Nucleon

Structure theory

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Pavia

understand the proton

quark-Higgs coupling

~ 9 MeV

938 MeV

understand the proton

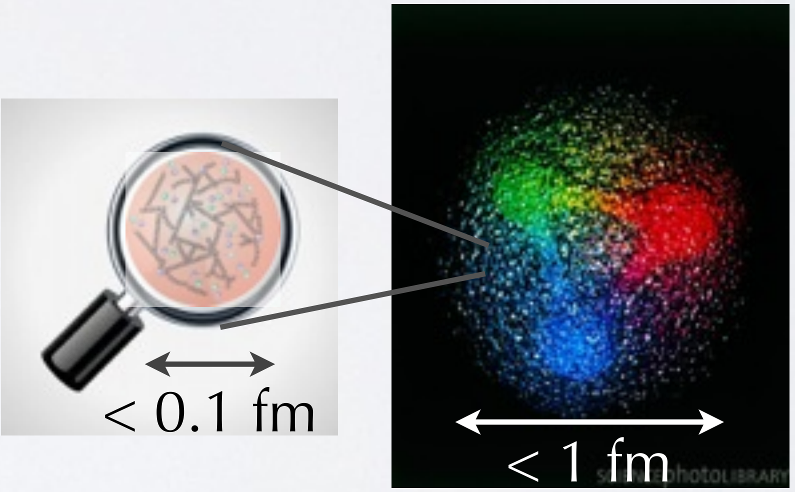
quark-Higgs coupling

~ 9 MeV

938 MeV

99% of proton mass is generated by dynamics of QCD confinement

&



lattice QCD

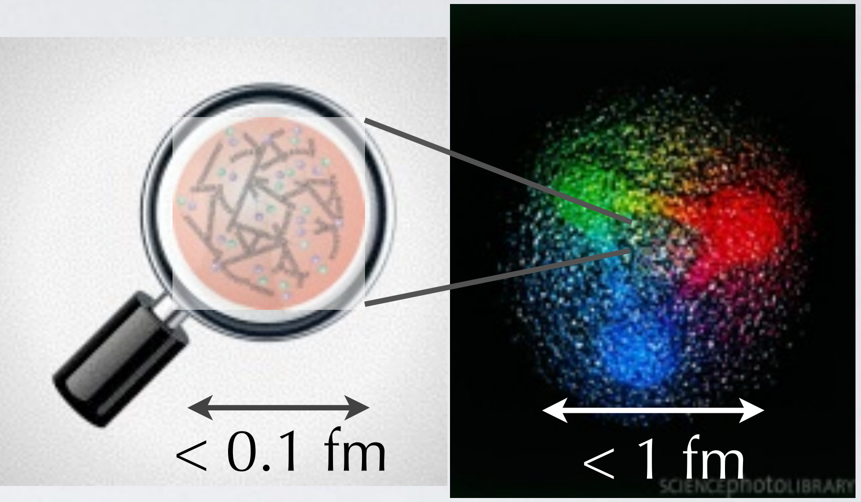
Hadron Physics

this talk

the Infinite Momentum Frame (IMF)

probe short distances

⇒ Deep-Inelastic (DIS) regime



IMF <=> Light-Cone (LC) kin.

DIS regime

$Q^2 \to \infty$


γ^*

virtual photon

N at rest

Lorentz boost

$p_\perp \ll p^+$

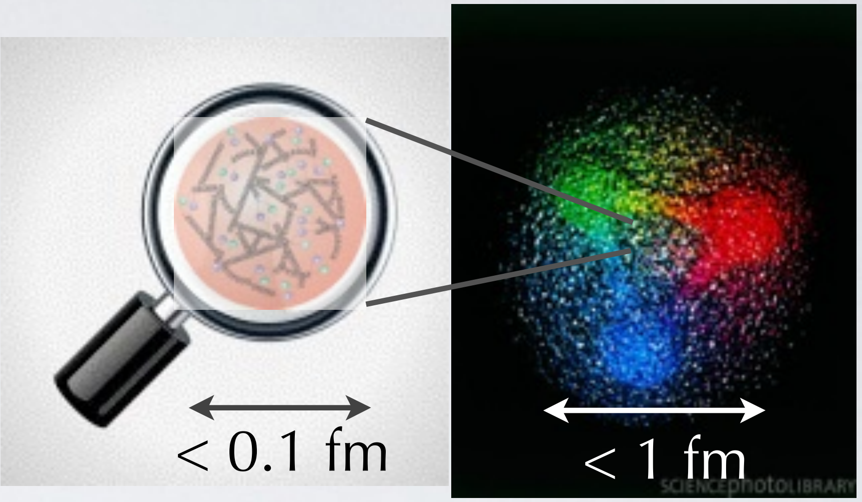
$p^+ = xP^+$

P^+

the Infinite Momentum Frame (IMF)

probe short distances

⇒ Deep-Inelastic (DIS) regime



DIS regime

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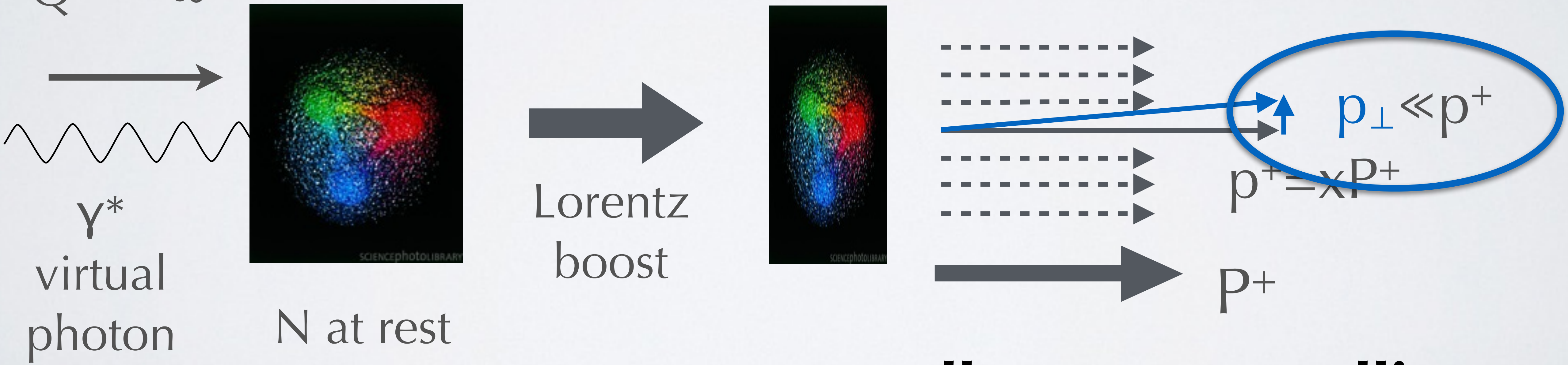
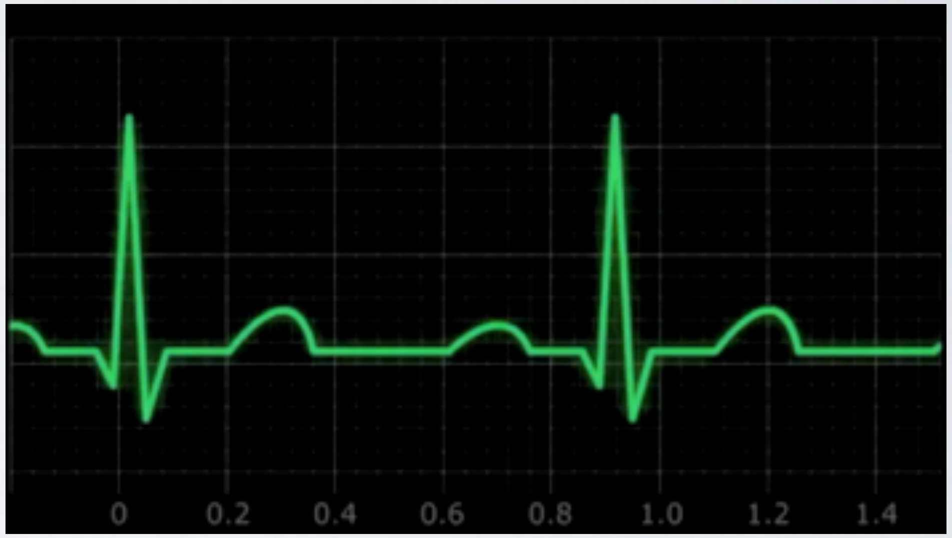


all partons ~ collinear

go beyond this approx.

main goal

the 3D-structure of the Nucleon

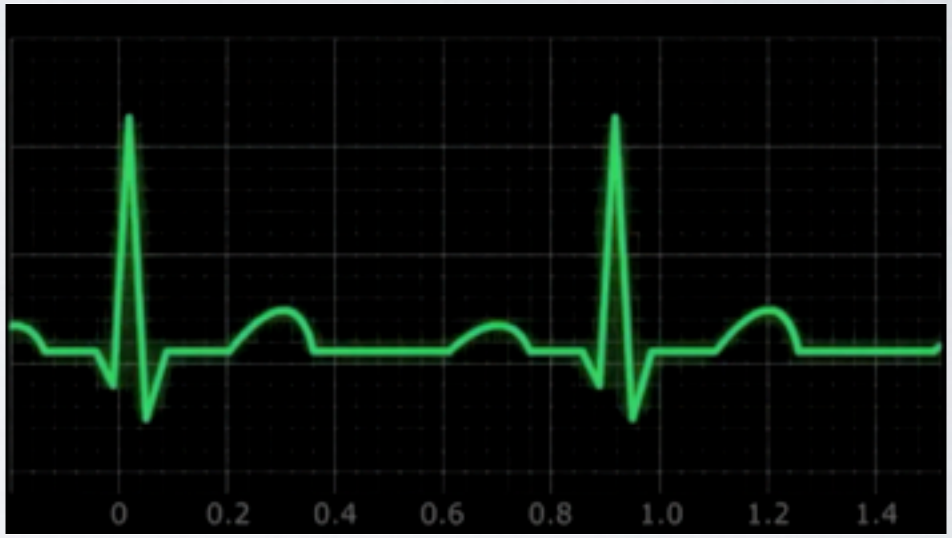


main goal

the 3D-structure of the Nucleon

mono-dim. info
on heart activity

ECG



main goal

the 3D-structure of the Nucleon


SCIENCEphotoLIBRARY

mono-dim. info
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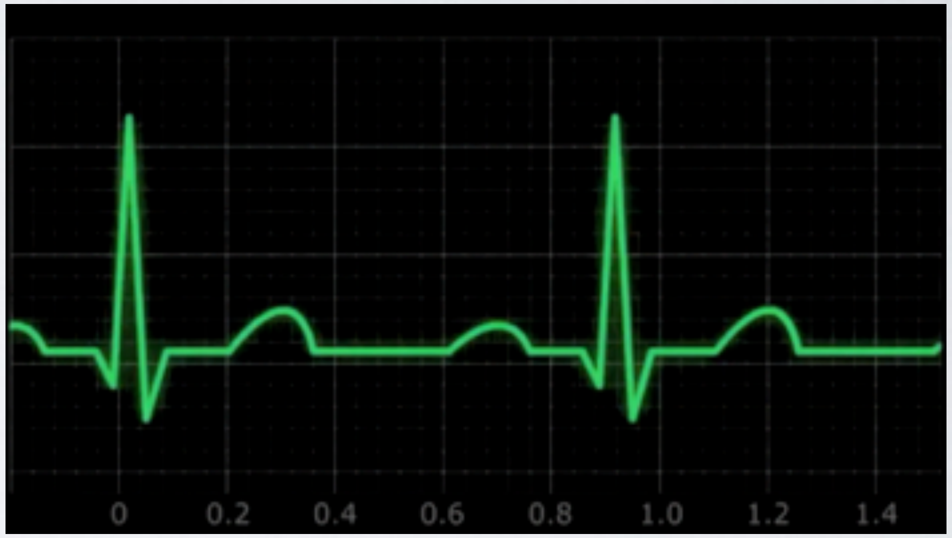
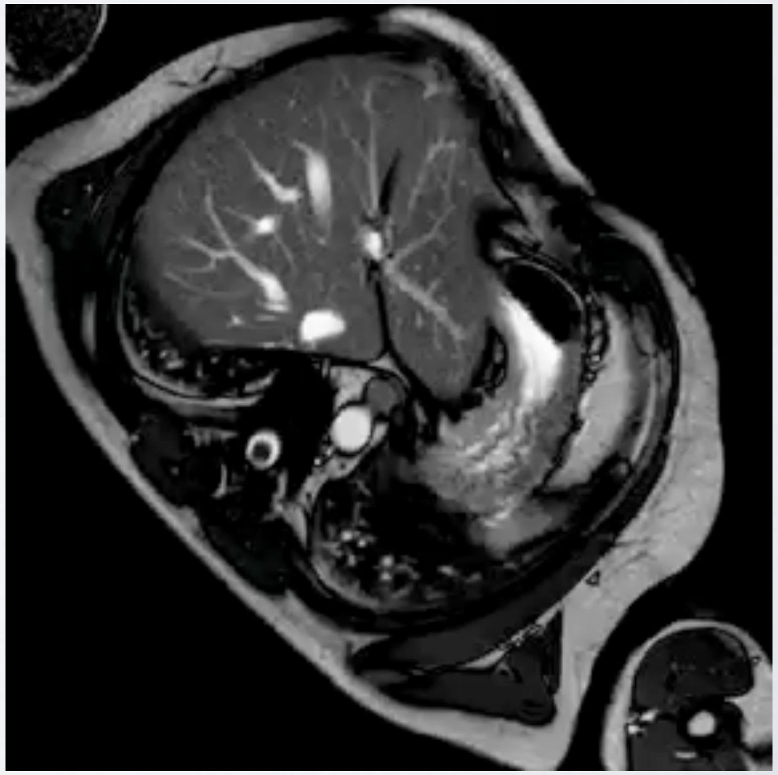


3-dim. tomography
of heart activity

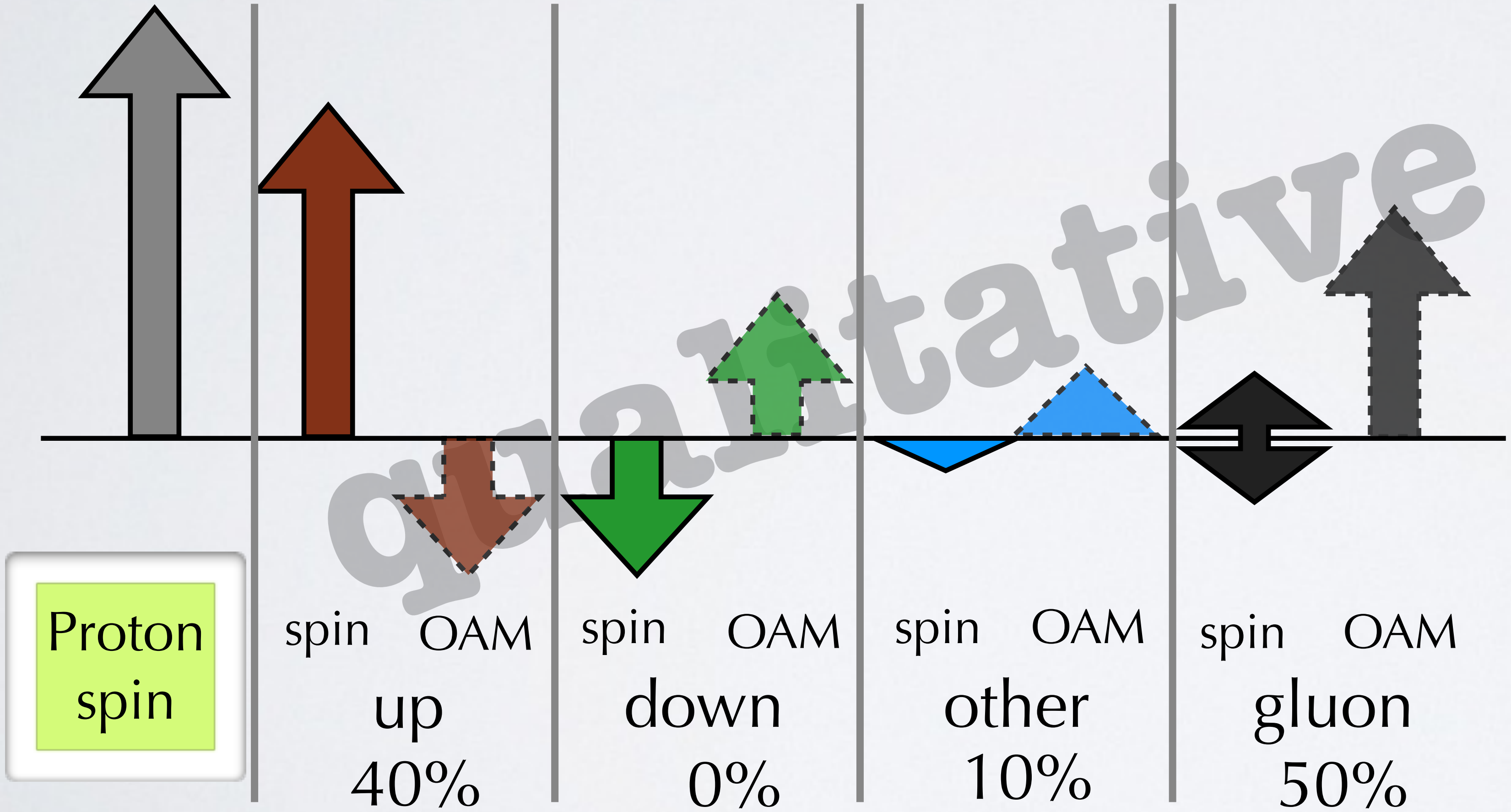
cardio
MR



the proton spin budget ?

