An influence of plasma on the wakefield amplitude excited in a dielectric structure by bunch train

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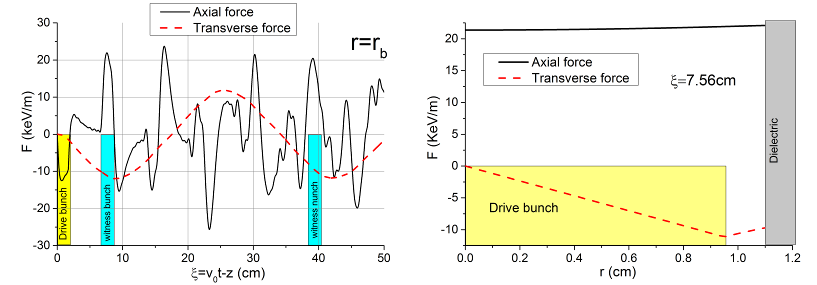
Focusing Property of PDWA

At the previous EAAC2013 we have reported that difficulties with stabilization of transverse motion of bunches in DWA can be resolved by filling of accelerated channel with a plasma [NIMA, V.740, p.124-129(2014)]



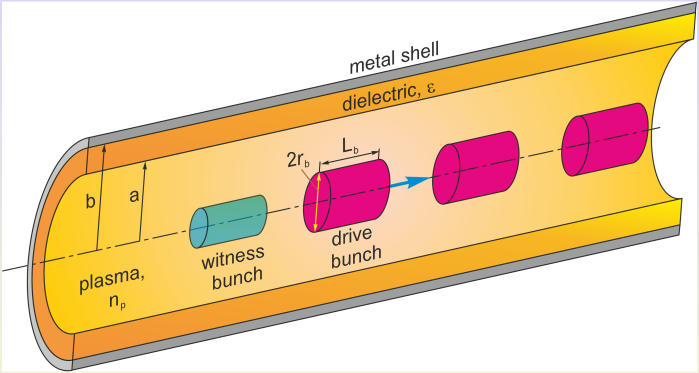
A GHz PDWA ($f_{01} \sim 2.71\,GHz$) suitable for an experiment at KIPT: OD=8.6cm, ID=2.2cm, $\varepsilon = 2.1$, $L_b = 1.7cm$, $2r_b = 1.9cm$, Bunch energy=5MeV, $Q_b = 0.32nC$, $f_p \approx 0.9\,GHz$ $\left(n_p = 10^{10} cm^{-3}\right)$; $n_b/n_p = 0.04$

Motivation

- At optimal focusing regime $f_{01} \gg f_p$ the amplitude of axial field is mainly formed by the dielectric wave (DW), and the amplitude of transverse field - by plasma wave (PW)
- When an increasing plasma density an axial electric field of PW increases, this could result in an increase the total accelerating field
- However, beforehand the optimal plasma density, when the maximum of an accelerating gradient is reached, can't be predicted:
  - An amplitude of the PW hasn't monotonically increasing character - it has the maximum when $k_p a = \omega_p a / v_0 \approx 2$
  - An amplitude of the DW at the accelerating channel axis decreases when a plasma density increases

The results of studies $E_z(n_p)$ are presented in this report

PDWA under investigation



PDWA unit. Isotropic plasma fills transport channel entirely. Drive bunch/bunch train is in magenta; test bunch is in dark green, dielectric tube is in green, and outer shell in grey is a conductor

Axial Field in Drift Channel

Having solved Maxwell equations we obtain next equations for the axial field excited by drive bunch train in drift channel:

$$E_z = -\frac{4Q_0}{a^2}\sum_{i=1}^{N_b}\sum_{s} R_s(r_b)e_z^s(r)\Psi_{||}^s\left[\tau-(i-1)T\right]$$

$$-\frac{4Q_0}{r_b L_b}e_z^p(r)\sum_{i=1}^{N_b}\Psi_{||}^p\left[\tau-(i-1)T\right]$$

$$e_z^s(r) = \left(\frac{a}{\omega_s D'(\omega_s)}\right)^{1/2}\frac{I_0(\kappa_p^s r)}{I_0(\kappa_p^s a)}, \quad R_s(r_b) = \frac{2}{\kappa_p^s r_b}e_z^s(r_b),$$

$$e_z^p(r) = \begin{cases} \frac{1}{k_p r_b} - \frac{I_0(k_p r)}{I_0(k_p a)}\Delta_1(k_p r_b, k_p a), & r < r_b \\ \frac{I_1(k_p r)}{I_0(k_p a)}\Delta_0(k_p a, k_p r_b), & r_b < r < a \end{cases}$$

$$\Psi_{||}^{p,s}(\tau) = \frac{1}{\omega_{p,s}\tau_b}\left[\sin(\omega_{p,s}\tau)\Theta(\tau) - sin[\omega_{p,s}(\tau-\tau_b)]\Theta(\tau-\tau_b)\right]$$

