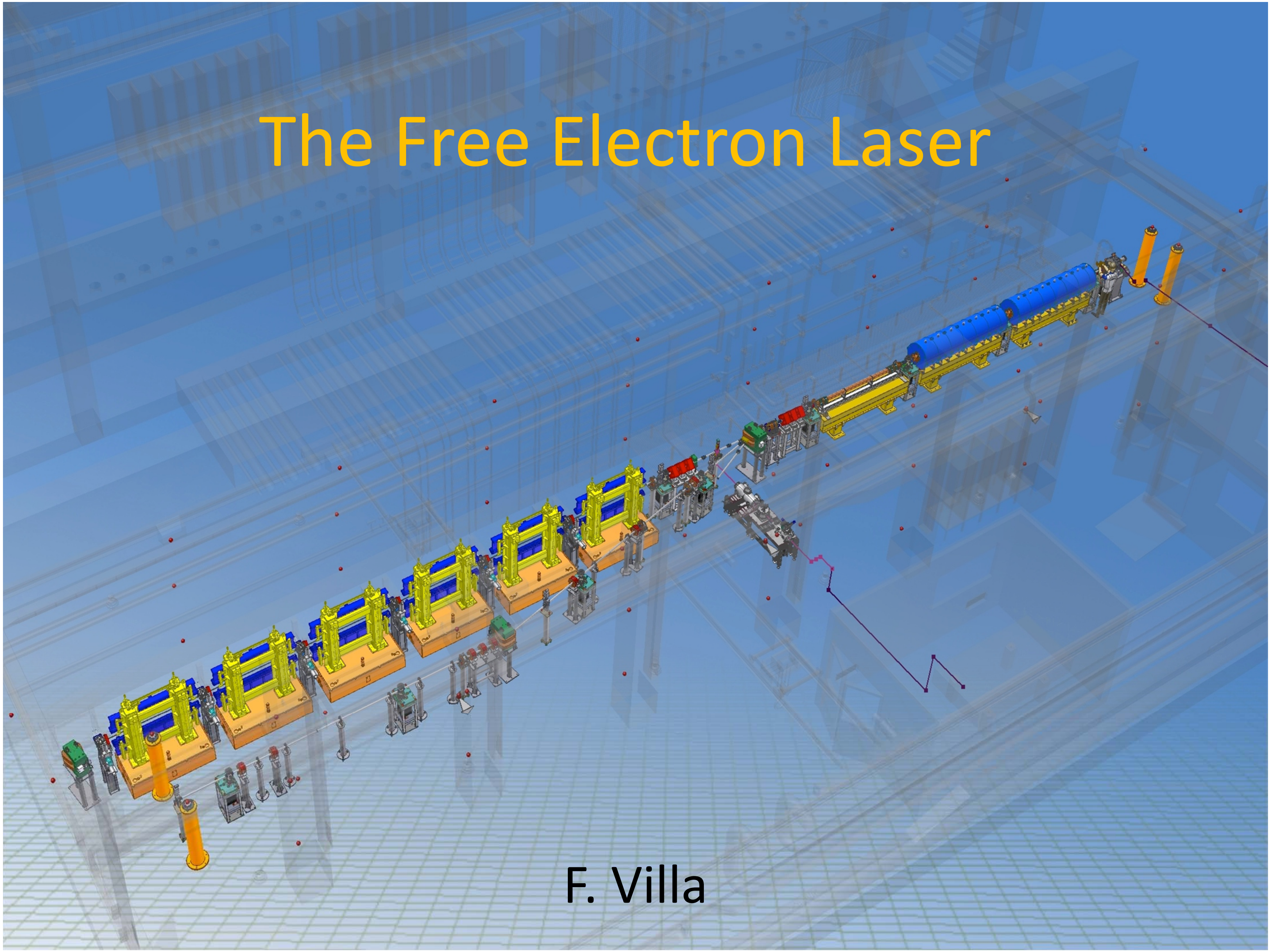
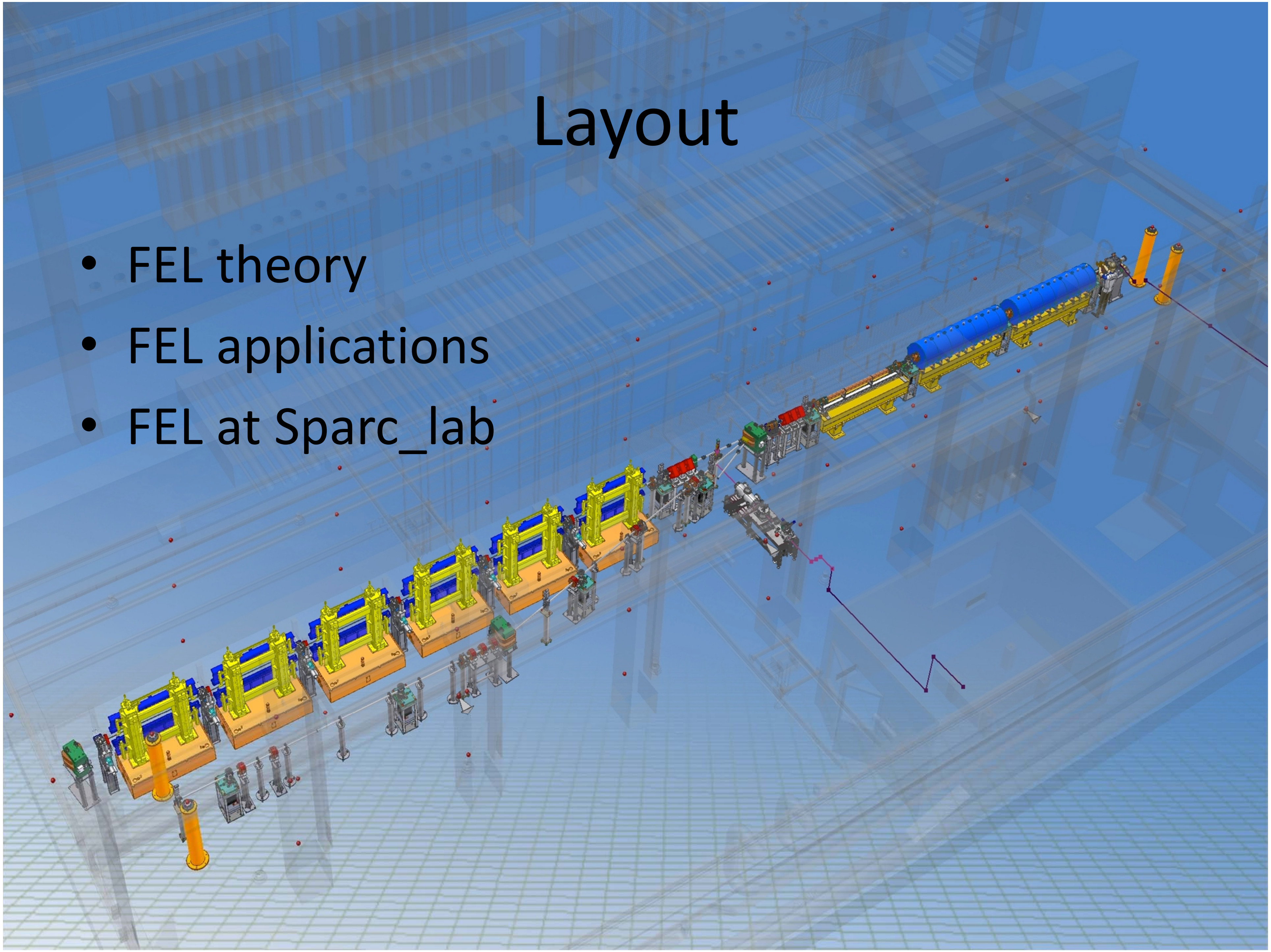


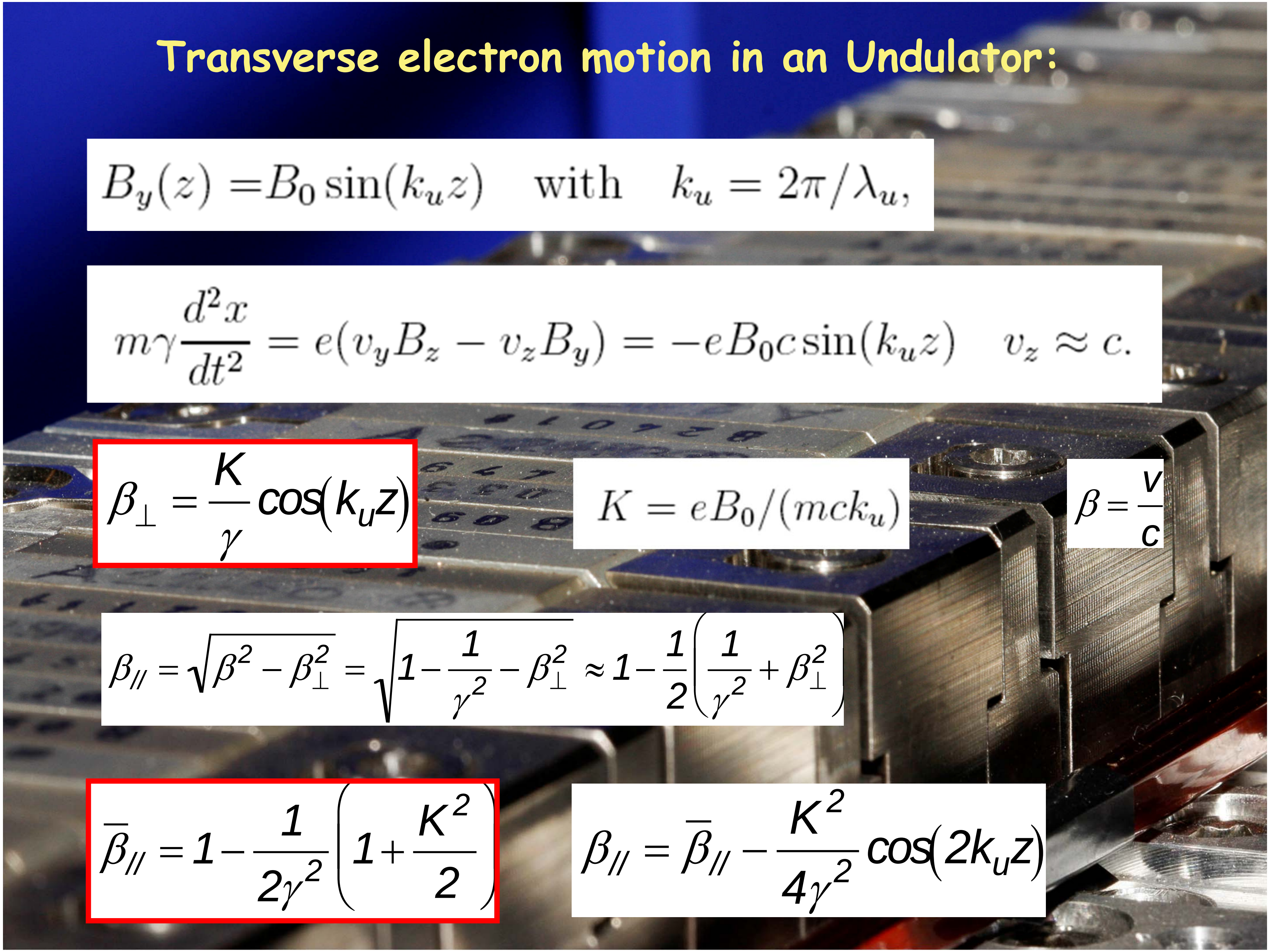
The Free Electron Laser

F. Villa

Layout

- FEL theory
- FEL applications
- FEL at Sparc_lab





Transverse electron motion in an Undulator:

$$B_y(z) = B_0 \sin(k_u z) \quad \text{with} \quad k_u = 2\pi/\lambda_u,$$

$$m\gamma \frac{d^2 x}{dt^2} = e(v_y B_z - v_z B_y) = -eB_0 c \sin(k_u z) \quad v_z \approx c.$$

$$\boxed{\beta_\perp = \frac{K}{\gamma}\cos(k_u z)}$$

$$K = eB_0/(mck_u)$$

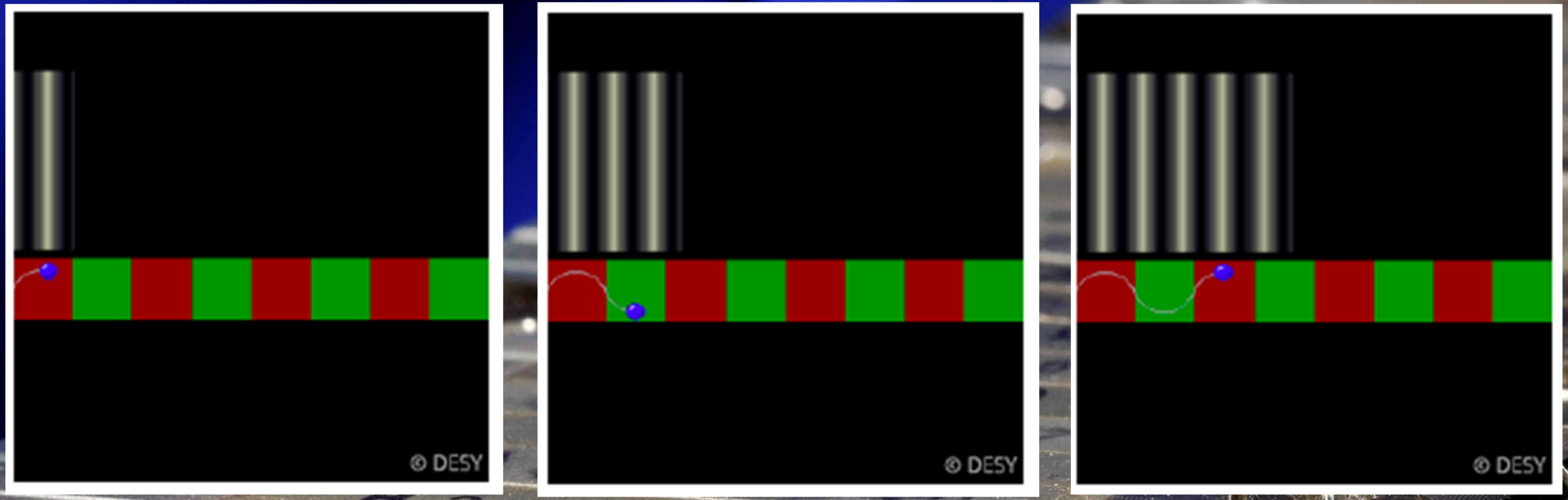
$$\beta = \frac{v}{c}$$

$$\beta_{//} = \sqrt{\beta^2 - \beta_\perp^2} = \sqrt{1 - \frac{1}{\gamma^2} - \beta_\perp^2} \approx 1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \beta_\perp^2\right)$$

$$\boxed{\bar{\beta}_{//} = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

$$\beta_{//} = \bar{\beta}_{//} - \frac{K^2}{4\gamma^2}\cos(2k_u z)$$

Transverse electron motion in an Undulator:slippage

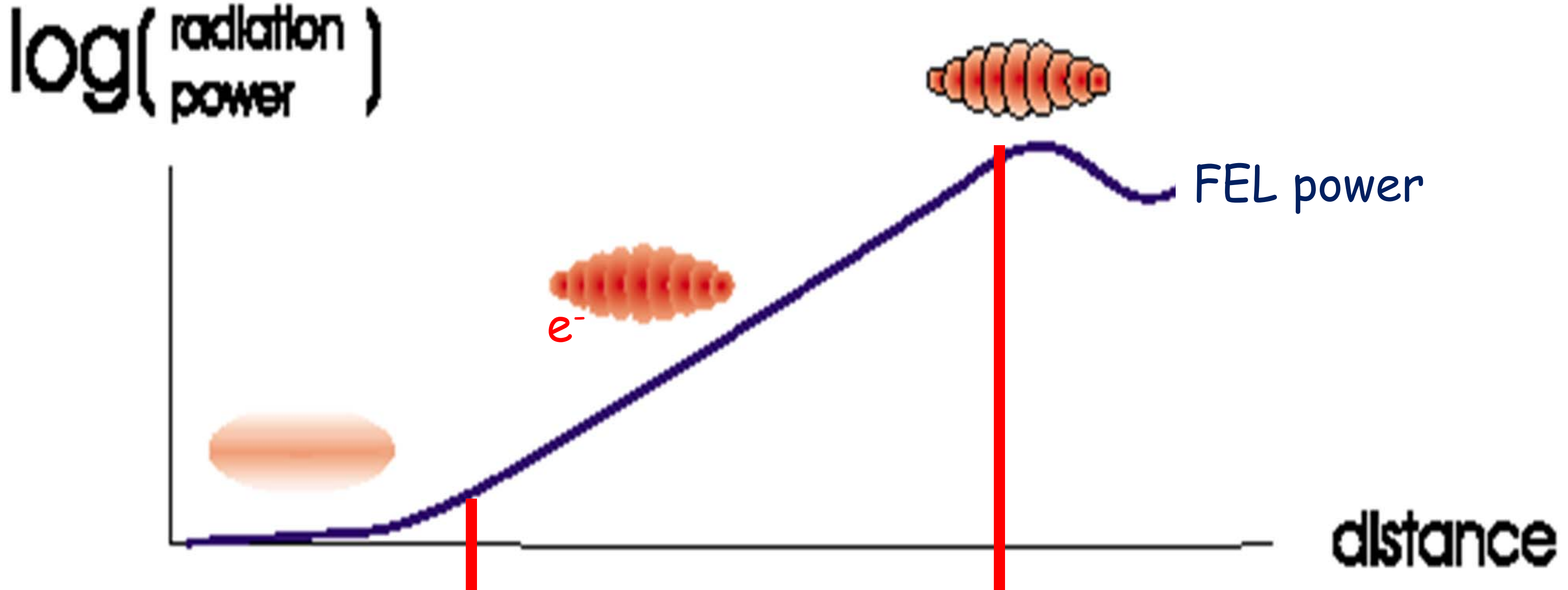


After one wiggler period the electron sees the radiation with the same phase if the flight time delay is exactly one radiation period: $\Delta t = t_e - t_{ph} = T_{rad}$

$$\Delta t = \frac{\lambda_u}{c\beta_{//}} - \frac{\lambda_u}{c} = \frac{\lambda_{rad}}{c} \longrightarrow \lambda_{rad} = \frac{1-\bar{\beta}_{//}}{\bar{\beta}_{//}}\lambda_u \xrightarrow{\bar{\beta}_{//}\approx 1} \lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