since EMC (1988, the “spin crisis”)
we can’t yet explain the proton spin
in terms of its constituents

$$1/2 = 1/2\,\Delta\Sigma + \mathrm{OAMq} + \Delta g + \mathrm{OAMg}$$



OAM = Orbital Angular Momentum

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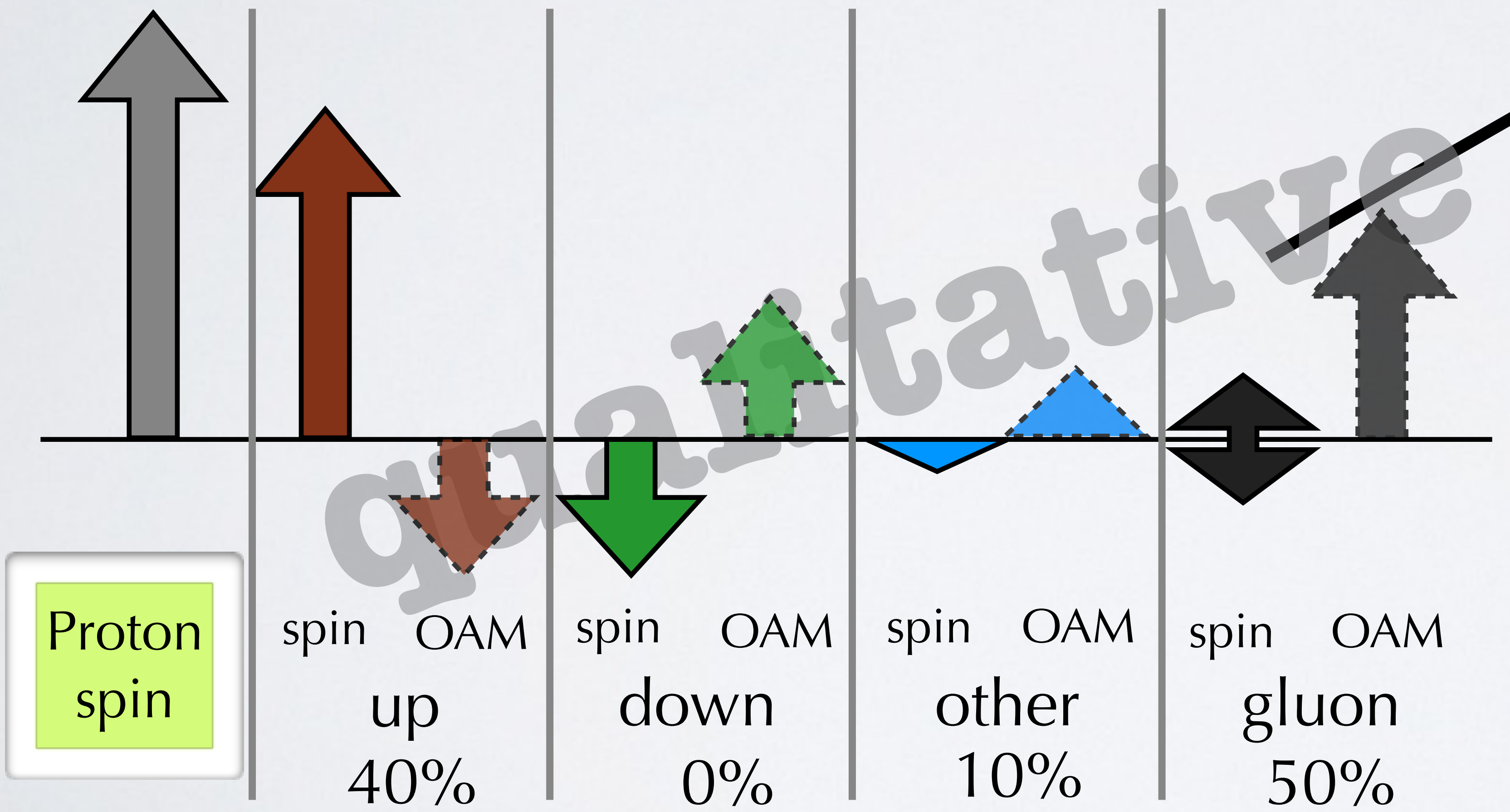
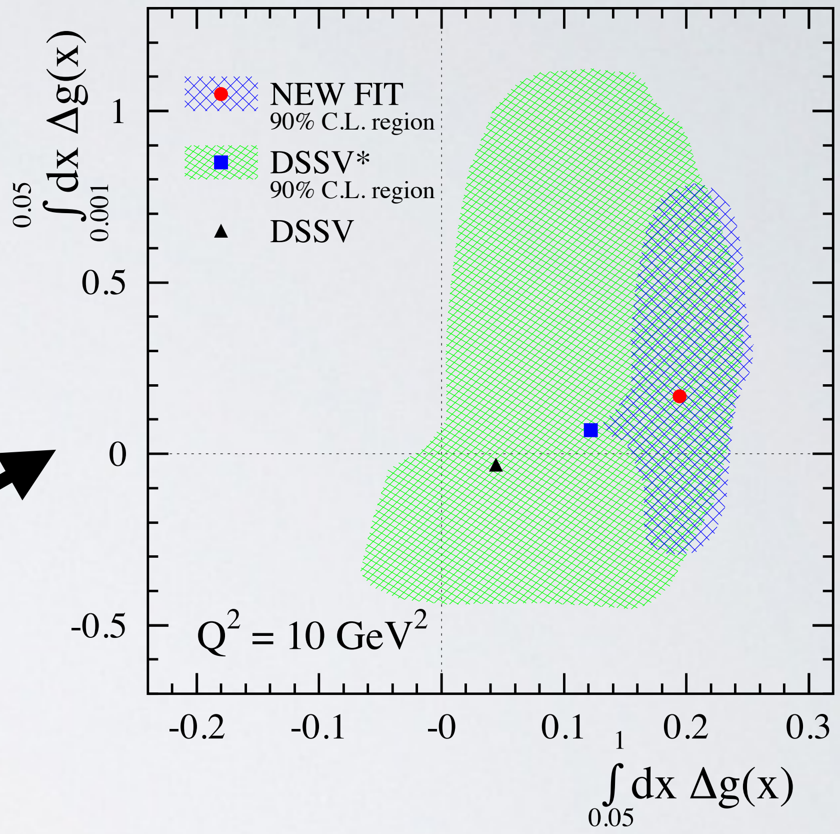


Proton spin

spin OAM	spin OAM	spin OAM	spin OAM
up	down	other	gluon
40%	0%	10%	50%

low x

De Florian et al., arXiv:1404.4293



valence

we don’t even know
the gluon helicity
-0.15 ≲ Δg ≲ 1

OAM = Orbital Angular Momentum

new tools needed

$$\bar{u}_{N'}\gamma^+ u_N \, F_1(t) + \bar{u}_{N'} \frac{i\sigma^{+\nu}\Delta_\nu}{2M} u_N \, F_2(t)$$

generalize to m-index operator

$$J_i = \varepsilon_{ijk}\left(x_j T^{0k} - x_k T^{0j}\right)$$

QCD
energy-momentum
tensor

*X. Ji, P.R.L. **78** (97) 610*

the Ji's sum rule

$$J_z^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x\, \left[H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2) \right]$$

total angular
momentum
of parton q

Generalized **P**arton **D**istributions
GPD(x, ξ, t; Q^2)

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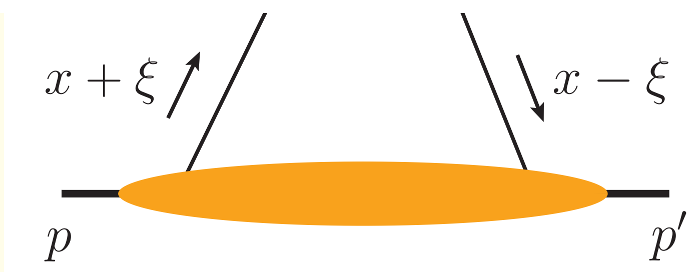

$$\xi \;=\; \frac{(P-P')^+}{(P+P')^+}$$

change in N
long. momentum

$$t = (P'-P)^2 = \Delta^2$$

non-diagonal (P′ ≠ P) hadronic matrix element

new tools needed

$$\bar{u}_{N'}\gamma^{+}u_{N}\,F_1(t) + \bar{u}_{N'}\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u_N\,F_2(t)$$

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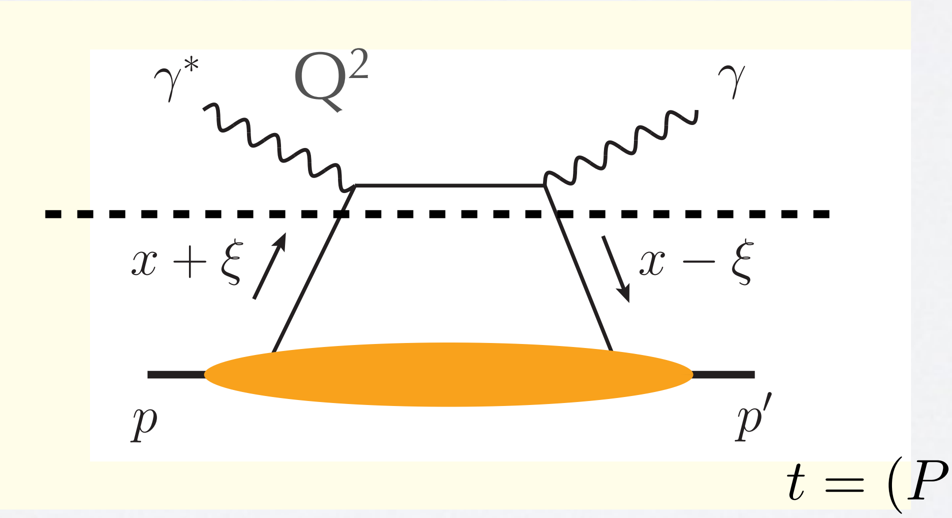
Generalized **P**arton **D**istributions
GPD(x, ξ, t; Q^2)

DVCS
factoriz. th.
for $Q^2 \gg t$



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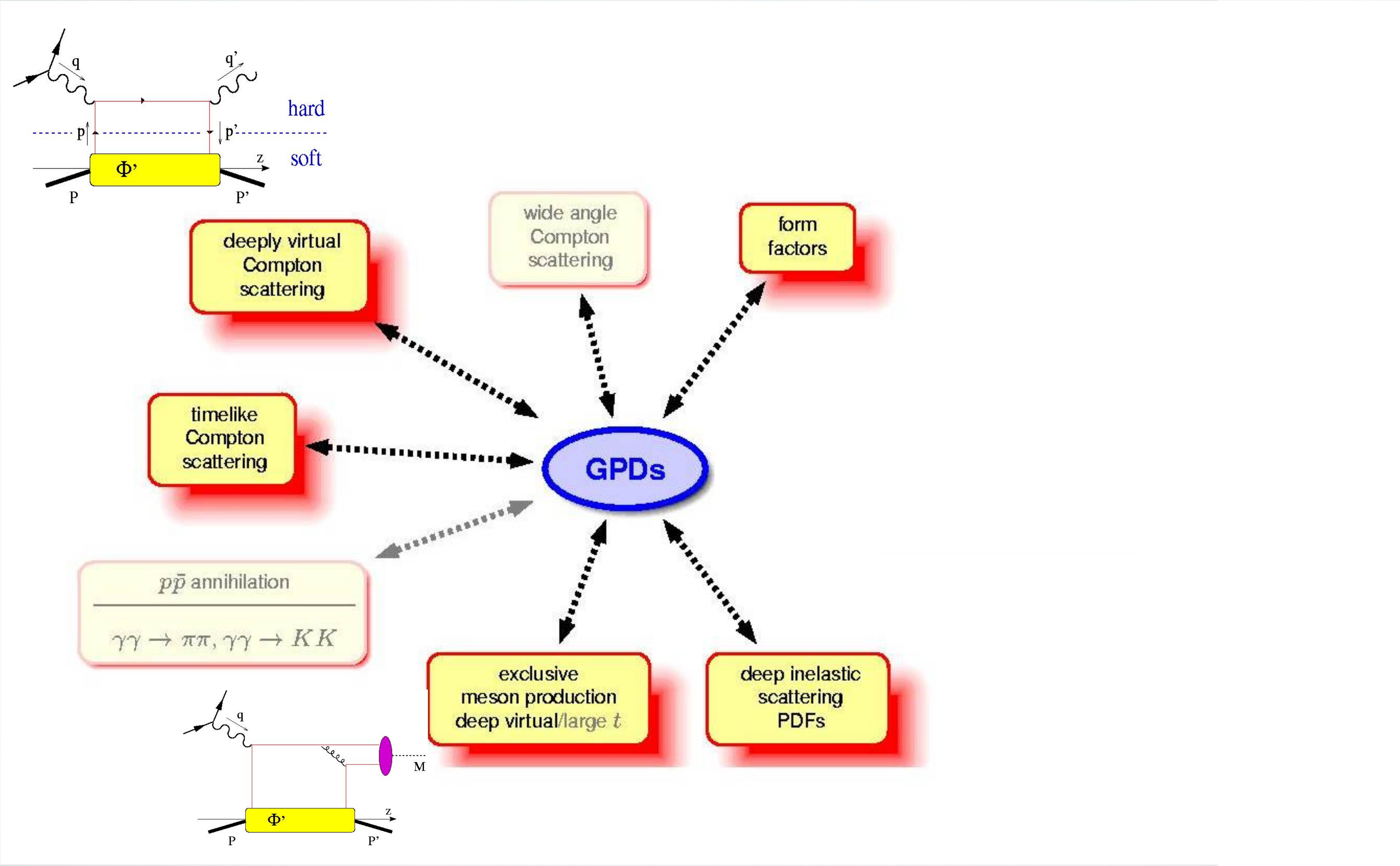
the GPD

GPD $(x, \xi, t; Q^2)$



deeply virtual Compton scattering

wide angle Compton scattering

form factors

timelike Compton scattering

GPDs

$p\bar{p}$ annihilation

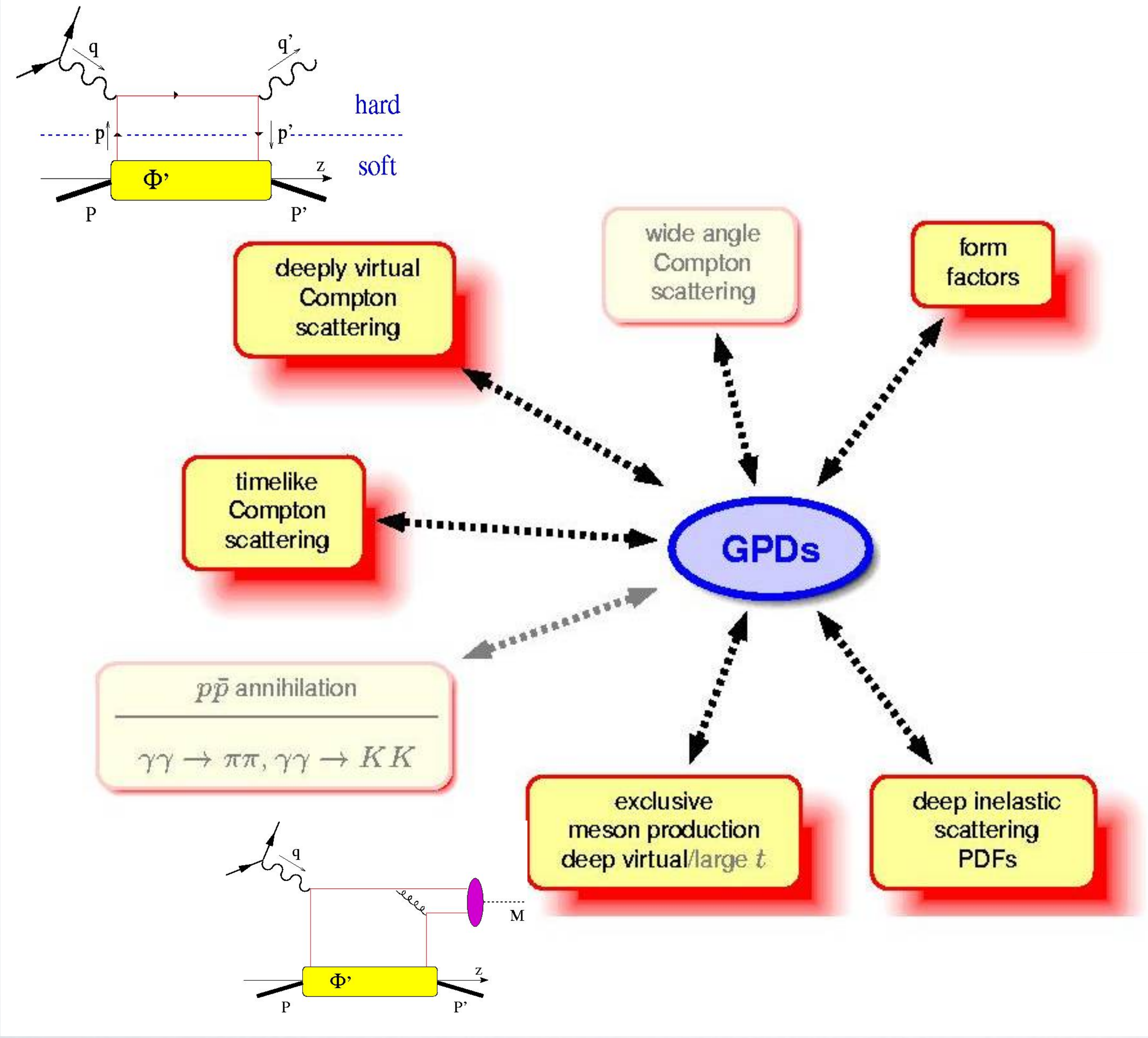
$\gamma\gamma \to \pi\pi, \gamma\gamma \to KK$

exclusive meson production deep virtual/large t

deep inelastic scattering PDFs

the GPD

GPD (x, ξ, t; Q^2)



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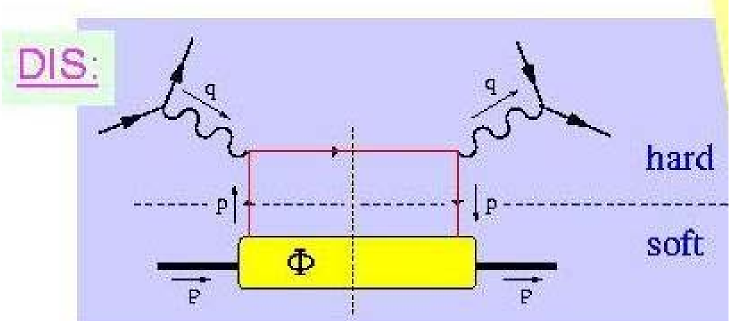
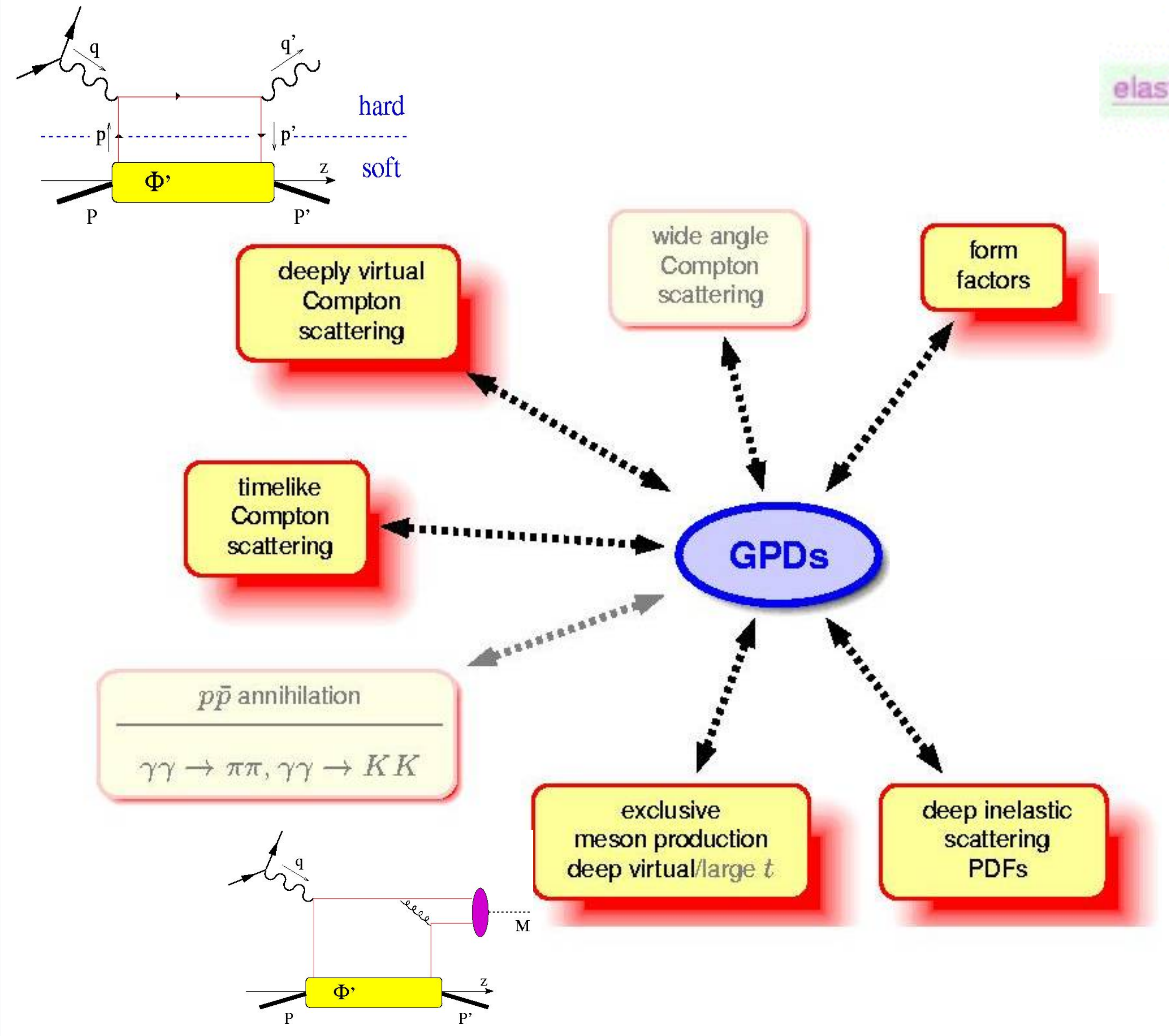
$$\lim_{\xi,t\to 0} GPD(x,\xi,t) = PDF(x)$$



the GPD

GPD (x, ξ, t; Q^2)



