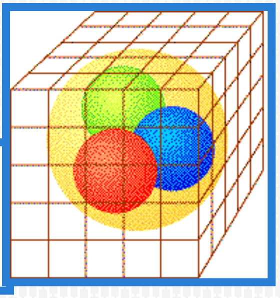
May 2014

Predicting the accuracy of Flavor Lattice inputs for What Next (~2025)

Cecilia Tarantino
Roma Tre University

The Present



Lattice QCD in Flavor Physics:

[Crucial role in the computation of long-distance QCD contributions]

We are in the era of

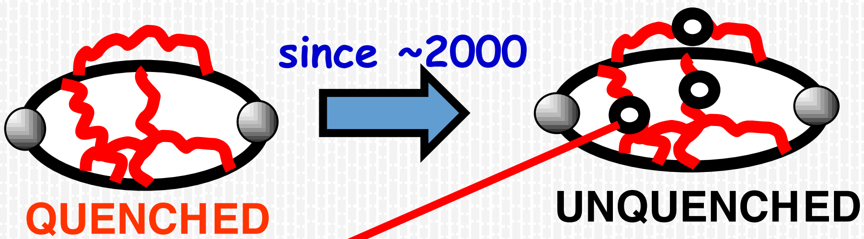
"PRECISION" LATTICE QCD

1) Increase of computational power

Unquenched simulations

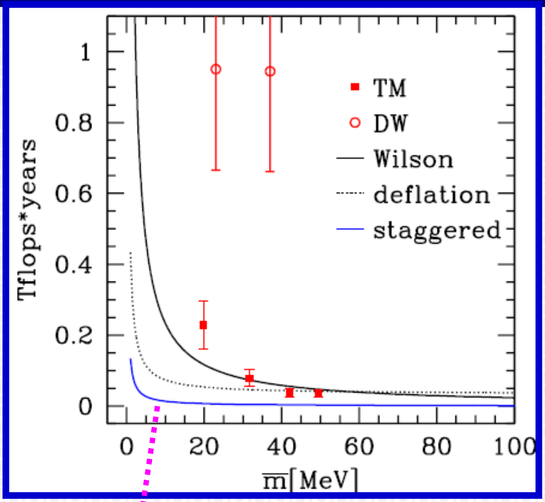
2) Algorithmic improvements:

Light quark masses in the ChPT regime



u,d (N_f=2)
u,d,s (N_f=2+1)
u,d,s,c (N_f=2+1+1) <-> Milc and ETM Collabs.

Members of the INFN I.S. LQCD123



The dependence of the computation cost on the quark mass is much smoother now!

since~2006

The last 10 year progress

Hadronic parameter	L.Lellouch ICHEP 2002 [hep-ph/0211359]	FLAG 2013 [1310.8555]
$f_+^{K\pi}(0)$	- First Lattice result in 2004 [0.9%]	[0.4%]
$\hat{B}_K$	[17%]	[1.3%]
f_{Bs}	[13%]	[2%]
f_{Bs}/f_B	[6%]	[1.7%]
$\hat{B}_{Bs}$	[9%]	[7%]
B_{Bs}/B_B	[3%]	[10%]
$F_{D*}(1)$	[3%]	[2%]
$B\to\pi$	[20%]	[10%]

The Future (~2025)

Before starting…

Take with caution
(the estimated accuracies are not accurate at all !!!)

Predictable improvements are taken into account,
what is unpredictable is NOT taken into account

Unpredictable effects are enhanced
in a 10-year prediction

I follow Vittorio Lubicz's Appendix in the SuperB CDR (2007 -> 2015)

(and Stephen Sharp's talk at *Lattice QCD: Present and Future* (Orsay, 2004))

Values of the simulation parameters (N_{conf}, a, m_l, L) to achieve a certain accuracy (1%, 0.5%, 0.1%)