Eigen Frequencies

$$\omega = \omega_p \Rightarrow \text{plasma wave}$$

$$D(\omega_s) \equiv \frac{\varepsilon_p(\omega_s)}{\kappa_p^s}\frac{I_1(\kappa_p^s a)}{I_0(\kappa_p^s a)} + \frac{\varepsilon}{\kappa_d^s}\frac{F_1(\kappa_d^s a, \kappa_d^s b)}{F_0(\kappa_d^s a, \kappa_d^s b)} = 0 \Rightarrow \text{dielectric wave}$$

$$\tau = t - z/v_0, \quad \tau_b = L_b/v_0, \quad k_p = \omega_p/v_0,$$

$$\kappa_p^s = \left[1 - \beta_0^2 \varepsilon_p(\omega_s)\right]^{1/2} \omega_s/v_0, \quad \kappa_d^s = \left[\beta_0^2 \varepsilon - 1\right]^{1/2} \omega_s/v_0,$$

$$\beta_0 = v_0/c, \quad \varepsilon_p(\omega) = 1 - \omega_p^2/\omega^2, \quad \omega_p^2 = 4\pi e^2 n_p/m,$$

$$\Delta_n(x,y) = I_n(x)K_0(y) - (-1)^n K_n(x)I_0(y),$$

$$F_n(x,y) = (-1)^n \left[J_n(x)Y_0(y) - Y_n(x)J_0(y)\right],$$

$$D'(\omega_s) = dD(\omega)/d\omega|_{\omega=\omega_s}.$$

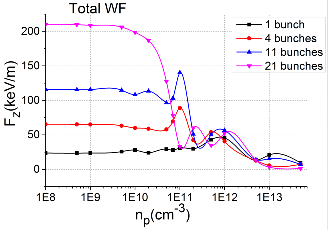
Initial Parameters of PWDA unit

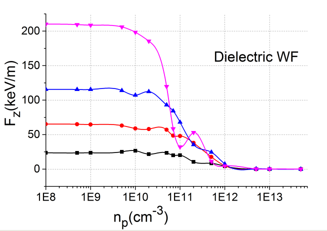
OD of dielectric tube	$8.6\, cm$
ID of dielectric tube	$2.2\, cm$
Relative dielectric constant, ε	2.1
Bunch energy	$5\, MeV$
Frequency of vacuum E_{01} mode, f_{01}	$2.71\, GHz$
Bunch repetition rate, f_m	$2.71\, GHz$
Bunch charge	$0.32\, nC$
Bunch diameter	$2.0\, cm$
Density of drive bunch electrons, n_b	$3.9 \cdot 10^8\, cm^{-3}$

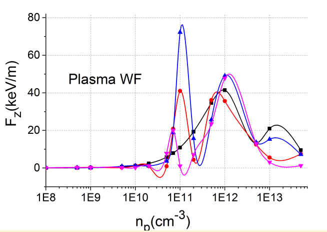
Plasma density will be changed during the next analysis.

The structure with above parameters provides not high accelerating gradient, however, it enables to track the base regularity how the wakefield amplitude depends from plasma density. High accelerating gradients can be obtained by a scaling of dimensions and plasma density.

$Fz(n_p)$ when keeping fixed dimensions and f_m — 1st case







$n_p (cm^{-3})$	$\omega_{01}/2\pi$ (MHz)	$\omega_p/2\pi$ (MHz)
$5\cdot10^8$	2710	200.8
10^9	2711	283.9
$5\cdot10^9$	2717	634.9
10^{10}	2725	897.9
$5\cdot10^{10}$	2782	2008
10^{11}	2846	2839
$5\cdot10^{11}$	3181	6349
10^{12}	3393	8979
$5\cdot10^{12}$	3837	20080
10^{13}	3976	28390.

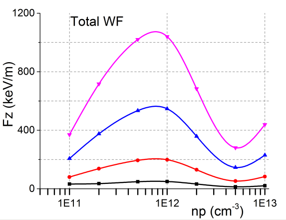
Reference case: Total WF = dielectric WF + plasma WF

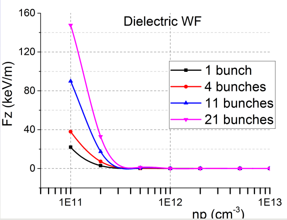
Conclusion for the 1st unit

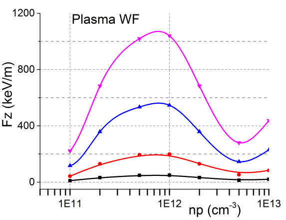
- When using single bunch for a generation of WF: a plasma filling of drift channel doesn't provide additional advantages in comparison with a pure dielectric structure of a pure plasma one
- When using bunch train: appreciable increasing of wakefield can be obtained at resonant plasma density ($\omega_p = \omega_{01}$) for not very long train
- The first reason of that behaviour is detuning between bunch repetition rate and eigen frequencies of the PDWA (dielectric and plasma)
- The second reason is detuning between plasma wave and dielectric wave