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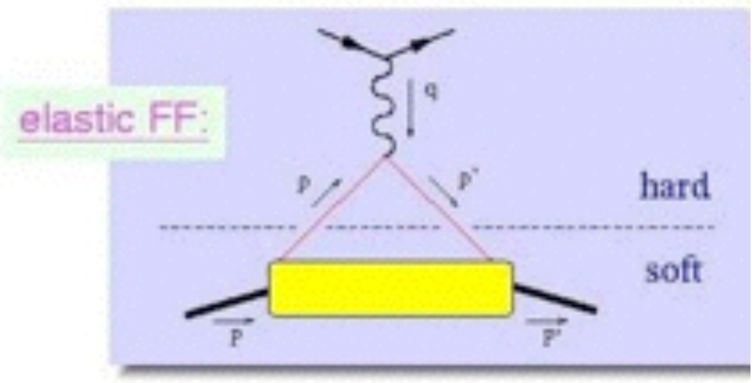
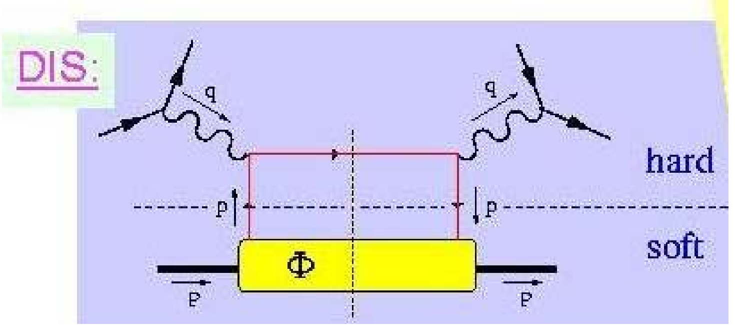


$\int$dx GPD(x,ξ,t) = FF(t)

$$\lim_{\xi,t\to 0} GPD(x,\xi,t) = PDF(x)$$



the GPD

GPD (x, ξ, t; Q²)

$$\lim_{\xi,t\to 0} GPD(x,\xi,t) = PDF(x)$$

$H^q\,(x,\ \xi\to 0,\ t\to 0)\ \Rightarrow\ f_1{}^q\,(x)$

not directly accessible

(E^q → N spin flip)

need model extrapolation

$$J_z^q \;=\; \frac{1}{2}\int dx\, x\,[\,H^q(x,0,0) + E^q(x,0,0)\,]$$

the GPD

GPD (x, ξ, t; Q^2)

$$\lim_{\xi,t\to 0} GPD(x,\xi,t) = PDF(x)$$

H^q (x, ξ→0, t→0) ⇒ $f_1{}^q$ (x)

not directly accessible

(E^q → N spin flip)

need model extrapolation

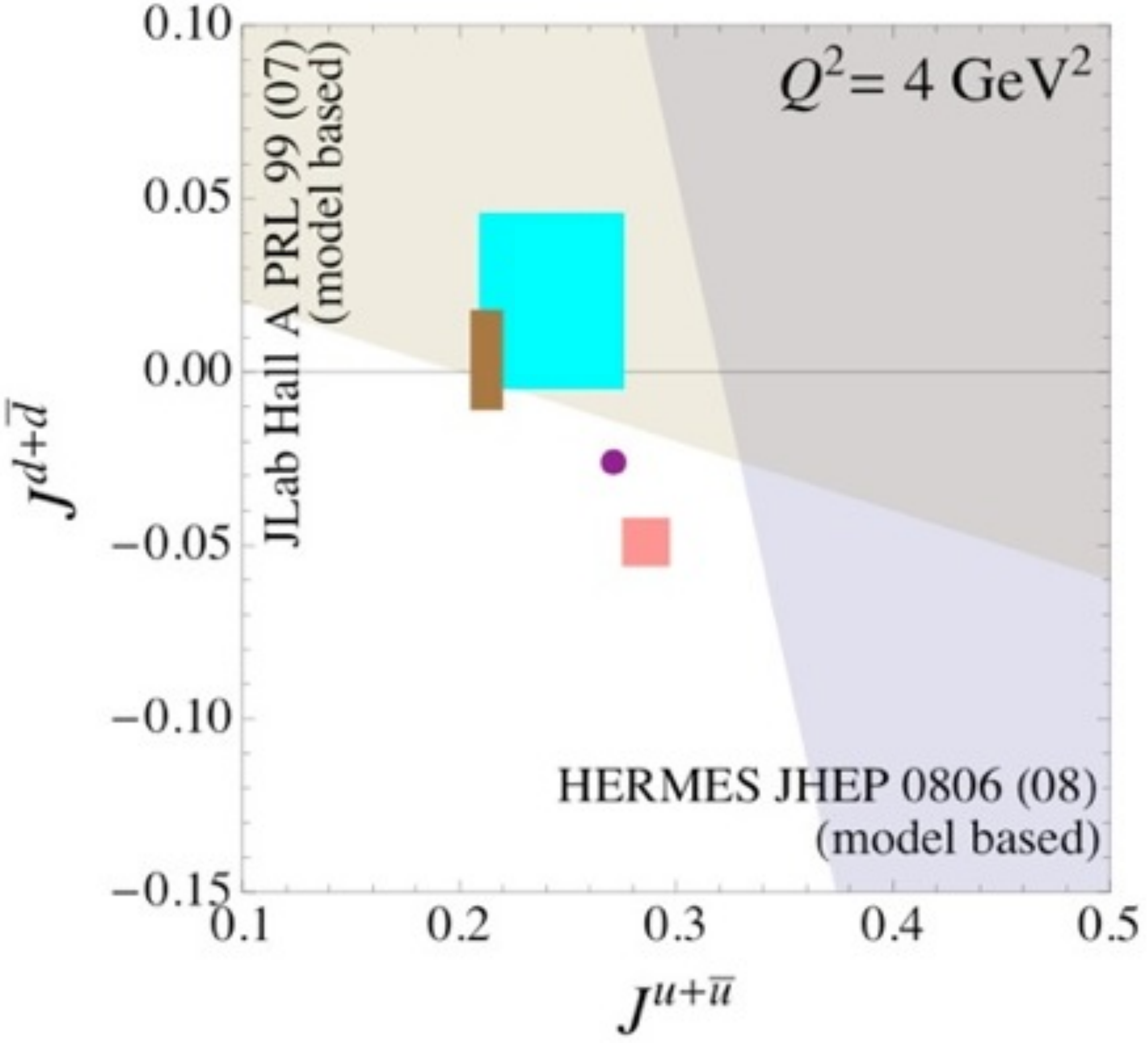
$$\begin{aligned} J_z^q &= \frac{1}{2}\int dx\, x\, [\, H^q(x,0,0) + E^q(x,0,0)\,] \\ &= \frac{1}{2}\, [\, A^q_{2,0}(0) \;+\; B^q_{2,0}(0)\,] \end{aligned}$$

moments of GPD

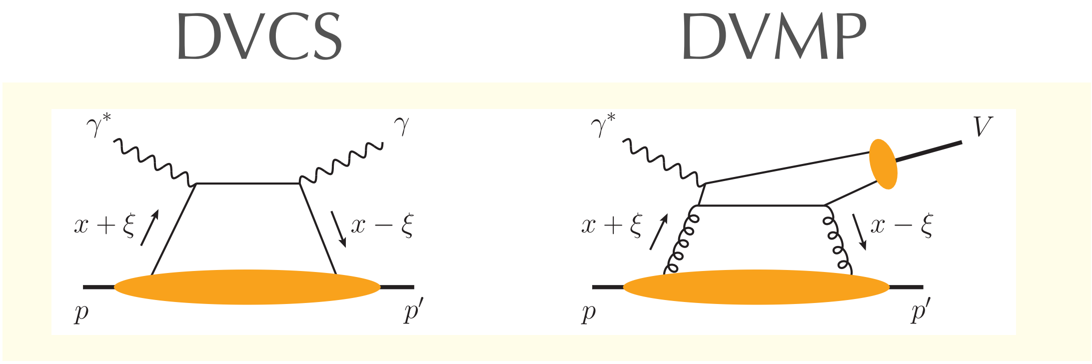
Generalized Form Factors **calculable on lattice**

$A_{1,0}$ (≡F_1), $B_{1,0}$ (≡F_2), **$A_{2,0}$** , **$B_{2,0}$** , $A_{3,0}$, $A_{3,2}$.. $B_{3,0}$, $B_{3,2}$..

J^q results
(model) params. of **GPD**



🟦 Goloskokov & Kroll, EPJ C59 (09) 809
🟫 Diehl et al., EPJ C39 (05) 1
🟪 Guidal et al., PR D72 (05) 054013
🟥 Liuti et al., PRD 84 (11) 034007



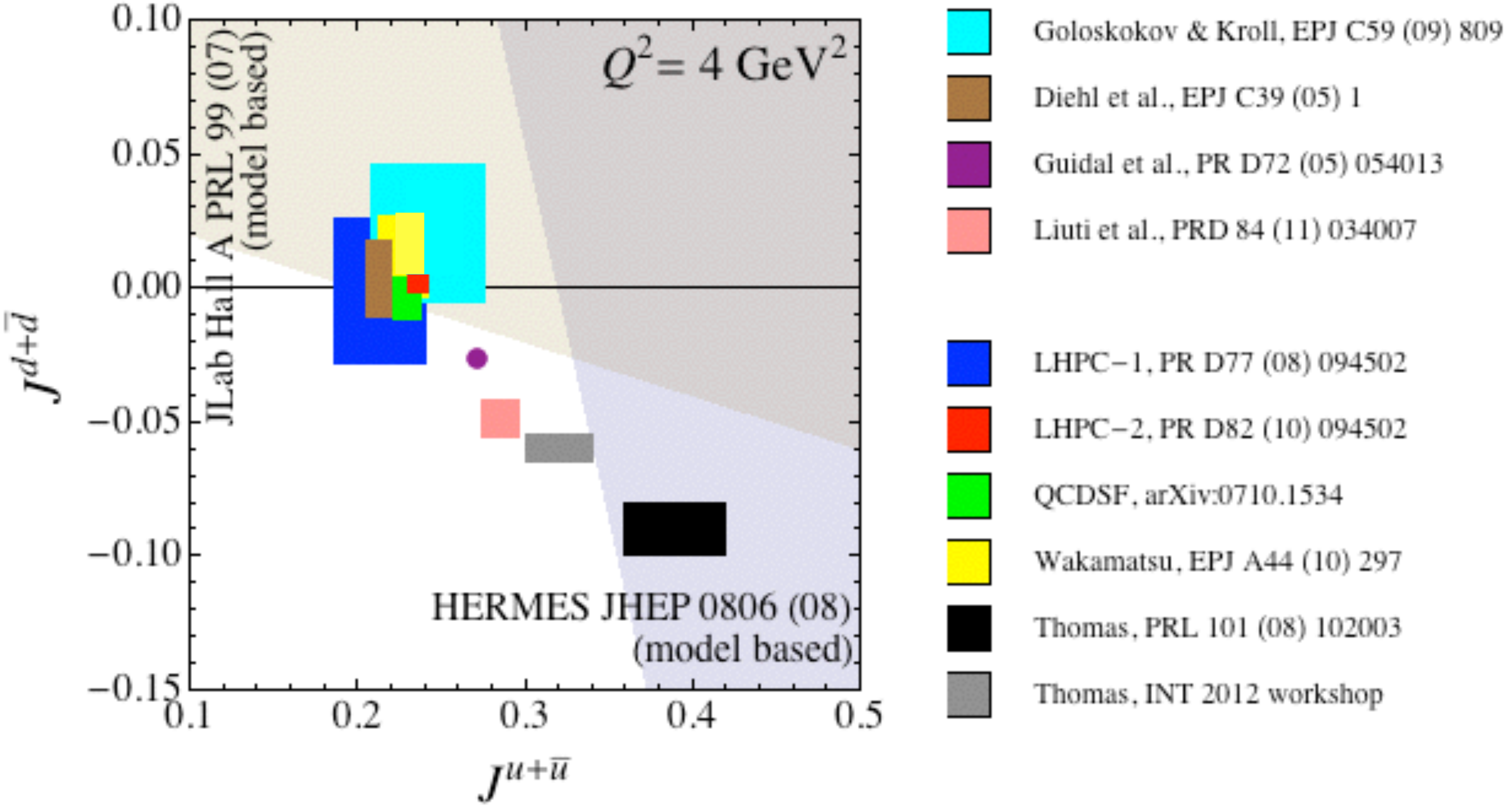
GPD convoluted in **C**ompton **F**orm **F**actors

$$d\sigma \sim \Sigma_n \; C_n(\,\mathrm{Re[CFF]},\mathrm{Im[CFF]}\,)\cos(n\Phi)$$
$$+ \; S_n(\,\mathrm{Re[CFF]},\mathrm{Im[CFF]}\,)\sin(n\Phi)$$

Φ azimuthal angle of γ / M

A. Bacchetta & M. Radici, arXiv:1206.2565 [hep-ph]
*“Physics Opportunities with the 12 GeV Upgrade at Jefferson Lab”, E.P.J. A**48** (12) 187*

J^q results compare with lattice QCD



} lattice

A. Bacchetta & M. Radici, arXiv:1206.2565 [hep-ph]
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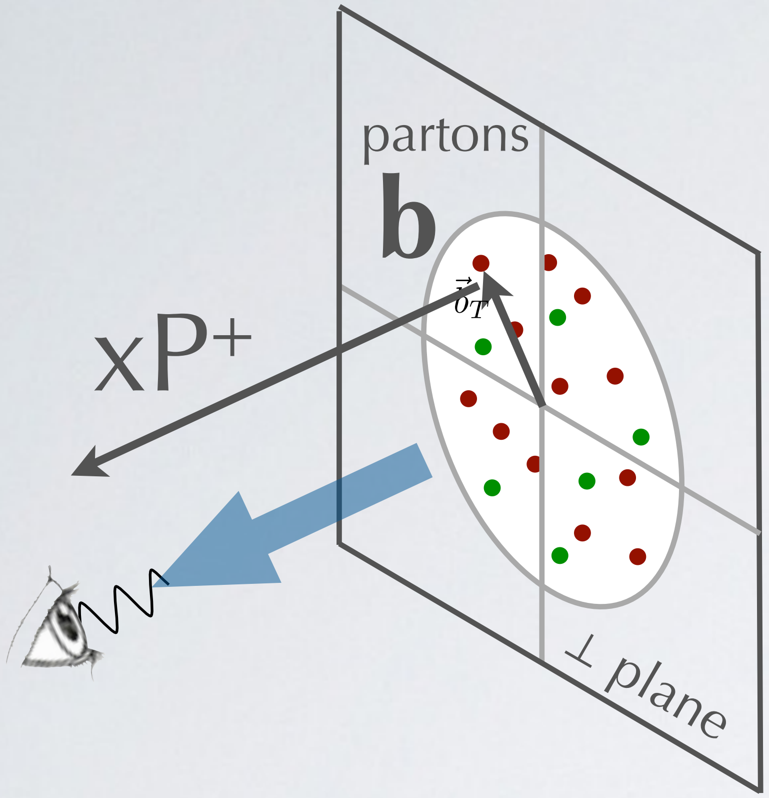
tomography of the Nucleon



GPD limit : $\xi \to 0$ ($P^+ = P'^+$) ; $t \to -(\mathbf{P'}_\perp - \mathbf{P}_\perp)^2 = -\mathbf{q}^2$

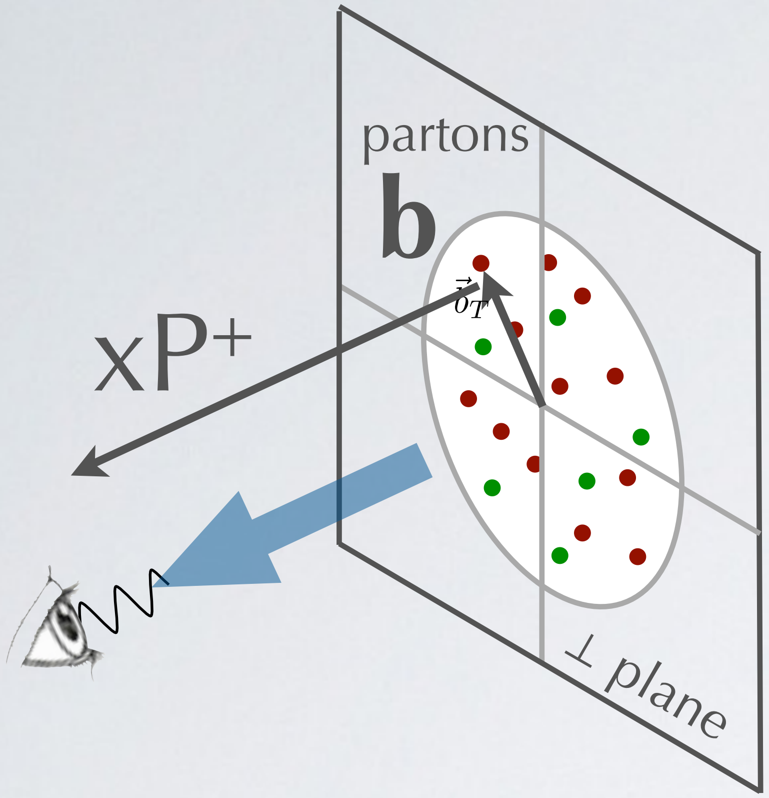
$$q(x, \mathbf{b}) \;=\; \int \frac{d\mathbf{q}}{(2\pi)^2}\, e^{i\mathbf{q}\cdot\mathbf{b}}\, H(x, 0, t = -\mathbf{q}^2)$$

q(x,**b**) is a density in **b** ↔ **q=P′$_\perp$-P$_\perp$**

\# density of partons with momentum x and position **b**

tomography of N

tomography of the Nucleon



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$$q(x, \mathbf{b}) \quad = \quad \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H(x, 0, t = -\mathbf{q}^2)$$

q(x,**b**) is a density in **b** ↔ $\mathbf{q} = \mathbf{P'}_\perp - \mathbf{P}_\perp$

density of partons with momentum x
and position **b**

tomography of N

valid for all x ⇒

$$\rho^0(\mathbf{b}) \quad = \quad \int dx \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H(x, 0, t = -\mathbf{q}^2)$$

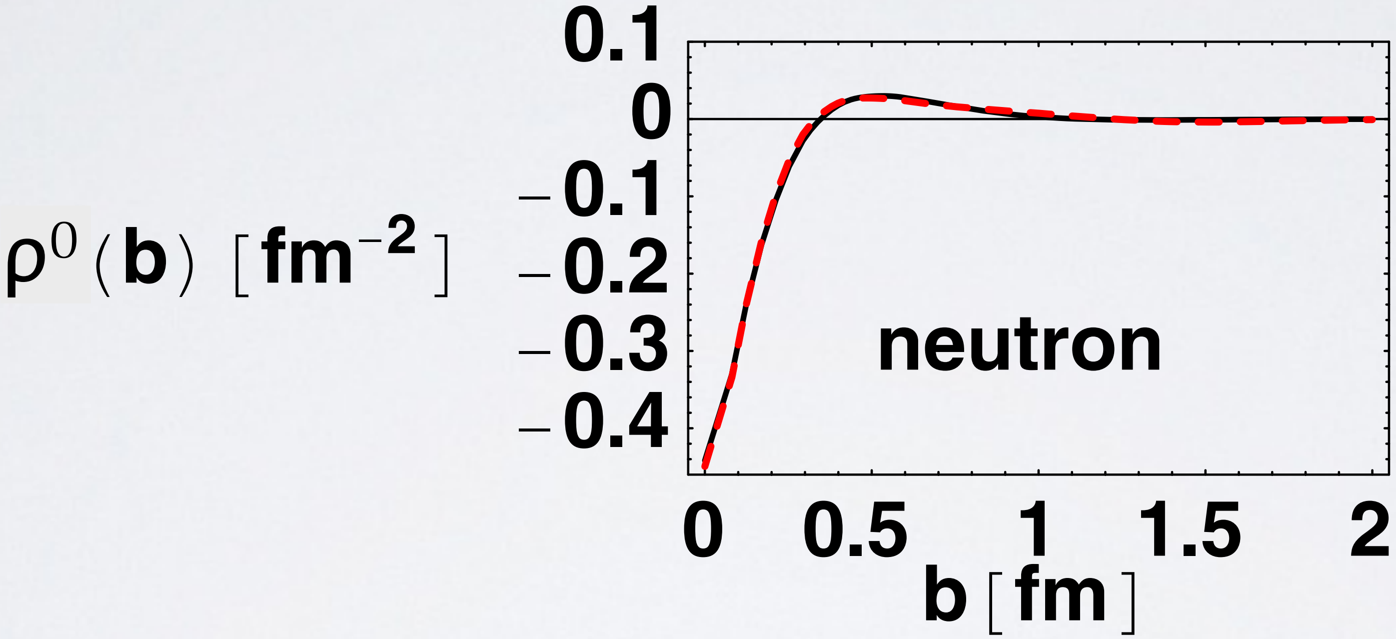
$$= \quad \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} F_1(t = -\mathbf{q}^2)$$

Dirac form factor

G.A. Miller, P.R.L. ***99*** *(07) 112001*

revolutionize the neutron

inside neutron



G.A. Miller,
*PRL***99** *(07) 112001*

neutron core with negative charge
plus π cloud with positive charge !

