Inflation, gravity waves & BICEP2

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Plan:

-What is inflation and why do we need it?

-Inflation and gravity waves

-Final remarks: Inflation and the multiverse

`The following should become part of common knowledge...'

(J. M. Maldacena, ICTP Trieste, april 3, 2014)

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i) Homogeneity/horizon problem:

- In cosmology, one assumes that the universe is spatially homogeneous (no preferred point in space) and isotropic (no preferred spatial direction).

Why is this a good assumption?

Inhomogeneities are gravitationally unstable

$\Rightarrow$ grow with time

From CMB observations (COBE, WMAP, Planck):

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Why do these causally separated patches show such similar physical conditions?

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A combination of CMB and LSS observations indicates that the spatial geometry of the universe is flat, $k = 0$.

$k = 0 \Rightarrow \rho = 3H^2 =: \rho_{\text{crit}}$

Define $\Omega := \frac{\rho}{\rho_{\text{crit}}}$ 1st Friedmann $\Rightarrow 1 - \Omega = -\frac{k}{(aH)^2}$

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In standard cosmology: $(aH)^{-1}$ ('comoving Hubble radius') grows with time $\Rightarrow |\Omega - 1|$ must diverge with time

$\Rightarrow$ the near-flatness observed today ($\Omega \approx 1$) requires extreme fine-tuning of Ω close to 1 in early universe!

E.g. $\quad |\Omega_{\mathrm{GUT}} - 1| \leq \mathcal{O}(10^{-55}), \quad |\Omega_{\mathrm{Pl}} - 1| \leq \mathcal{O}(10^{-61})$

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iii) Magnetic monopoles:

Grand unified theories (from which the standard model $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ arose by breaking of a larger symmetry group) predict magnetic monopoles.

Why do we not observe them today?

Way out:

Look at 1st Friedmann: $1 - \Omega = -\frac{k}{(aH)^2}$

If $(aH)^{-1}$ decreases instead of growing with time:

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Can show: $\frac{d}{dt}\frac{1}{aH} < 0 \quad \Rightarrow \ddot{a} > 0 \quad \underset{\text{Friedm. eqns.}}{\overset{\uparrow}{\Rightarrow}} \rho + 3p < 0$

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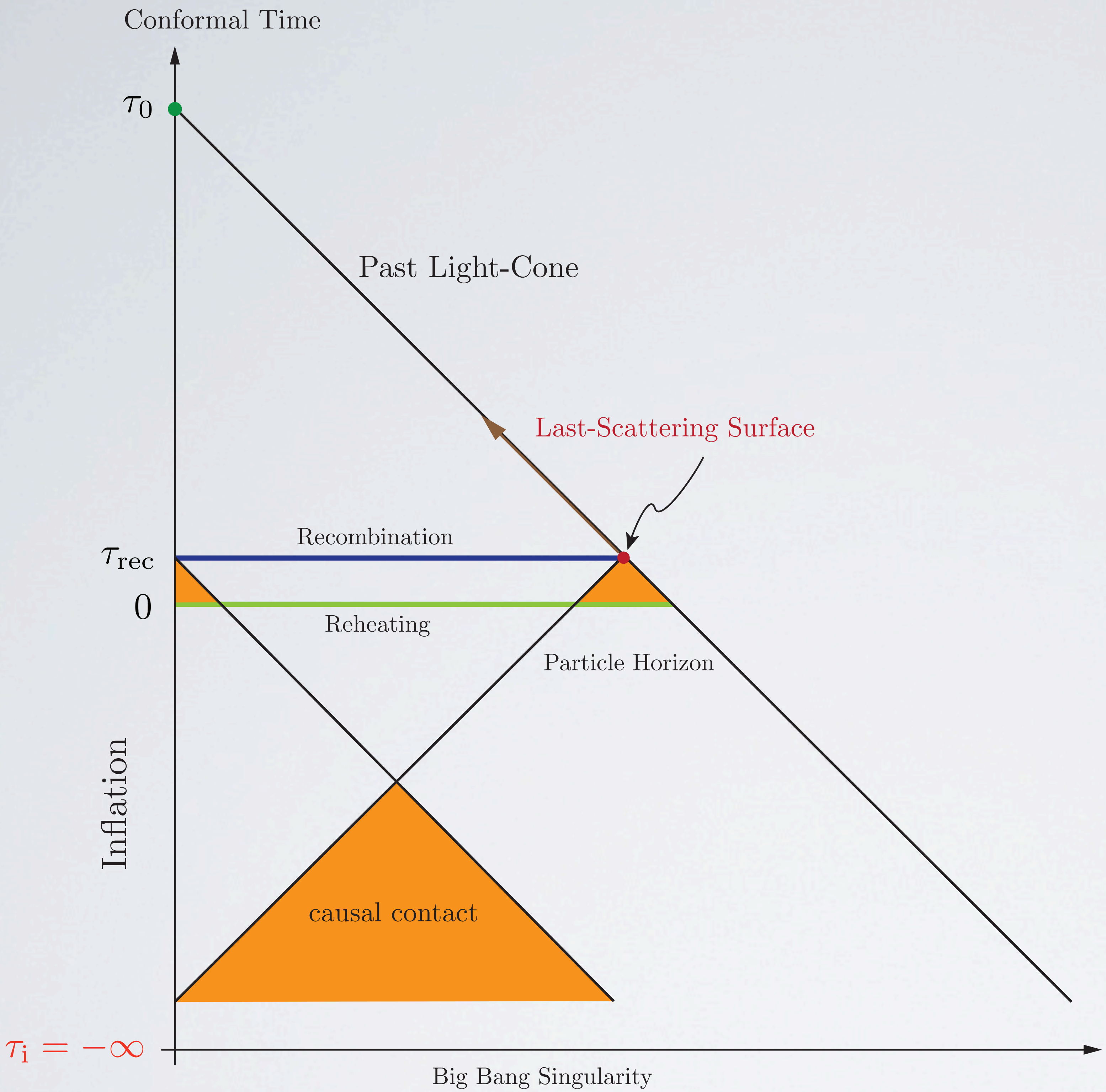
Note: This solves also problems i) and iii), since:

