Semi-analytical fluid study of the propagation of an ultrastrong femtosecond laser pulse in a plasma with ultrarelativistic electron jitter

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Abstract

The interaction of a multi-petawatt, pancake-shaped laser pulse with an unmagnetized plasma is studied analytically and numerically in a regime with ultrarelativistic electron jitter velocities, in which the plasma electrons are almost completely expelled from the pulse region. The study is applied to a laser wakefield acceleration scheme with specifications that may be available in the next generation of Ti:Sa lasers and with the use of recently developed pulse compression techniques. A set of novel nonlinear equations is derived using a three-timescale description, with an intermediate timescale associated with the nonlinear phase of the electromagnetic wave and with the spatial bending of its wave front. They describe, on an equal footing, both the strong and the moderate laser intensity regimes, pertinent to the core and to the edges of the pulse.

1. Introduction

- Our investigation is aimed for the advancement of the laser wakefield acceleration scheme [1-4].
- The analysis is based on the Lorentz-Maxwell fluid model in the fully relativistic regime taking the pancake approximation, developed earlier.
- In our recent paper [5] we studied in detail only the WIR (*Weak Intensity Regime*) and MIR (*Moderate Intensity Regime*). The SIR (*Strong Intensity Regime*) was discussed in [5] only qualitatively and it was pointed out that it was fundamentally different from MIR, since it involved vastly different scalings in the core and at the edges of such pulse.
- In SIR the electrons are almost completely expelled from the core by a very strong ponderomotive force, creating a vacuum channel. An electromagnetic wave packet is imbedded in such vacuum channel and features (almost) linear properties, as if it was propagating in vacuum. Conversely, the edges of the pulse (most importantly, the leading edge) operate within the MIR, and the sort of nonlinear self-organization described in Ref. [5] is expected to occur there.
- In order to study the propagation of a **very large amplitude pulse**, we need a general description that includes both the **(quasi)linear bahavior inside the vacuum channel** and the **proper boundary conditions at its edges**, including the creation of such vacuum channel by the electron expulsion at the leading edge of the pulse.

- We consider the **SIR laser intensities of the order of 10[20] W cm[2]**, which are **30-50 times bigger than those attainable nowadays**. With the currently available laser energies, the **maximum electron beam energy reached in laser-plasma accelerators (LPAs)** is **> 1 GeV** [6], and a fundamental limitation to reach higher beam energies is set by the **pump depletion**. A simple arithmetics shows that **to produce a 10 GeV electron bunch with a charge of 1 nC, holding 10 J of kinetic energy, with a laser to particle beam efficiency 1-10%, laser energy of 100-1000 J is needed**, i.e. **P = 40-400 PW**, if the pulse duration is 25 fs.
- To enable the predictions for the multi-petawatt laser pulse behavior, **we derive a novel mathematical model that describes both the moderate and the strong intensity regimes. In the classical picture of a slowly varying amplitude of the laser pulse, based on a two-timescale description, this is not possible because the dispersion characteristics of electromagnetic waves in MIR and SIR are too different from each other and can not be described on a common footing.**
- **In the core of a very strong (i.e. SIR) pulse, the electromagnetic wave practically propagates in a vacuum. Such wave is not dispersive, i.e. its group velocity is constant and coincides with its phase velocity.**
- Conversely, **at the edges of such pulse the amplitude is smaller and the wave is dispersive**. Under such conditions, the simple envelope description used previously in the MIR, breaks down [7].
- Our model is derived using **a three-timescale description**, with an **intermediate timescale associated with the nonlinear, intensity-dependent, phase of the electromagnetic pulse.** The Schroedinger equation for the phase is considerably simplified under the physical conditions of the FLAME laser system (such as the laser frequency, pulse duration and spot size, plasma density etc.). For a laser power of order 1020 W/cm², our equation for the phase can be solved within the **WKB** (Wentzel-Kramers-Brillouin) **approximation**.

2. Mathematical Model

- The analytic studies of the laser-plasma interaction with intensities suitable for LPA have been attempted hitherto only for quasi 1-D, pancake-shaped pulses, using the "***quasistatic***" approximation and in a **cold-fluid description**, see the classical papers [8-11] and references therein.
- In the ***mildly relativistic regime***, the evolution of the plasma wake and of the laser pulse (depletion, frequency redshifting) was satisfactorily described using a reduced wave equation and a quasistatic plasma response [12], with a good agreement with full Maxwell-fluid results.
- Such fluid calculations provide a valuable insight also into kinetic phenomena, e.g. by **establishing the thresholds for the wave breaking that results in the electron trapping**.