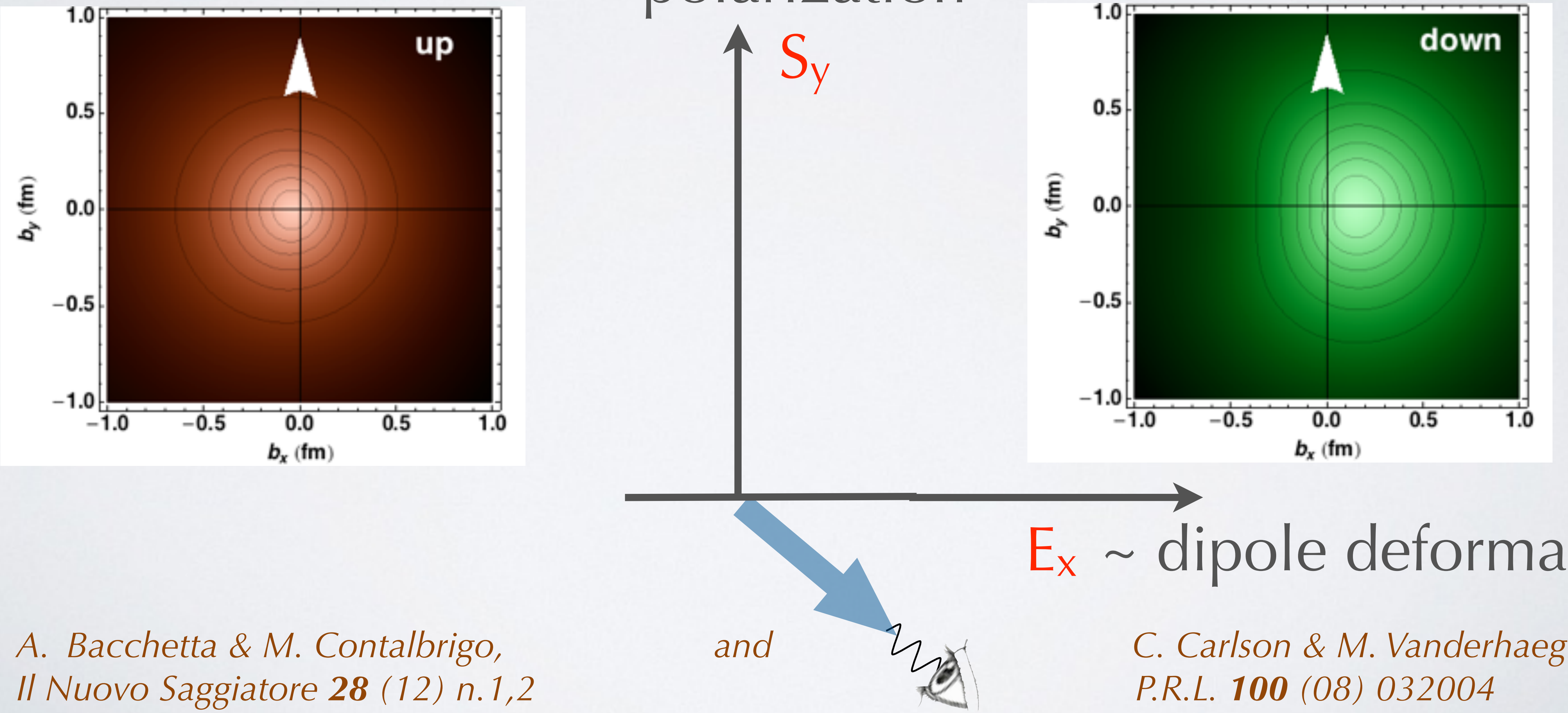
polarized N → deformation

polarization S_y → spin-flip $E(x,0,-\mathbf{q}^2)$ → b_x deformation

$\mathbf{b}$ = b (cosΦ$_b$, sinΦ$_b$)

$$\rho(\mathbf{b}) = \rho^0(\mathbf{b}) \ + \ \cos\phi_b \int_0^\infty \frac{d|\mathbf{q}|}{2\pi} \, \frac{\mathbf{q}^2}{2M} \, J_1(|\mathbf{q}|b) \, F_2(Q^2 = \mathbf{q}^2)$$

proton polarization



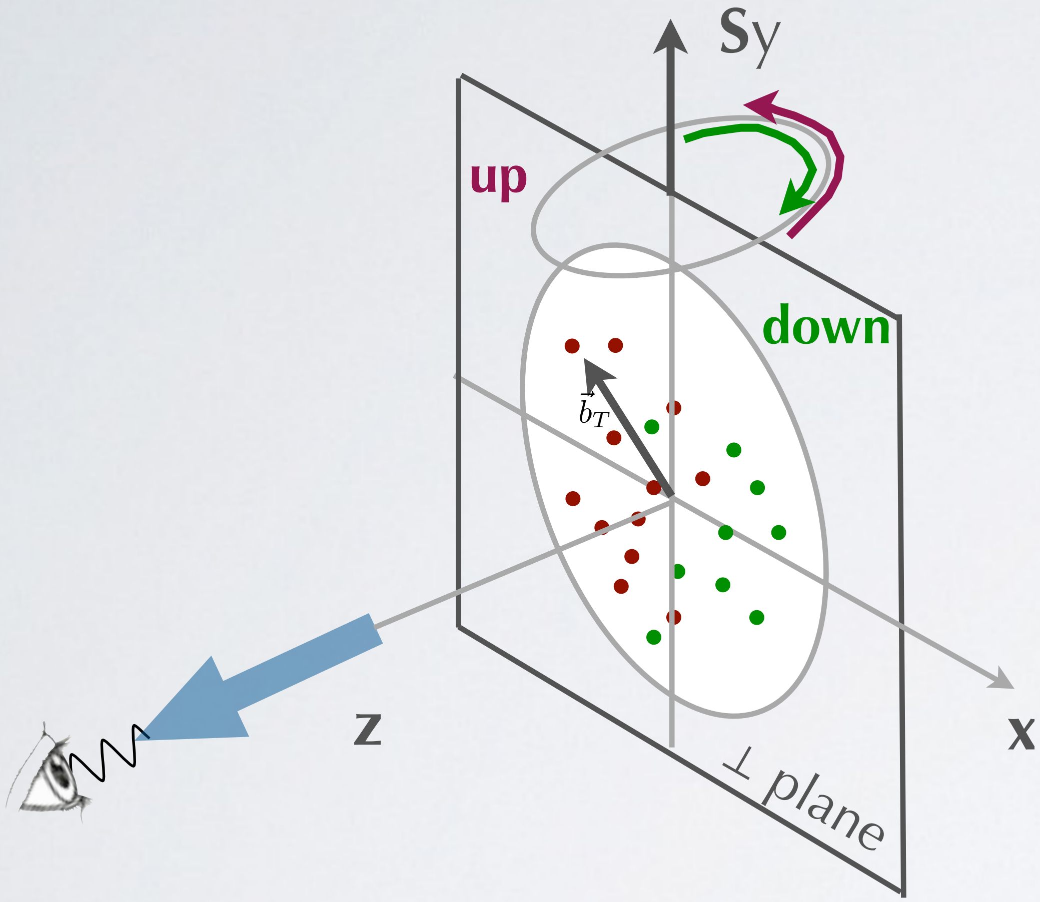
E_x ~ dipole deformation

A. Bacchetta & M. Contalbrigo, Il Nuovo Saggiatore ***28*** *(12) n.1,2*

and

C. Carlson & M. Vanderhaeghen, P.R.L. ***100*** *(08) 032004*

parton **O**rbital **A**ngular **M**omentum



N$^\uparrow$ polarization along **y**
gives a twist along **x**
to parton charge densities
because of their
Orbital **A**ngular **M**omentum
(OAM)

how to define it ?

(gauge-inv. definition is
common problem for
gauge field th.'s)

definition #1 of **OAM**

from Ji's sum rule :

OAM = total J – helicity

$$\begin{aligned} L_z^q(Q^2) &\equiv J_z^q(Q^2) \quad \left\{ = \frac{1}{2} \int dx\, x \left[f_1^q(x;Q^2) + E^q(x,0,0;Q^2) \right] \right\} \\ &- S_z^q(Q^2) \quad \left\{ = \int dx\, g_1(x;Q^2) \right\} \end{aligned}$$

gauge invariant

measurable (DIS → f_1, g_1 ; DVCS → E)

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$$
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\end{aligned}
$$

gauge invariant

measurable (DIS → f_1, g_1 ; DVCS → E)

but L^q does not satisfy canonical relations

alternatives?…

for a review *E. Leader & C. Lorcé, arXiv:1309.4235 [hep-ph]*

the latest scenario from lattice

M. Deka et al. (χQCD), arXiv:1312.4816 [hep-lat]

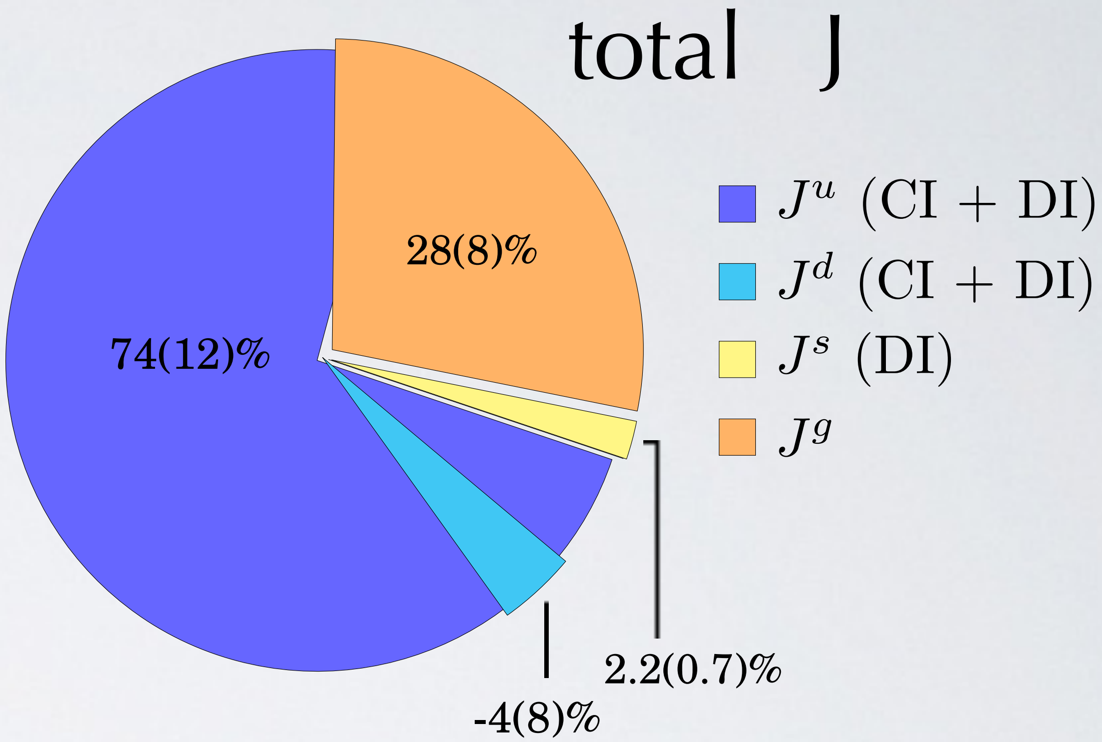
Connected **Insertions** **Disconnected**

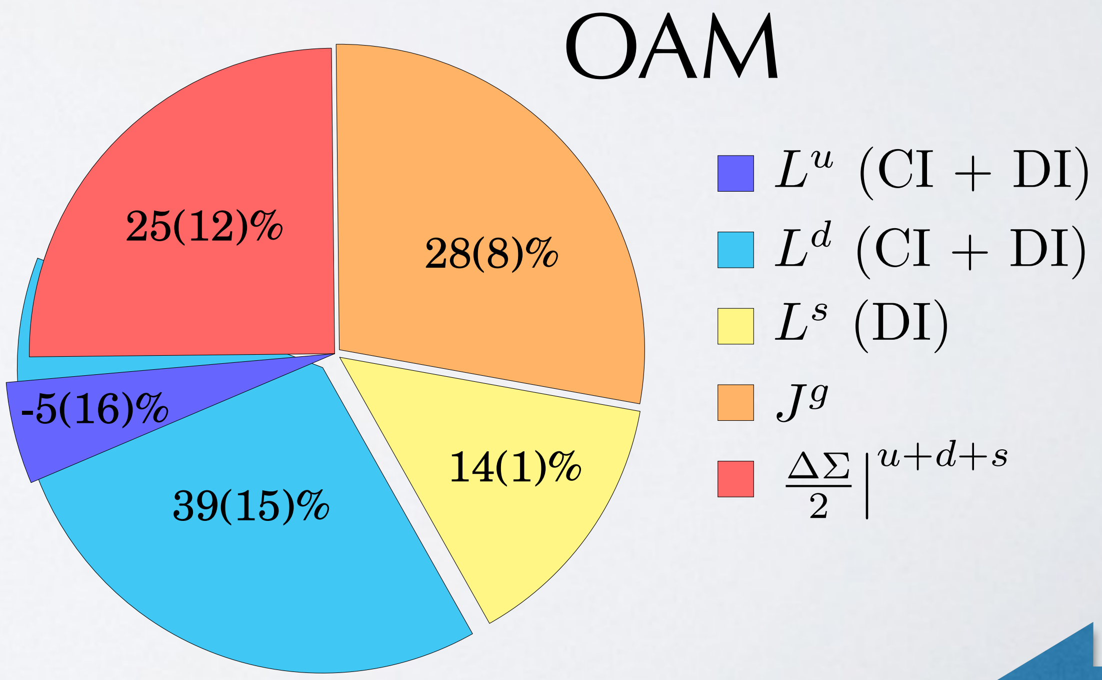
	CI(u)	**CI(d)**	**CI(u+d)**	**DI(u/d)**	**DI(s)**	**Glue**
$\langle x \rangle$	0.416(40)	0.151(20)	0.567(45)	0.037(7)	0.023(6)	0.334(56)
$T_2(0)$	0.283(112)	-0.217(80)	0.061(22)	-0.002(2)	-0.001(3)	-0.056(52)
$2J$	0.704(118)	-0.070(82)	0.629(51)	0.035(7)	0.022(7)	0.278(76)
g_A	0.91(11)	-0.30(12)	0.62(9)	-0.12(1)	-0.12(1)	–
$2L$	-0.21(16)	0.23(15)	0.01(10)	0.16(1)	0.14(1)	–

TABLE III. Renormalized values in $\overline{MS}$ scheme at $\mu = 2$ GeV.

$$g_A^0 = \Delta u + \Delta d + \Delta s$$

$$2J - g_A^0 = 2L$$

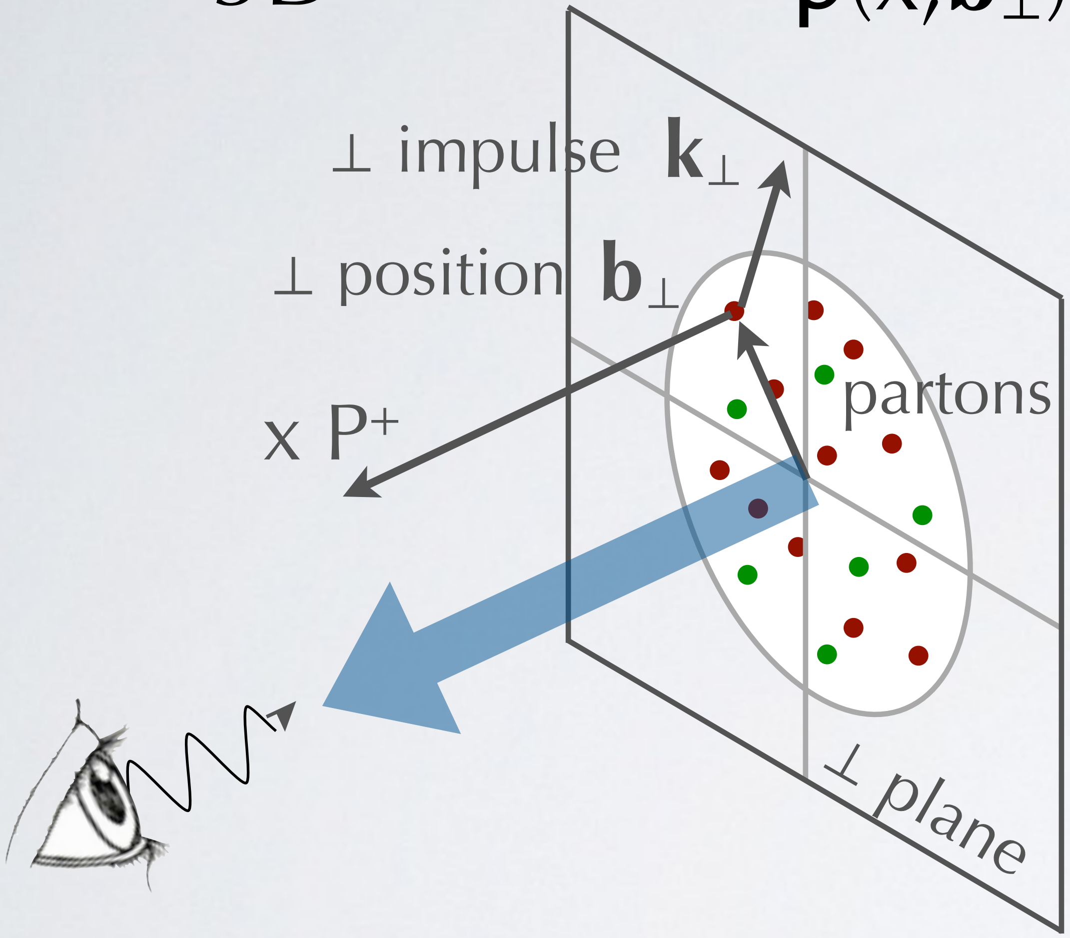




Wigner Distribution

5D —— $W_{S_N,S_q}(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

3D —— $\rho(x,\mathbf{b}_\perp)$



C. Lorcé, B. Pasquini, M. Vanderhaeghen,
JHEP 1105 (11) 041

The "mother" distribution

correlation of quark ⊥ **momentum** and **position** for **S_N and S_q polarizations**

