Cooling of relativistic electron beams in intense laser pulses: chirps and radiation spectra

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Outline

1. Introduction to radiation reaction
2. Importance and inclusion of quantum effects
3. Electron beam cooling in a collision with an intense laser pulse
4. Chirped laser pulses
5. Stochastic single-photon-emission model
6. Conclusions and future work.

Classical radiation reaction

- The motion of a charged particle in an external electromagnetic field is governed by the Lorentz force,

$$\ddot{x}^a = -\frac{q}{m} F^a{}_b \, \dot{x}^b, \qquad \text{or} \qquad \frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{1}$$

- However, an accelerating charge radiates energy (and momentum) — *How does this emission affect the dynamics of the particle?*
- The *radiation reaction* force responsible for the particle's recoil is typically very small compared to the applied force, and so neglected.

Note: Work in Heaviside-Lorentz units with $\epsilon_0 = 1$, and take $c = 1$.

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- However, an accelerating charge radiates energy (and momentum) — *How does this emission affect the dynamics of the particle?*
- The *radiation reaction* force responsible for the particle's recoil is typically very small compared to the applied force, and so neglected.
- ***But***... not always:
  As the field becomes strong, the charge radiates more and radiation reaction may become important.
  - Future high-intensity laser facilities (such as ELI).

Note: Work in Heaviside-Lorentz units with $\epsilon_0 = 1$, and take $c = 1$.

Classical descriptions of radiation reaction

- Lorentz–Abraham–Dirac equation:

$$\ddot{x}^a = -\frac{q}{m} F^a{}_b \dot{x}^b + \tau \Delta^a{}_b \dddot{x}^b, \tag{2}$$

where $\tau = q^2/6\pi m \simeq 6 \times 10^{-24}$ s is the characteristic time of the electron, and $\Delta^a{}_b = \delta^a_b + \dot{x}^a \dot{x}_b$ preserves the mass-shell condition.

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- **Landau–Lifshitz**: Treat radiation reaction as a small perturbation:

$$\ddot{x}^a = -\frac{q}{m} F^a{}_b \, \dot{x}^b - \tau \frac{q}{m} \left[\dot{x}^c \partial_c F^a{}_b \, \dot{x}^b - \frac{q}{m} \Delta^a{}_b F^b{}_c \, F^c{}_d \, \dot{x}^d \right]. \qquad (3)$$

Good: No runaway solutions or preacceleration issues.
Bad: Purely classical description.

- Often claimed that Landau–Lifshitz is valid provided only that quantum effects can be ignored [Spohn 2000; Kravets *et al.* 2013].

Importance of quantum effects

- Quantum effects can typically be ignored provided that the observed field $\hat{E}$ is much smaller than the critical field $E_S = 1.3 \times 10^{18}$ V/m,

$$\chi = \frac{\hat{E}}{E_S} \ll 1. \tag{4}$$

- Upcoming facilities (such as ELI) will produce extremely strong fields in which both *RR* and quantum effects will play a dominant role.

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- In the **quantum** picture, must radiate entire quanta of energy.
  - Limits max. photon energy and suppresses high-frequency emission.
- Expected that classical theories overestimate radiation reaction in regimes where quantum effects become important (as they contain emission at all frequencies).

Semi-classical extension to Landau–Lifshitz

- **Semi-classical model:**
  Scale the radiation reaction force to compensate for this overestimation as χ increases [Kirk, Bell & Arka 2009]

$$\tau \to g(\chi)\tau, \tag{5}$$

  where $g(\chi)$ involves a non-trivial integral over Bessel functions.

- Use the approximation $g(\chi) = (1 + 12\chi + 31\chi^2 + 3.7\chi^3)^{-4/9}$ found by Thomas *et al.* (2012).

- Expect semi-classical model to be valid provided that quantum effects remain weak, $\chi^2 \ll 1$.