↓

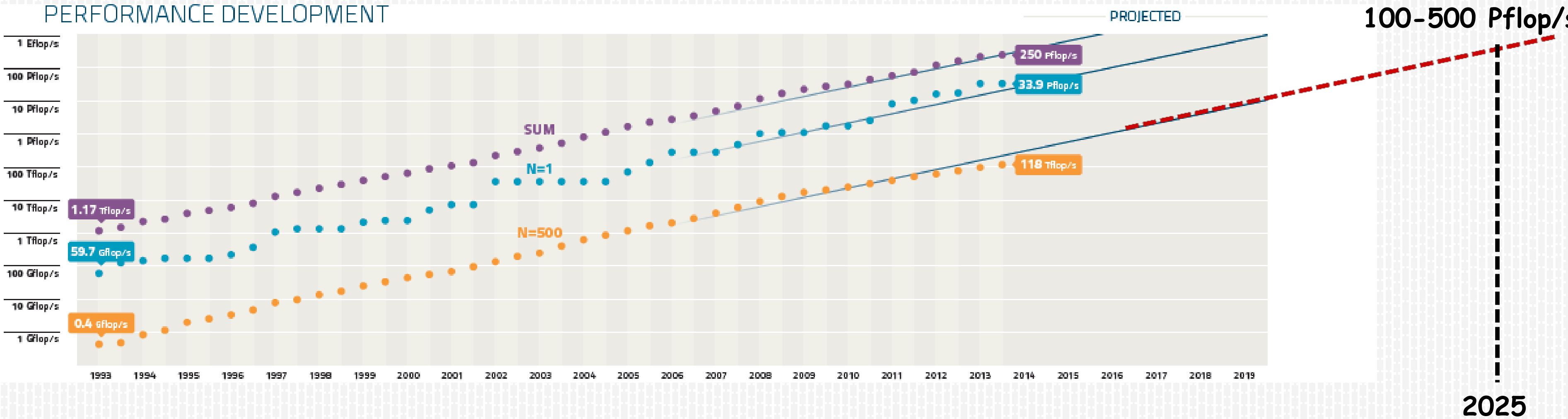
Computational cost of the corresponding simulation

↓

Comparison to the expected future computational power

History (and prediction) of the computational power
from Moore's Law (1965):
The number of transistors on integrated circuits doubles approximately every two years
(thanks to miniaturization)

Performance improvement of O(10^3) every 10 years



Lattice collaborations typically have at hand per year
a computational power similar to the 500° most powerful computer
(0.1-0.5 Pflops-years in 2014 → 100-500 Pflops-years in 2025)

Ultimate limits of the Law

Gordon Moore's interview (2005):
In terms of size you can see that we're approaching the size of atoms, which is a fundamental barrier, but it'll be two or three generations before we get that ... We have another 10 to 20 years before we reach a fundamental limit.

In 2008 it was noted that for the last 30 years it has been predicted that Moore's law would last at least another decade.
"Moore's Law: "We See No End in Sight," Says Intel's Pat Gelsinger". SYS-CON. 2008-05-01. Retrieved 2008-05-01

There exist different estimates for the ultimate limit...
2025 is, nowadays, safe according to essentially *everybody*

Computational cost of a Lattice Simulation as a function of the parameter values (e.g. Wilson-like fermions, N_f=2)

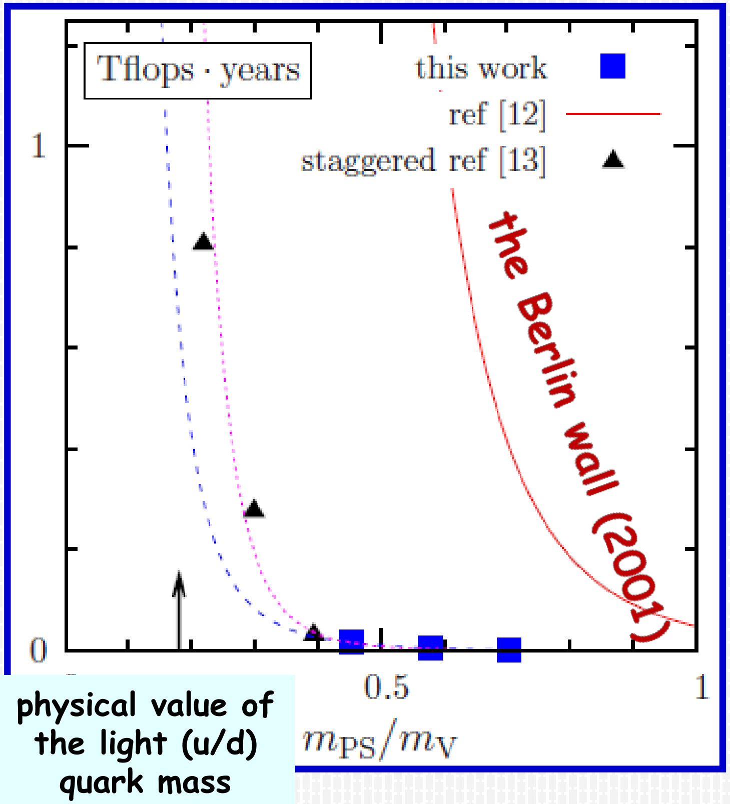
Del Debbio, Giusti, Luscher, Petronzio, Tantalo, hep-lat/0610059

$$\text{TFlops} - \text{years} \simeq 0.03 \left(\frac{N_{\text{conf}}}{100}\right)\left(\frac{L_s}{3\ \text{fm}}\right)^5\left(\frac{L_t}{2L_s}\right)\left(\frac{0.2}{\hat{m}/m_s}\right)\left(\frac{0.1\ \text{fm}}{a}\right)^6$$