The relative slippage of the radiation envelope through the electron beam can be neglected, provided that $l_b \gg N_u\lambda_r$ (Steady State Regime)

FEL light generation



Letargy

Spontaneous Emission

Low Gain

Slow Bunching

Exponential Growth

Stimulated emission

High Gain

Enhanced Bunching

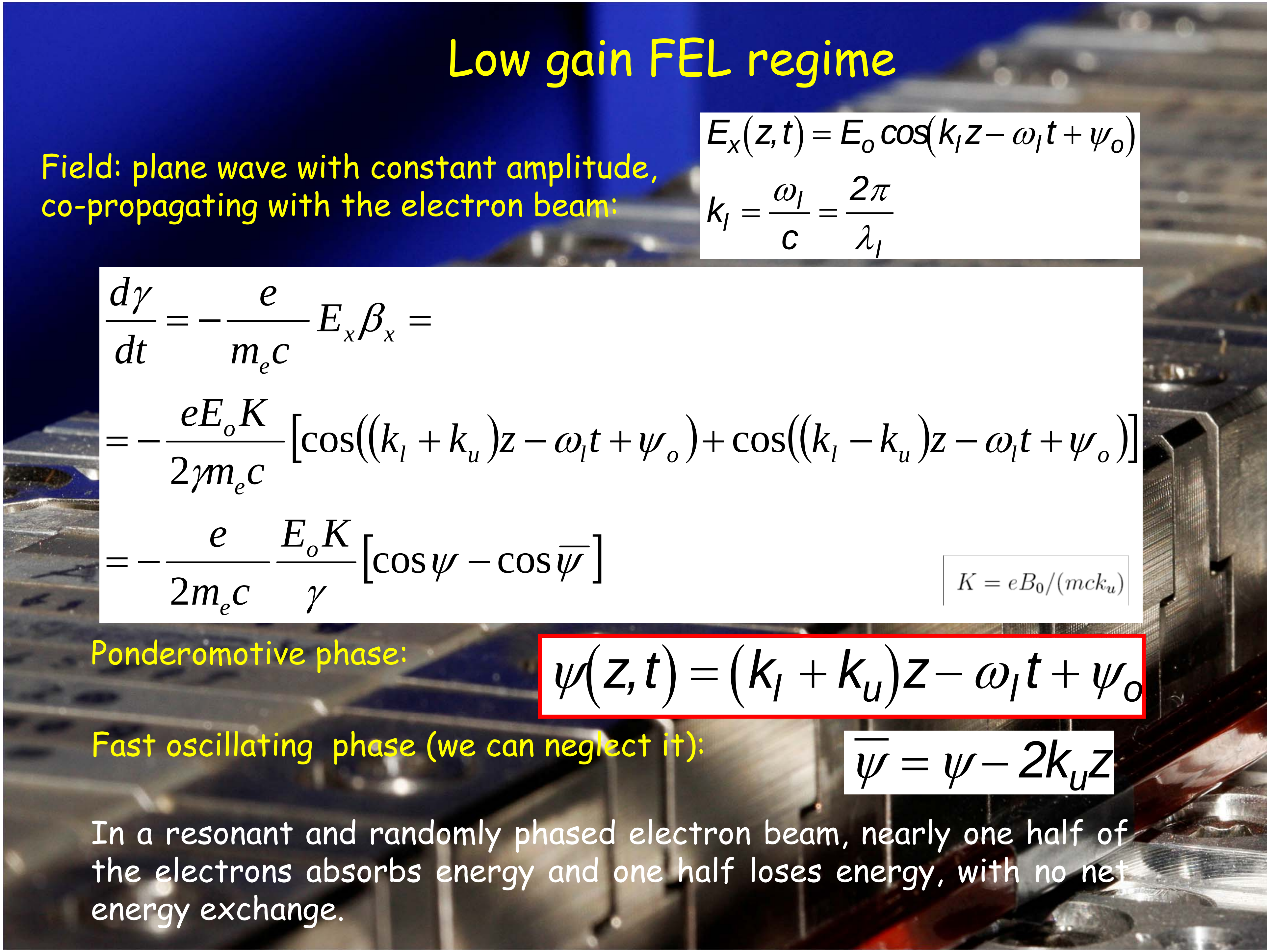
Saturation

Absorption

No Gain

Debunching

Low gain FEL regime



Field: plane wave with constant amplitude, co-propagating with the electron beam:

$$E_x(z,t) = E_o \cos(k_l z - \omega_l t + \psi_o)$$

$$k_l = \frac{\omega_l}{c} = \frac{2\pi}{\lambda_l}$$

$$\frac{d\gamma}{dt} = -\frac{e}{m_e c} E_x \beta_x =$$

$$= -\frac{eE_o K}{2\gamma m_e c}\left[\cos((k_l + k_u)z - \omega_l t + \psi_o) + \cos((k_l - k_u)z - \omega_l t + \psi_o)\right]$$

$$= -\frac{e}{2m_e c}\frac{E_o K}{\gamma}\left[\cos\psi - \cos\overline{\psi}\right]$$

$$K = eB_0/(mck_u)$$

Ponderomotive phase:

$$\boxed{\psi(z,t) = (k_l + k_u)z - \omega_l t + \psi_o}$$

Fast oscillating phase (we can neglect it):

$$\overline{\psi} = \psi - 2k_u z$$

In a resonant and randomly phased electron beam, nearly one half of the electrons absorbs energy and one half loses energy, with no net energy exchange.

The “ponderomotive” (radiation + undulator) field, propagates in forward direction with a phase velocity that corresponds to the velocity of the resonant particle: $\psi(z,t)=(k_l+k_u)z-\omega_l t+\psi_o$

$$\bar{\beta}_{//}=1-\frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$\frac{d\psi}{dt}=(k_l+k_u)\bar{v}_z-k_l c\equiv 0 \longrightarrow \bar{v}_z\equiv\frac{k_l c}{k_l+k_u}=c\left(1-\frac{1}{2\gamma_r^2}\left(1+\frac{K^2}{2}\right)\right)$$

For particles with off resonance energy $\gamma\neq\gamma_r$ the ponderomotive phase is no longer constant

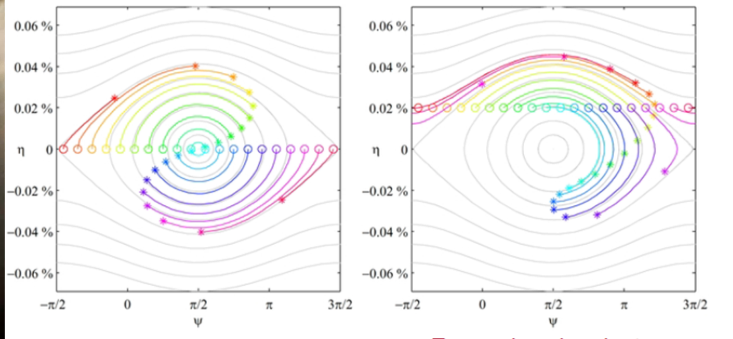
$$\frac{d\psi}{dt}=(k_l+k_u)\bar{v}_z-k_l c\overset{k_u<<k_l}{\approx}k_l c\left(\frac{k_u}{k_l}-\frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right)\right)=\frac{k_l c}{2}\left(\frac{1}{\gamma_r^2}-\frac{1}{\gamma^2}\right)\left(1+\frac{K^2}{2}\right)$$

Two coupled first order differential equations

$$\eta=\frac{\gamma-\gamma_r}{\gamma_r}<<1$$

$$\frac{d\psi}{dt}\approx k_u c\frac{\gamma^2-\gamma_r^2}{\gamma_r^2}\approx 2k_u c\frac{\gamma-\gamma_r}{\gamma_r}=2k_u c\eta$$

$$\frac{d\eta}{dt}=\frac{1}{\gamma_r}\frac{d\gamma}{dt}=-\frac{eE_oK}{2\gamma_r^2 m_e c}\cos\psi$$

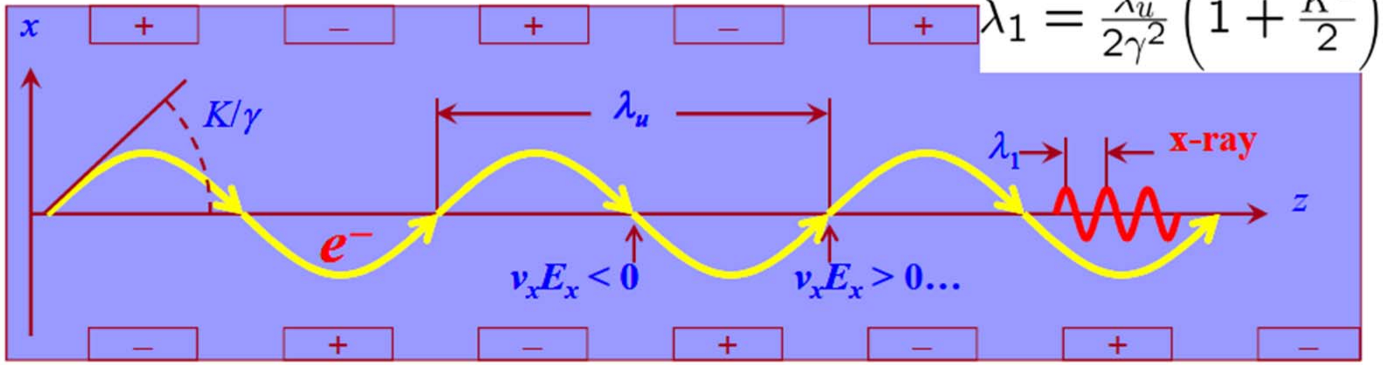


No net energy transfer

Energy loss by electrons

High gain FEL regime

- Electrons **slip** behind EM wave by λ_1 per undulator period (λ_u)



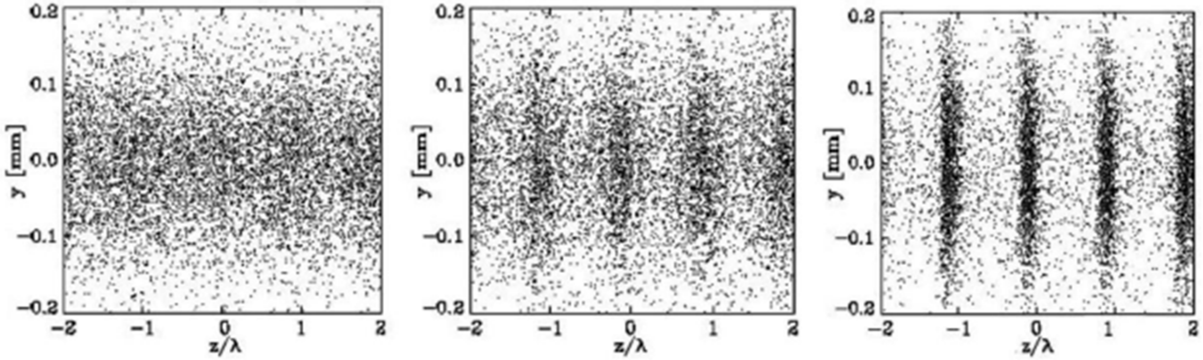
$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

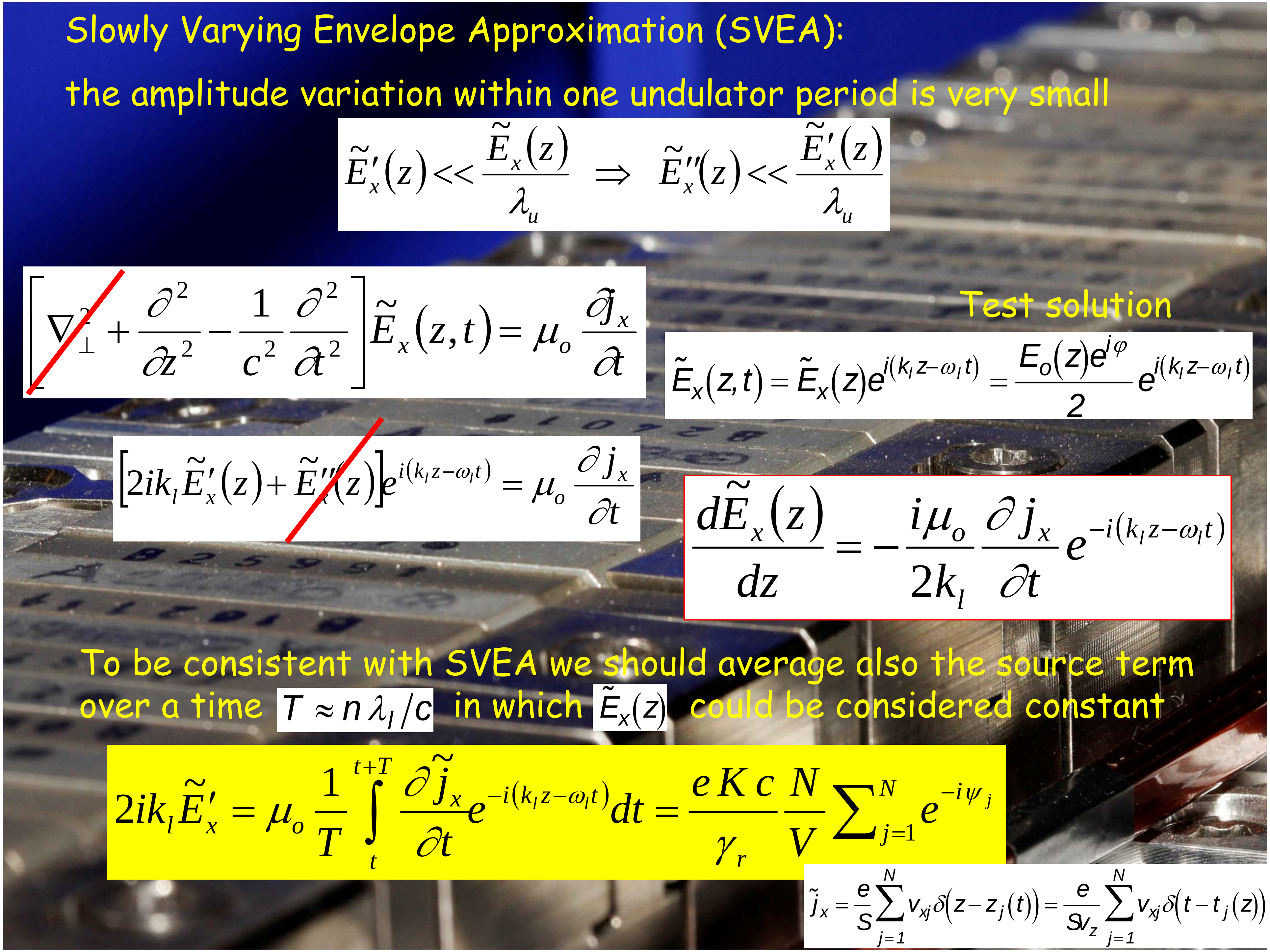
- Due to sustained interaction, some electrons lose energy, while others gain ➔ **energy modulation at λ_1**

- e^- losing energy slow down, and e^- gaining energy catch up ➔ **density modulation at λ_1 (microbunching)**

- Microbunched beam radiates coherently at λ_1, enhancing the process ➔ **exponential growth of radiation power**

Raubenheimer, SSSEPB13





Slowly Varying Envelope Approximation (SVEA):

the amplitude variation within one undulator period is very small

$$\tilde{E}_x'(z) << \frac{\tilde{E}_x(z)}{\lambda_u} \quad \Rightarrow \quad \tilde{E}_x''(z) << \frac{\tilde{E}_x'(z)}{\lambda_u}$$

$$\left[\cancel{\nabla_\perp^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\tilde{E}_x(z,t) = \mu_o \frac{\partial \tilde{j}_x}{\partial t}$$

Test solution

$$\tilde{E}_x(z,t) = \tilde{E}_x(z)e^{i(k_l z-\omega_l t)} = \frac{E_o(z)e^{i\varphi}}{2}e^{i(k_l z-\omega_l t)}$$