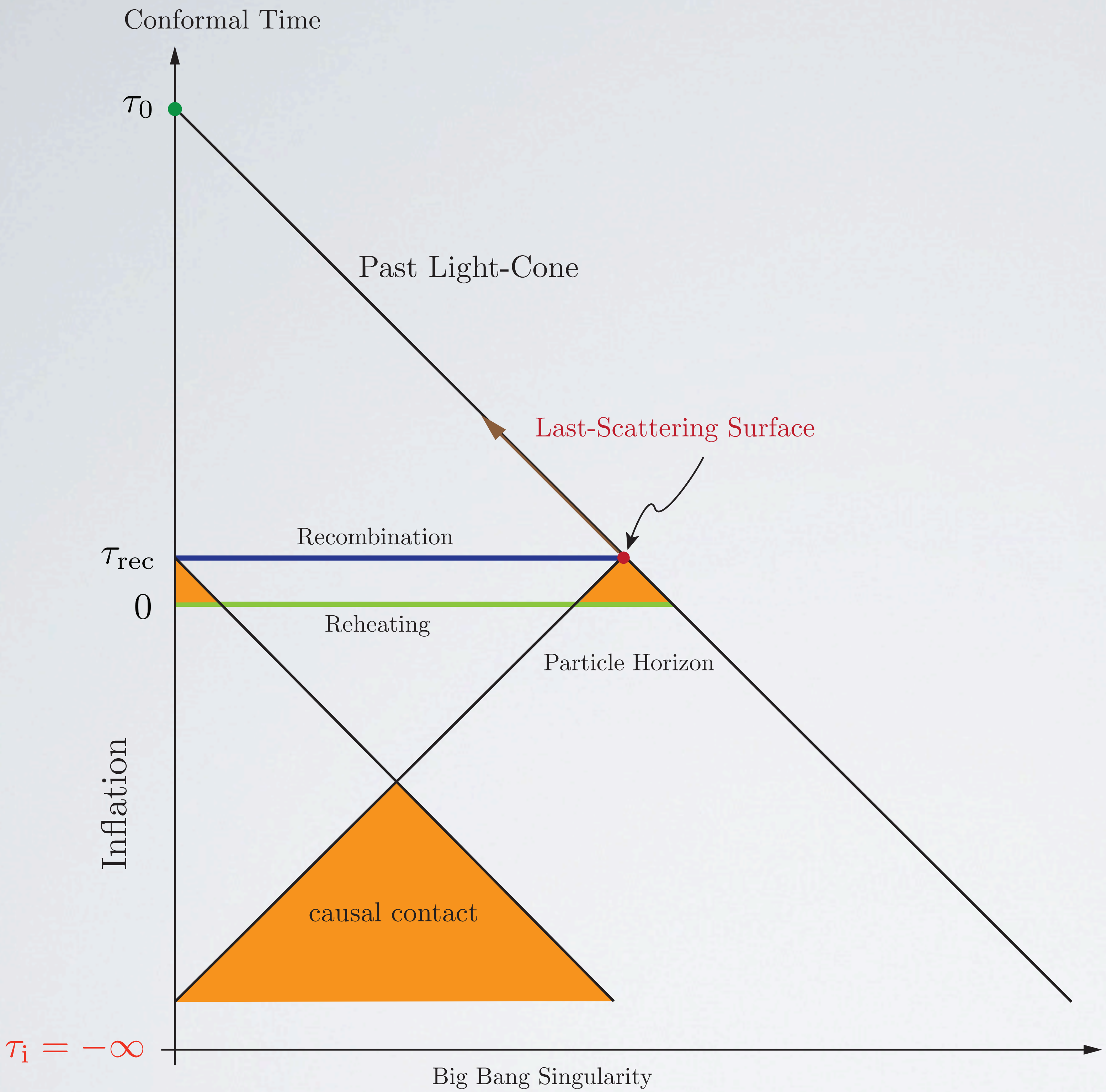
i) light cones are determined by conformal time