Let's start the tuning

$Fz(n_p)$ when changing ID — 2nd case





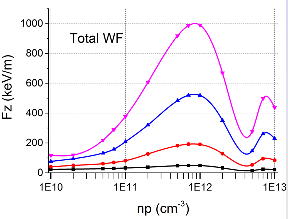


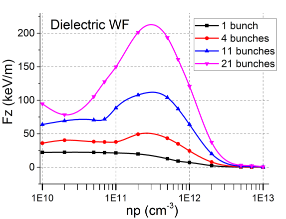
$n_p(cm^{-3})$	a (cm)
$5\cdot10^{10}$	-
10^{11}	1.085
$2\cdot10^{11}$	2.2805
$5\cdot10^{11}$	3.0937
10^{12}	3.4658
$5\cdot10^{12}$	3.9362
10^{13}	4.0442

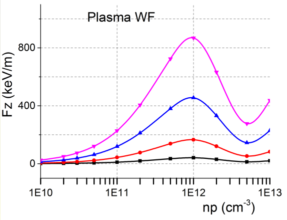
Total WF (top left) = dielectric WF (top right) + plasma WF (bottom)

$$b = 4.3\,cm,\ f_{01} = 2.84 \div 28.4\,GHz$$

$Fz(n_p)$ when changing OD — 3rd case





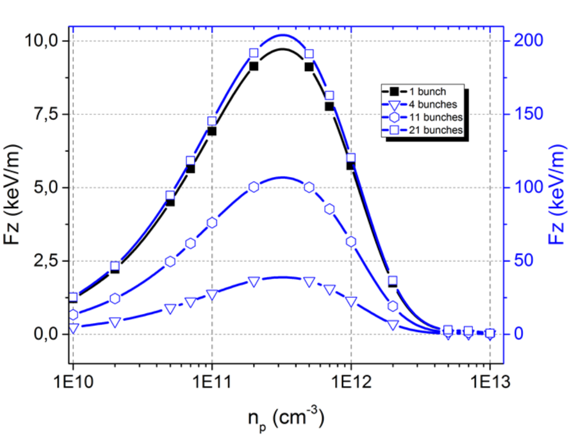


$n_p (cm^{-3})$	b (cm)
10^{10}	12.406
$5\cdot10^{10}$	5.815
10^{11}	4.31
$2\cdot10^{11}$	3.281
$5\cdot10^{11}$	2.4115
10^{12}	1.997
$5\cdot10^{12}$	1.479
10^{13}	1.3638

Total WF (top left) = dielectric WF (top right) + plasma WF (bottom)

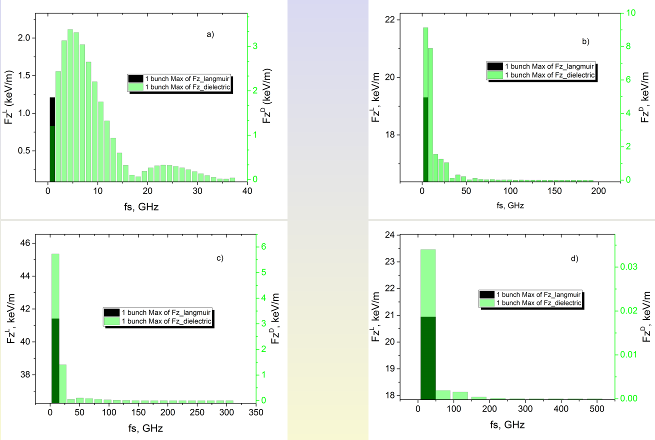
$$a = 1.1\,cm,\ f_{01} = 0.9 \div 28.4\,GHz$$

$Fz(n_p)$ for the 1st DW mode when changing OD



Dielectric WF of the first radial mode E_{01}. Left axis is the F_z for single bunch

Spectra of wakefields from single bunch — 3rd case



a) – $n_p = 10^{10}$, b) – $n_p = 2 \cdot 10^{11}$, c) – $n_p = 10^{12}$, d) – $n_p = 10^{13}\ cm^{-3}$

Conclusion

- Amplitude of wakefield in PDWA can increased by tuning eigen frequencies to bunch repetition rate by the way changing of inner or outer diameters of dielectric tube
- Maximum of wakefield amplitude is reached near the maximum of plasma wave amplitude
- There is a wide interval of plasma density where dielectric waves get an appreciable input in total wakefield
- For the case of changing of ID this interval is at low plasma density, far from optimal one where maximum of total wakefield is reached
- For the case of changing of OD this interval is wider and input of dielectric wave is significant even at total wakefield maximum

Thank you for your attention!