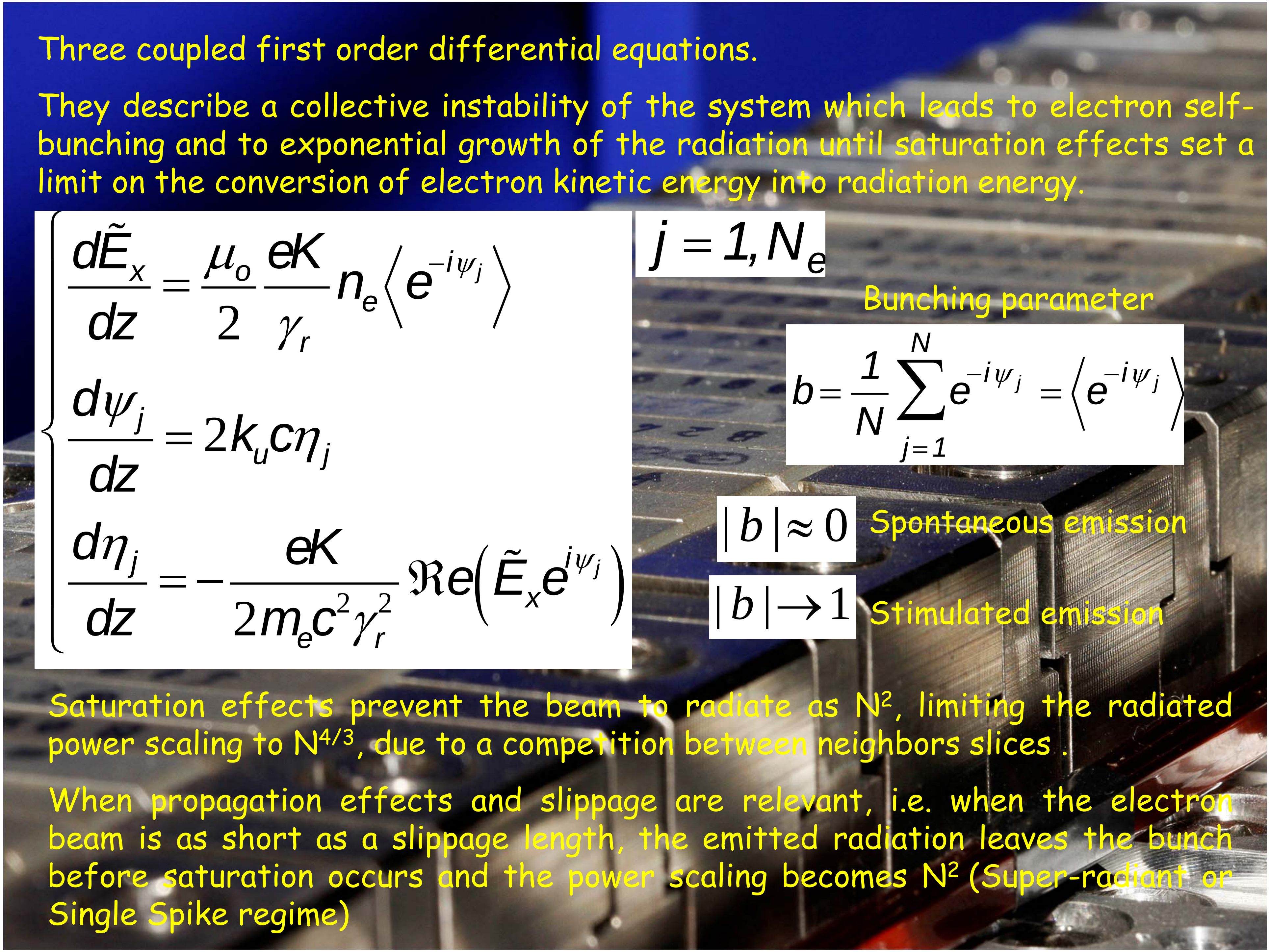
$$\left[2ik_l\tilde{E}_x'(z) + \cancel{\tilde{E}_x''(z)}\right]e^{i(k_l z-\omega_l t)} = \mu_o \frac{\partial j_x}{\partial t}$$

$$\frac{d\tilde{E}_x(z)}{dz} = -\frac{i\mu_o}{2k_l}\frac{\partial j_x}{\partial t}e^{-i(k_l z-\omega_l t)}$$

To be consistent with SVEA we should average also the source term over a time $T \approx n\lambda_l/c$ in which $\tilde{E}_x(z)$ could be considered constant

$$2ik_l\tilde{E}_x' = \mu_o\frac{1}{T}\int_t^{t+T}\frac{\partial \tilde{j}_x}{\partial t}e^{-i(k_l z-\omega_l t)}dt = \frac{e\,K\,c}{\gamma_r}\frac{N}{V}\sum_{j=1}^{N}e^{-i\psi_j}$$

$$\tilde{j}_x = \frac{e}{S}\sum_{j=1}^{N}v_{xj}\delta\left(z-z_j(t)\right) = \frac{e}{Sv_z}\sum_{j=1}^{N}v_{xj}\delta\left(t-t_j(z)\right)$$



Three coupled first order differential equations.

They describe a collective instability of the system which leads to electron self-bunching and to exponential growth of the radiation until saturation effects set a limit on the conversion of electron kinetic energy into radiation energy.

$$\begin{cases} \dfrac{d\tilde{E}_x}{dz} = \dfrac{\mu_o}{2}\dfrac{eK}{\gamma_r} n_e \left\langle e^{-i\psi_j} \right\rangle \\ \dfrac{d\psi_j}{dz} = 2k_u c\eta_j \\ \dfrac{d\eta_j}{dz} = -\dfrac{eK}{2m_e c^2 \gamma_r^2} \Re e\left(\tilde{E}_x e^{i\psi_j}\right) \end{cases}$$

$$j = 1, N_e$$

Bunching parameter

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\psi_j} = \left\langle e^{-i\psi_j} \right\rangle$$

$|b| \approx 0$ Spontaneous emission

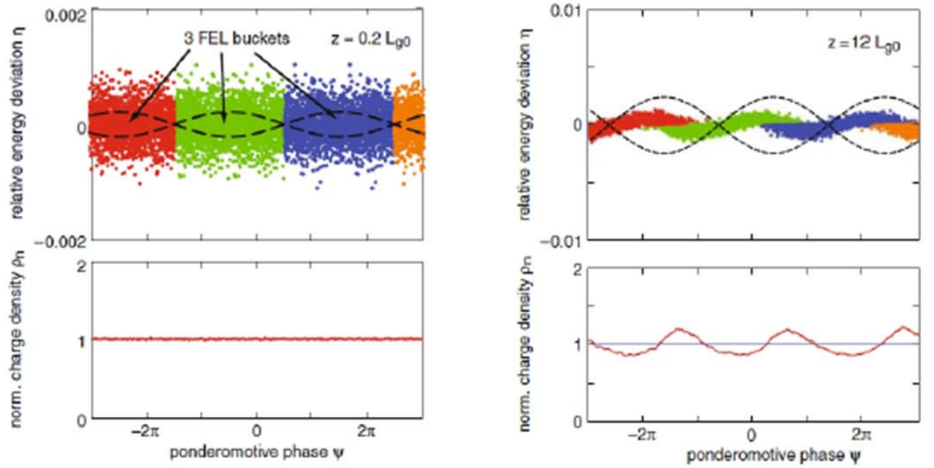
$|b| \to 1$ Stimulated emission

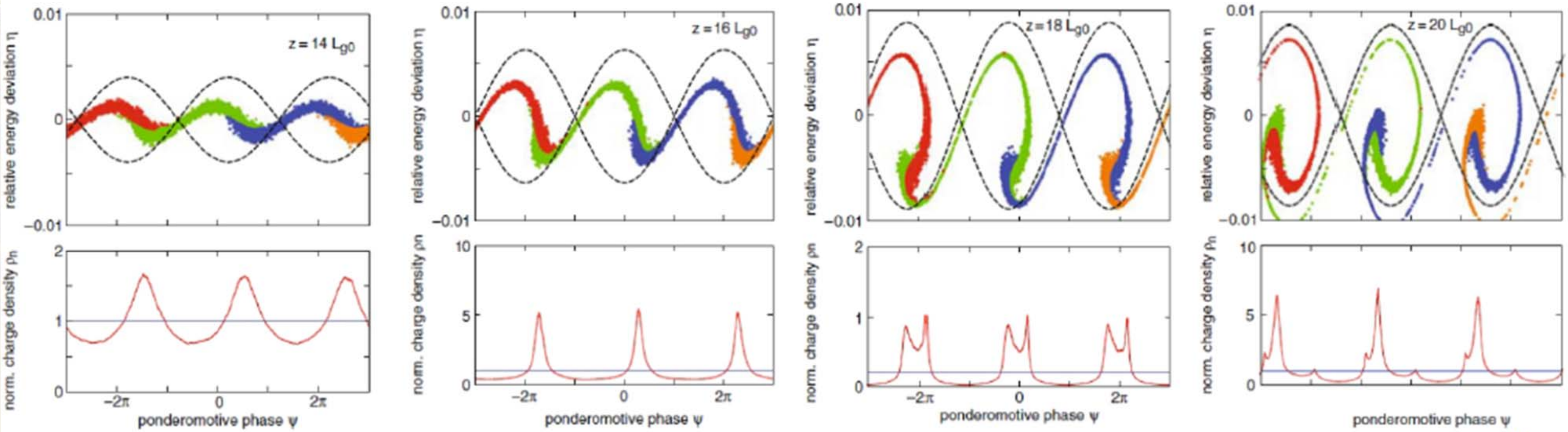
Saturation effects prevent the beam to radiate as N^2, limiting the radiated power scaling to $N^{4/3}$, due to a competition between neighbors slices .

When propagation effects and slippage are relevant, i.e. when the electron beam is as short as a slippage length, the emitted radiation leaves the bunch before saturation occurs and the power scaling becomes N^2 (Super-radiant or Single Spike regime)

Evolution of FEL microbunching

FEL bucket grows rapidly as the beam begins to bunch and radiate coherently. After a ¼ synchrotron oscillation the bunching begins to decrease again and the FEL saturates

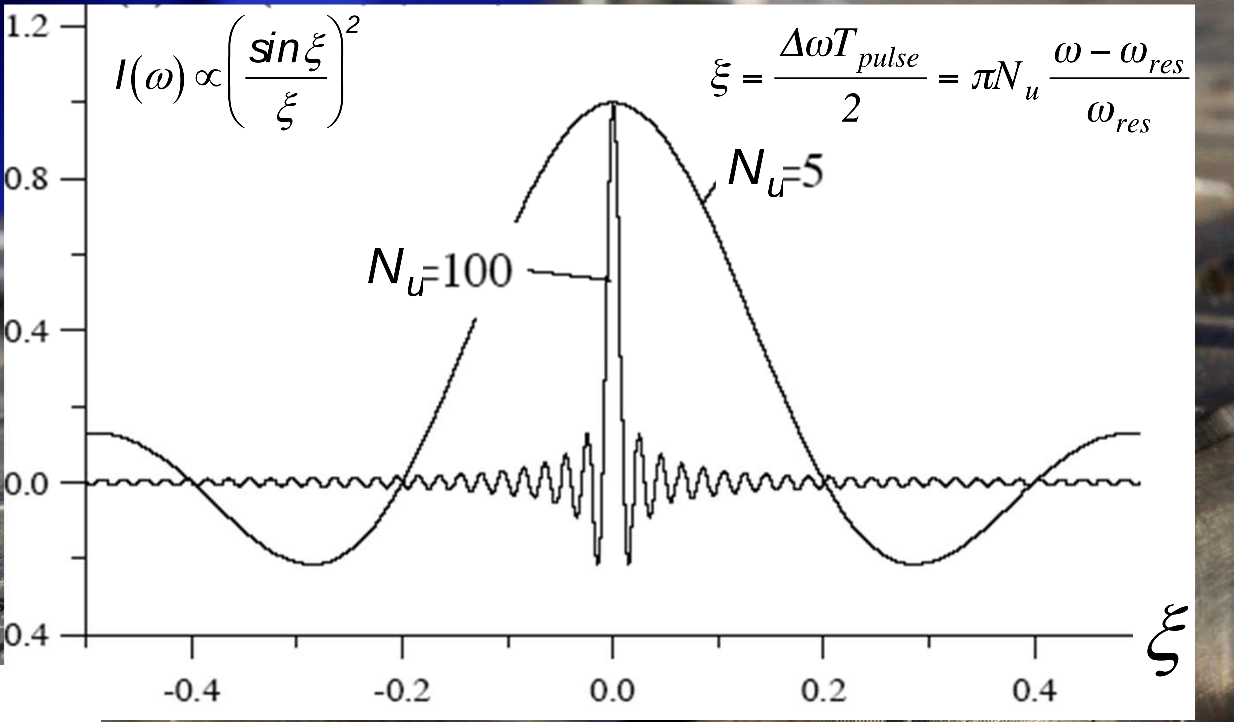




From P. Schmuser et al.

Spectral Intensity

$$\lambda_{rad} \approx \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\vartheta^2\right)$$



$$\frac{\Delta\lambda_{rad}}{\lambda_{rad}} = \frac{\lambda_{rad}(\vartheta)-\lambda_{rad}(0)}{\lambda_{rad}(0)}$$

$$= \frac{\gamma^2\vartheta^2}{1+\frac{K^2}{2}} = \frac{\lambda_u}{2\lambda_{rad}}\vartheta^2$$

$$= \frac{\lambda_u}{\lambda_{rad}}\sigma_{x'}^2 = \frac{\lambda_u}{\lambda_{rad}}\frac{\varepsilon^2}{\sigma_x^2} \approx \frac{\varepsilon_n^2}{\sigma_x^2}$$

==>

$$\vartheta \approx \sqrt{\frac{1}{\gamma^2 N_u}\left(1+\frac{K^2}{2}\right)} \approx \frac{1}{\gamma\sqrt{N_u}}$$

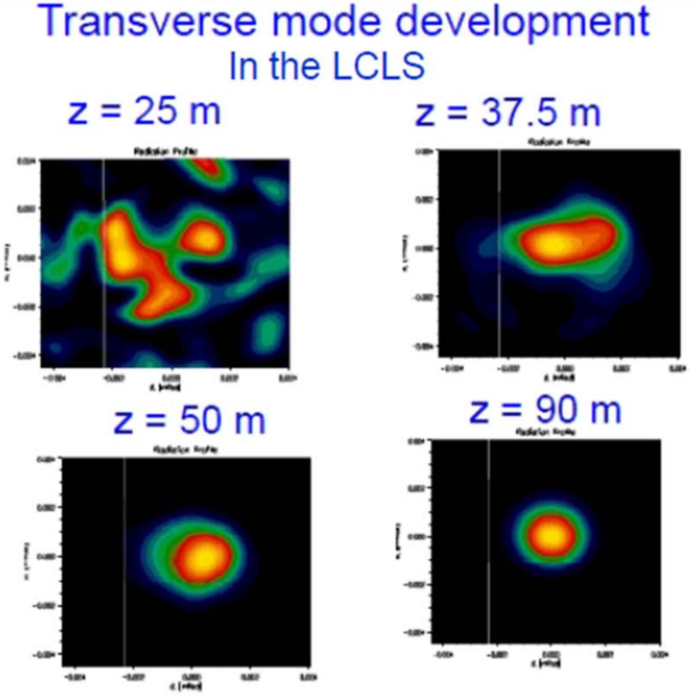
Line width

Angular width

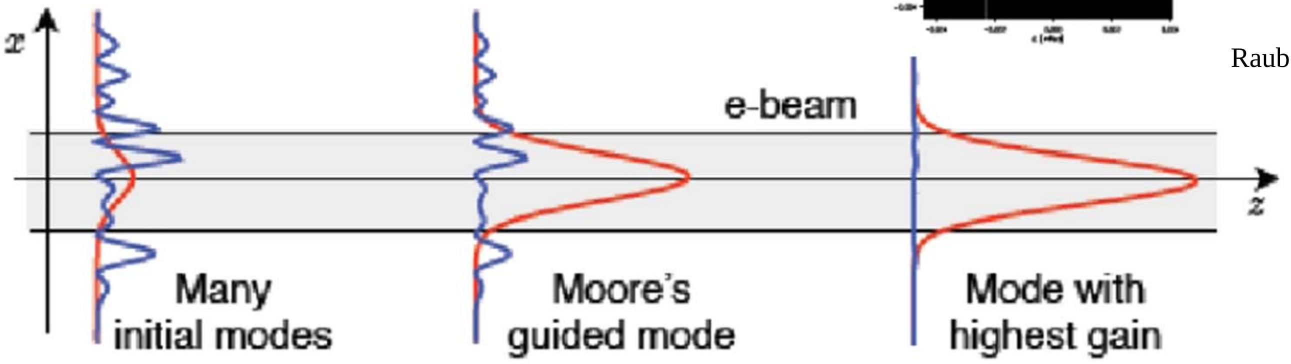
$$\frac{\Delta\omega}{\omega} \approx \frac{1}{N_u}$$

Transverse coherence

Radiation can be decomposed into transverse modes. The TEM_{00} has the highest intensity on axis while other modes extend out radially. The fundemental TEM_{00} mode grows fastest and rapidly dominates although the other modes can catch up once into saturation

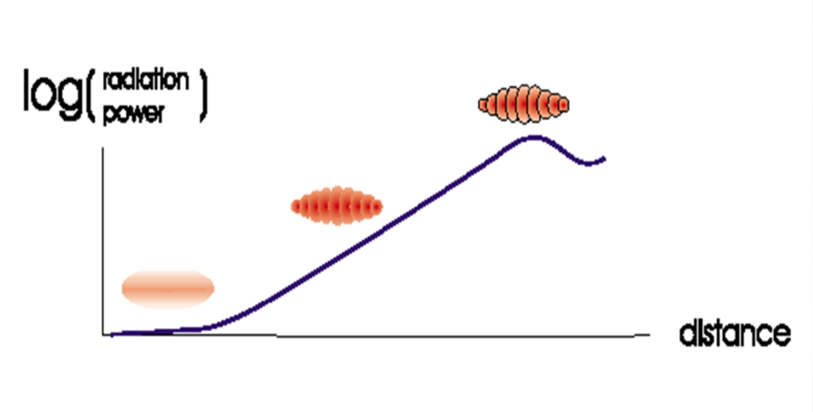

Transverse mode development
In the LCLS

Raubenheimer, SSSEPB13



FEL require a very intense, high quality e-beam

- FEL Parameter $$\rho = 0.136 \frac{1}{\gamma_r} J^{1/3} B_u^{2/3} \lambda_u^{4/3}$$
- Exponential growth $$P(z) = \frac{P_0}{9} \exp\left(\frac{z}{L_G}\right)$$
- Gain Length $$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$
- Saturation power $$P_{sat} = \rho P_{beam} \propto N_e^{4/3}$$
- Constraint on emittance $$\boxed{\varepsilon = \frac{\varepsilon_n}{\gamma} < \frac{\lambda_0}{4\pi}}$$
- Constraint on energy spread $$\boxed{\Delta\gamma/\gamma < \rho}$$
- Relative bandwidth $$\frac{\Delta\omega}{\omega} = \sqrt{\frac{\rho}{N_u}}$$