Collision with a high-intensity plane-wave laser

- Define the basis vectors $\{k, \epsilon, \lambda, \ell\}$, where k is the laser (null) wavevector and ϵ, λ are orthogonal polarisation vectors.
- Work with the coordinates

$$\underbrace{\phi = -k \cdot x = \omega t - \mathbf{k} \cdot \mathbf{x},}_{\text{phase}} \quad \underbrace{\xi = \epsilon \cdot x, \quad \sigma = \lambda \cdot x,}_{\text{transverse coordinates}} \quad \psi = -\ell \cdot x. \tag{6}$$

- Electromagnetic field tensor (arbitrary polarisation):

$$\frac{q}{m} F^a{}_b = a_\epsilon(\phi)\big(\epsilon^a k_b - k^a \epsilon_b\big) + a_\lambda(\phi)\big(\lambda^a k_b - k^a \lambda_b\big). \tag{7}$$

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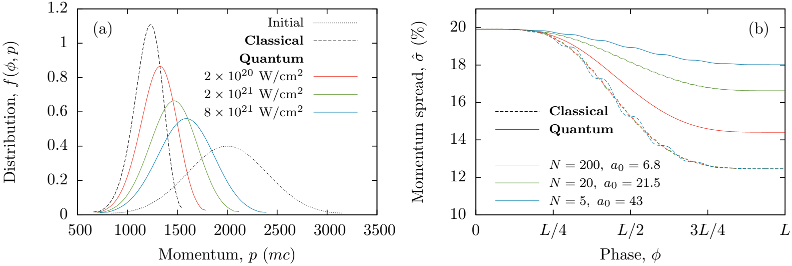
- Linearly-polarised, N-cycle plane-wave pulse (length $L = 2\pi N$):

$$a_\epsilon(\phi) = \begin{cases} a_0 \sin(\phi) \sin^2(\pi\phi/L) & \text{for } 0 < \phi < L, \\ 0 & \text{otherwise,} \end{cases} \qquad a_\lambda(\phi) = 0. \tag{8}$$

Collision with a high-intensity laser pulse

Results



Electron beam: Initial Gaussian with 20% spread around ~ 1 GeV.

Laser: $Na_0^2 = 9248$

e.g. $N = 20$ ($a_0 = 21.5$) and $\lambda = 800$ nm: 27 fs (FWHM), $I = 2 \times 10^{21}$ W/cm^2.

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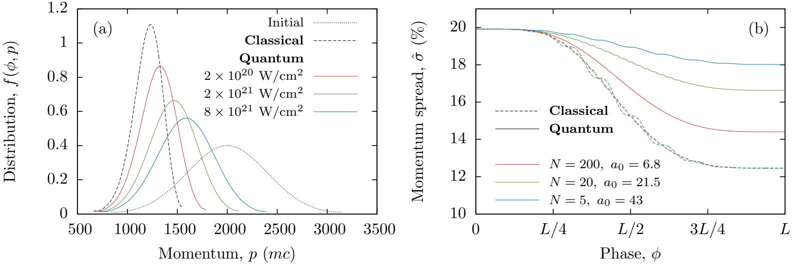

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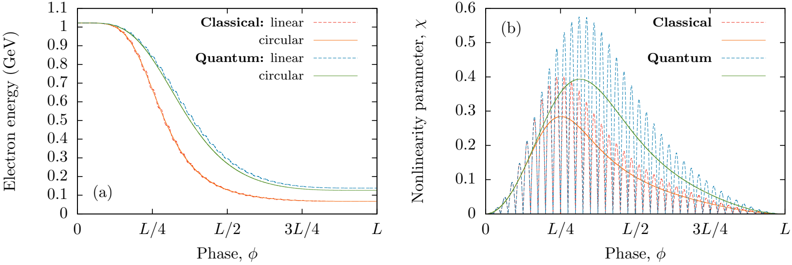
Classical predictions depend only on the fluence $\mathcal{E} \propto Na_0^2$ [Neitz & Di Piazza 2014].
Semi-classical predictions sensitive to a_0 directly.

Linear *vs.* circular polarisation

$N = 20$, $a_0 = 100$: $I_{\text{lin}} = 4.28 \times 10^{22}$ W/cm^2 and $I_{\text{circ}} = 2.14 \times 10^{22}$ W/cm^2



- Circular: $a_\epsilon(\phi) = \frac{a_0}{\sqrt{2}} \sin(\phi) \sin^2(\frac{\pi\phi}{L})$ and $a_\lambda(\phi) = \frac{a_0}{\sqrt{2}} \cos(\phi) \sin^2(\frac{\pi\phi}{L})$. Reduced peak intensity, but same fluence.
- Classical: Final state prediction insensitive to polarisation change.
- Semi-classical: Reduced peak intensity $\longrightarrow$ less reduction in cooling.