Following these works, we assume that:

- The transverse variations are much smaller than the longitudinal ones: $\nabla_\perp \ll \partial/\partial z$
- the solution is slowly varying in the frame that moves with the velocity u $\mathbf{e}_z$ we have derived our system ***wave equation + Poisson's equation***.
- The latter involves **fully relativistic electrons** and the details can be seen e.g. in our recent paper [5]. The equations are valid also for ultrarelativistic electrons, $p_{\perp_0} \gg m_0 c$.
- Being affected by the return electron current, **the wake is inherently electromagnetic**, but for **sufficiently broad pulses**, both pancake-shaped [5, 12] and spherical [13], the electromagnetic effects are weak and **the wake may be considered as purely electrostatic**.
- These equations are valid in an **unmagnetized** plasma, and are written in the following dimensionless quantities (with obvious meaning of the symbols)

$$t' = \omega_{pe} t, \quad \vec{r}\,' = \frac{\omega_{pe}}{c}(\vec{r} - \vec{e}_z\, ut), \quad n' = \frac{n}{n_0}, \quad u' = \frac{u}{c},$$
$$\vec{p}' = \frac{\vec{p}}{m_0 c}, \quad \vec{v}' = \frac{\vec{v}}{c}, \quad \phi' = \frac{q\phi}{m_0 c^2}, \quad \vec{A}' = \frac{q\vec{A}}{m_0 c},$$

Maxwell's eqs. parallel and perpendicular to the direction of the e.m. wave propagation (in terms of the vector potential and the scalar potential)

$$\left[\frac{\partial^2}{\partial t^2} - 2u\,\frac{\partial^2}{\partial z\,\partial t} - (1-u^2)\frac{\partial^2}{\partial z^2} - \nabla_\perp^2\right]\vec{A}_\perp + \nabla_\perp\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right)\phi = \vec{v}_\perp n,$$
$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2}\right)\phi = 1 - n.$$
$$\gamma = (1 + \vec{p}^{\,2}/m_0^2 c^2)^{\frac{1}{2}}$$

Electron continuity eq., longitudinal and perpendicular components of the momentum equations

$$\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right)n + \nabla\cdot(n\vec{v}) = 0,$$
$$\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z} + \vec{v}_\perp\cdot\nabla_\perp\right)(p_z + A_z) - \vec{v}_\perp\,\frac{\partial}{\partial z}(\vec{p}_\perp + \vec{A}_\perp) + \frac{\partial}{\partial z}(\gamma + \phi) = 0,$$
$$\left[\frac{\partial}{\partial t} + (v_z - u)\frac{\partial}{\partial z} + \vec{v}_\perp\cdot\nabla_\perp\right](\vec{p}_\perp + \vec{A}_\perp) - v_i\nabla_\perp(p_i + A_i) + \nabla_\perp(\gamma + \phi) = 0,$$

3. Quasistatic regime

- The solution of the hydrodynamic equations is sought in a quasistatic regime, i.e. when it is only slowly varying in the moving reference fran$\partial/\partial t \ll u\,\partial/\partial z$
- In the approximate expressions for the charge and current densities, we use the leading order solution of the electron hydrodynamic equations, which is found as **a stationary 1-D solution that is propagating with the speed of light**, setting $\partial/\partial t = \nabla_\perp = 1 - u = 0$

$$\frac{\partial}{\partial z}[(v_z - 1)n] = 0,$$
$$\frac{\partial}{\partial z}(-p_z + \gamma + \phi) = 0,$$
$$\frac{\partial}{\partial z}(\vec{p}_\perp + \vec{A}_\perp) = 0,$$

the leading parts of motion Eqs. are obtained in a simple form

while from $\nabla\cdot\vec{A} = 0$, within the same accuracy, we have $\partial A_z/\partial z = 0$.

for $z \longrightarrow \pm\infty$, we have $\phi = A = \vec{v} = \vec{p} = 0$ $\gamma = n = 1$, and using $\gamma = (1 + p_z^2 + \vec{p}_\perp{}^2)^{\frac{1}{2}}$

$$n = \frac{(\phi-1)^2 + \vec{A}_\perp{}^2 + 1}{2(\phi-1)^2}, \qquad \vec{v}_\perp n = \frac{\vec{A}_\perp}{\phi - 1}$$

$$\left[\frac{\partial^2}{\partial t^2} - 2u\,\frac{\partial^2}{\partial t\,\partial z} - (1-u^2)\frac{\partial^2}{\partial z^2} - \nabla_\perp^2 + \frac{1}{1-\phi}\right]\vec{A}_\perp = -\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right)\nabla_\perp\phi,$$
$$\frac{\partial^2\phi}{\partial z^2} = \frac{(\phi-1)^2 - 1 - \vec{A}_\perp{}^2}{2(\phi-1)^2}.$$

4. Modulational assumption

For both MIR and SIR, we seek the solution of the wave equation in the moving frame as the sum of a slowly varying component and a modulated electromagnetic wave, including a phase that is varying on **an intermediate scale**, viz.