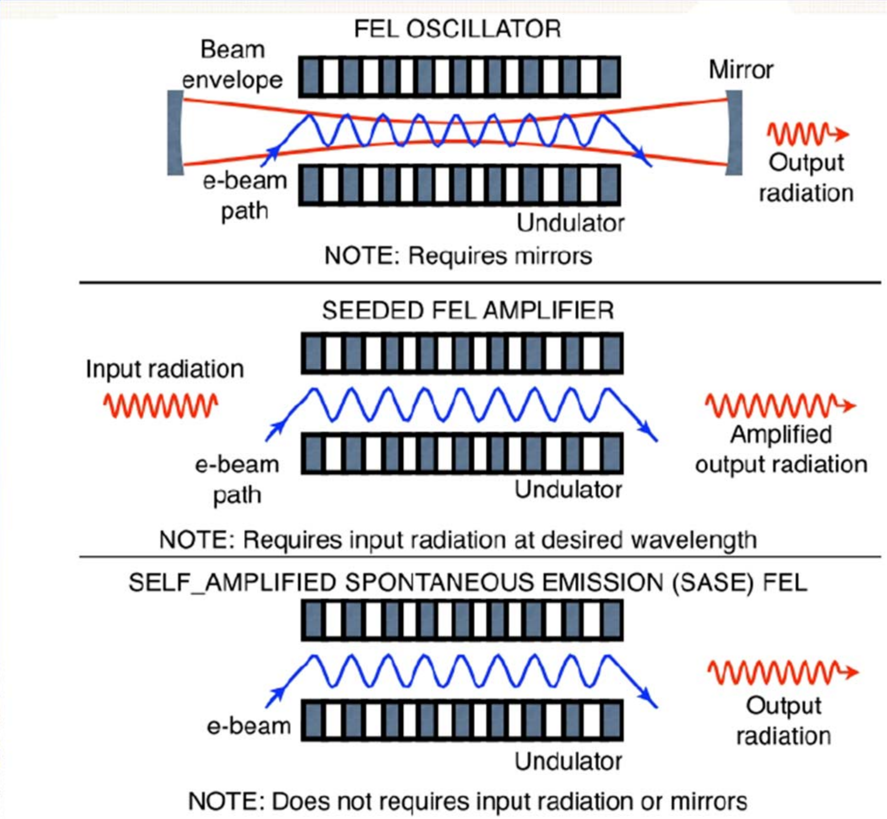


Schmuser, et al., *Ultraviolet and Soft X-Ray Free-Electron Lasers: Introduction to Physical Principles, Experimental Results, Technological Challenges*, STMP 229 (Springer, Berlin Heidelberg 2008), DOI 10.1007/ 978-3-540-79572-8

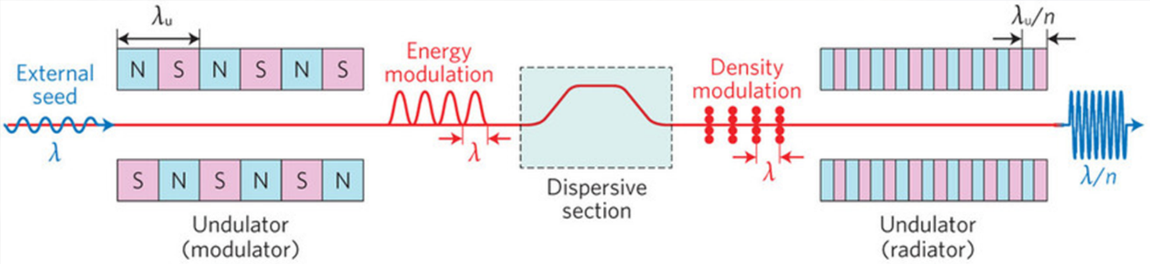
FEL operations

Oscillator
Seeding
SASE (Self Amplified Spontaneous Emission)
HGHG (High Gain Harmonic Generation)
Self seeding

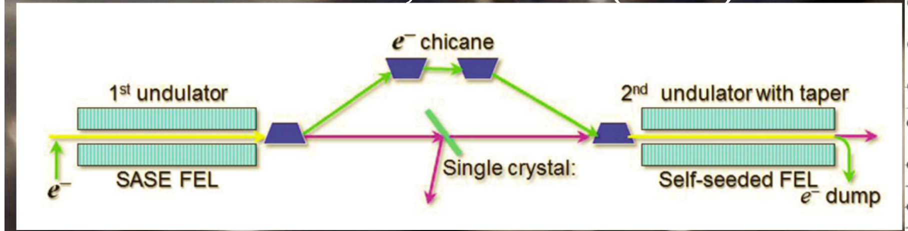
and many more:
p-SASE, i-SASE, HB-SASE, EEHG, …



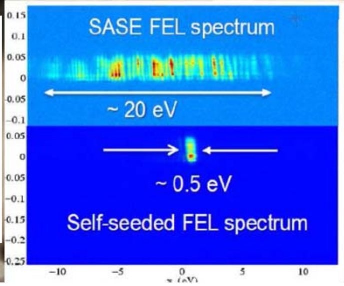
Raubenheimer, SSSEPB13



Nature Photonics 7, 852–854 (2013)

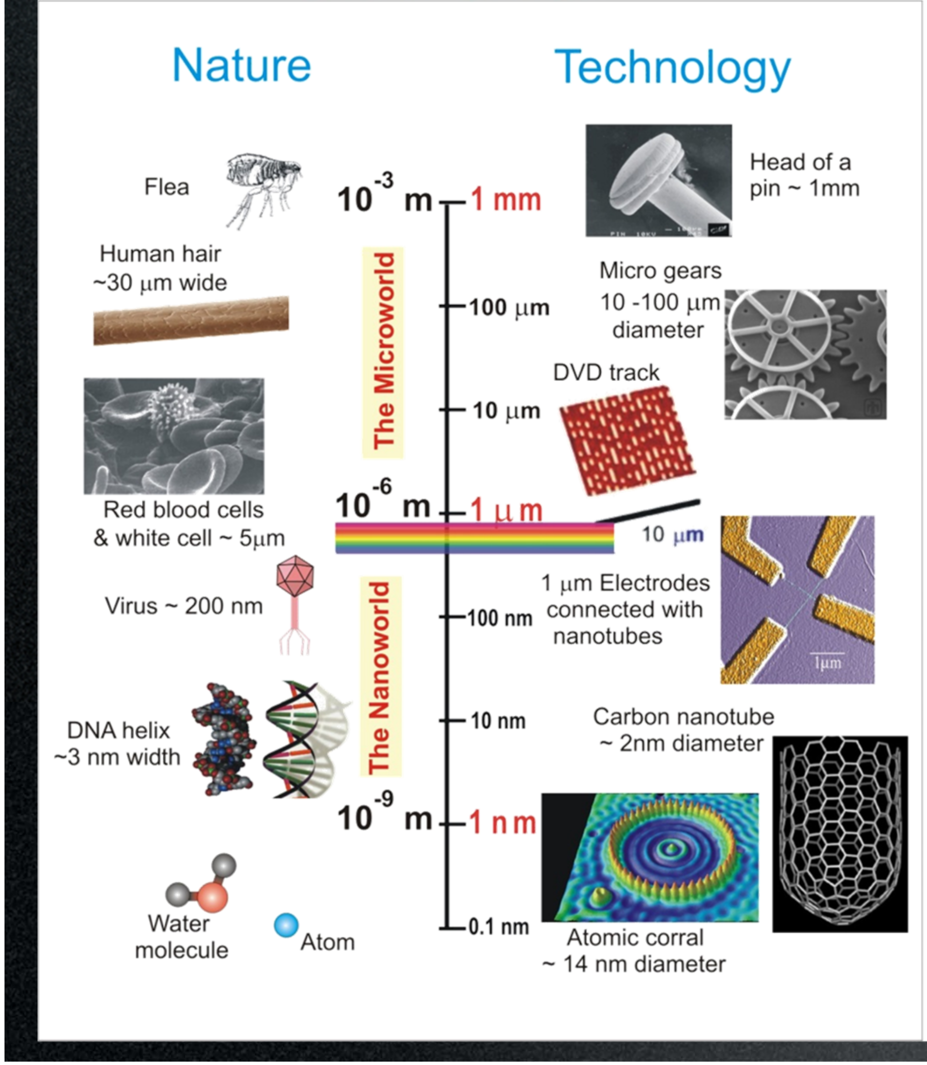


F.-J. Decker (SLAC)

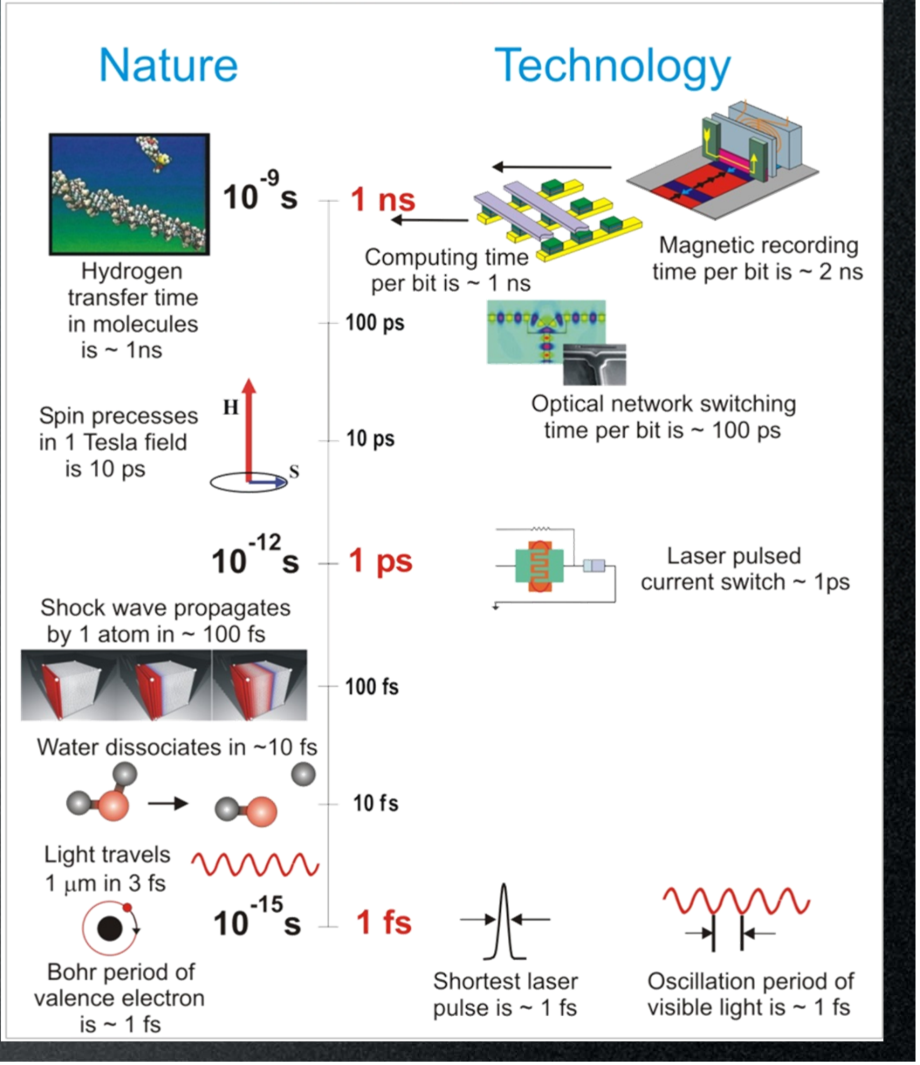


FEL applications

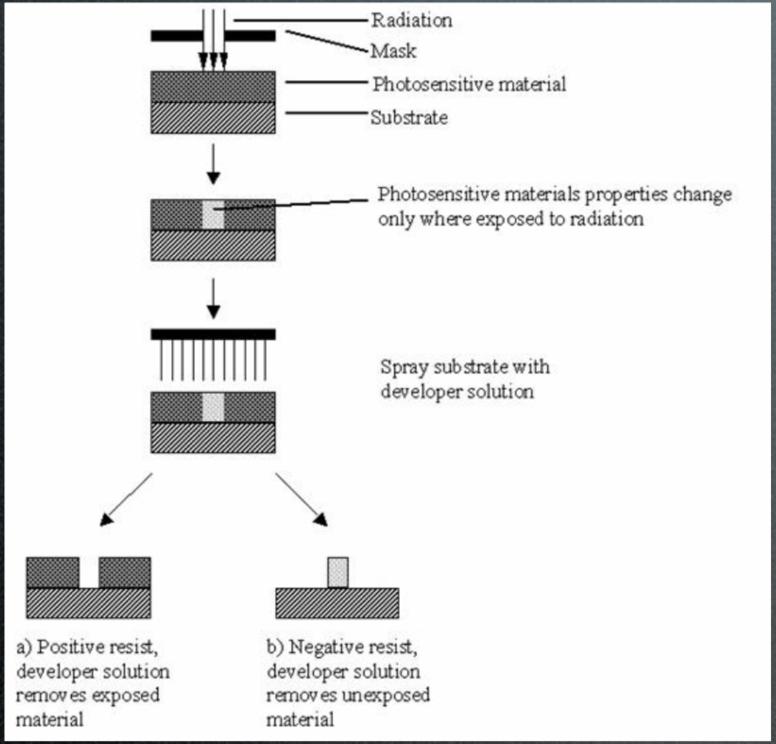
Ultra-Small



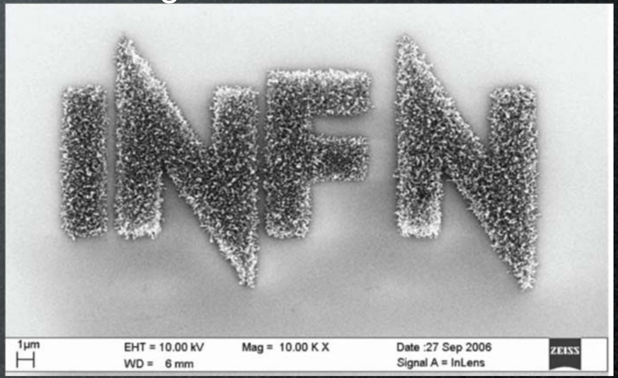
Ultra-Fast



Nano-lithography



Michelangelo Ambrosio INFN-GINT

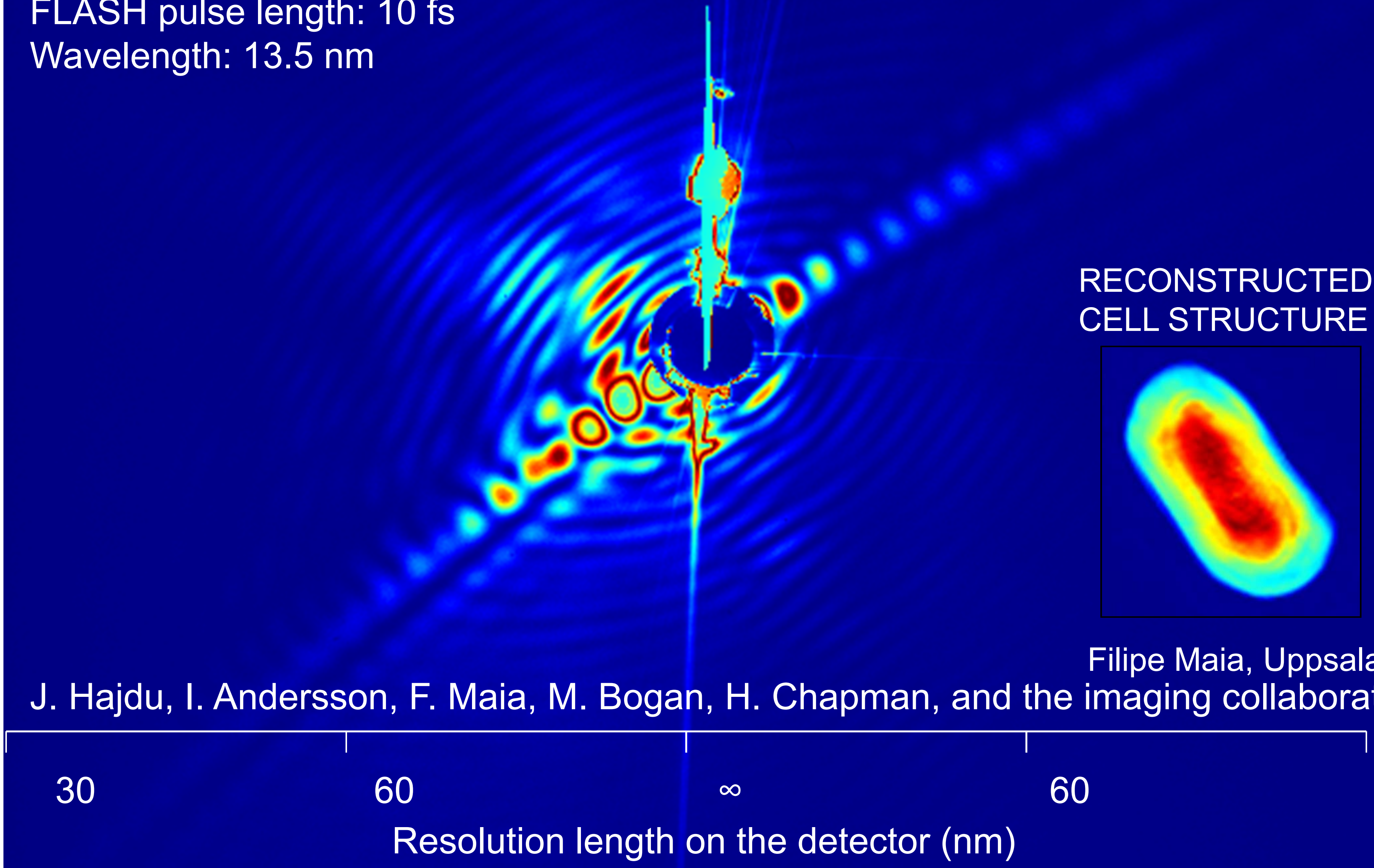


- Extreme UV Lithographt is the candidate technology with <50-35 nm
- Cost effective solutions based on FEL sources can be foreseen

FIRST FLASH DIFFRACTION IMAGE OF A LIVING CELL

FLASH soft X-ray laser, Hamburg, Germany

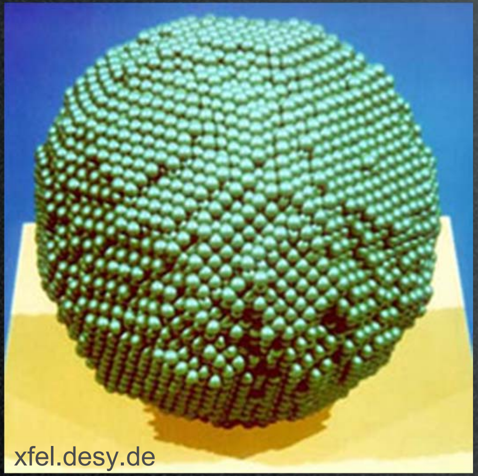
FLASH pulse length: 10 fs
Wavelength: 13.5 nm



Filipe Maia, Uppsala

J. Hajdu, I. Andersson, F. Maia, M. Bogan, H. Chapman, and the imaging collaboration

Cluster and nanoparticle


xfel.desy.de

Clusters are small bits of matter composed of anywhere from a few to tens of thousands of atoms.