<u>not positive-definite</u>

but $\mathbf{b} \leftrightarrow \mathbf{q}=\mathbf{P}'_\perp-\mathbf{P}_\perp$

no constraint from Heisenberg principle

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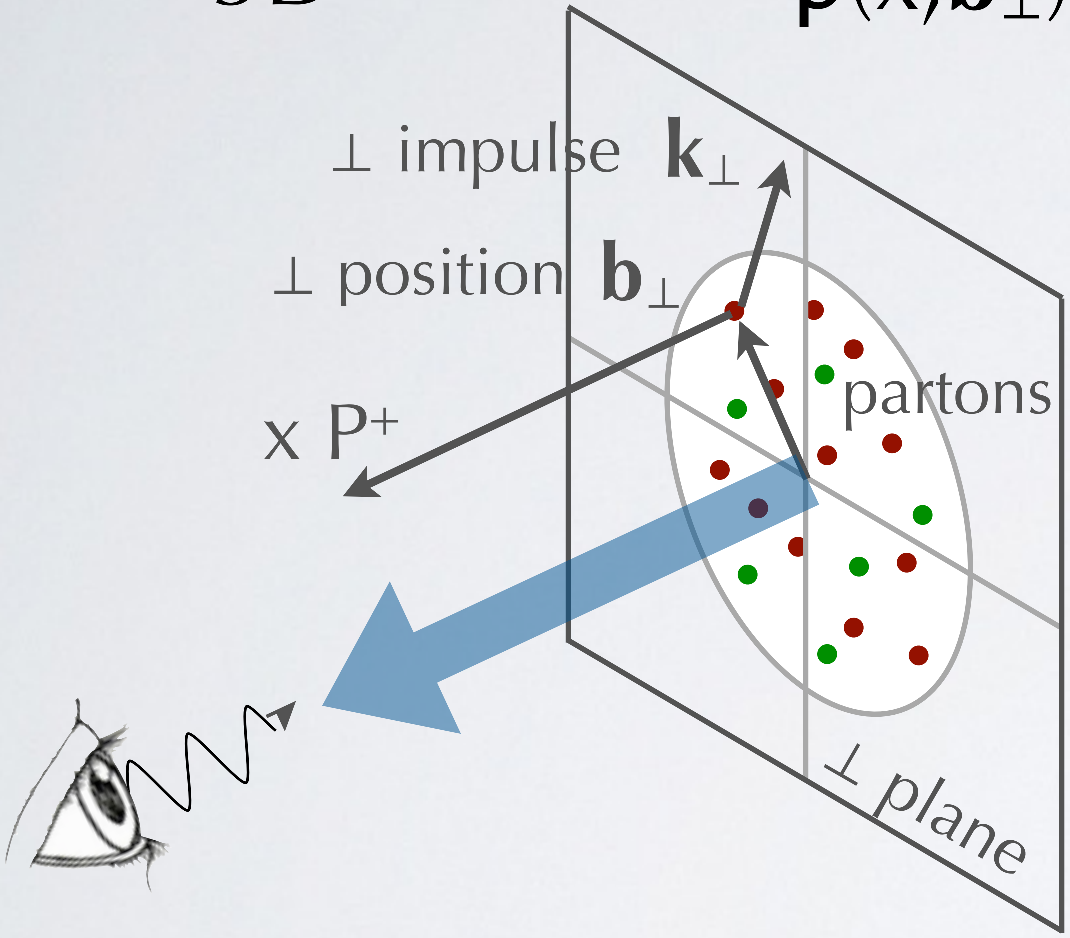

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C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11) 041

$$\boxed{\int d\mathbf{k}_\perp\, W(x,\mathbf{k}_\perp,\mathbf{b}_\perp) \rightarrow q(x,\mathbf{b}_\perp) \rightarrow \mathrm{GPD}}$$

Transverse Mom. Distributions (TMD)

5D ——— $\int d\mathbf{b}_\perp \; W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

3D ——— $q(x,\mathbf{k}_\perp)$ **TMD** parton density in k-space

is not the F.T. of $q(x,\mathbf{b}_\perp)$

Transverse **M**om. **D**istributions (TMD)

5D ——— $\int d\mathbf{b}_\perp \, W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

$\searrow$

3D ——— $q(x,\mathbf{k}_\perp)$ **TMD**

parton density in k-space
is not the F.T. of $q(x,\mathbf{b}_\perp)$

leading twist: 8 **TMDs**

quark pol.

		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Transverse **M**om. **D**istributions (TMD)

5D ——— $\int d\mathbf{b}_\perp\, W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

↘

3D ——— $q(x,\mathbf{k}_\perp)$ **TMD**

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	U	L	T
U	f_1		$h_1^\perp$
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T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

** Anselmino et al., P.R. D**87** (13) 094019*

** A. Bacchetta, A. Courtoy, M. Radici, JHEP **03** (13) 119*

$$\int d\mathbf{k}_\perp\, \mathbf{TMD}(x,\mathbf{k}_\perp) \rightarrow \mathbf{PDF}(x)$$

Transverse **M**om. **D**istributions (TMD)

5D ——— $\int d\mathbf{b}_\perp \, W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

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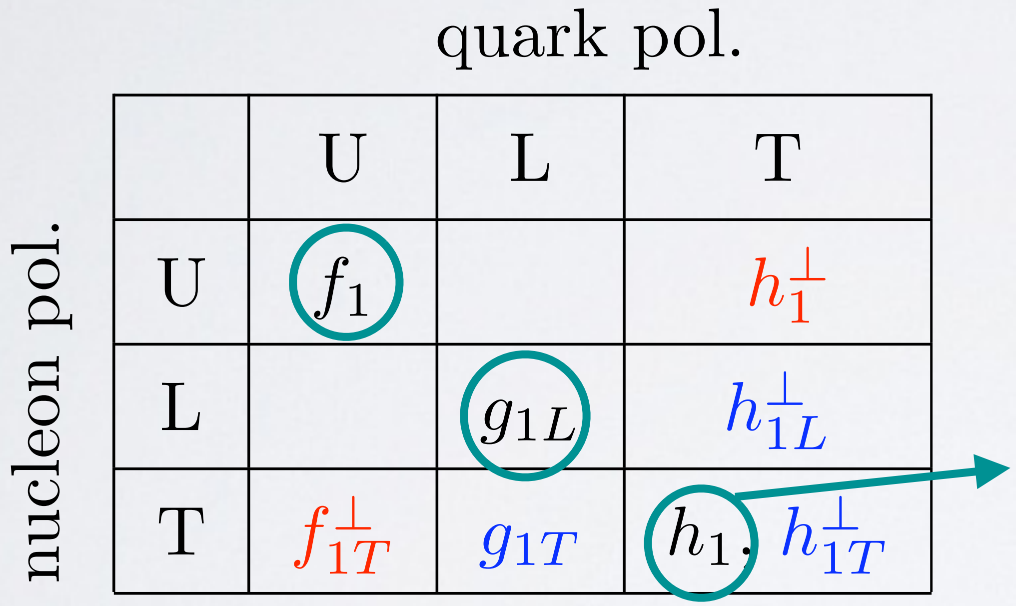


		quark pol.		
		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

$\int dx \, h_1(x)$ = tensor charge

** Anselmino et al., P.R. D**87** (13) 094019*

** A. Bacchetta, A. Courtoy, M. Radici, JHEP **03** (13) 119*

$\int d\mathbf{k}_\perp \, \mathbf{TMD}(x,\mathbf{k}_\perp) \rightarrow \mathbf{PDF}(x)$

$\int dk_\perp$ TMD$(x, k_\perp)$ → PDF(x) ?

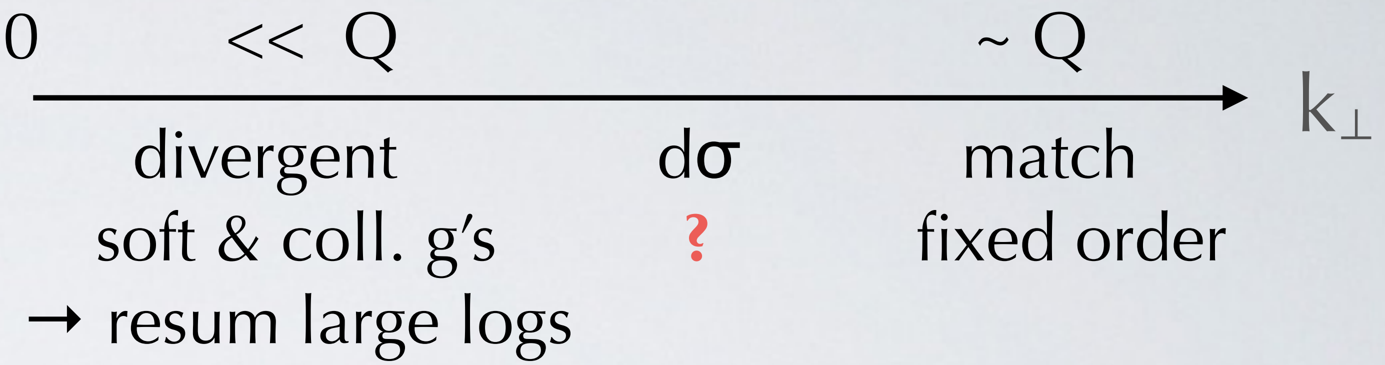
quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

$$\int_0^\infty dk_\perp$$



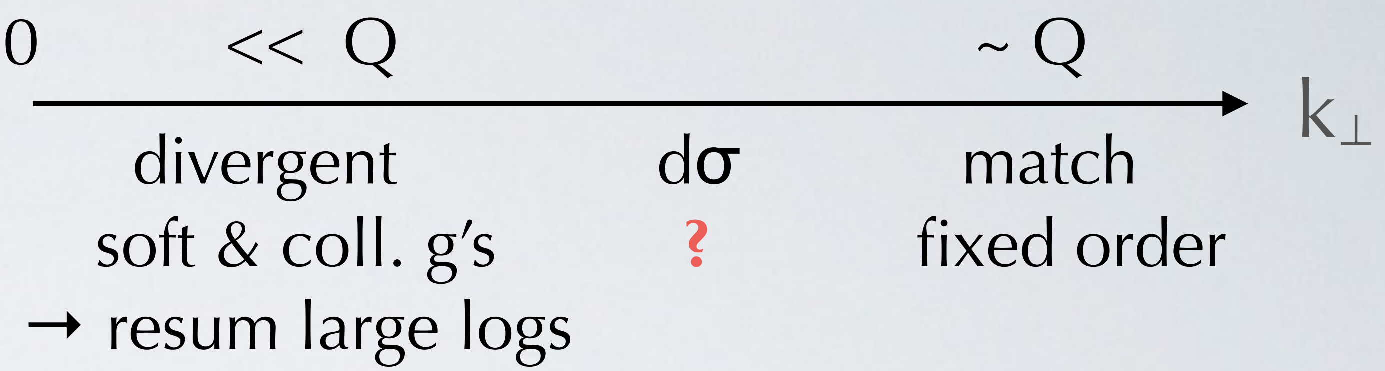
Collins, Soper, Sterman, N.P. ***B250*** *(85) 199*
Collins, "Foundations of perturb. QCD" (C.U.P.,11)
Echevarria et al., E.P.J. ***C73*** *(13) 2636 .....*

$\int dk_\perp$ TMD$(x,k_\perp)$ → PDF(x) ?

		quark pol.		
		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

$$\int_0^\infty dk_\perp$$



divergent
soft & coll. g's
→ resum large logs

dσ
?

match
fixed order

Collins, Soper, Sterman, N.P. ***B250*** *(85) 199*
Collins, "Foundations of perturb. QCD" (C.U.P.,11)
Echevarria et al., E.P.J. ***C73*** *(13) 2636 .....*

in $\mathbf{b}_\perp$ space

$$f_1^q(x,\mathbf{b}_\perp;Q^2) = \sum_i \left[C_{q\,i}\otimes f_1^i\right]\left(x;\frac{c_0^2}{b_*^2}\right)\, e^{S_P(b_*;Q)}\; e^{S_{NP}(\mathbf{b}_\perp)\,\log Q/Q_0}\; f_1^q(x,\mathbf{b}_\perp;Q_0^2)$$

hard coeffs.   PDF   perturb. Sudakov

non-perturb. Sudakov   non-perturb. input TMD

all divergent for b⊥ → ∞   (k⊥ → 0)

depend
on **parameters**
to be fitted

prescription: b⊥ ⇒ $b_* = \dfrac{b_\perp}{\sqrt{1+\dfrac{b_\perp^2}{b_{\max}^2}}}$

$f_1^q(x,\mathbf{k}_\perp)$ → LHC

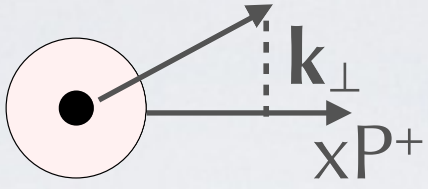
quark pol.

nucleon pol.

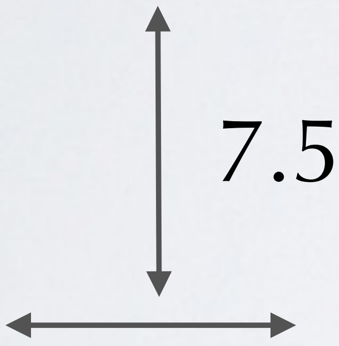
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

$f_1 =$



impact of TMD on
Z^0 peak → W mass

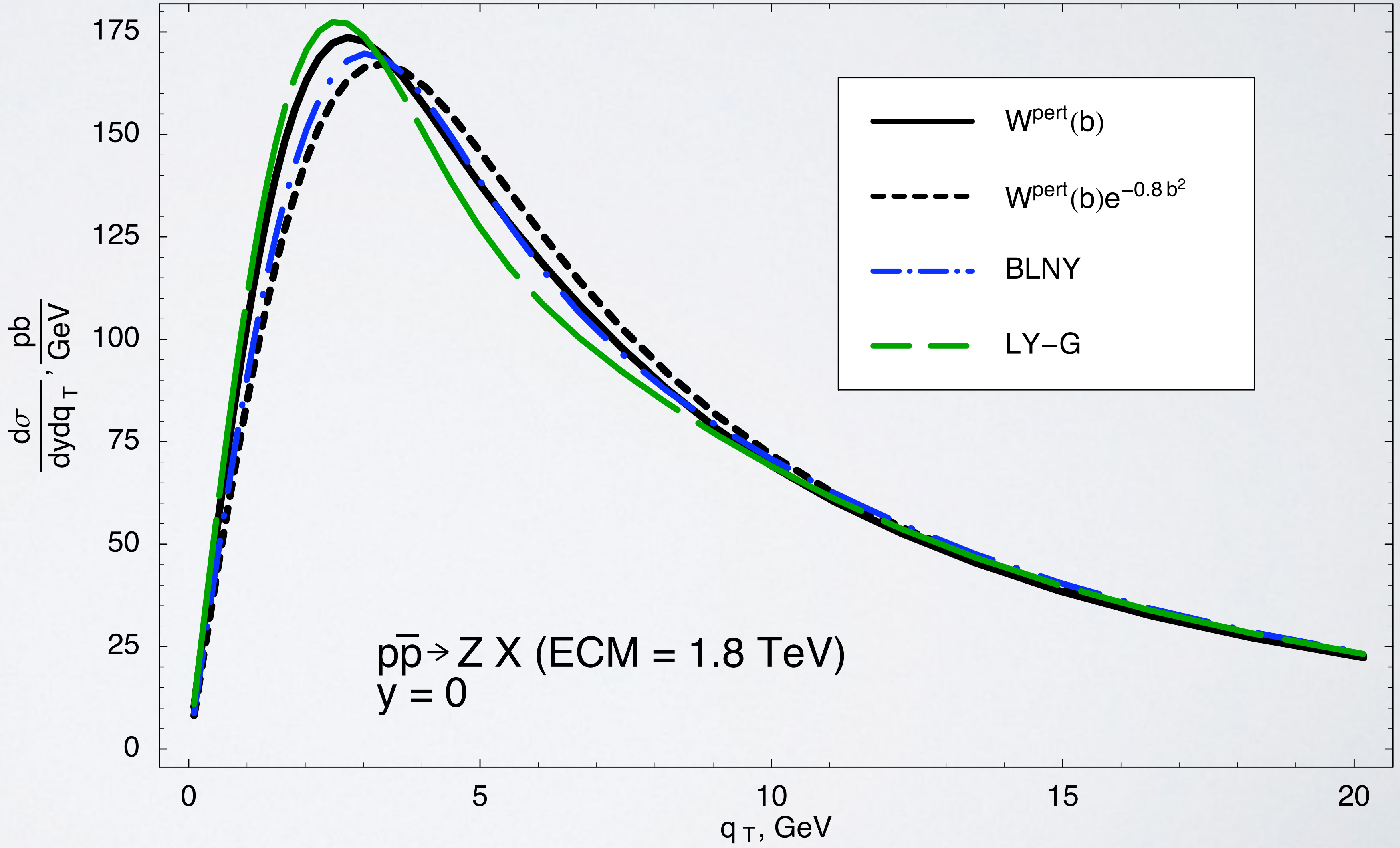


7.5%

30%

uncertainty

P. Nadolski, hep-ph/0412146



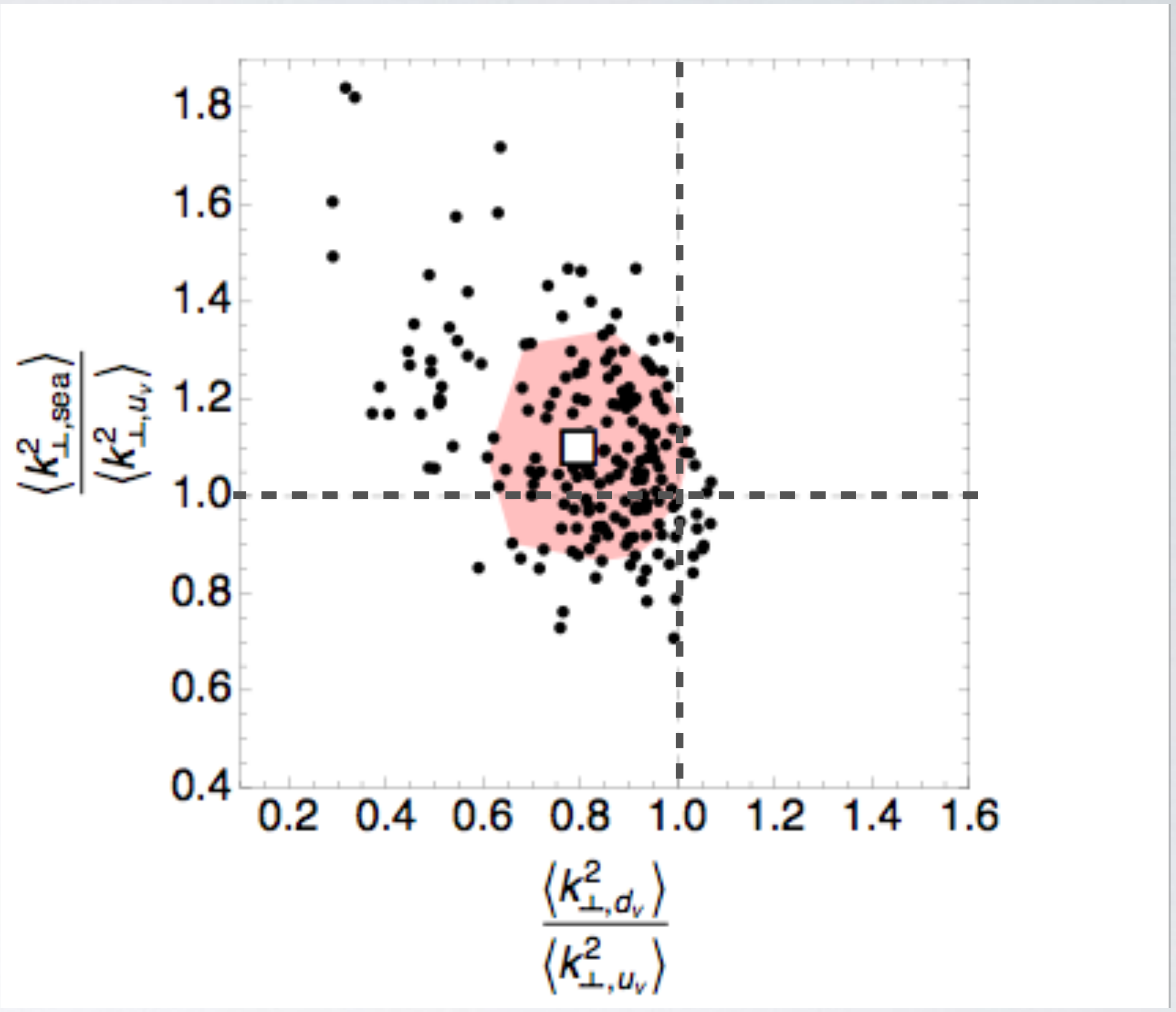
flavor analysis of TMD(x,$\mathbf{k}_\perp$)

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

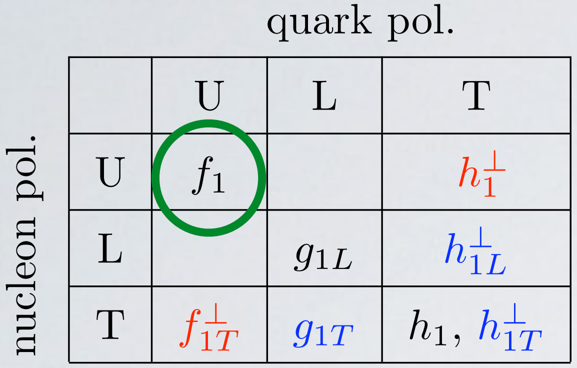
Twist-2 TMDs

fit SIDIS
multiplicity
from HERMES



A. Signori et al., JHEP ***1311*** *(13) 194*

flavor analysis of TMD$(x,\mathbf{k}_\perp)$



quark pol.

