c F^c_d \dot{x}^d \right].
\]

**Good:** No runaway solutions or preacceleration issues.

**Bad:** Purely classical description.

▶ Often claimed that Landau–Lifshitz is valid provided only that quantum effects can be ignored [Spohn 2000; Kravets et al. 2013].
Importance of quantum effects

Quantum effects can typically be ignored provided that the observed field $\hat{E}$ is much smaller than the critical field $E_S = 1.3 \times 10^{18} \text{ V/m}$,

$$\chi = \frac{\hat{E}}{E_S} \ll 1.$$  \hspace{1cm} (4)

Upcoming facilities (such as ELI) will produce extremely strong fields in which both $RR$ and quantum effects will play a dominant role.

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- **Classically**, can radiate small amounts of energy at *all* frequencies.

- In the **quantum** picture, must radiate entire quanta of energy.
  - Limits max. photon energy and suppresses high-frequency emission.

- Expected that **classical theories overestimate radiation reaction** in regimes where quantum effects become important (as they contain emission at all frequencies).
Semi-classical extension to Landau–Lifshitz

- **Semi-classical model:**
  Scale the radiation reaction force to compensate for this overestimation as $\chi$ increases [Kirk, Bell & Arka 2009]

  $$\tau \rightarrow g(\chi)\tau,$$  \hspace{1cm} (5)

  where $g(\chi)$ involves a non-trivial integral over Bessel functions.

- Use the approximation $g(\chi) = (1 + 12\chi + 31\chi^2 + 3.7\chi^3)^{-4/9}$ found by Thomas *et al.* (2012).

- Expect semi-classical model to be valid provided that quantum effects remain weak, $\chi^2 \ll 1$. 
Collision with a high-intensity plane-wave laser

- Define the basis vectors \( \{k, \epsilon, \lambda, \ell\} \), where \( k \) is the laser (null) wavevector and \( \epsilon, \lambda \) are orthogonal polarisation vectors.

- Work with the coordinates

\[
\phi = -k \cdot x = \omega t - k \cdot x, \quad \xi = \epsilon \cdot x, \quad \sigma = \lambda \cdot x, \quad \psi = -\ell \cdot x. \tag{6}
\]

- Electromagnetic field tensor (arbitrary polarisation):

\[
\frac{q}{m} F^{a}_{\ b} = a_\epsilon(\phi)(\epsilon^{a} k_{b} - k^{a} \epsilon_{b}) + a_\lambda(\phi)(\lambda^{a} k_{b} - k^{a} \lambda_{b}). \tag{7}
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- Linearly-polarised, \( N \)-cycle plane-wave pulse (length \( L = 2\pi N \)):

\[
a_\epsilon(\phi) = \begin{cases} 
  a_0 \sin(\phi) \sin^2(\pi \phi/L) & \text{for } 0 < \phi < L, \\
  0 & \text{otherwise},
\end{cases} \quad a_\lambda(\phi) = 0. \tag{8}
\]
Collision with a high-intensity laser pulse

Results

Electron beam: Initial Gaussian with 20% spread around $\sim 1$ GeV.
Laser: $Na_0^2 = 9248$

$\text{e.g. } N = 20 \text{ (}a_0 = 21.5\text{)} \text{ and } \lambda = 800 \text{ nm}: 27 \text{ fs (FWHM)}, I = 2 \times 10^{21} \text{ W/cm}^2.$
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Classical predictions depend only on the fluence \( \mathcal{E} \propto Na_0^2 \) [Neitz & Di Piazza 2014].

Semi-classical predictions sensitive to \( a_0 \) directly.
Linear vs. circular polarisation

$N = 20$, $a_0 = 100$: $I_{\text{lin}} = 4.28 \times 10^{22}$ W/cm$^2$ and $I_{\text{circ}} = 2.14 \times 10^{22}$ W/cm$^2$

- Circular: $a_\epsilon(\phi) = \frac{a_0}{\sqrt{2}} \sin(\phi) \sin^2\left(\frac{\pi \phi}{L}\right)$ and $a_\lambda(\phi) = \frac{a_0}{\sqrt{2}} \cos(\phi) \sin^2\left(\frac{\pi \phi}{L}\right)$. Reduced peak intensity, but same fluence.

- Classical: Final state prediction insensitive to polarisation change.

- Semi-classical: Reduced peak intensity $\rightarrow$ less reduction in cooling.
Chirped laser pulses

- Semi-classical beam cooling sensitive to how energy is distributed within the pulse, not just the total energy (as in the classical case)

- Chirps occur in both the production of high-intensity pulses (CPA) and the propagation of pulses in media
  → Investigate their effect on beam dynamics

- Chirped pulse length for $N$-cycle pulse: $L_\Delta = \frac{2\pi N}{1 + \Delta/2}$.