Small particles are different from bulk matter; finite size effects influence all properties of matter.

Examples are tiny carbon spheres and carbon tubes that are considered promising candidates for use as nanotechnological components.
(17 000 copper atoms in the picture on the right).

Limited photon energy of standard laser systems prevents measuring the full valence electron structure as well or performing photon energy dependent spectroscopy across shallow core edges

The beam intensities available at 3rd generation synchrotron radiation facilities are still far below what is required for meaningful gas phase experiments.

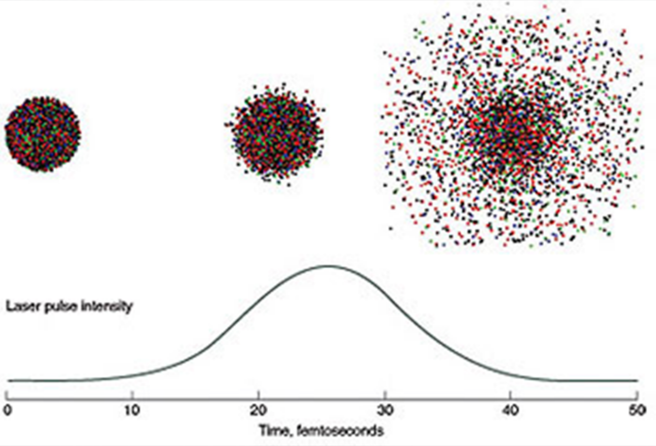
Protein imaging

bioMolecules.mov

Using extremely short and intense X-ray pulses to capture images of objects such as proteins before the X-rays destroy the sample.

Single-molecule diffractive imaging with an X-ray free-electron laser.

Individual biological molecules will be made to fall through the X-ray beam, one at a time, and their structural information recorded in the form of a diffraction pattern.



Lawrence Livermore National Laboratory (LLNL)

The pulse will ultimately destroy each molecule, but not before the pulse has diffracted from the undamaged structure.

The patterns are combined to form an atomic-resolution image of the molecule.

The speed record of 25 femtoseconds for flash imaging was achieved.

Models indicate that atomic-resolution imaging can be achieved with pulses shorter than 20 femtoseconds.

Make a movie of chemical reactions

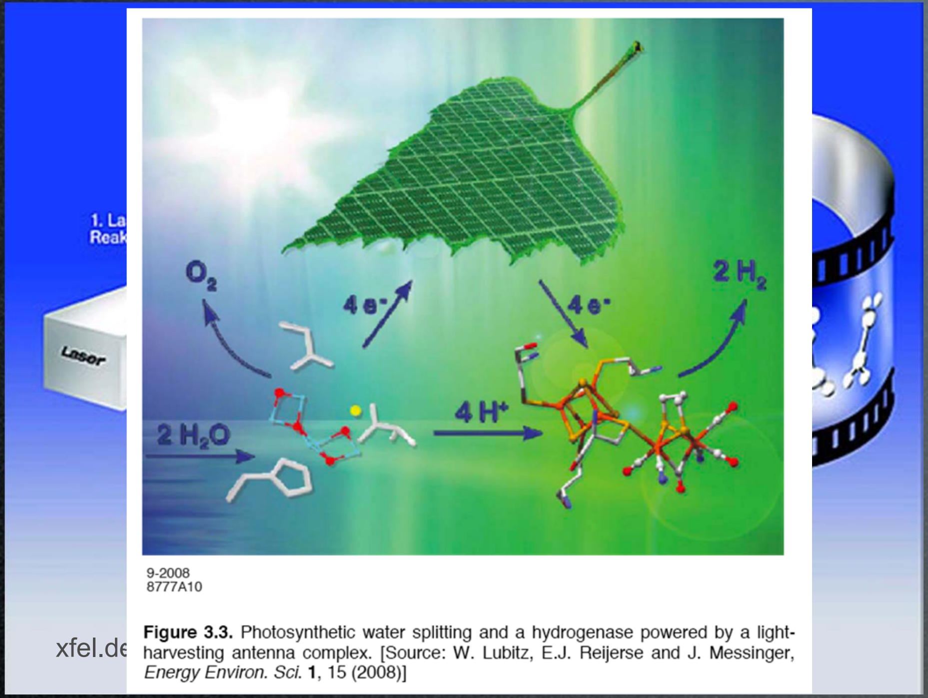


Figure 3.3. Photosynthetic water splitting and a hydrogenase powered by a light-harvesting antenna complex. [Source: W. Lubitz, E.J. Reijerse and J. Messinger, *Energy Environ. Sci.* **1**, 15 (2008)]

Chemical reactions often take place incredibly quickly: orders of magnitude of femtosecond are not rare. The atomic changes that occur when molecules react with one another take place in moments that brief.

The XFEL X-ray laser flashes make it possible to film these rapid processes with an unprecedented level of quality.

Since the flash duration is less than 100 femtoseconds, images can be made in which the movements of detail are not blurred.

And thanks to the short wavelength, atomic details become visible in the films.

To film a chemical reaction, one needs a series of pairs of X-ray laser flashes.

The first flash in each pair triggers the chemical reaction. With the second flash, a snapshot is then made.

The delay between the two flashes can be precisely modified to within femtosecond and a series of snapshots can be made at various times following the start of the reaction.

In each case, the images are of different molecules, but these images can be combined into a film.

X-FEL user facilities

Under construction and planned xFEL user facility projects

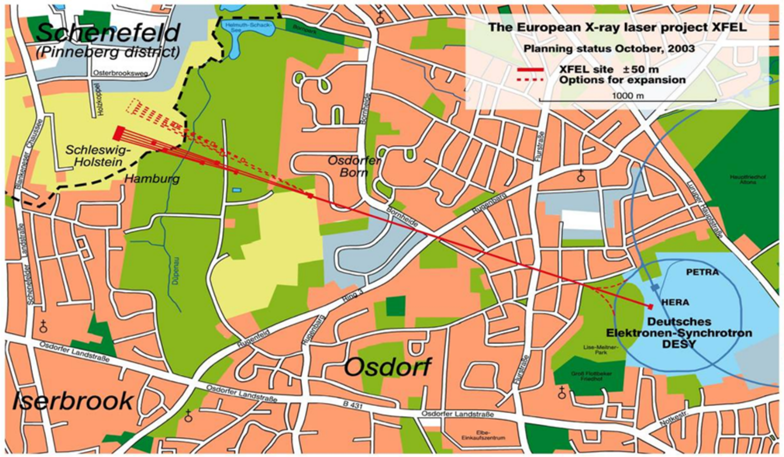
European XFEL; SwissFEL; PAL-XFEL; SXFEL; NGLS

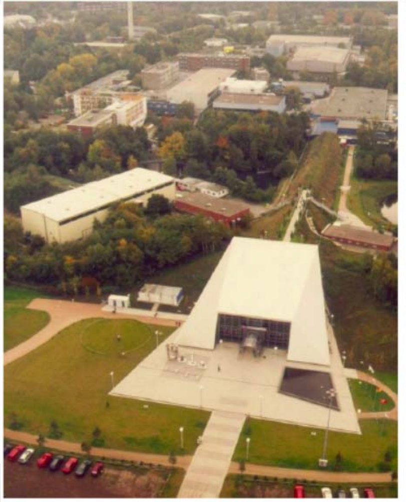
Other proposals under development
LUNEX-5, MaRIE...

Many FEL R&D projects and facilities
APEX, ATF, CLARA, JLAB, MAX-IV, NLCTA, SCSS, SDL, SDUV FEL, WiFEL, SPARC, FELcats....

E
FALSH – DESY

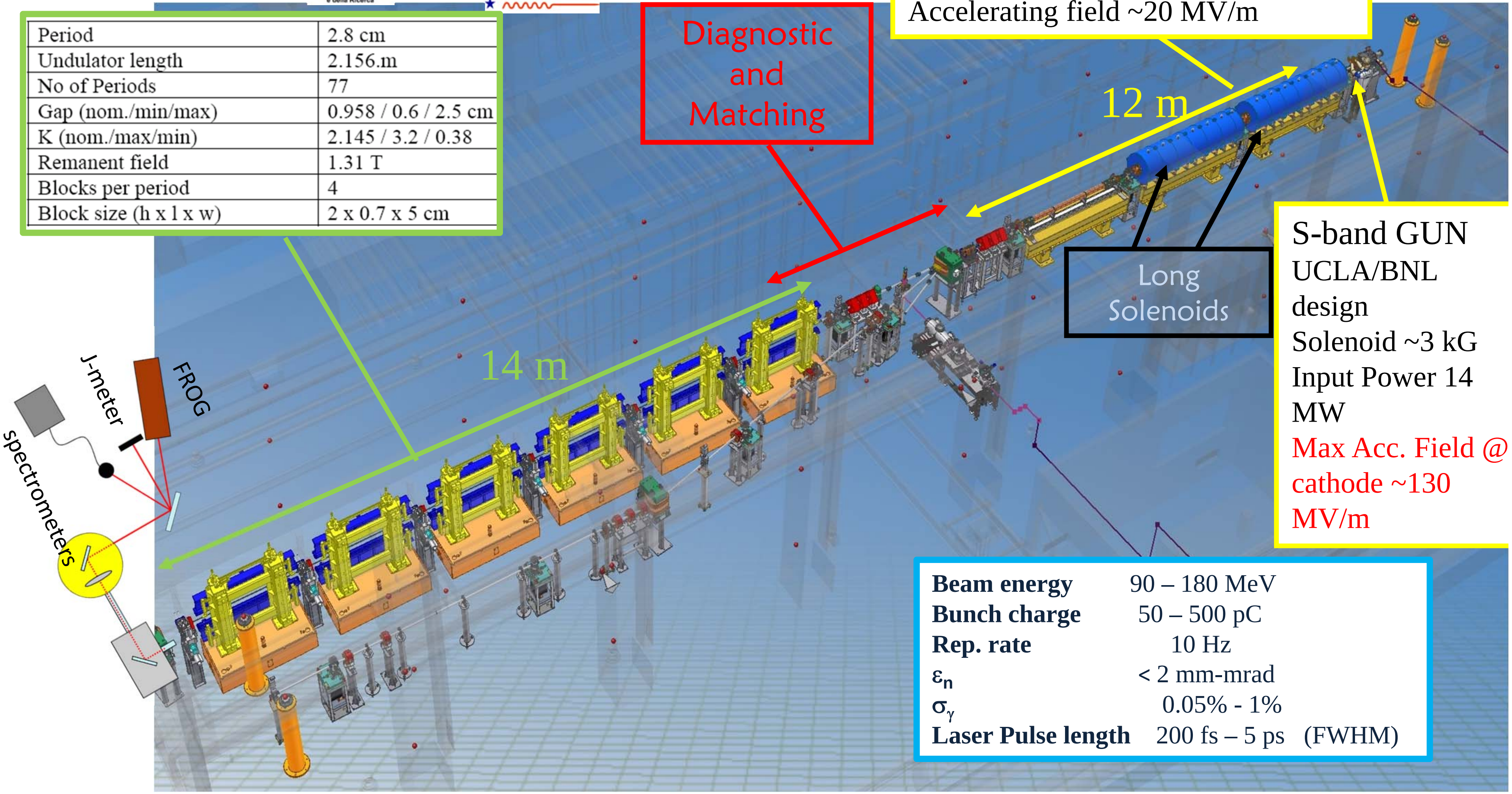
USA
LCLS - SLAC





FERMI– Elettra

The SPARC Free Electron Laser



Period	2.8 cm
Undulator length	2.156.m
No of Periods	77
Gap (nom./min/max)	0.958 / 0.6 / 2.5 cm
K (nom./max/min)	2.145 / 3.2 / 0.38
Remanent field	1.31 T
Blocks per period	4
Block size (h x l x w)	2 x 0.7 x 5 cm

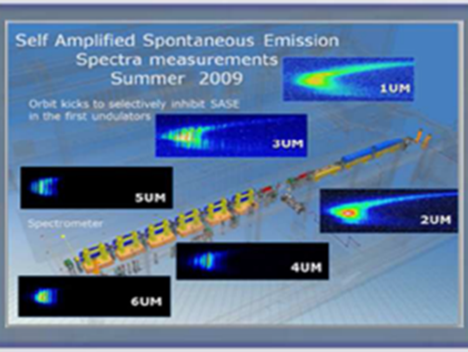
180 MeV S-band linac
SLAC constant gradient design
Solenoid ~300 G
Accelerating field ~20 MV/m

S-band GUN
UCLA/BNL design
Solenoid ~3 kG
Input Power 14 MW
Max Acc. Field @ cathode ~130 MV/m

Beam energy	90 – 180 MeV
Bunch charge	50 – 500 pC
Rep. rate	10 Hz
ε_n	< 2 mm-mrad
σ_γ	0.05% - 1%
Laser Pulse length	200 fs – 5 ps (FWHM)

Past FEL experiments

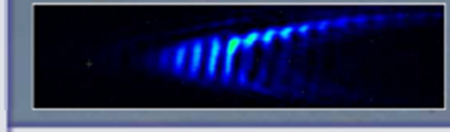
SASE

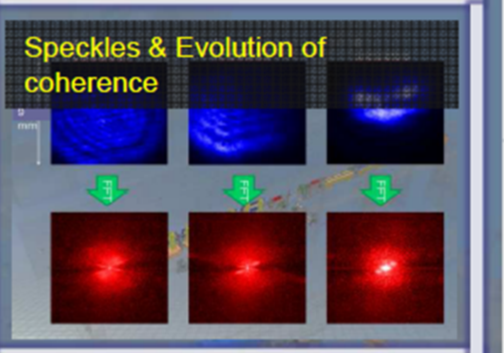

Self Amplified Spontaneous Emission
Spectra measurements
Summer 2009
Orbit kicks to selectively inhibit SASE in the first undulators

SINGLE SPIKE (e-beam chirp & taper)

Substantial increase of Pulse energy
Energy 140 uJ (max 380 uJ) Rel Linewidth 0.8% rms

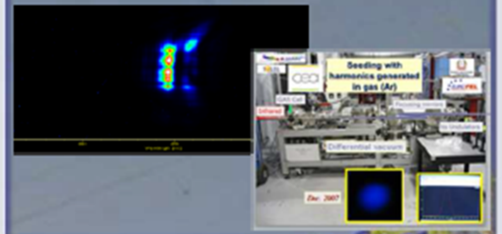
TWINS***
(Two simultaneous spikes ~560 fs separation)



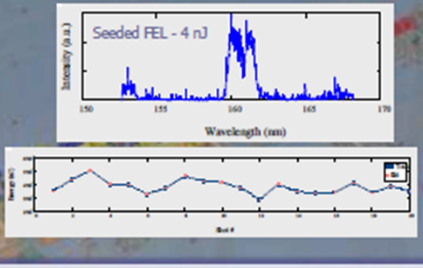

Speckles & Evolution of coherence

HHG Seeding

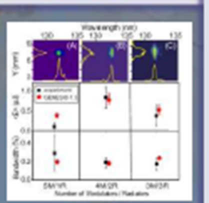
3° harmonic direct seeding & FEL harmonic generation with HHG