Slowly varying vector potential corresponding to the selfgenerated quasistationary magnetic field.

$$\vec{A}_\perp = \vec{A}_\perp^{(0)}(t_2, \vec{r}_2) + \{\vec{A}_{\perp_0}(t_2, \vec{r}_2)\; e^{i[\varphi(t_1,\vec{r}_1) - \omega' t + k'(z+ut)]} + c.c.\}.$$

$$\omega' = \frac{\omega}{\omega_{pe}}, \quad k' = \frac{ck}{\omega_{pe}} = \frac{d_e}{\lambda}$$
$$\omega = \sqrt{c^2k^2 + \omega_{pe}^2}$$

Regime in which the self-generated magnetic field can be neglected:

$$[\alpha_{Re}(\phi) + i\,\alpha_{Im}(\phi)]A_{\perp_0} - 2\,i\,\epsilon^2\,\frac{\partial A_{\perp_0}}{\partial t_2} - 2\,i\,\epsilon(\nabla_\rho\varphi\cdot\nabla_2)A_{\perp_0} - \epsilon^2\,\nabla_2^2 A_{\perp_0} = 0$$
$$\frac{\partial^2\phi}{\partial z_2^2} = \frac{(\phi-1)^2 - 1 - |A_{\perp_0}|^2}{2(\phi-1)^2},$$
$$(\nabla_\rho\varphi)^2 = \kappa^2(\phi),$$

$$\alpha_{Re}(\phi) = \phi/(1-\phi) + \kappa^2(\phi),$$
$$\alpha_{Im}(\phi) = -\nabla_1^2\varphi + \frac{\partial^2\varphi}{\partial t_1^2} = -\nabla_\rho^2\varphi + \mathcal{O}(\epsilon^4).$$

ZAKHAROV - TYPE SYSTEM OF EQUATIONS TO DESCRIBE PARAMETRIC PROCESSES

5. Some numerical results

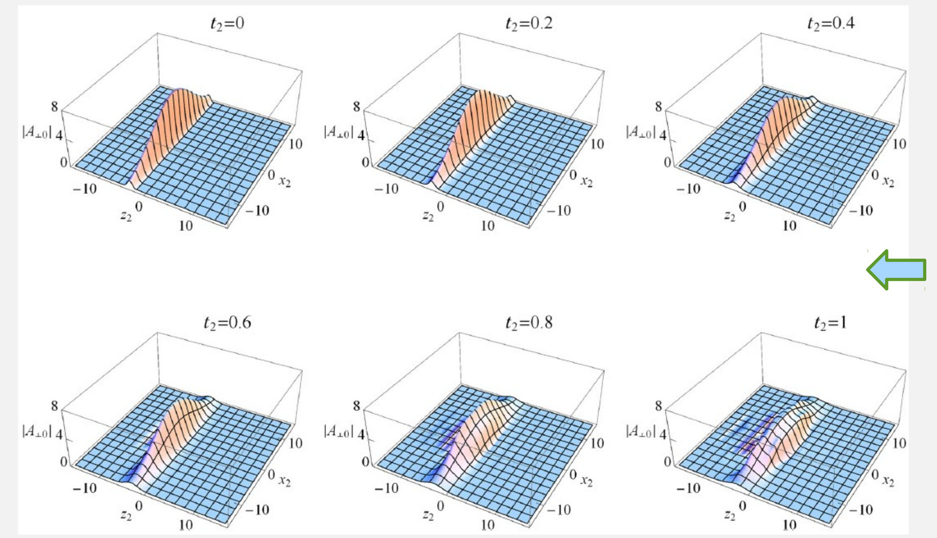


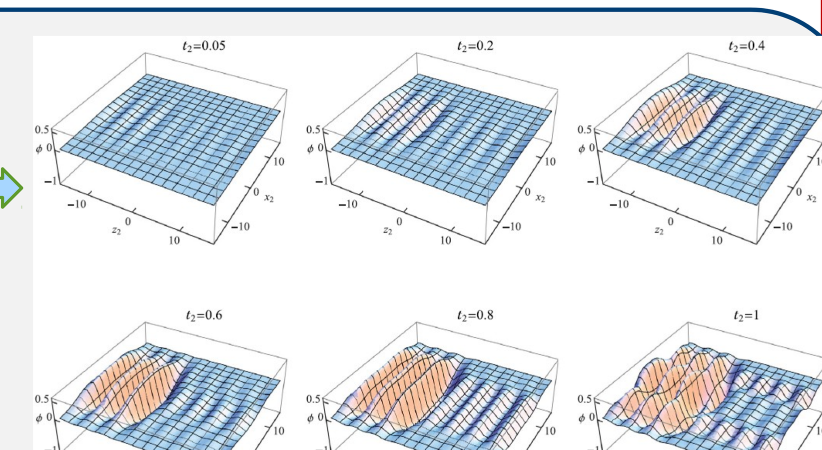
Fig. 1 - Evolution of the envelope of the pancake laser pulse with an amplitude that is expected to be used in a future accelerator scheme (SIR). The initial condition was:

$$A_{\perp_0}(x_2, z_2, 0) = 0.6\, a_L(z_2/L_z)\;\exp(-x_2^2/2L_x^2)\;\exp(i\,\delta k\, z_2)$$
$L_z = 1.6$ and $L_x = 7.5.$ $\delta k = 0.5$

which gave the maximum stability. The initial electrostatic potential and initial nonlinear phase were adopted to be zero.

In the physical (non-scaled) variables, these initial pulse length and width are 1.8 μm and 300 μm, respectively. Likewise, the dimensionless time t_{2max} = 1.1 corresponds, in physical units, to 9.69 10^{-12} s, during which time the pulse travels 3 mm. (color online).

Fig. 2 - Evolution of the electrostatic wake potential $\phi(x_2,z_2,t_2)$, produced by the laser pulse dusplayed in Fig. 1. A very large localized negative potential is created, with $|\phi| \approx 1$, which indicates the almost complete expulsion of electrons in the vicinity of the laser pulse.



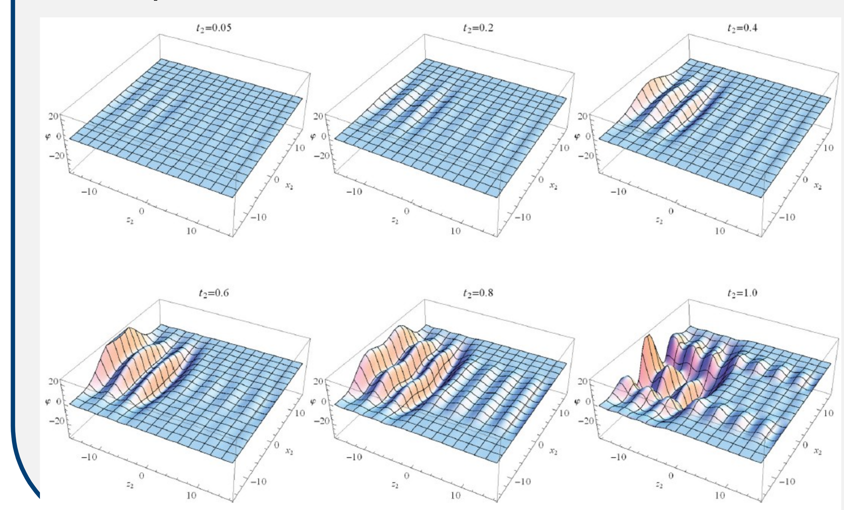


Fig. 3 - Evolution of the nonlinear phase $\varphi(x_2, z_2, t_2)$ of the laser pulse displayed in Fig. 1. A noticeable bending of the wave front occurs for $t_2 > 0.5$, simultaneously with the emergence of an electrostatic wake.

6. Conclusions and Remarks

- **We have studied the SIR** (also called the ultrarelativistic regime) of pancake-shaped laser pulses through an unmagnetized plasma, by using a (semi)analytic hydrodynamic description.
- We have derived nonlinear equations that appropriately describe all the three intensity regimes, discussed earlier [5].
- In the classical picture of the slowly varying amplitude of a laser pulse, based on a two-timescale description, it is not possible to study the strong and moderate-intensity regimes simultaneously, because the dispersion characteristics of electromagnetic waves in MIR and SIR are too different from each other and can not be described on a common footing.
- In the core of a very strong (i.e., SIR) pulse, the electromagnetic wave practically propagates in a vacuum. Such wave is not dispersive, i.e., its group velocity is constant and coincides with its phase velocity. Conversely, at the edges of such pulse, the amplitude is smaller and the wave is dispersive. Under such conditions, the simple envelope description, suitable for the MIR, breaks down.
- We derived novel model equations, based on a three-timescale description, that account for the evolution of the nonlinear phase of the laser wave. At very large laser intensities, this gives a smooth transition to a nondispersive e.m. wave and the saturation of the nonlocal nonlinearity.
- Kinetic effects, e.g., plasma wave-breaking, trapping of resonant particles, and their subsequent acceleration, are not included in the present analysis. They are the subject of our study in progress that will be presented later.

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