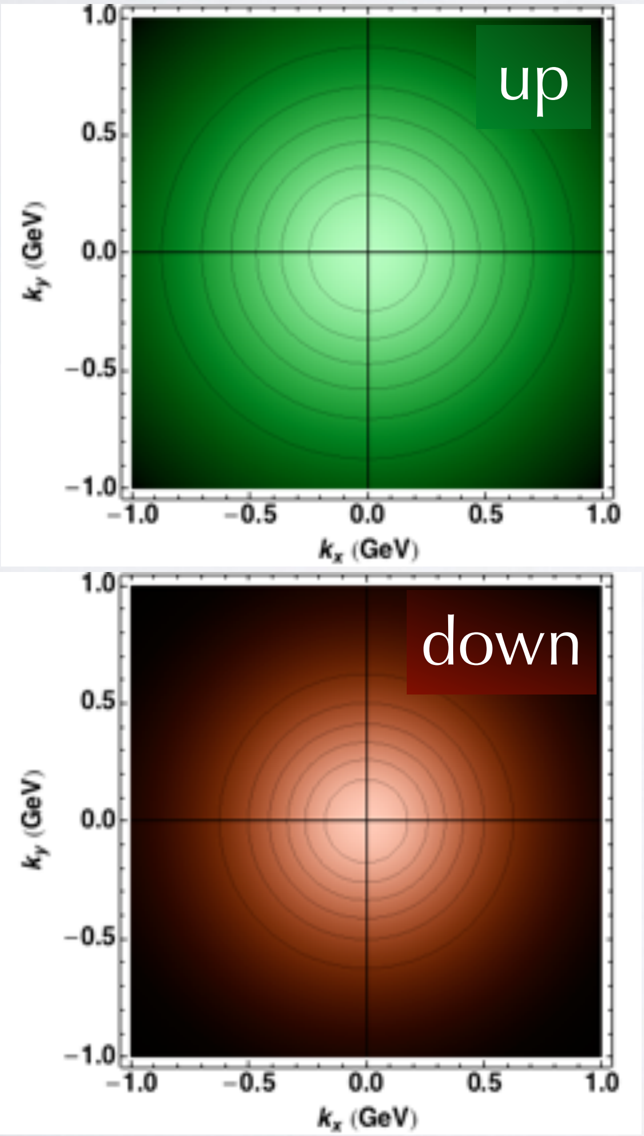
nucleon pol.

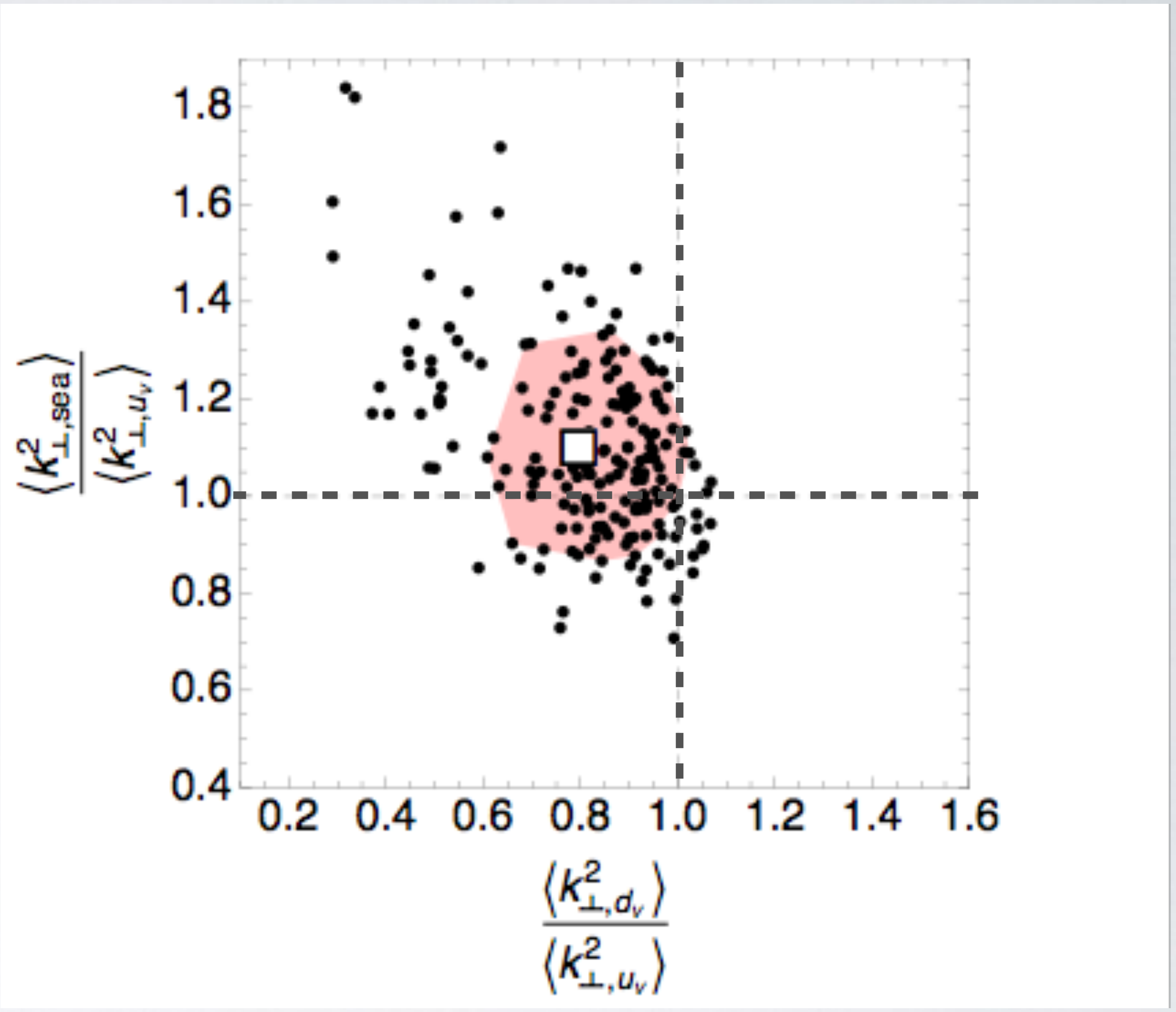
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

fit SIDIS
multiplicity
from HERMES

x=0.1





*A. Signori et al., JHEP**1311** (13) 194*

down < up < sea ?

the Sivers effect

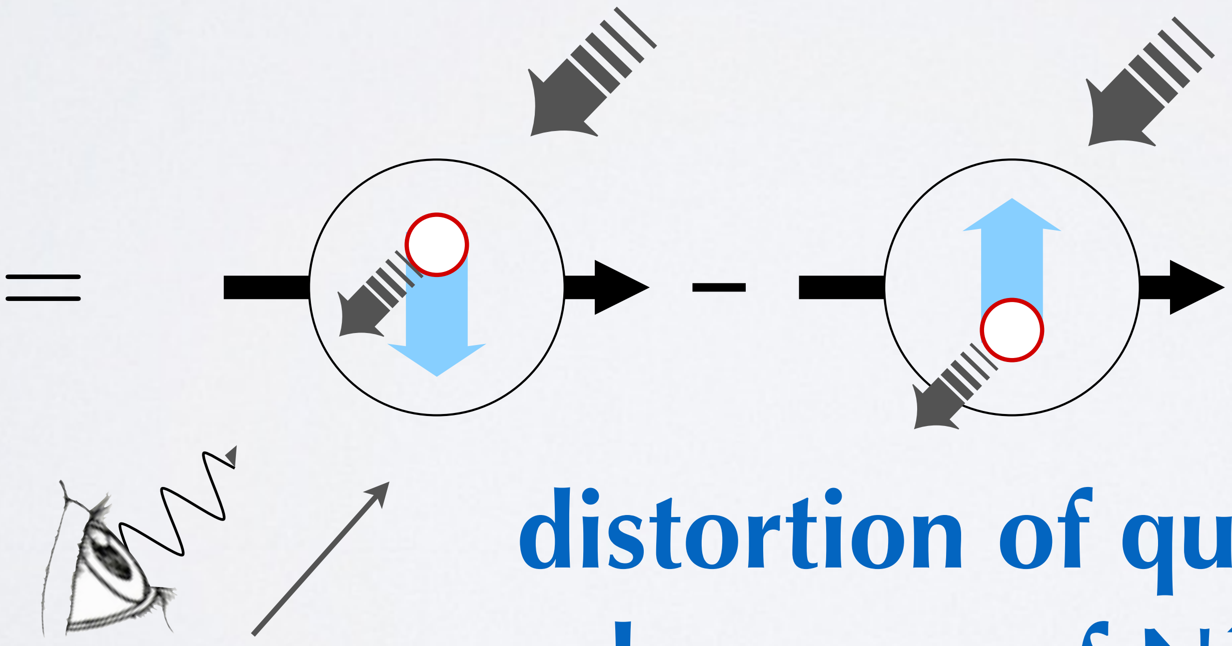
leading twist: 8 **TMDs**

		quark pol.		
		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$\boxed{f_{1T}^\perp}$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Sivers function

$f_{1T}^\perp =$



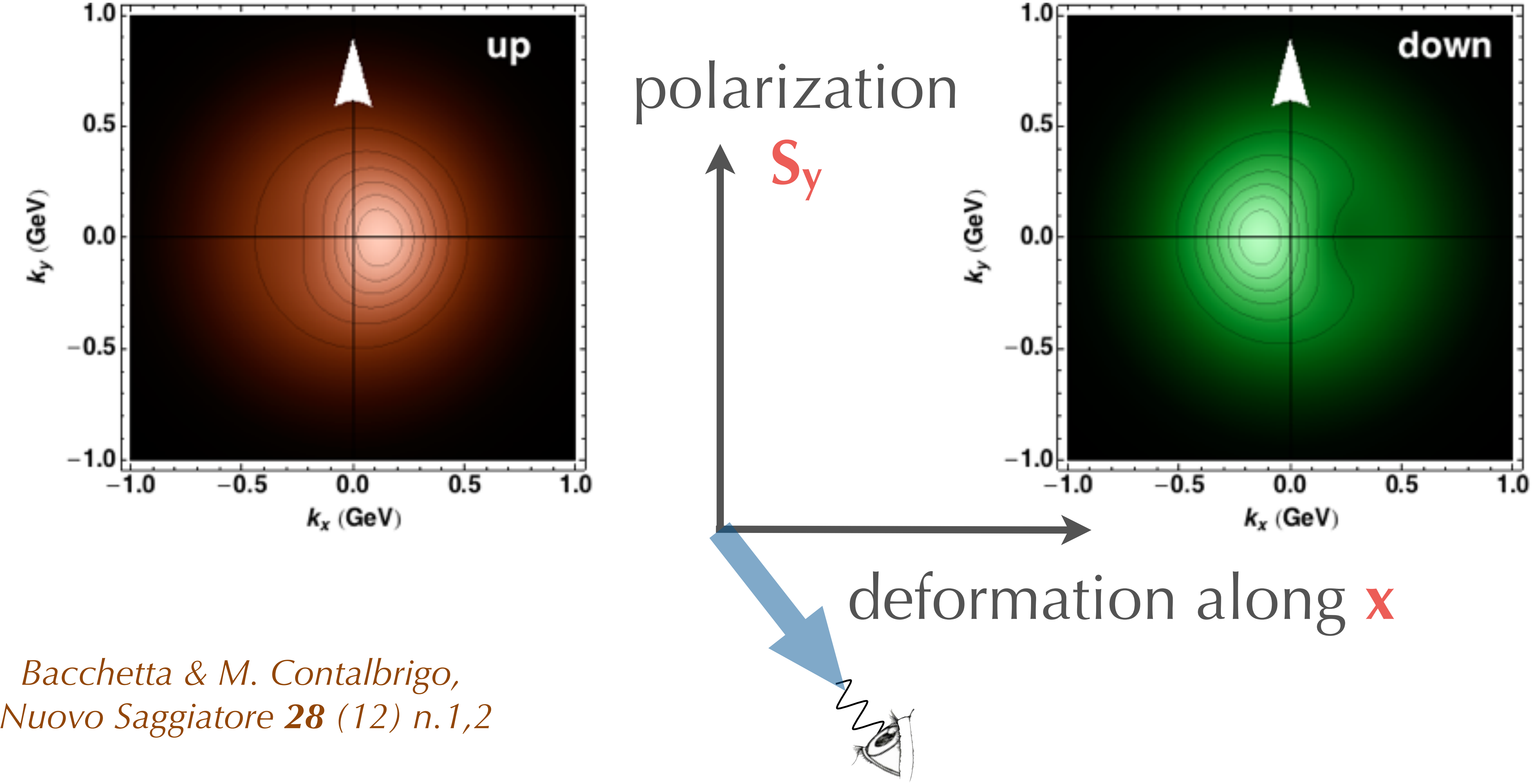
distortion of quark distribution because of N$^\uparrow$ polarization

*D. Sivers, P.R. D**41** (90) 83*

flavor dependence of Sivers effect

distribution of unpolarized q in polarized P↑

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\,\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot\mathbf{S}}{M}$$