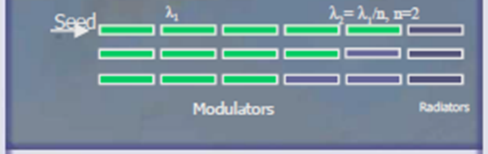


5° harmonic direct seeding



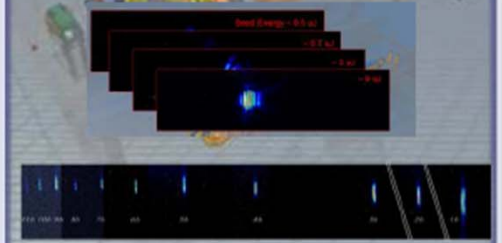
HGHG with HHG



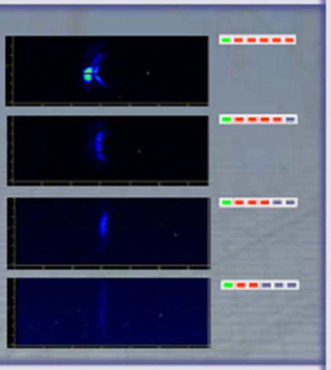


Seeding in superadiance

Emission of harmonics in superradiance

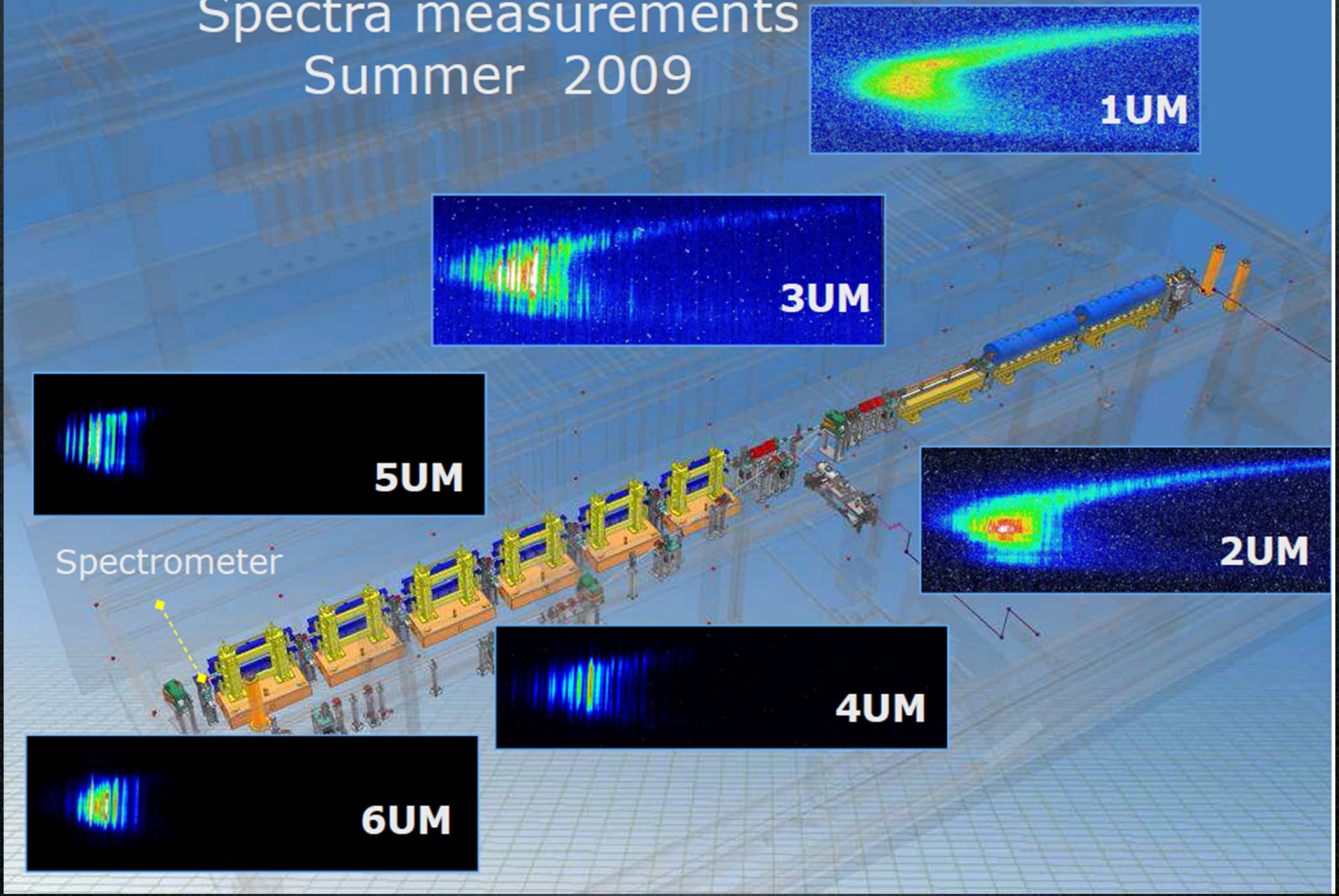


Superradiant cascade

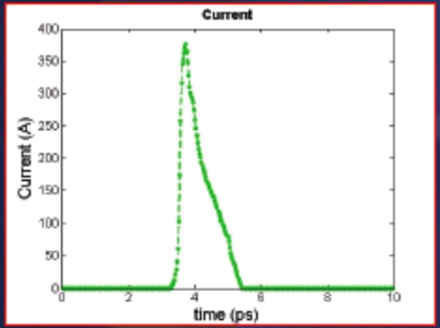


F. Ciocci 2012

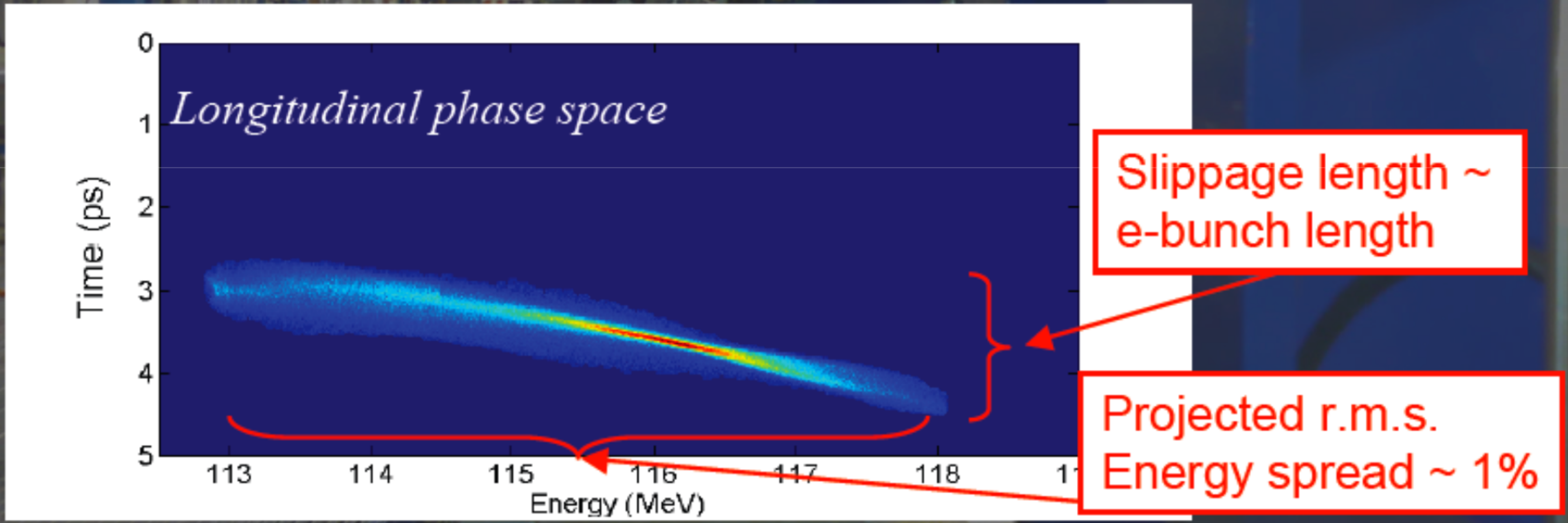
Self Amplified Spontaneous Emission
Spectra measurements
Summer 2009



- Compression with “Velocity Bunching”
  - High peak current (up to 380A)



Strong chirp / energy spread in the longitudinal phase space



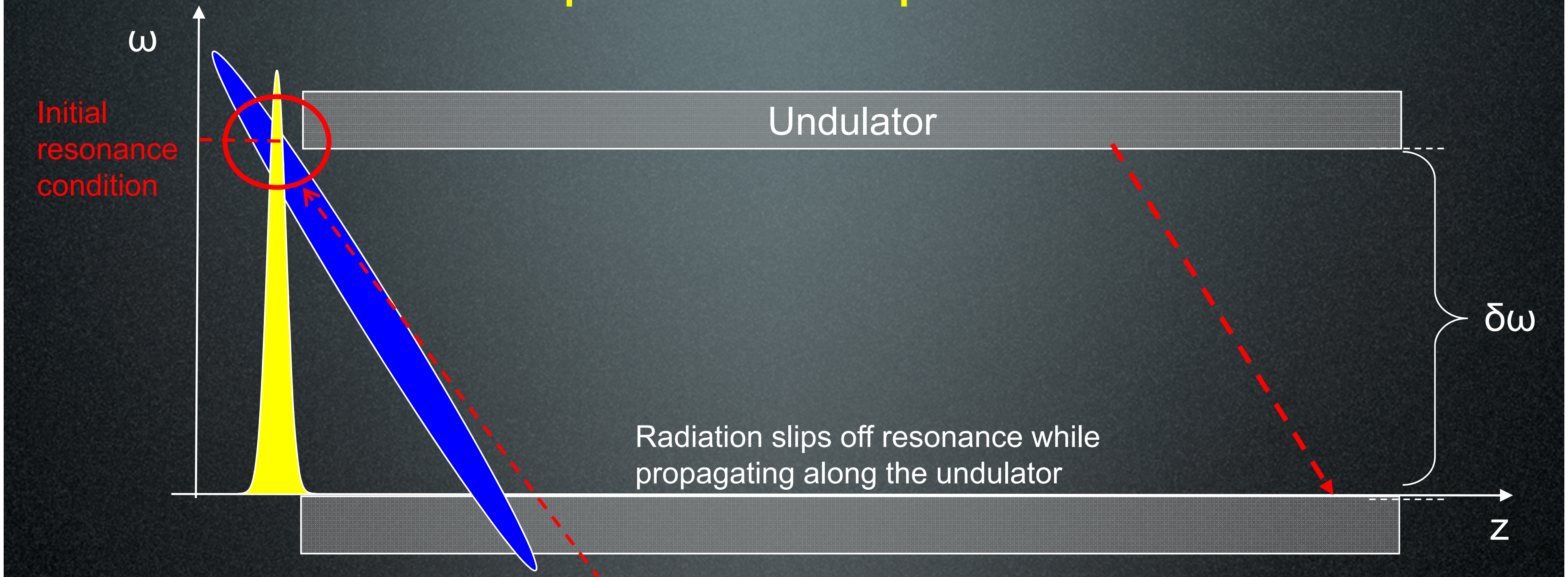
Slippage length ~ e-bunch length

Projected r.m.s. Energy spread ~ 1%

Average beam energy	115 MeV
Energy spread	1.33 % (0.7 MeV) [-8.7 keV/μm]

rms bunch length	0.42 ps
Peak current	380 A
Transv. emittance	2.7/3 μm

Chirped Beam Spectrum

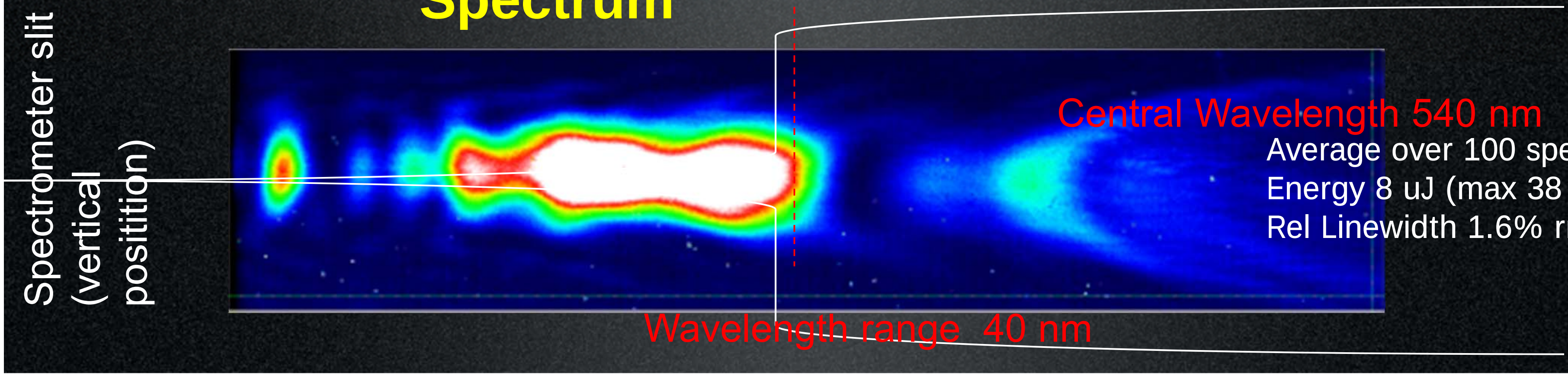


Initial resonance condition

Radiation slips off resonance while propagating along the undulator

Resonance condition is a function of
Beam energy (chirp) / Undulator K (untapered)

Spectrum



Central Wavelength 540 nm

Average over 100 spectra
Energy 8 uJ (max 38 uJ)
Rel Linewidth 1.6% rms

Wavelength range 40 nm

Compensation with Undulator taper

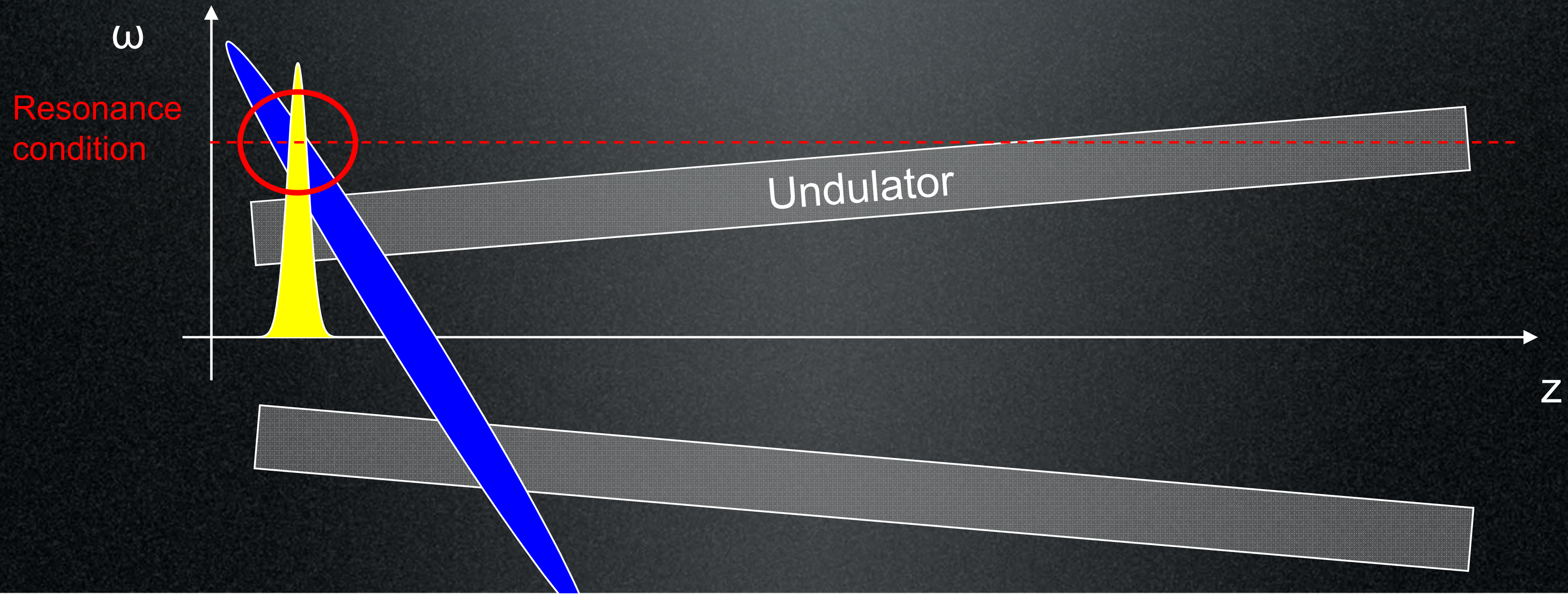
$$\omega_r = \frac{2\gamma^2}{1+\frac{K^2}{2}}\omega_u$$

$$\omega_u = \frac{2\pi c}{\lambda_u}$$

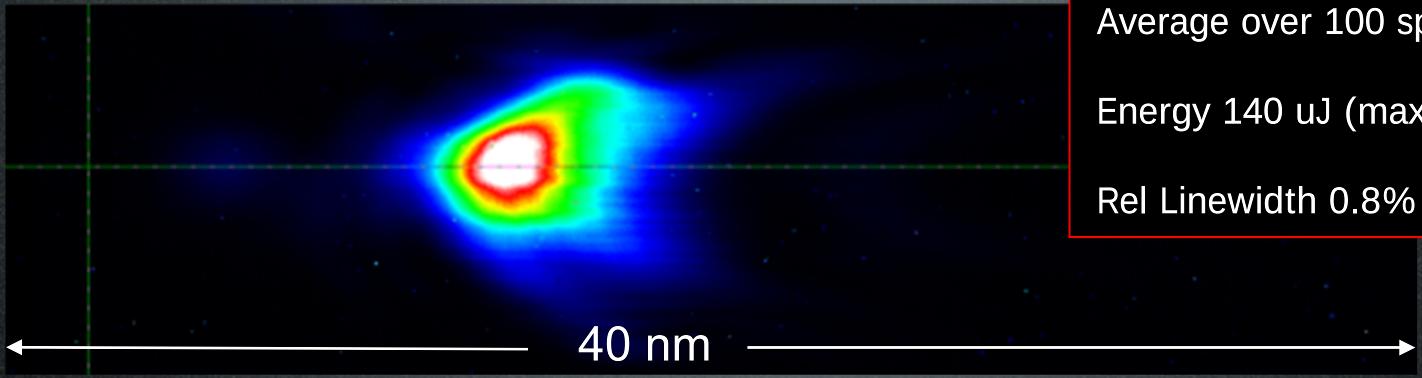
Chirp $\bar{\gamma} = \bar{\gamma}(s) = \gamma_0 + \alpha(s - s_0)$

Taper $K = K(z) = K_0 + \alpha_k(z - z_0)$

Resonance is maintained by tuning the undulator taper



Single Spike observed in many spectra

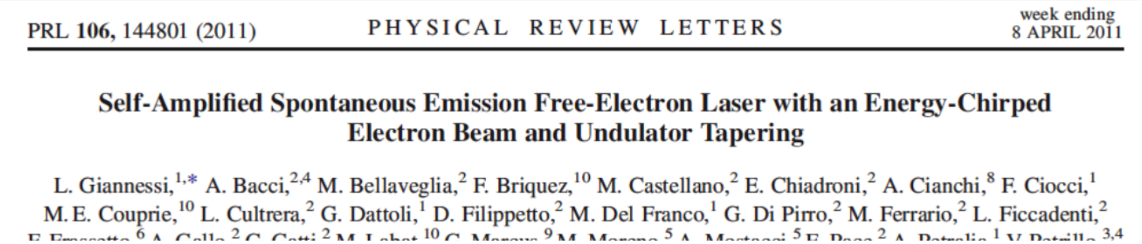


Average over 100 spectra:

Energy 140 uJ (max 380 uJ)

Rel Linewidth 0.8% rms

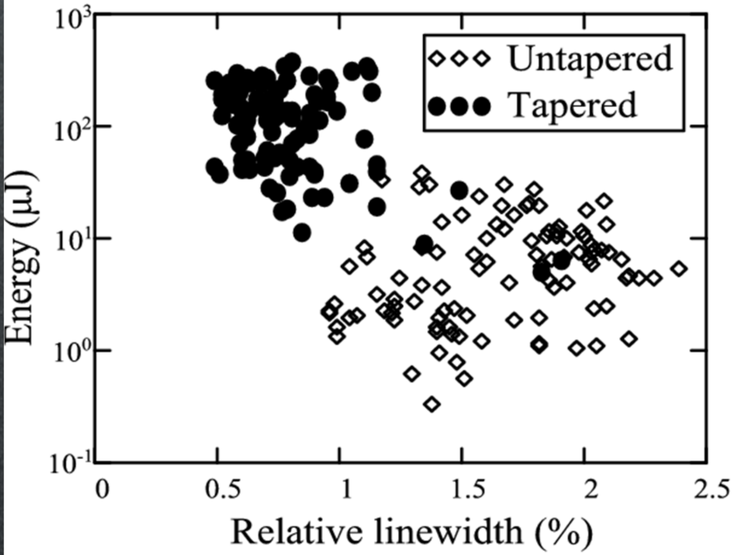
Average energy per pulse 18 times higher in a narrower bandwidth



PRL **106,** 144801 (2011) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2011

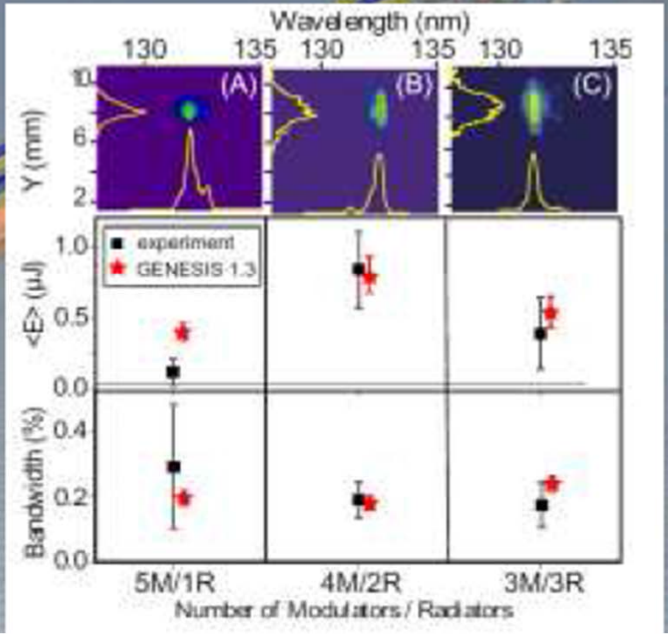
Self-Amplified Spontaneous Emission Free-Electron Laser with an Energy-Chirped Electron Beam and Undulator Tapering

L. Giannessi,[1,*] A. Bacci,[2,4] M. Bellaveglia,[2] F. Briquez,[10] M. Castellano,[2] E. Chiadroni,[2] A. Cianchi,[8] F. Ciocci,[1] M. E. Couprie,[10] L. Cultrera,[2] G. Dattoli,[1] D. Filippetto,[2] M. Del Franco,[1] G. Di Pirro,[2] M. Ferrario,[2] L. Ficcadenti,[2]



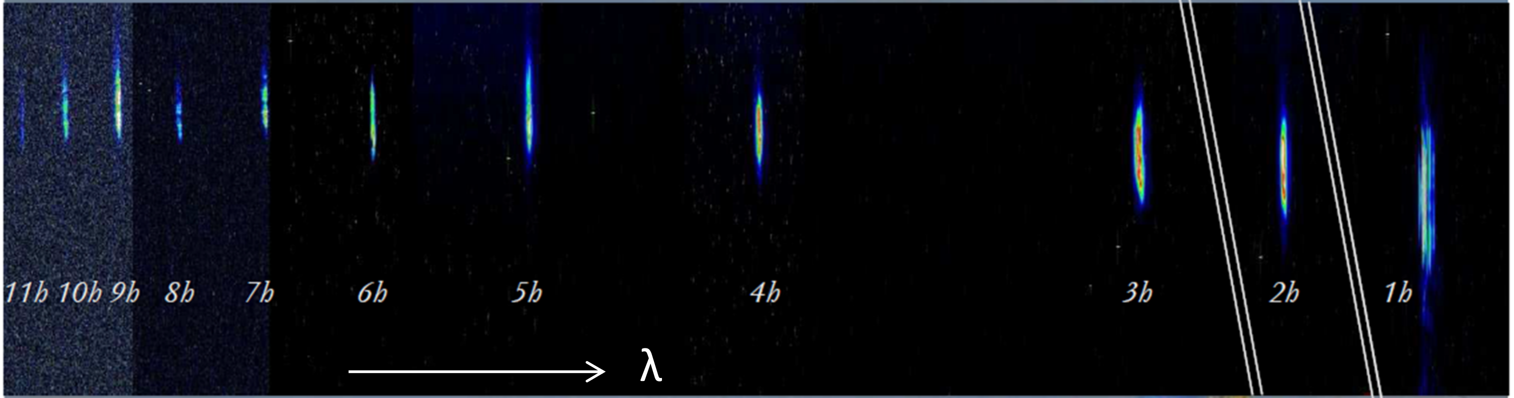
Cascaded FEL seeded with harmonics generated in gas

- Seed @ 266 nm / ~50 nJ
- 5-4-3 UM tuned @ 266 nm / 1-2-3 UM tuned @133 nm

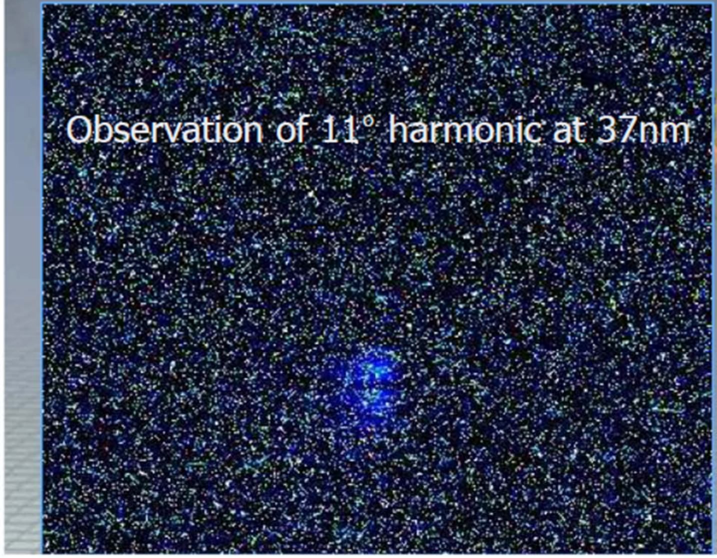


PRL 107, 224801 (2011)

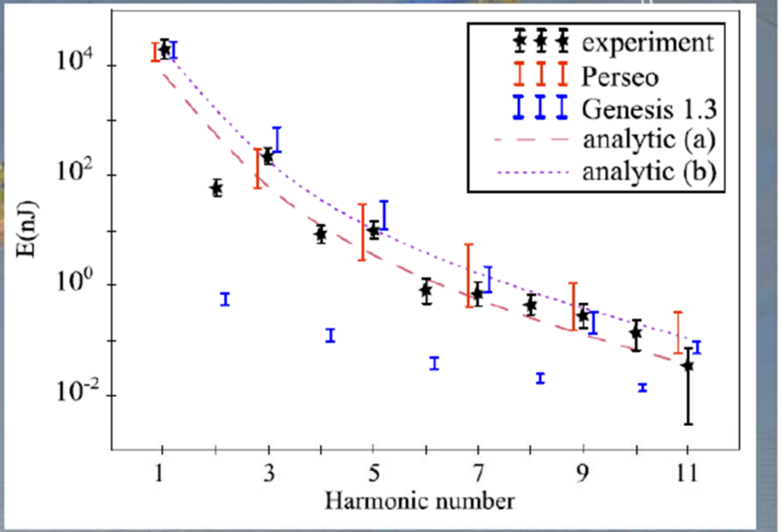
Emission of harmonics in superradiance



PRL 108, 164801 (2012)

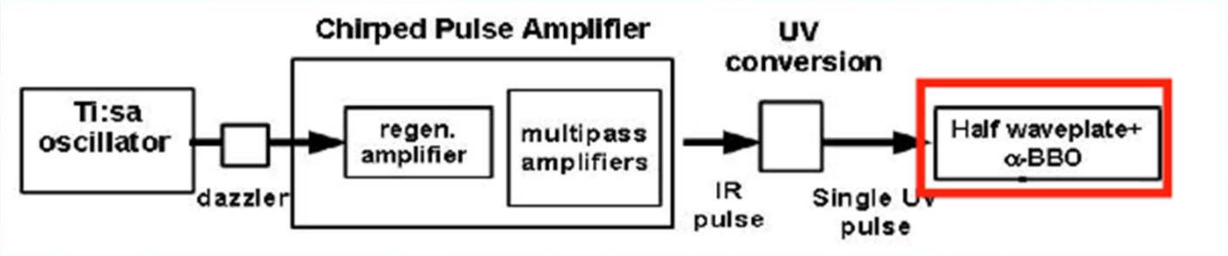


Observation of 11° harmonic at 37nm

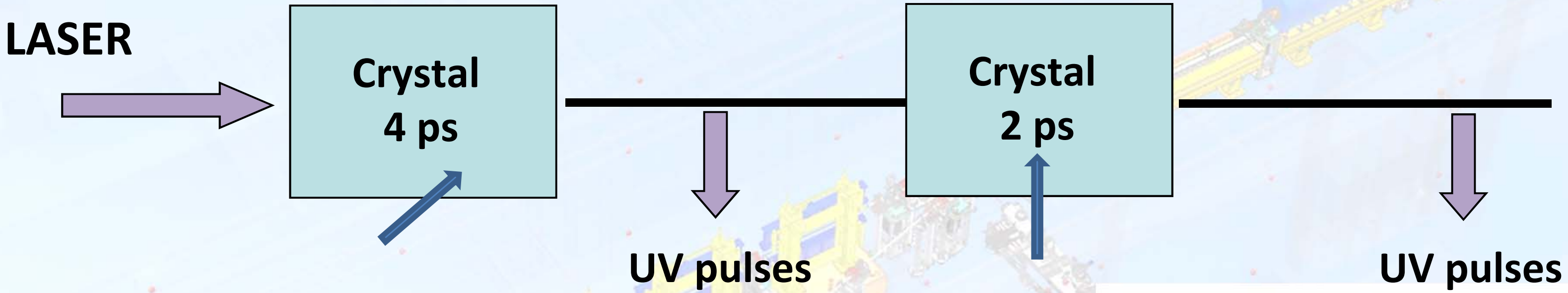


Analytical model: G. Dattoli and P. L. Ottaviani, J. Appl. Phys. 86, (1999)

Laser Pulse Train Generation



$$\Delta\tau = \left|\frac{1}{v_{ge}} - \frac{1}{v_{go}}\right| L_{crystal}$$



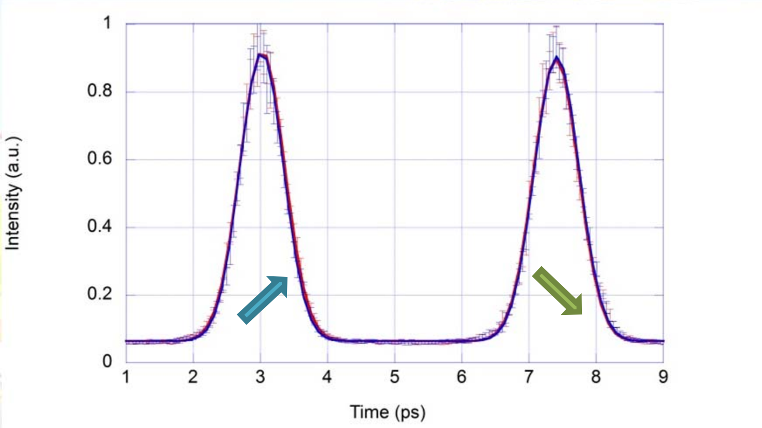
LASER

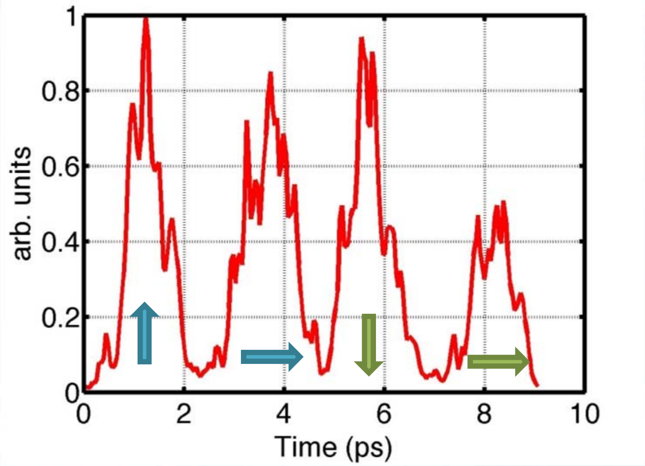
Crystal 4 ps

Crystal 2 ps

UV pulses

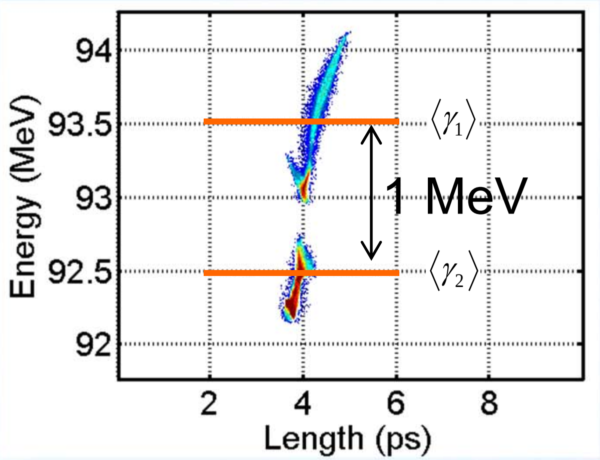
UV pulses





F. Villa et al., NIM A 740, 188 (2014)

Two color SASE FEL



two bunches with
a two-level energy distribution
and time overlap (Laser COMB tech.)

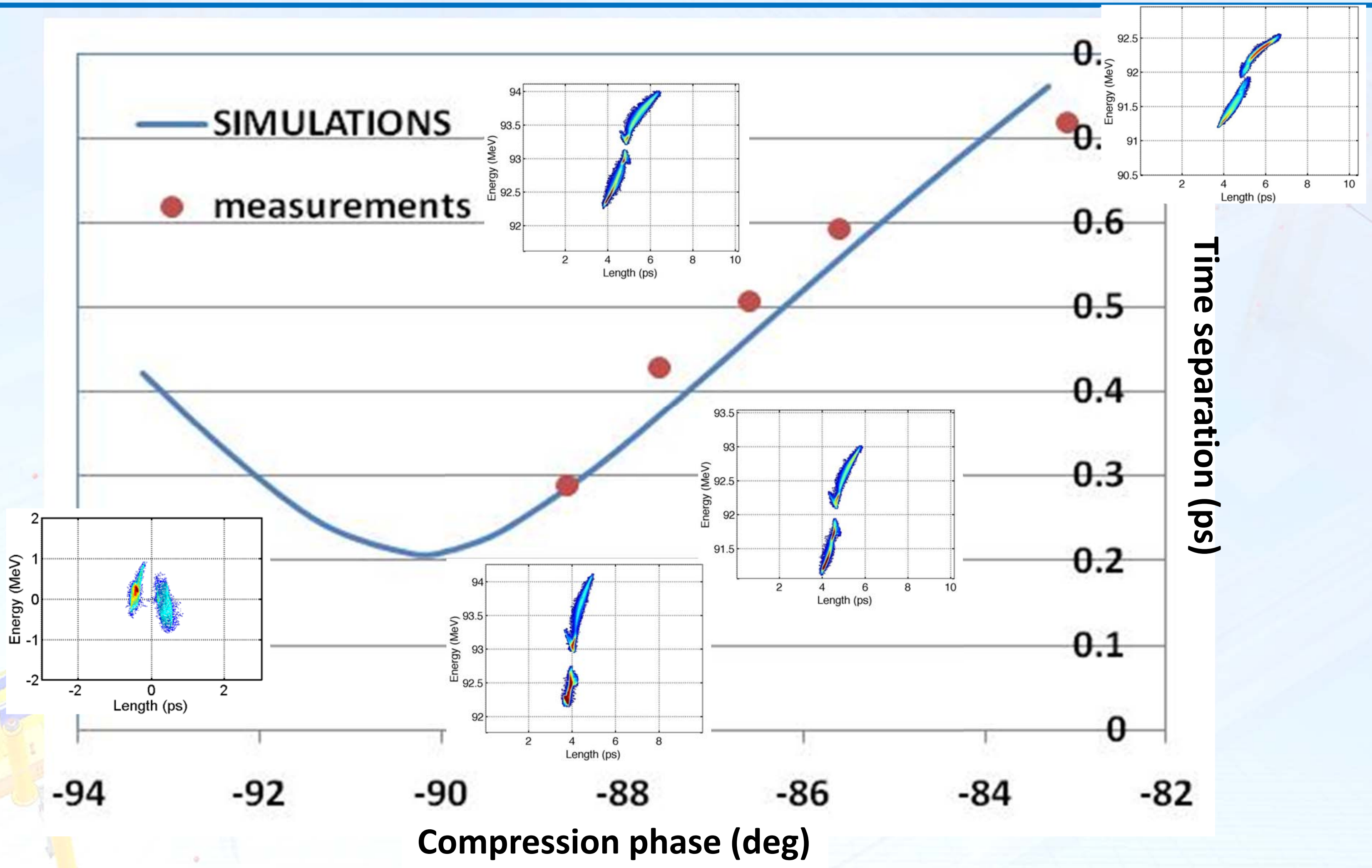
- Lasing condition: $\frac{\delta\gamma_{1,2}}{\langle\gamma_{1,2}\rangle} < \rho$
- To prevent mode competition: $\frac{\langle\gamma_1\rangle - \langle\gamma_2\rangle}{\langle\gamma\rangle} > \rho$
- Single spike condition: $l_b \approx L_{coop} = \frac{\lambda_r}{4\pi\sqrt{3}\rho}$