0.03 →0.1 [N_f=2+1]
→0.05 [O(a)-improved]
→0.3-1.0 [Ginsparg-Wilson]

x3 of overhead (less expensive simulations to perform continuum extrapolation…)

(We will see if a more detailed study of recent simulations provides a more optimistic estimate)



The wall fall ($1/m_l^3 \rightarrow 1/m_l$) is an important example of how unpredictable (theoretical and algorithmic) developments can have a significant impact

Values of the simulation parameters (N_{conf}, a, m_l, L) to achieve a certain accuracy ε= 1%, 0.5%, 0.1%

Statistical uncertainty

$$\varepsilon_{stat} \propto \frac{1}{\sqrt{N_{conf}}}$$

$$\varepsilon_{stat} \approx 1\% \leftrightarrow N_{conf} \approx 10^2$$

$$\varepsilon_{stat} \approx 0.5\% \leftrightarrow N_{conf} \approx 4 \cdot 10^2$$

$$\varepsilon_{stat} \approx 0.1\% \leftrightarrow N_{conf} \approx 10^4$$

Systematic uncertainties:

- Discretization effects → a
- Chiral extrapolation → (m_l)
- Finite volume effects → ($M_\pi \cdot L$)→L

Discretization effects $\rightarrow$ a

$$Q_{\rm latt} = Q_{\rm cont}\left[1 + (a\Lambda_2)^2 + (a\Lambda_n)^n + \ldots\right]$$

4 $\leftrightarrow$ O(a)-improved
3 $\leftrightarrow$ unimproved

The uncertainty due to the continuum extrapolation can be estimated in a simplified (but conservative) way by assuming that:

- Two lattice spacings are available ($a_{min}, \sqrt{2}a_{min}$)
- A linear fit in a^2 is performed
- An estimate of the error is given by the difference between the result obtained from the linear fit and the determination from the complete formula

$$\varepsilon \equiv \delta Q_{\rm cont}/Q_{\rm cont} \simeq \left(2^{n/2} - 2\right)\,(a_{\rm min}\Lambda_n)^n$$

$\Lambda_n \sim \Lambda_{QCD} \sim 0.8$GeV for light Physics ($\pi$,K)
$\Lambda_n \sim m_c \sim 1.5$GeV for charm Physics (D,D_s)

$\Lambda_n \sim m_b \sim 4.5$GeV for b Physics (B,$B_s$) [simulating m_b^{phys}]
$\Lambda_n \sim 2m_c \sim 3.0$GeV for b Physics [simulating around the charm + extrapolation]

- The error introduced by the $1/m_h$ extrapolation has to be taken into account (again comparing an approximated fit in $1/m_h$ and a more complete formula)
- There are smart methods to reduce this uncertainty (ratio method, effective actions,…)

Values required for a [in fm] (for O(a)-improved actions)

	1%	0.5%	0.1%
Λ_{QCD}~0.8GeV	0.065	0.055	0.037
m_c~1.5GeV	0.035	0.029	0.020
$2m_c$~3.0GeV	0.018	-	-
m_b~4.5GeV	0.012	0.010	0.005

Typical finest a at present in π/K simulations

The error due to the $1/m_h$ extrapolation becomes dominant (higher simulated masses would be needed to get 0.1-0.5%)

Achieved by MILC

Chiral extrapolation → (m_l) [similar procedure]

$$Q_{\rm latt} = Q_{\rm phys}\left[1 + c_1\,(m_P/m_V)^2 + c_2\,(m_P/m_V)^4 + \ldots\right]$$

→

$$\varepsilon \equiv \delta Q_{\rm phys}/Q_{\rm phys} \simeq 2\,c_2\,(m_P/m_V)^4_{\rm min}$$

related to m_l/m_s by ChPT

(m_l/m_s)	1%	0.5%	0.1%
	0.08	0.05	0.02

$(m_l/m_s)^{phys} \approx 0.04$

Simulations at the light physical point are required to go below 0.5%

First simulations and results are available
[All collaborations are going to the physical point]