$$\tau = \int \frac{dt}{a(t)} \,.$$

Inflation pushes big bang singularity to 'infinite past', $\tau \rightarrow -\infty$.

$\Rightarrow$ light cones of apparently causally disconnected regions actually intersect at an earlier time (if inflation lasts long enough)


Conformal Time
τ_0
Past Light-Cone
Last-Scattering Surface
Recombination
τ_{rec}
0
Reheating
Particle Horizon
Inflation
causal contact
$\tau_{\mathrm{i}} = -\infty$
Big Bang Singularity



iii) Magnetic monopoles are created before (or during) inflation, so that the rapid expansion thereafter dilutes their density to unobservably low levels.

-Inflation started at $\sim 10^{-36}$s after big bang. Within a fraction of a second (until $\sim 10^{-34} - 10^{-32}$s after big bang) the universe grew exponentially (~ 60 e-foldings). This requires negative pressure source. Such a source can be provided by a scalar field ('inflaton') that slowly rolls down its potential:

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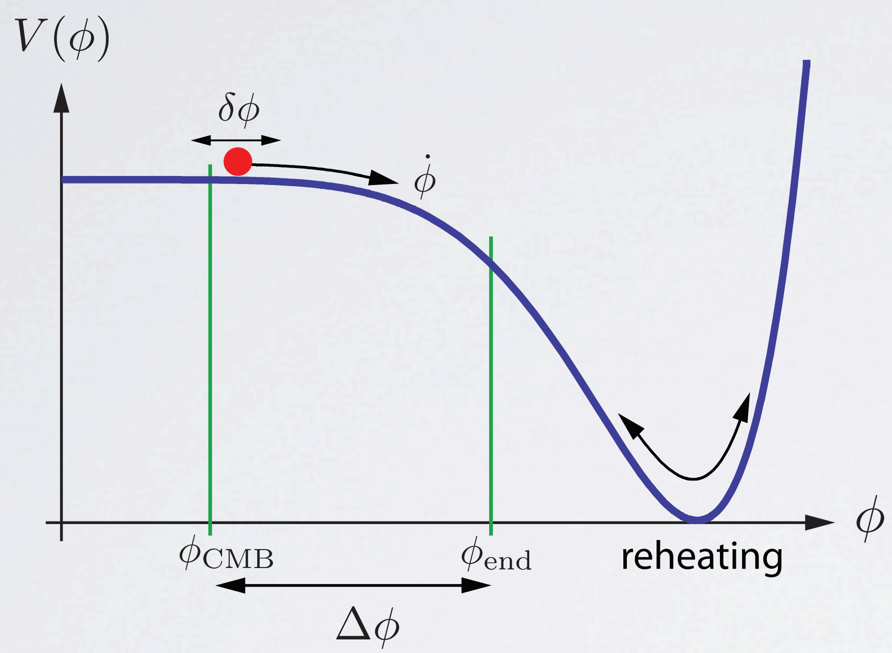