Chirped laser pulses

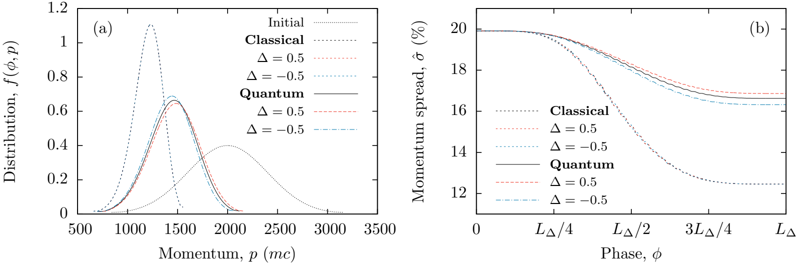
- Semi-classical beam cooling sensitive to how energy is distributed within the pulse, not just the total energy (as in the classical case)
- Chirps occur in both the production of high-intensity pulses (CPA) and the propagation of pulses in media
$\longrightarrow$ *Investigate their effect on beam dynamics*
- Chirped pulse length for N-cycle pulse: $L_\triangle = 2\pi N/(1+\Delta/2)$.
- Linearly chirped phase: $\eta(\phi;\Delta) = \phi\big(1+\phi\Delta/2L_\triangle\big)$.
- Pulse shape function generalised to include a chirp:

$$a_\epsilon(\phi;\Delta) = \begin{cases} a_0\sqrt{1+\Delta/2}\,\sin(\eta)\sin^2(\pi\phi/L_\triangle) & \text{for } 0<\phi<L_\triangle. \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Factor $\sqrt{1+\Delta/2}$ ensures that chirp rate Δ does not change fluence.

Chirped laser pulses

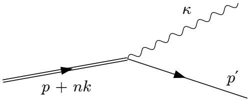
Results for $N = 20$ ($a_0 = 21.5$) pulses with chirp rate $\Delta = \pm 0.5$



- Confirms classical prediction for final state independent of chirp.
- Positive chirp: shorter duration so peak intensity increases

  Higher χ $\longrightarrow$ increased *RR* suppression $\longrightarrow$ less beam cooling
- Negative chirp: peak intensity decreases ...
- *Chirping the laser pulse contributes a smaller effect than going from a classical to a semi-classical description* [SRY *et al.* 2015].

Stochastic single-photon-emission model



- Electron continuously absorbing and emitting laser photons (k)

$$p^a + n_{\text{abs}} k^a = p'^a + n_{\text{em}} k^a + \kappa^a \quad \Longrightarrow \quad \Omega = \Omega' + \tilde{\Omega}. \tag{10}$$

($n = n_{\text{abs}} - n_{\text{em}}$ is the net number of *laser* photons absorbed.)

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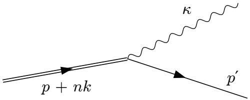

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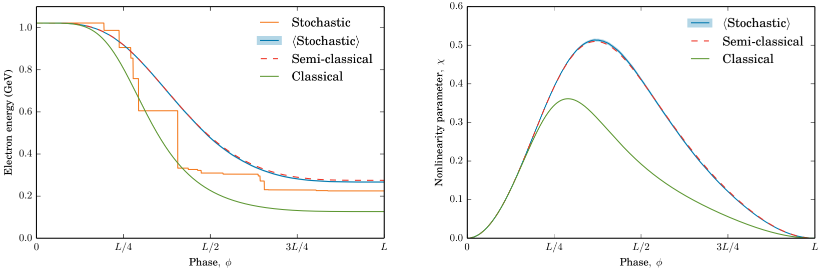
1. Diff. probability $dW = \Gamma d\phi$ [Ritus 1985; Green & Harvey 2014]:

$$\Gamma = \int_0^{\Omega} d\tilde{\Omega}\; P(\Omega, \tilde{\Omega}), \quad \text{where} \quad \Omega = -\frac{k^a p_a}{m} \quad \text{and} \quad \tilde{\Omega} = -\frac{k^a \kappa_a}{m}.$$

   (Emission probability $P(\Omega, \tilde{\Omega})$ depends on the field strength.)
2. Propagate with Lorentz force; emit photon if $r \in [0, 1) < dW$.
3. Find $\tilde{\Omega}$ such that $\int_0^{\tilde{\Omega}} dx\; P(\Omega, x) = \zeta\Gamma$, with $\zeta \in [0, 1)$.