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K_{rms}^2}{2}\right)$$

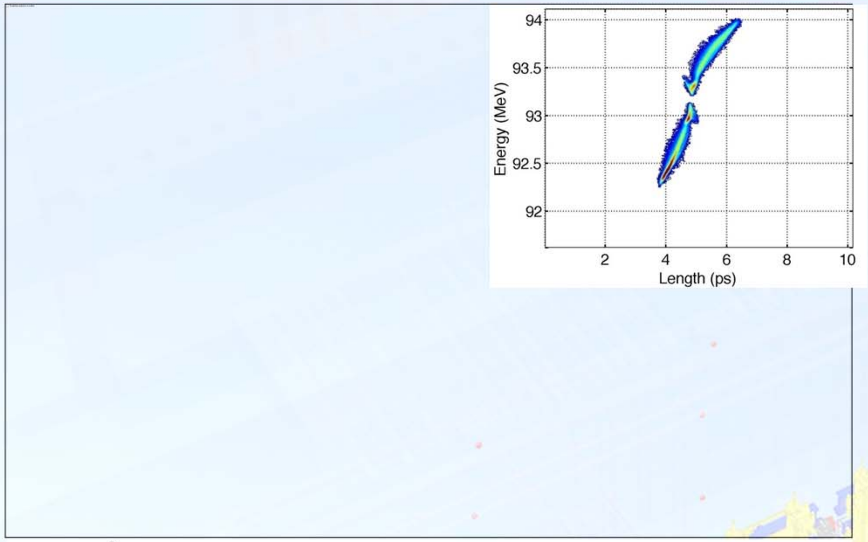
$$\frac{\Delta\lambda_r}{\langle\lambda_r\rangle} = 2\frac{\langle\gamma_1\rangle - \langle\gamma_2\rangle}{\langle\gamma\rangle}$$

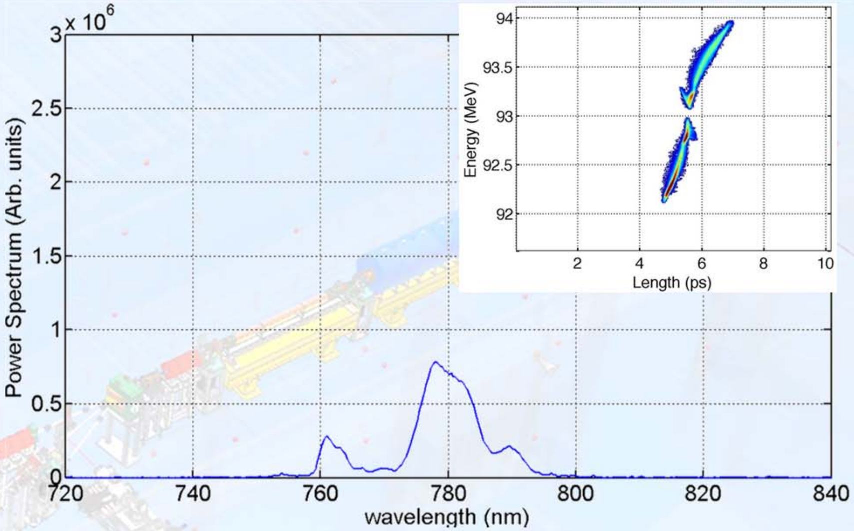
F. Ciocci et al., PRL 111, 264801 (2013)

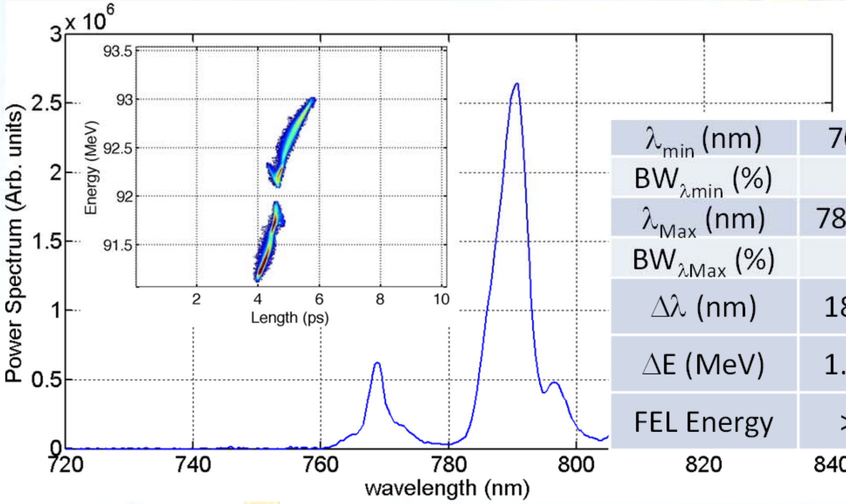
Experimental Compression Curve



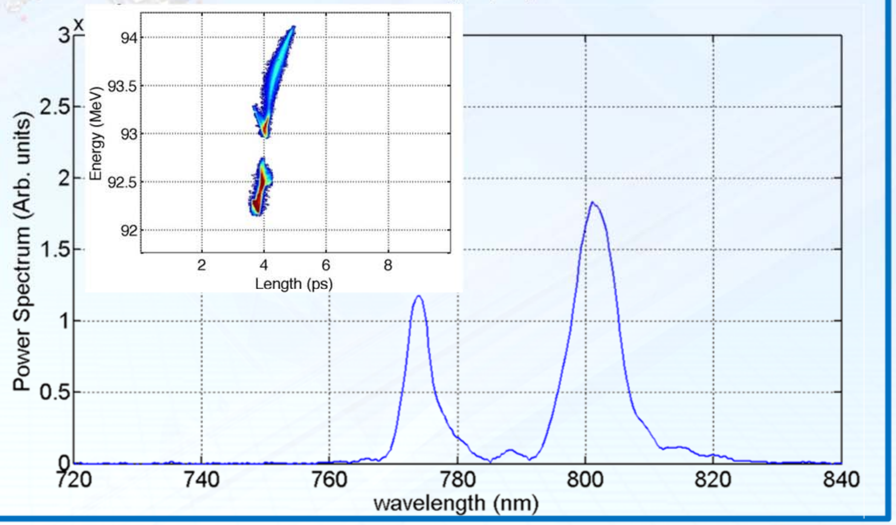
FEL Experiments: Two-levels radiation spectra



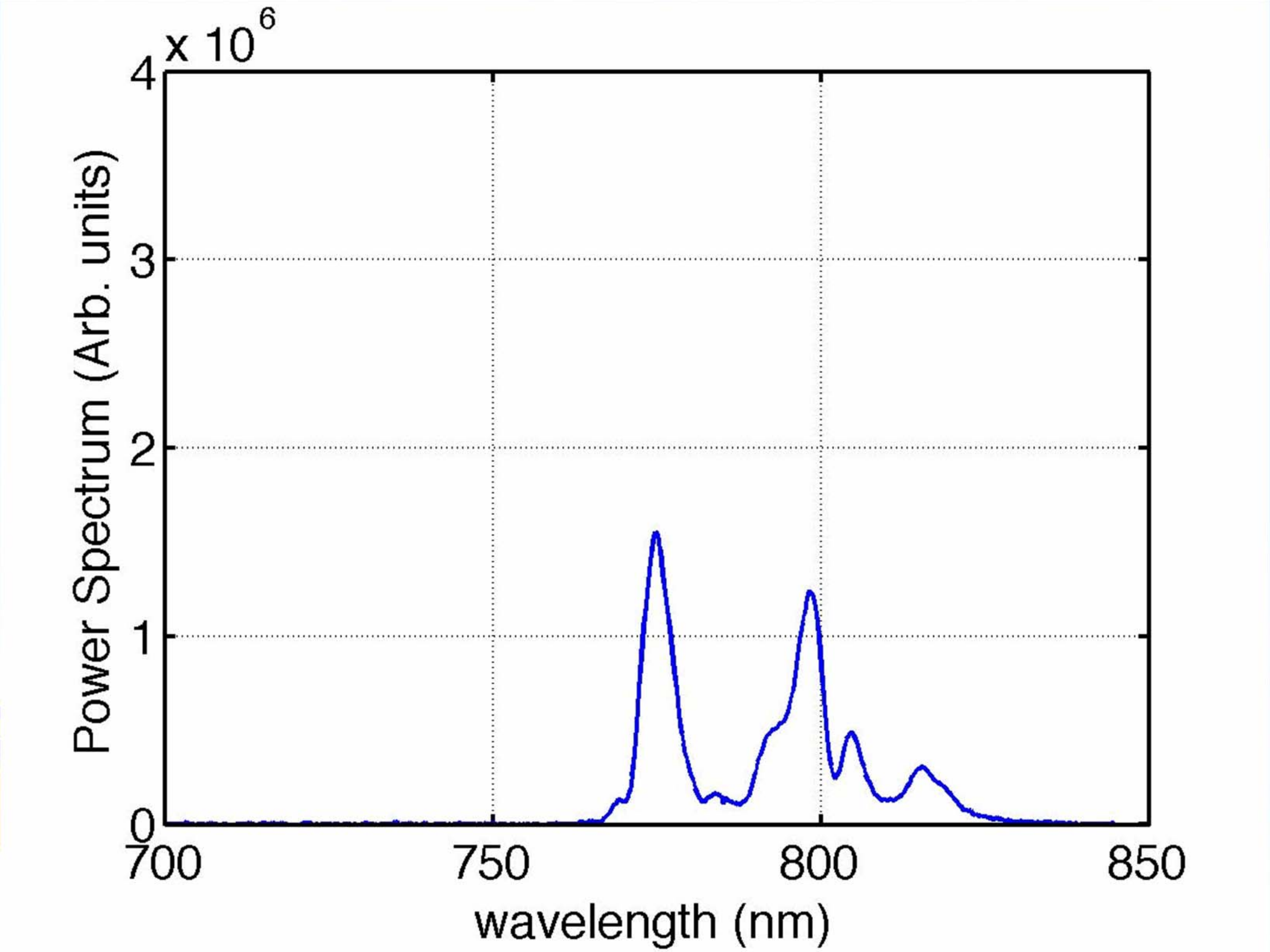




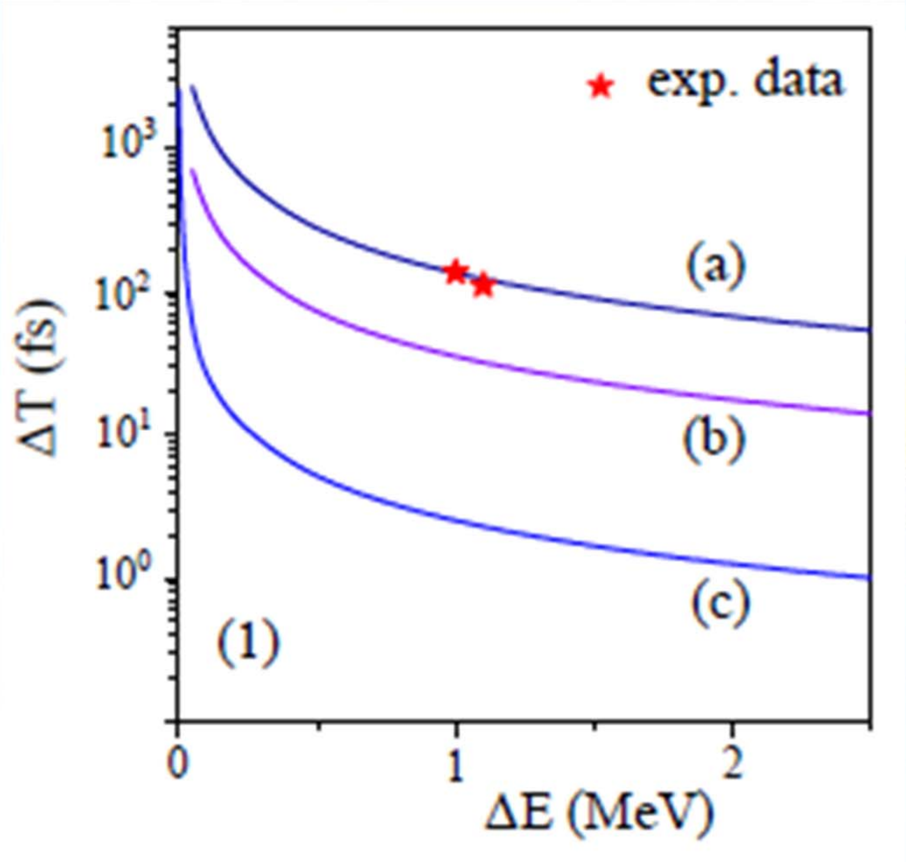
λ_{min} (nm)	769.8 (2)
$BW_{\lambda min}$ (%)	0.5
λ_{Max} (nm)	788.6 (1.3)
$BW_{\lambda Max}$ (%)	0.7
$\Delta\lambda$ (nm)	18.8 (2.9)
ΔE (MeV)	1.1 (0.17)
FEL Energy	> 37 μJ



SASE fluctuations



Expected time modulation at shorter wavelength



$$\Delta t = \frac{\lambda^2}{c(\lambda_2 - \lambda_1)} = \frac{\lambda_u(1 + K_w^2/2)}{4c\gamma\Delta\gamma}$$

(a) SPARC case.

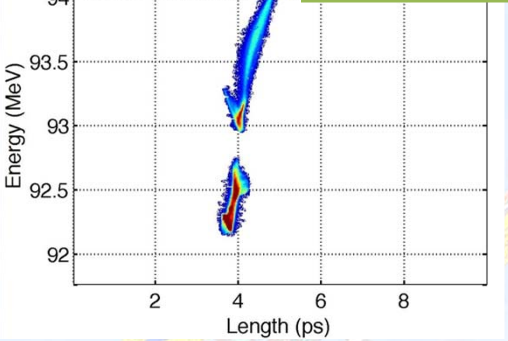
(b) $\lambda = 30\,nm$

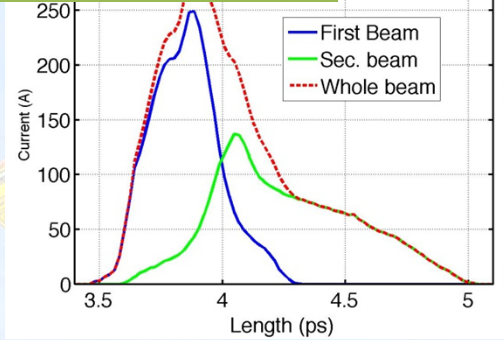
(c) $\lambda = 0.15\,nm$

FEL Experiments: Time-modulated pulses

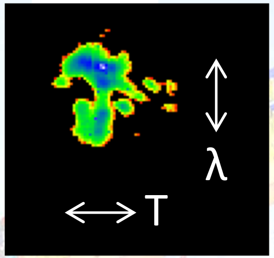
	Energy (MeV)	En. Spread (%)	Length (ps)	Charge (pC)
First Beam	92.515 (0.033)	0.174 (0.005)	0.147 (0.002)	82.15 (1.58)
Second Beam	93.588 (0.033)	0.317 (0.005)	0.283 (0.003)	77.85 (1.56)
Whole Beam	93.038 (0.032)	0.631 (0.003)	0.305 (0.004)	160.00 (3.10)

Energy Separation (MeV)	1.07 (0.05)
Time Separation (ps)	0.42 (0.03)



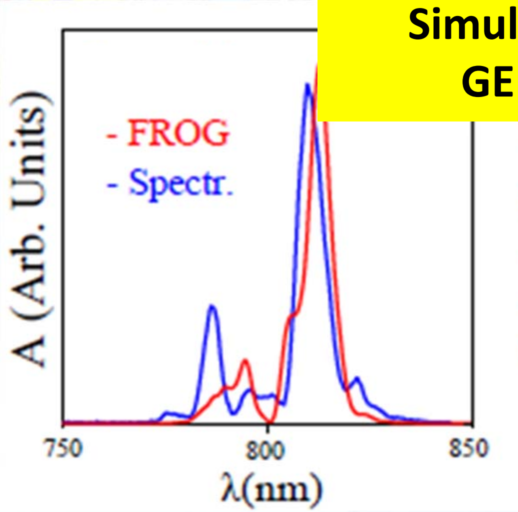


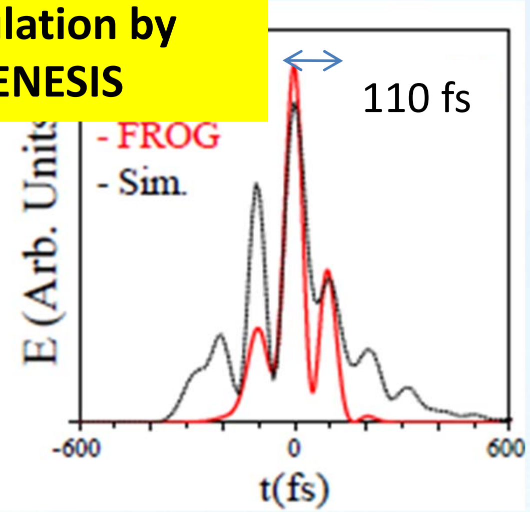
FROG traces



Δλ (nm)	BW (%)	RMS Time duration (fs)	Time separation (fs)
18	0.86	80	110

Simulation by GENESIS





V. Petrillo et al., PRL 111, 114802 (2013)