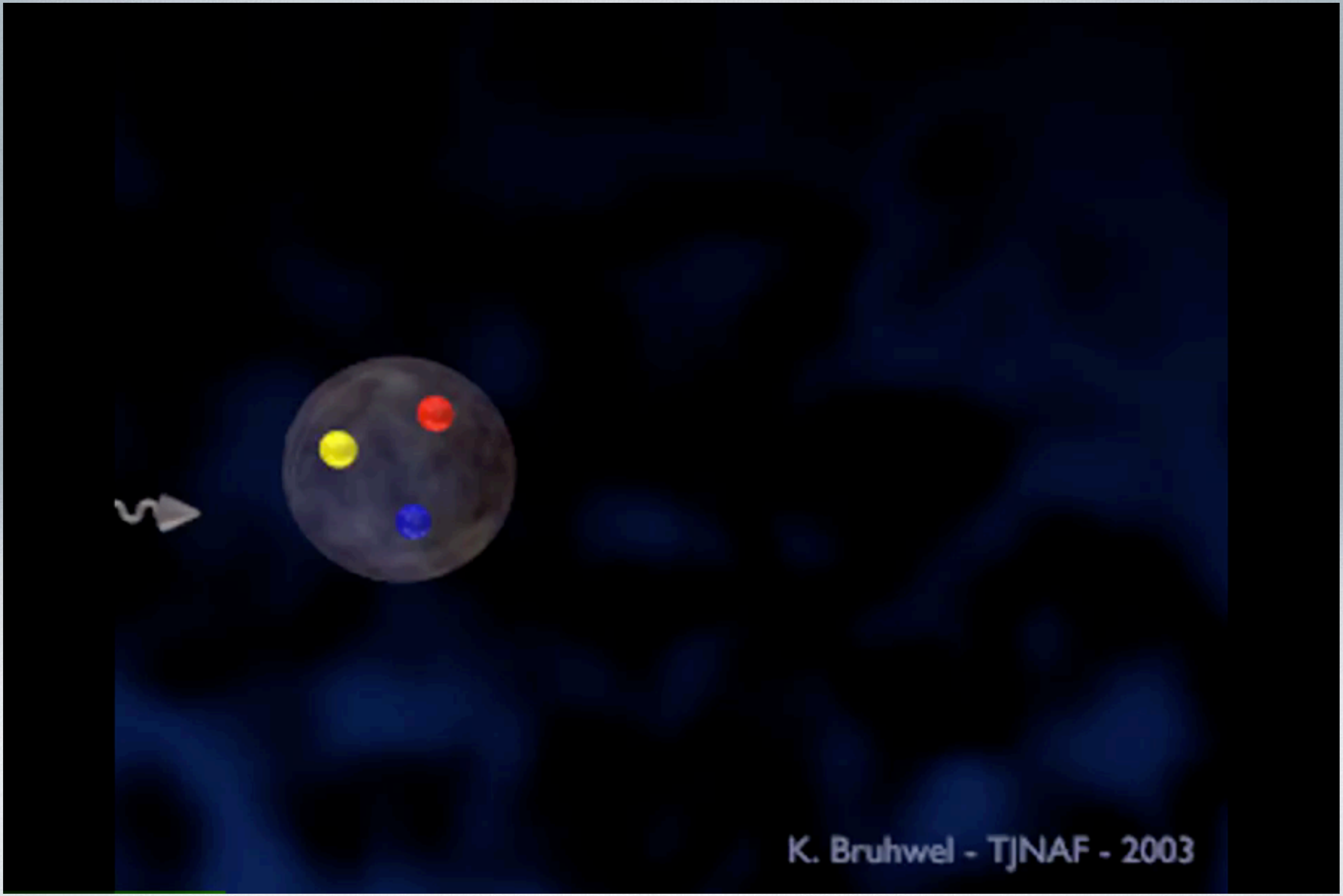
polarization

S_y

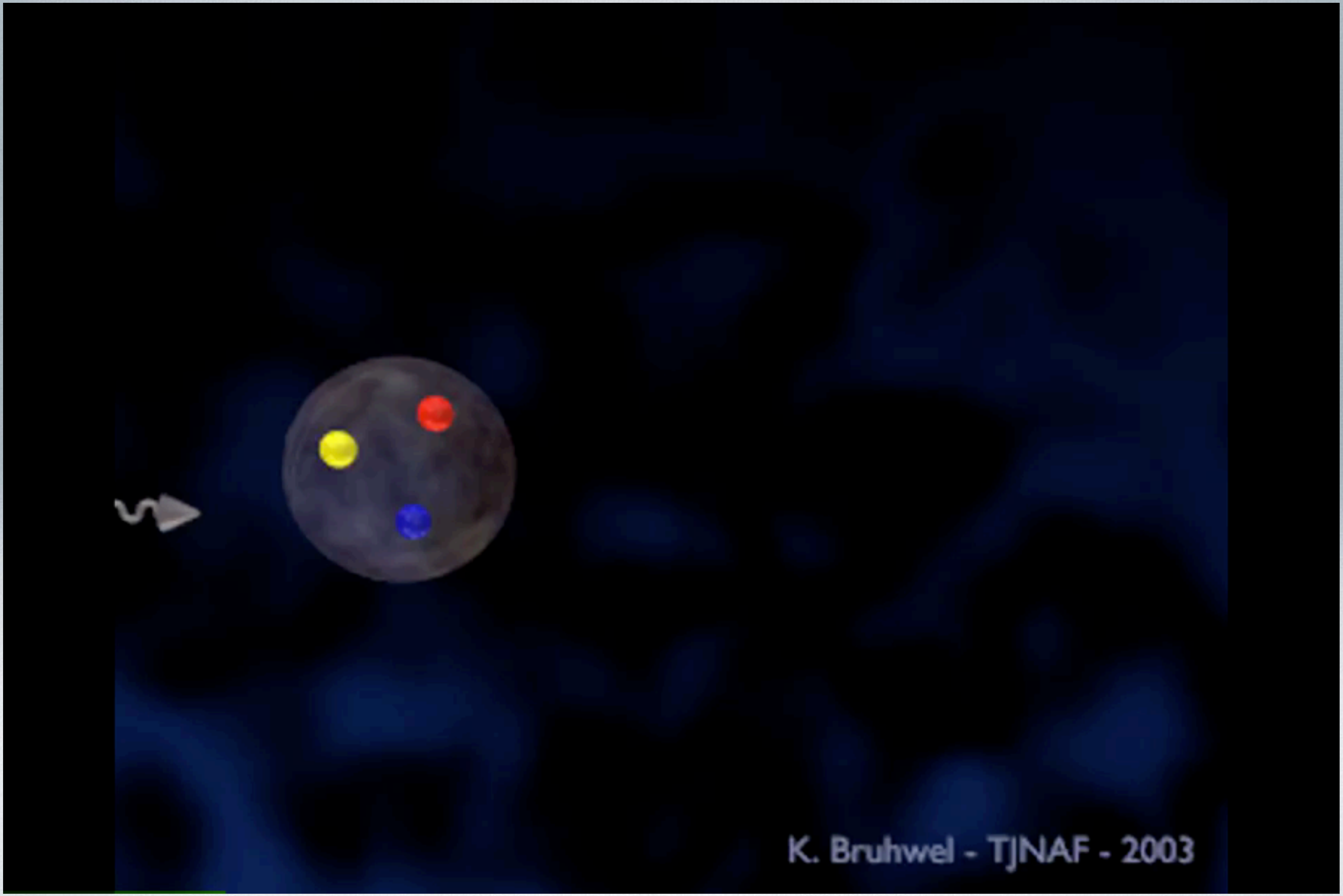
deformation along **x**

A. Bacchetta & M. Contalbrigo,
Il Nuovo Saggiatore ***28*** *(12) n.1,2*

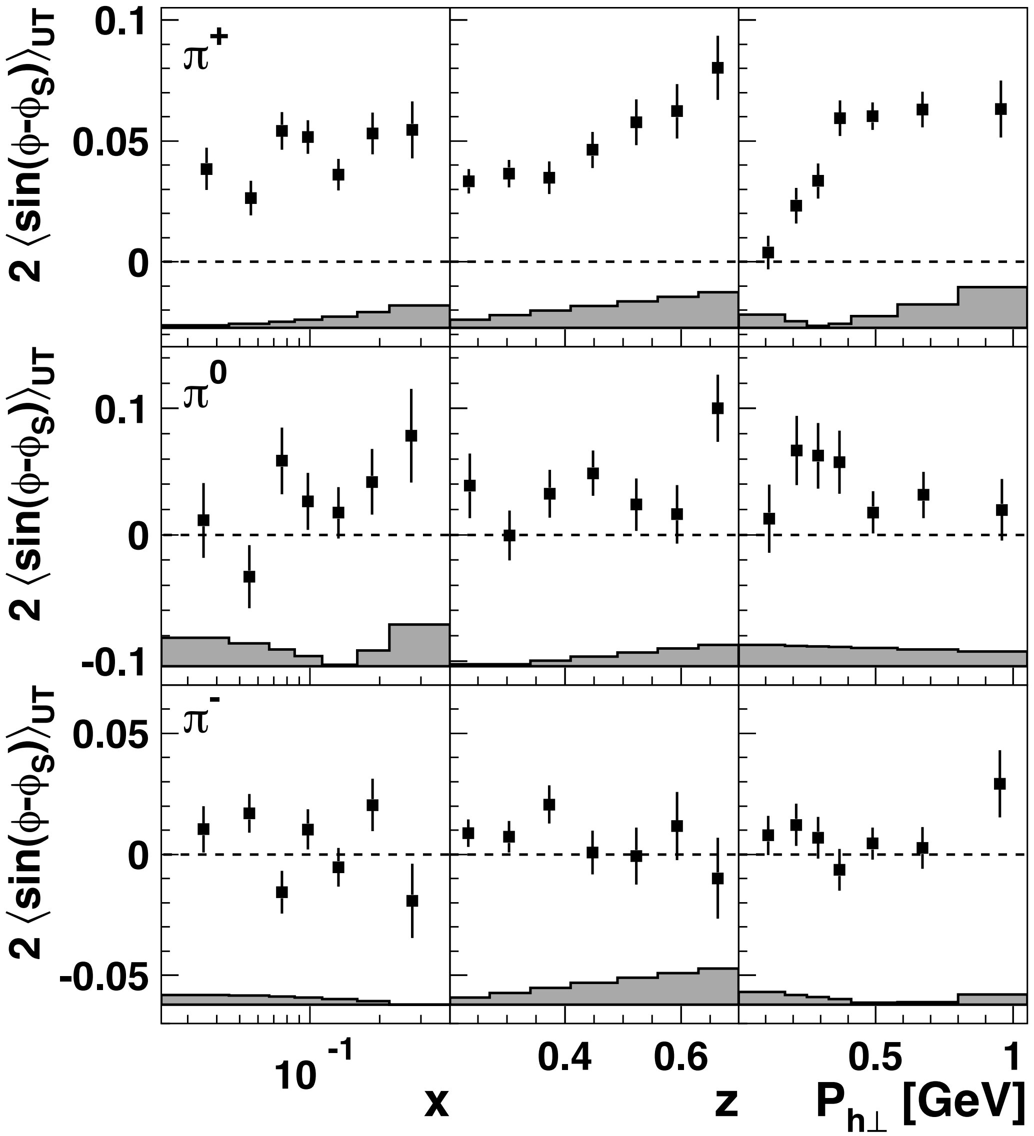
the Sivers effect in semi-incl. DIS (SIDIS)



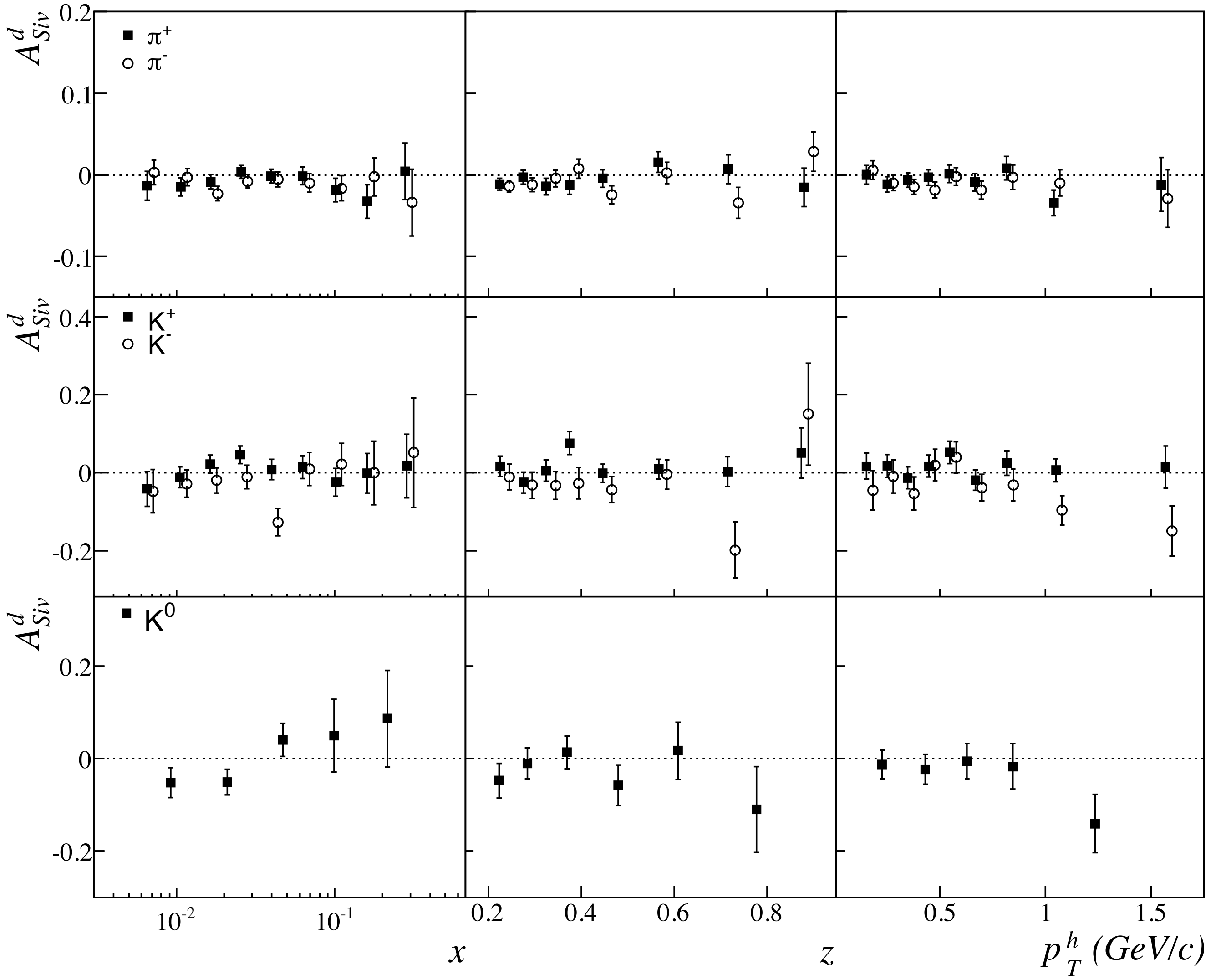
the Sivers effect in semi-incl. DIS (SIDIS)

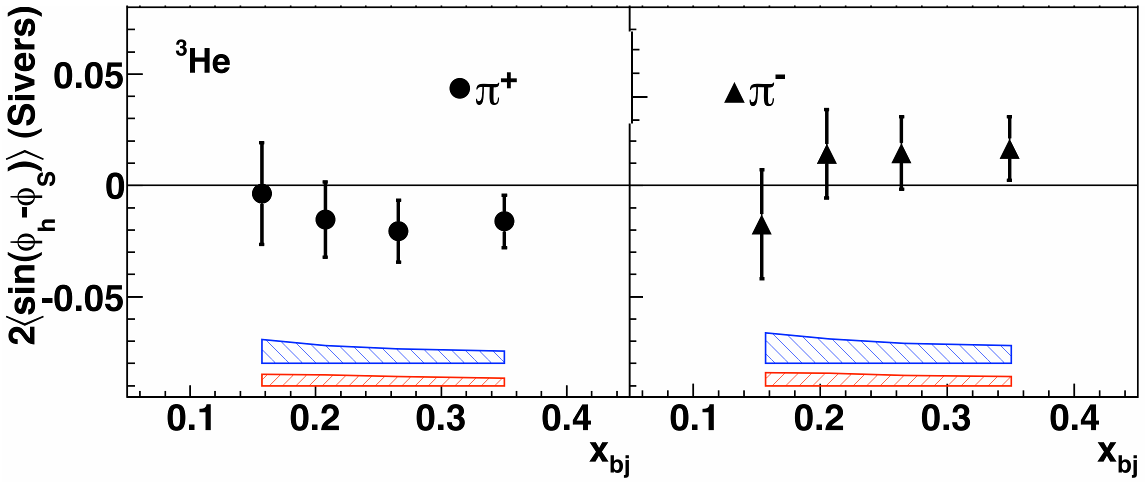


*PRL**103** (09) 152002*



*PL B**673** (09) 127*





Jefferson Lab

Hall A

*PRL**107** (11) 072003*

Sivers effect has been measured in $N^{\uparrow}(e,e'\pi)$!

parametrizations of Sivers function

leading twist: 8 **TMDs**

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

** M. Anselmino et al, E.P.J. A**39** (09) 89*

** M. Anselmino, M.E. Boglione, S. Melis,*
*P.R. D**86** (12) 014028*

** A. Bacchetta & M. Radici,*
*P.R.L. **107** (11) 212001*

** W. Vogelsang & F. Yuan,*
*P.R. D**72** (05) 054028*

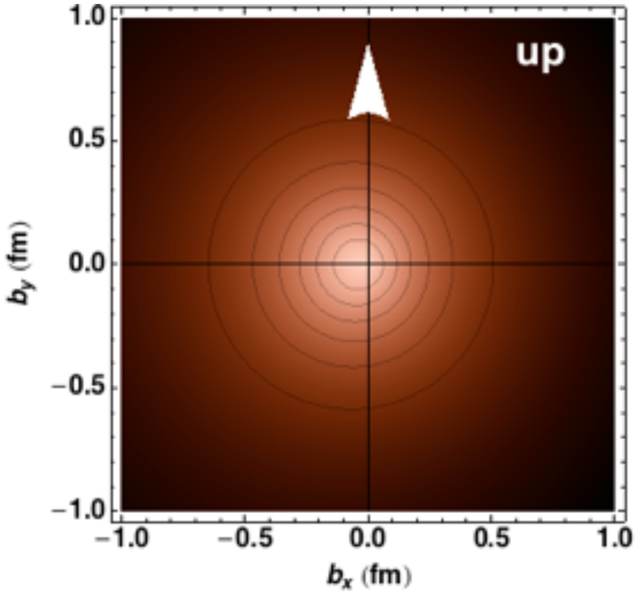
** J.C. Collins et al.,*
*P.R. D**73** (06) 014021*

** S.M. Aybat, A. Prokudin, T.C. Rogers,*
*P.R.L. **108** (12) 242003*

** P. Sun & F. Yuan,*
*P.R. D**88** (13) 034016*

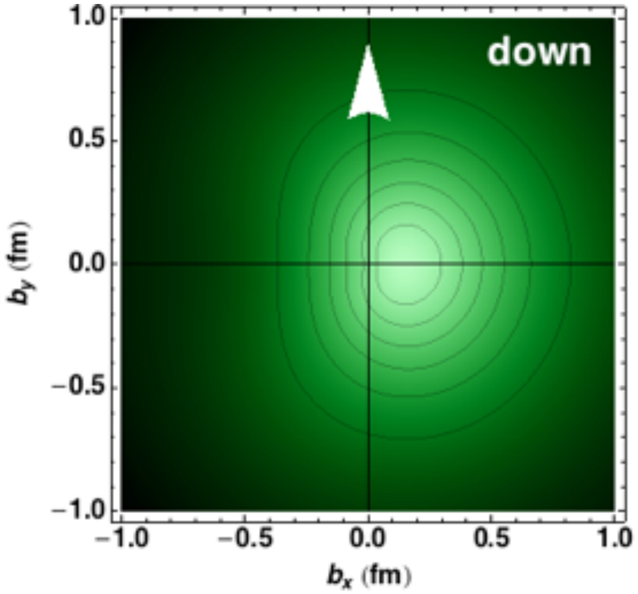
** D. Boer,*
*N.P. **B874** (13) 217*

GPD E



$N^{\uparrow}$
polarization

y



in **b** space

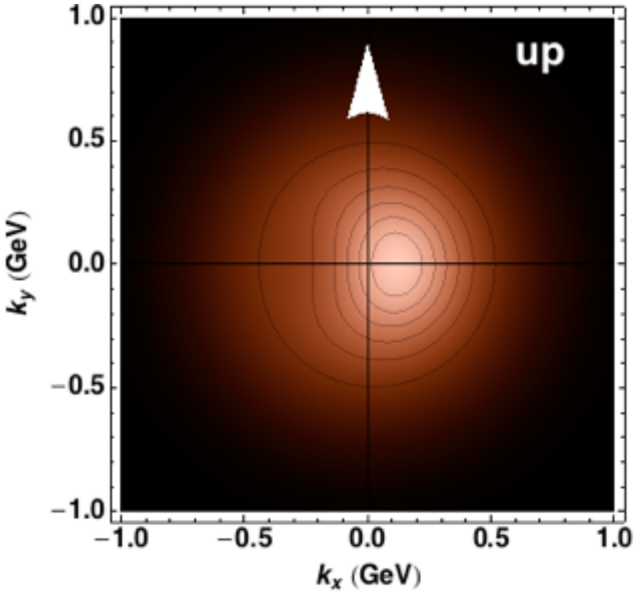
x

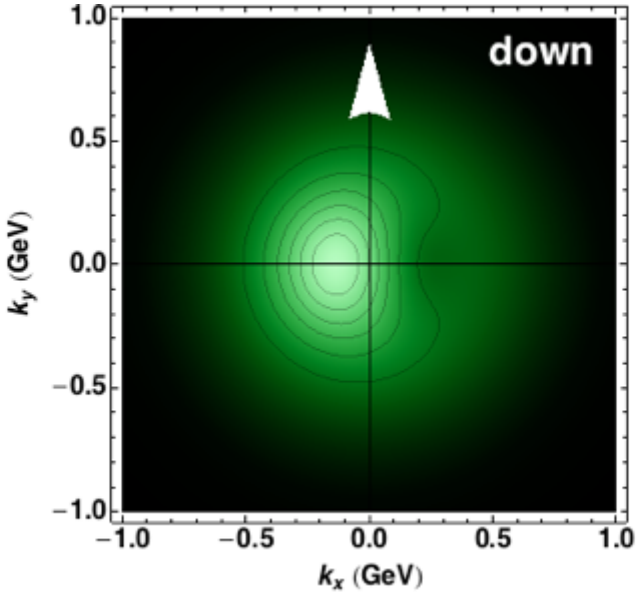
deformation

in **k** space

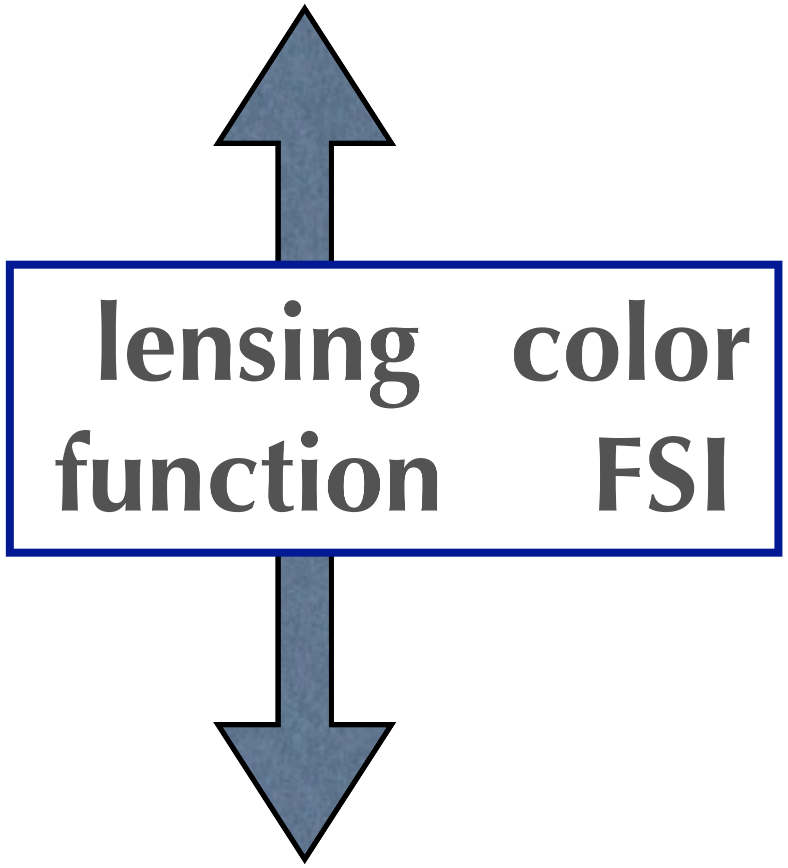
TMD $f_{1T}^{\perp}$

Sivers
effect





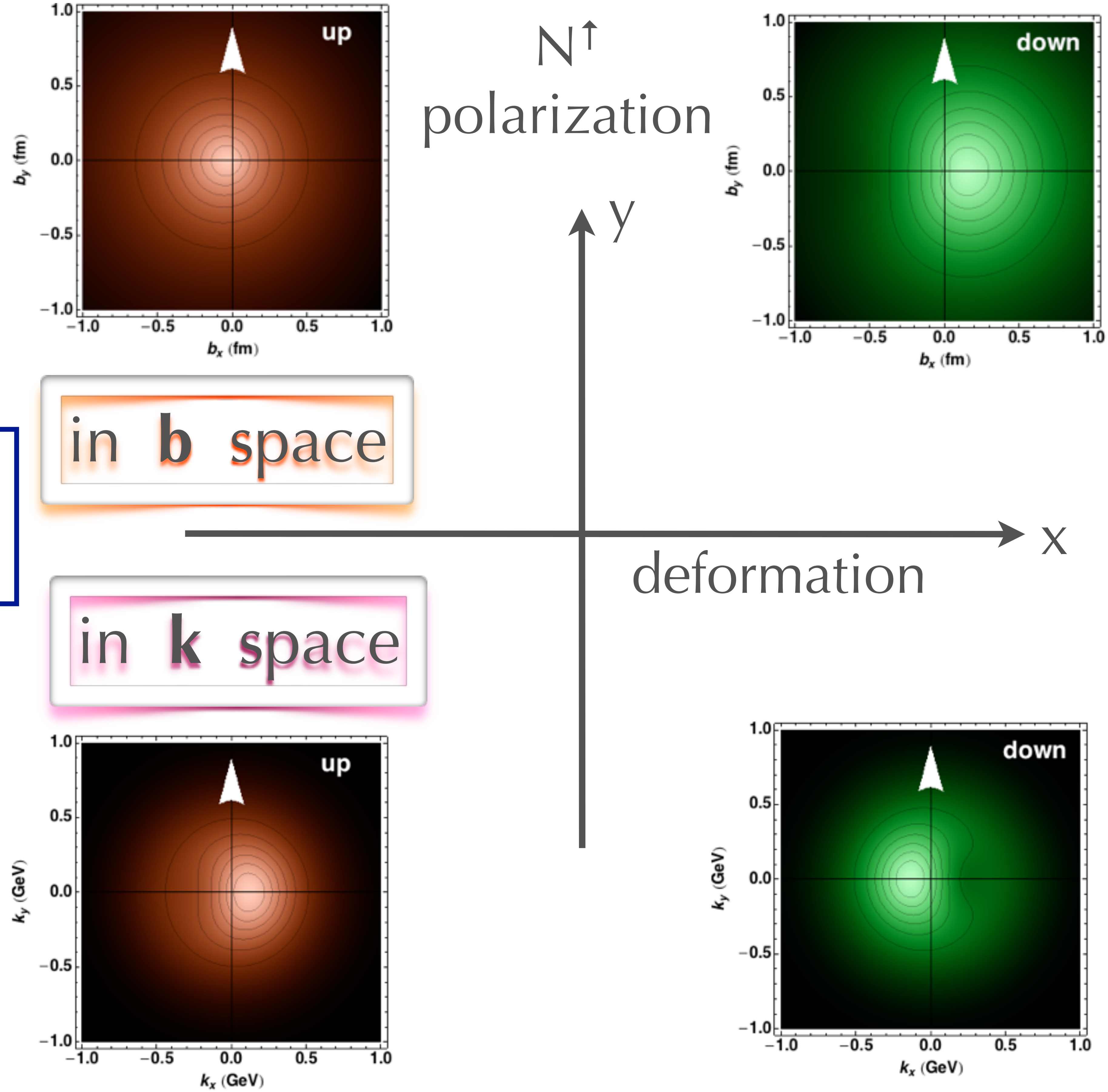
GPD E



lensing function **color FSI**

TMD $f_{1T}^{\perp}$

Sivers effect



$N^{\uparrow}$ polarization

y

x

deformation

in **b** space

in **k** space

*M. Burkardt, P.R. D**66** (02) 114005*

Ji's sum rule

$$J_z^q(Q^2) \;=\; \frac{1}{2}\int_0^1 dx\, x\, \left[\, H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2)\,\right]$$

not accessible

assumption

$$f_{1T}^{\perp(0)q}(x;Q_L^2) = -L(x)E^q(x,0,0;Q_L^2)$$

lensing funct. (at some Q_L)