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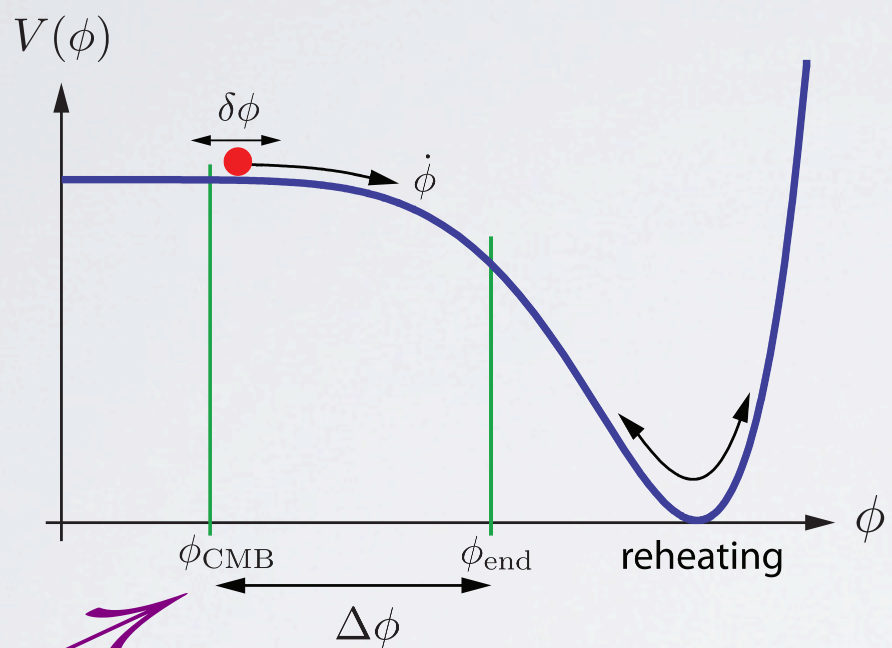

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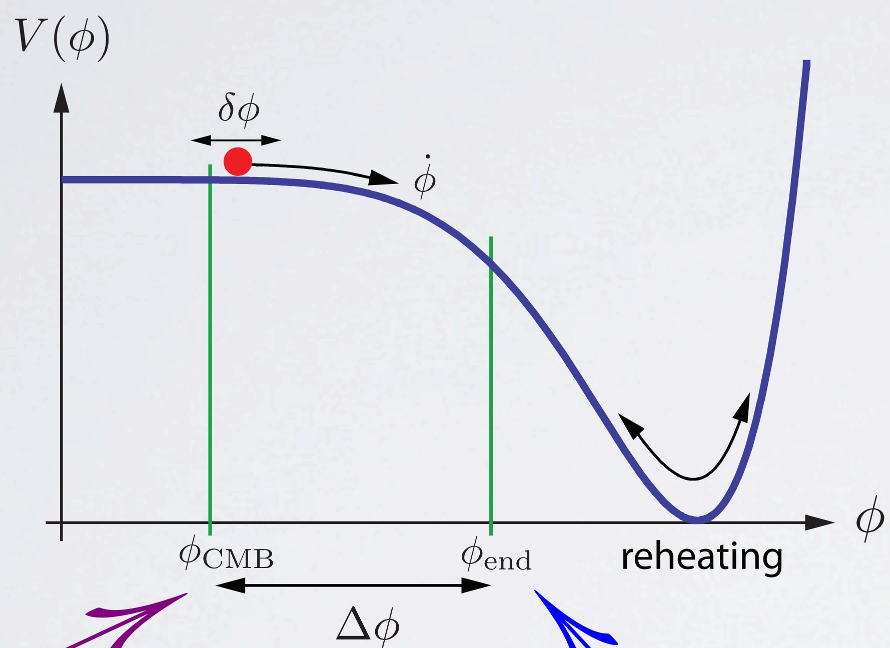