Finite volume effects → $(M_\pi \cdot L) \to L$ [similar procedure]

$$\varepsilon \equiv \delta Q_{\rm phys}/Q_{\rm phys} \sim C_Q(m_\pi, L)\, \exp(-m_\pi L)$$

	1%	0.5%	0.1%
$(M_\pi \cdot L)$	4.6	5.3	6.9

Present state of the art from the FLAG13 color code

Finite-volume effects:
★ $M_{\pi,\min} L > 4$ or at least 3 volumes
○ $M_{\pi,\min} L > 3$ and at least 2 volumes
■ otherwise

With $M_\pi = M_\pi^{phys}$
(as we expect for all light Physics simulations)

	1%	0.5%	0.15
L [fm]	6.5	7.5	9.7

What is the computational cost with these simulation parameters?

Del Debbio, Giusti, Luscher, Petronzio, Tantalo, hep-lat/0610059

$$\text{TFlops} - \text{years} \simeq 0.03 \left(\frac{N_{\text{conf}}}{100}\right) \left(\frac{L_s}{3\ \text{fm}}\right)^5 \left(\frac{L_t}{2L_s}\right) \left(\frac{0.2}{\hat{m}/m_s}\right) \left(\frac{0.1\ \text{fm}}{a}\right)^6$$

0.3

factor 1.5 from improvement,
factor 3 from Nf=2+1,
factor 2-3 from overhead

N.B.

- The required a is different for π/K, $D_{(s)}$, $B_{(s)}$
- Small m_l/m_s and large L are required for π/K, D and B (not for D_s and B_s)

Pflops-years

	1%	0.5%	0.1%
π/K	0.5	15	$4 \cdot 10^4$
D	20	$7 \cdot 10^2$	$2 \cdot 10^6$
D_s	0.2	2	$5 \cdot 10^2$
B	10^3	-	-
B_s	20	$4 \cdot 10^2$	$3 \cdot 10^5$

Naïve estimate:
There are smart methods
to reduce discretization effects
(ratio method, effective actions, …)

From Moore's Law 100-500 Pflops-years will be available for LatticeQCD

1° observation

Take with caution estimates below 1%:
Isospin breaking and electromagnetic effects
become relevant and have to be taken into account

$Q_u \neq Q_d$: $O(\alpha_{e.m.}) \approx 1/100$	"electromagnetic"
$m_u \neq m_d$: $O[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$	"strong"

Other small effects, now well under control,
can start contributing to the uncertainty
(suppression of the excited states,
determination of the lattice spacing from different observables,…)

2° observation

Pflops-years

	1%	0.5%	0.1%
π/K	0.5	15	$4\cdot10^4$
D	20	$7\cdot10^2$	$2\cdot10^6$
D_s	0.2	2	$5\cdot10^2$
B	10^3	-	-
B_s	20	$4\cdot10^2$	$3\cdot10^5$

Naïve estimate:
There are smart methods
to reduce discretization effects
(ratio method, effective actions,…)

Different hadronic quantities for a given sector
have a different degree of difficulty:

Given estimates are for the simplest quantities like
decay constants and B-parameters (determined from 2-point
correlators or ratios of correlators)

Form factors (requiring more noisy 3-point correlators and
an extrapolation in q^2) are more expensive

For $K \to \pi\, l\, \nu$ and $B \to D/D^*\, l\, \nu$, however, one measures on the Lattice
the difference of the f.f. from 1, so that the uncertainty
on the f.f. turns out to be smaller

Therefore, my tentative (INACCURATE!) estimates are:

Hadronic parameter	L.Lellouch ICHEP 2002 [hep-ph/0211359]	FLAG 2013 [1310.8555]	2025 [What Next]
$f_+^{K\pi}(0)$	- First Lattice result in 2004 [0.9%]	[0.4%]	[0.1%]
$\hat{B}_K$	[17%]	[1.3%]	[0.1-0.5%]
f_{Bs}	[13%]	[2%]	[0.5%]
f_{Bs}/f_B	[6%]	[1.8%]	[0.5%]
$\hat{B}_{Bs}$	[9%]	[5%]	[0.5-1%]
B_{Bs}/B_B	[3%]	[10%]	[0.5-1%]
$F_{D^*}(1)$	[3%]	[1.8%]	[0.5%]
$B\to\pi$	[20%]	[10%]	[$\gtrsim$1%]

More unpredictable but more surprising progresses can occur for the observables that today are very difficult (or infeasible): $K\to\pi\nu\bar{\nu}$, $K\to\pi l^+ l^-$, $K\to\pi\pi$, Δm_K