- Linearly chirped phase: $\eta(\phi; \Delta) = \phi \left( 1 + \frac{\phi \Delta}{2 L_\Delta} \right)$.

- Pulse shape function generalised to include a chirp:

$$a_\epsilon(\phi; \Delta) = \begin{cases} a_0 \sqrt{1 + \Delta/2} \sin(\eta) \sin^2 \left( \frac{\pi \phi}{L_\Delta} \right) & \text{for } 0 < \phi < L_\Delta. \\ 0 & \text{otherwise.} \end{cases}$$

(9)

Factor $\sqrt{1 + \Delta/2}$ ensures that chirp rate $\Delta$ does not change fluence.
Chirped laser pulses

Results for $N = 20$ ($a_0 = 21.5$) pulses with chirp rate $\Delta = \pm 0.5$

- Confirms classical prediction for final state independent of chirp.
- **Positive chirp:** shorter duration so peak intensity increases
  
  Higher $\chi \rightarrow$ increased $RR$ suppression $\rightarrow$ less beam cooling
- **Negative chirp:** peak intensity decreases ...

- **Chirping the laser pulse contributes a smaller effect than going from a classical to a semi-classical description** [SRY et al. 2015].
Electron continuously absorbing and emitting laser photons ($\kappa$)

$$p^a + n_{\text{abs}} k^a = p'^a + n_{\text{em}} k^a + \kappa^a \implies \Omega = \Omega' + \tilde{\Omega}.$$  (10)

($n = n_{\text{abs}} - n_{\text{em}}$ is the net number of laser photons absorbed.)
Stochastic single-photon-emission model

Electron continuously **absorbing** and **emitting** laser photons \( (\kappa) \)

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p^a + n_{\text{abs}} \kappa^a = p'^a + n_{\text{em}} \kappa^a + \kappa^a \implies \Omega = \Omega' + \tilde{\Omega}. \tag{10}
\]

\((n = n_{\text{abs}} - n_{\text{em}} \text{ is the net number of } laser \text{ photons absorbed.})\)

1. Diff. probability \( dW = \Gamma \, d\phi \) [Ritus 1985; Green & Harvey 2014]:

\[
\Gamma = \int_{0}^{\Omega} d\tilde{\Omega} \, P(\Omega, \tilde{\Omega}), \quad \text{where} \quad \Omega = -\frac{k^a p_a}{m} \quad \text{and} \quad \tilde{\Omega} = -\frac{k^a \kappa_a}{m}.
\]

(Emission probability \( P(\Omega, \tilde{\Omega}) \) depends on the field strength.)

2. Propagate with Lorentz force; emit photon if \( r \in [0, 1) < dW \).

3. Find \( \tilde{\Omega} \) such that \( \int_{0}^{\tilde{\Omega}} dx \, P(\Omega, x) = \zeta \Gamma, \) with \( \zeta \in [0, 1) \).
Stochastic single-photon-emission model

Model validation: $N = 10$ and $a_0 = 100$ [27 fs with peak $I_{\text{circ}} = 2.14 \times 10^{22} \text{ W/cm}^2$]

- Ensemble of 15 000 identical initial electrons (with $\gamma_0 = 2 000$).
- Despite $\chi \simeq 0.5$, the semi-classical prediction is in good agreement with the average of the stochastic model.
- Single stochastic trajectory shows discrete photon emission.
Stochastic single-photon-emission model

Backscattered radiation: \( N = 10 \) and \( a_0 = 100 \) with circular polarisation

- **Statistics:** Ensemble of 15 000 electrons; sample of 299 412 photons.
- Compatible with \( 2\gamma^2 \approx 10^7 \) laser frequency upshift.
- Peak average photon energy \( \sim 70 \text{ MeV} \).
Stochastic model with chirped pulses
Backscattered radiation: $N = 10$ and $a_0 = 100$ with circular polarisation and $|\Delta| = 0.5$

- **Statistics**: Sample of 263,031 and 352,065 photons.
- **Negative chirp**: Increased photon number, reduced photon energy.
- **Chirp** does not alter the spectral cutoff.
Conclusions

- Future laser facilities will operate in regimes where not only radiation reaction but quantum effects will also play a role.
- Semi-classical extension to $LL$ predicts reduced beam cooling.
- Quantum models of radiation reaction are sensitive to variation in peak intensity, not just total fluence.
- Modified energy distribution using chirps and polarisation — Only a small change.
- Stochastic model shows good agreement to semi-classical model.
- Chirp does not alter radiation frequency cutoff, but...
- Negative chirp shown to increase photon emission and cooling.

**Future work:**
- Electron beam cooling with stochastic model.
- Include two-photon emission and pair production.
- Transverse pulse structure.
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http://phys.strath.ac.uk/alpha-x/pub/People/people.html
References