Stochastic single-photon-emission model

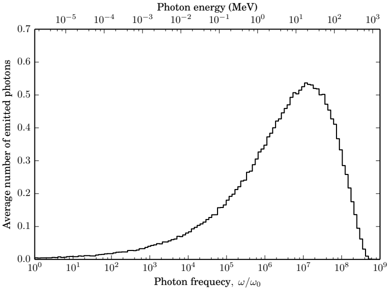
Model validation: $N = 10$ and $a_0 = 100$ $\left[27\text{ fs with peak } I_{\text{circ}} = 2.14 \times 10^{22}\text{ W/cm}^2\right]$

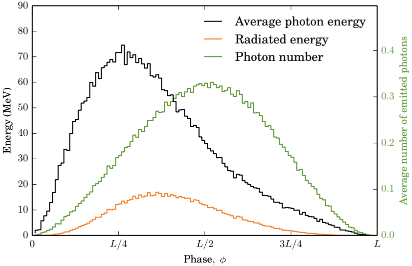


- Ensemble of 15 000 identical initial electrons (with $\gamma_0 = 2\,000$).
- Despite $\chi \simeq 0.5$, the semi-classical prediction is in good agreement with the average of the stochastic model.
- Single stochastic trajectory shows discrete photon emission.

Stochastic single-photon-emission model

Backscattered radiation: $N = 10$ and $a_0 = 100$ with circular polarisation

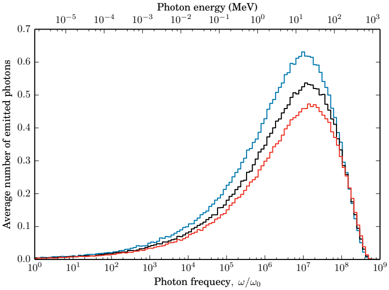


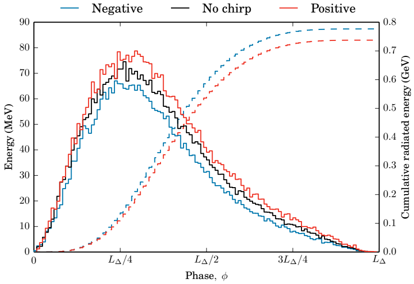


- **Statistics**: Ensemble of 15 000 electrons; sample of 299 412 photons.
- Compatible with $2\gamma^2 \simeq 10^7$ laser frequency upshift.
- Peak average photon energy ~ 70 MeV.

Stochastic model with chirped pulses

Backscattered radiation: $N = 10$ and $a_0 = 100$ with circular polarisation and $|\Delta| = 0.5$





- **Statistics**: Sample of 263 031 and 352 065 photons.
- **Negative chirp:** Increased photon number, reduced photon energy.
- Chirp does not alter the spectral cutoff.

Conclusions

- Future laser facilities will operate in regimes where not only radiation reaction but quantum effects will also play a role.
- Semi-classical extension to *LL* predicts reduced beam cooling.
- Quantum models of radiation reaction are sensitive to variation in peak intensity, not just total fluence.
- Modified energy distribution using chirps and polarisation
  — Only a small change.
- Stochastic model shows good agreement to semi-classical model.
- Chirp does not alter radiation frequency cutoff, but...
- Negative chirp shown to increase photon emission and cooling.
- **Future work:**
  - Electron beam cooling with stochastic model.
  - Include two-photon emission and pair production.
  - Transverse pulse structure.

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The ALPHA-X collaboration

http://phys.strath.ac.uk/alpha-x/pub/People/people.html

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