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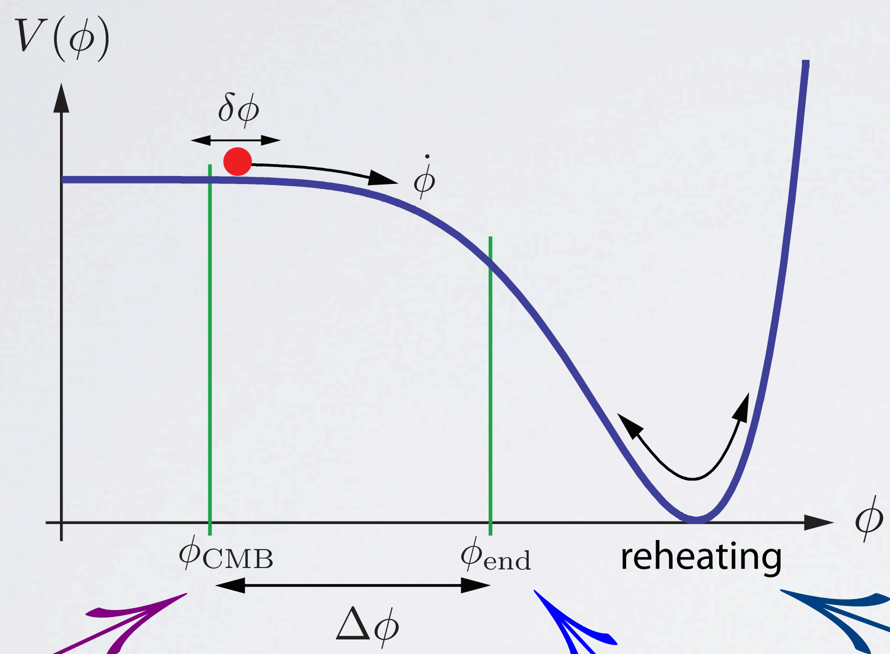

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inflation ends

scalar field oscillates around minimum of potential. Inflationary energy is converted into standard model dof's and hot Big Bang starts

-slow roll parameters:

$$\epsilon := \frac{M_{\rm Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

(potential nearly constant, such that potential energy dominates over kinetic energy ('slow roll'); needed for negative pressure)

$$\eta := M_{\rm Pl}^2 \frac{V''(\phi)}{V} \; ; \quad |\eta| \ll 1$$

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Further inflation pioneers:
Andrei Linde, Andreas Albrecht and Paul Steinhardt

II) Inflation and gravity waves

-Take Einstein eqns. sourced by scalar field (inflaton),

$$\underset{\text{geometry}}{G_{\mu\nu}} = 8\pi G \underset{\text{energy-momentum tensor of inflaton}}{T_{\mu\nu}}\,,$$

and linearize around background solution $\phi = \phi(t)$,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)\,.$$

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$\Rightarrow 1 + 10 - 4 - 4 = 3$ effective degrees of freedom:

1 scalar $\delta\phi$

2 tensor h_{ij} $(i,j = 1,2,3)$ 6 dof's

$$\frac{\partial h_{xj}}{\partial x} + \frac{\partial h_{yj}}{\partial y} + \frac{\partial h_{zj}}{\partial z} = 0 \qquad \text{(transverse)} \qquad -3$$

$$h_{xx} + h_{yy} + h_{zz} = 0 \qquad \text{(traceless)} \qquad \underline{-1}$$

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These are free fields (since we linearized Einstein eqns. $\Rightarrow$ quadratic action for $\delta\phi$, h^s)

$\Rightarrow$ Have collection of harmonic oscillators $\delta\phi_{\vec{k}}(t)$, $h^s_{\vec{k}}(t)$

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-Compute power spectrum $\langle 0|\delta\phi_{\vec{k}}(t)\delta\phi_{\vec{k}'}(t)|0\rangle$, and similar for $h^s_{\vec{k}}(t)$

$\uparrow$

vacuum state

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- tensor to scalar ratio: $r = \dfrac{\text{amplitude of tensor fluctuations}}{\text{amplitude of scalar fluctuations}}$

r is a direct measure of the energy scale of inflation (i.e., potential V of inflaton), since one can show that

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} \cdot 10^{16}\,\mathrm{GeV}$$

BICEP2: $r \sim 0.2$ (Planck: $r < 0.11$ at 95% confidence)

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- Measuring r can also be used to rule out/confirm certain inflaton potentials $V(\phi)$ (from which r can be computed) predicted by string theory

V) Final Remarks: Inflation and the multiverse

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(When the universe grew exponentially in the first tiny fraction of a second after the Big Bang, some parts of space-time expanded more quickly than others. This could have created `bubbles' of space-times that then developed into other universes.)