*A. Bacchetta, F. Conti, M. Radici, P.R. D**78** (08) 074010*

Ji's sum rule

$$J_z^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x\,\left[H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2)\right]$$

not accessible

assumption

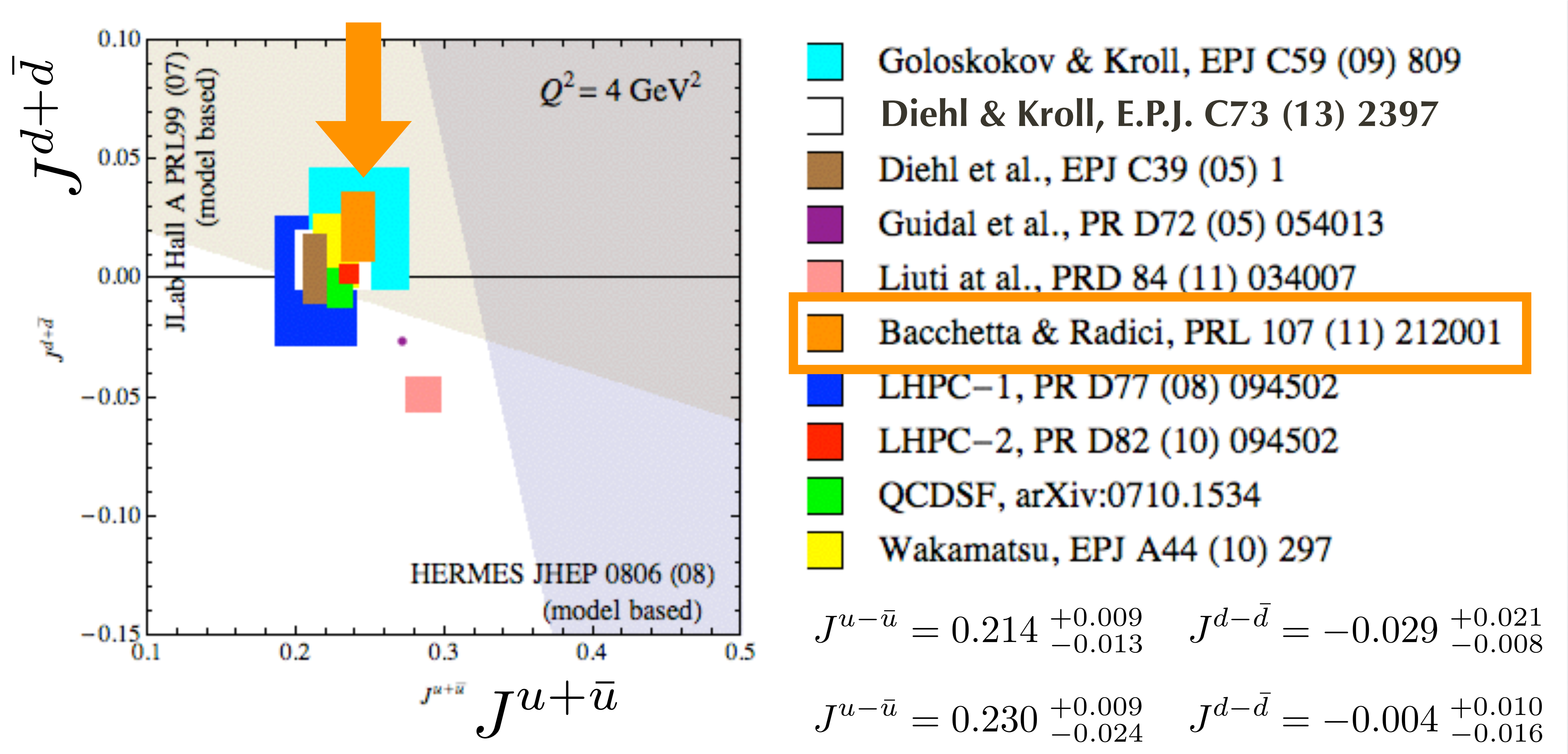
$$f_{1T}^{\perp(0)q}(x;Q_L^2) = -L(x)E^q(x,0,0;Q_L^2)$$

lensing funct.

(at some Q_L)

*A. Bacchetta, F. Conti, M. Radici, P.R. D**78** (08) 074010*

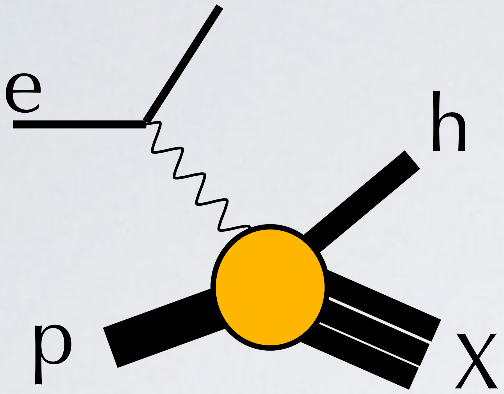
comparison with other GPD extractions and lattice results



$J^{u-\bar u} = 0.214^{+0.009}_{-0.013}$ $\quad J^{d-\bar d} = -0.029^{+0.021}_{-0.008}$ 🟧

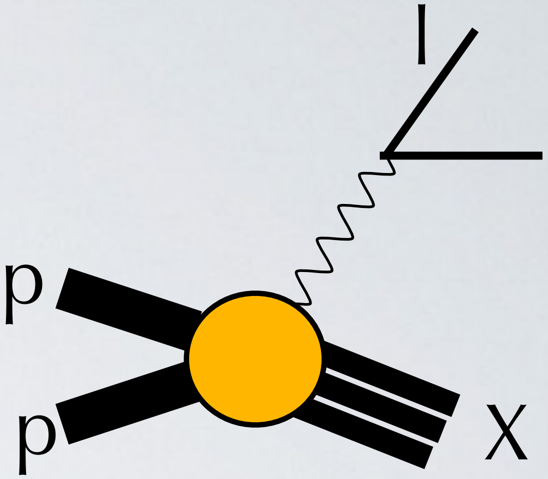
$J^{u-\bar u} = 0.230^{+0.009}_{-0.024}$ $\quad J^{d-\bar d} = -0.004^{+0.010}_{-0.016}$ ☐

Reason #2 for the Sivers function

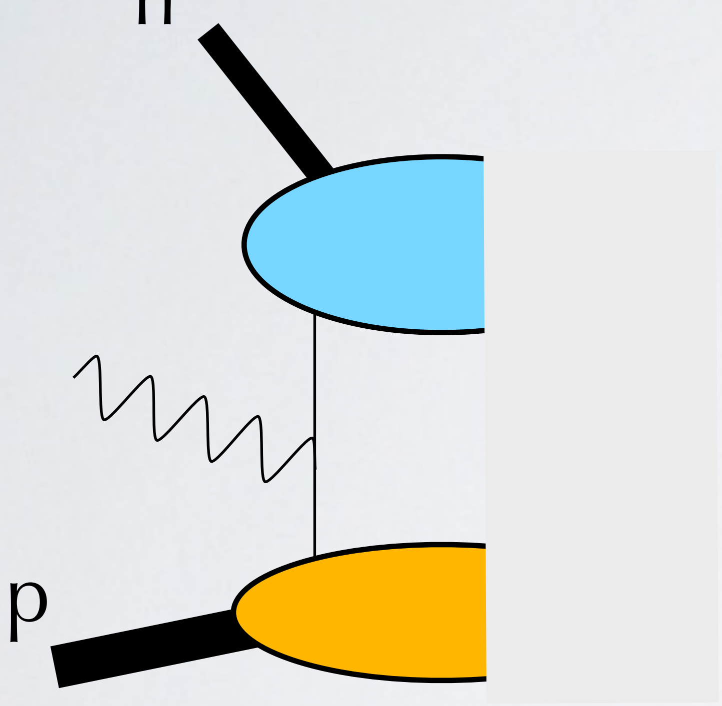


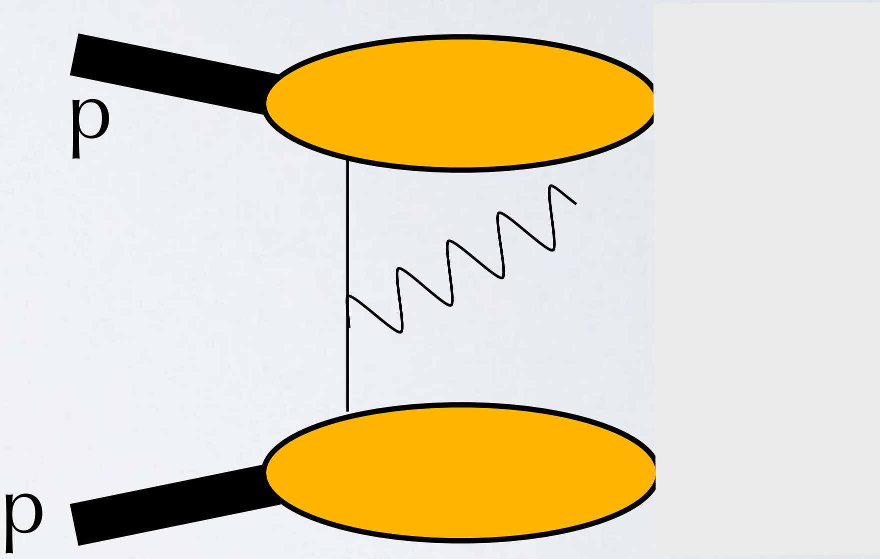
SIDIS

Drell-Yan

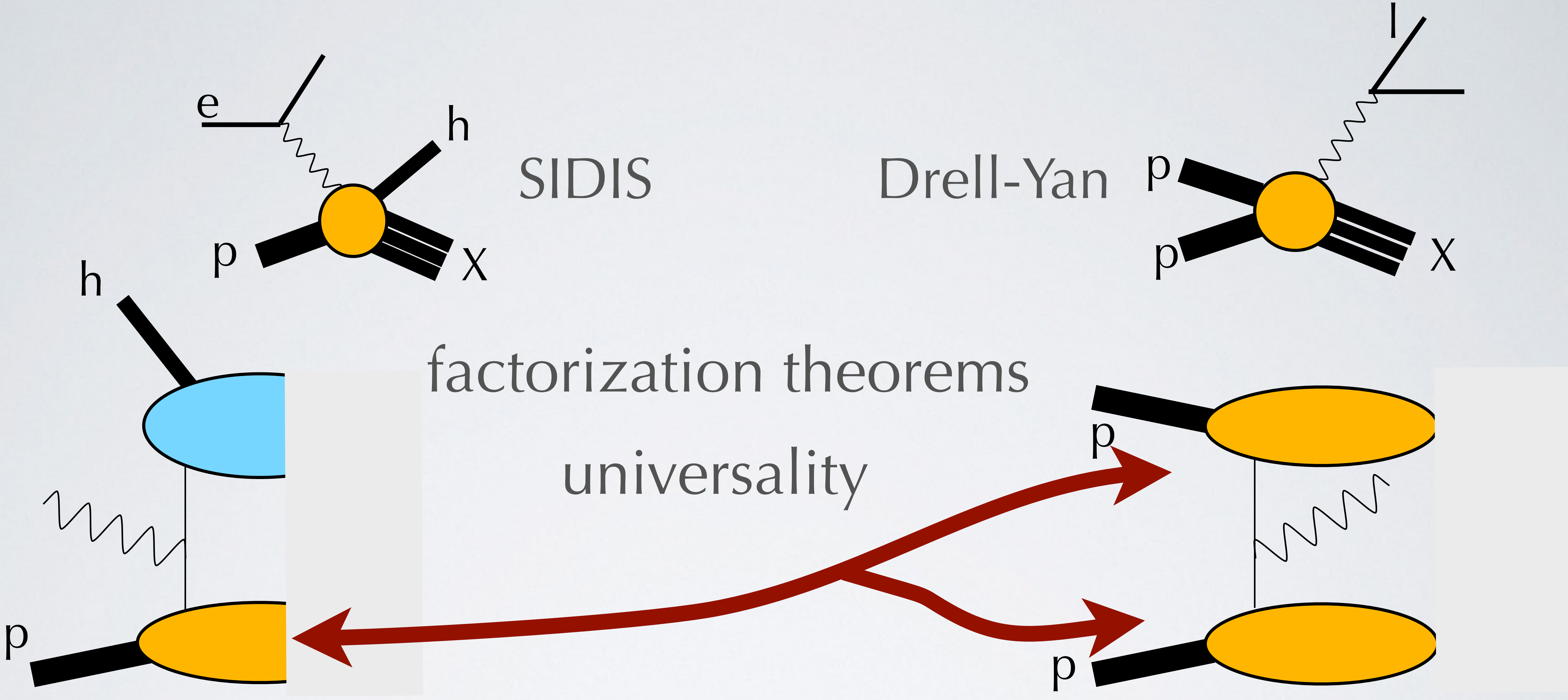


factorization theorems





Reason #2 for the Sivers function



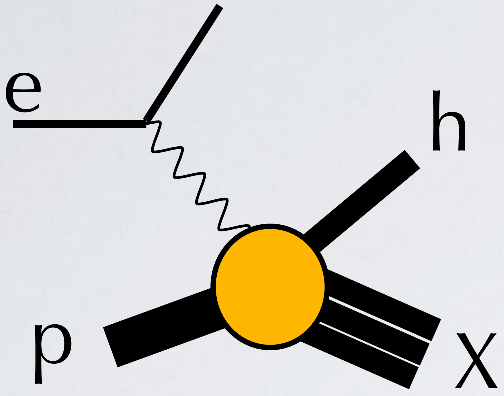
SIDIS

Drell-Yan

factorization theorems

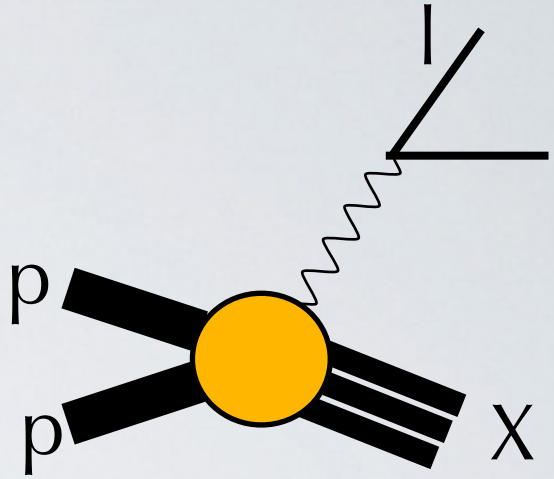
universality

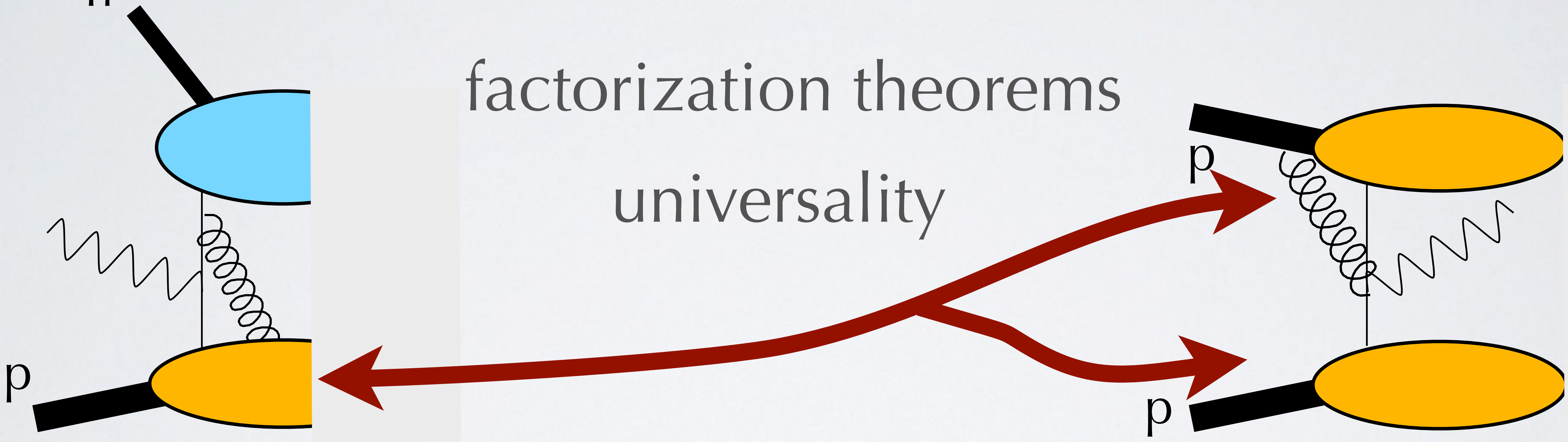
Reason #2 for the Sivers function



SIDIS

Drell-Yan





factorization theorems

universality

“Final” residual color interactions “Initial”

QCD prediction to be tested:
(at COMPASS)

$$\text{Sivers}\Big|_{\text{SIDIS}} = -\text{Sivers}\Big|_{\text{D-Y}}$$

*J.C. Collins, P.L. B**536** (02)*

“roadmap” to a multi-dim. picture of N

“Electron Ion Collider: the Next QCD Frontier”
arXiv:1212.1701 [nucl-ex]

GPD(x, ξ=0, t=-$\mathbf{q}^2$)

“roadmap” to a multi-dim. picture of N

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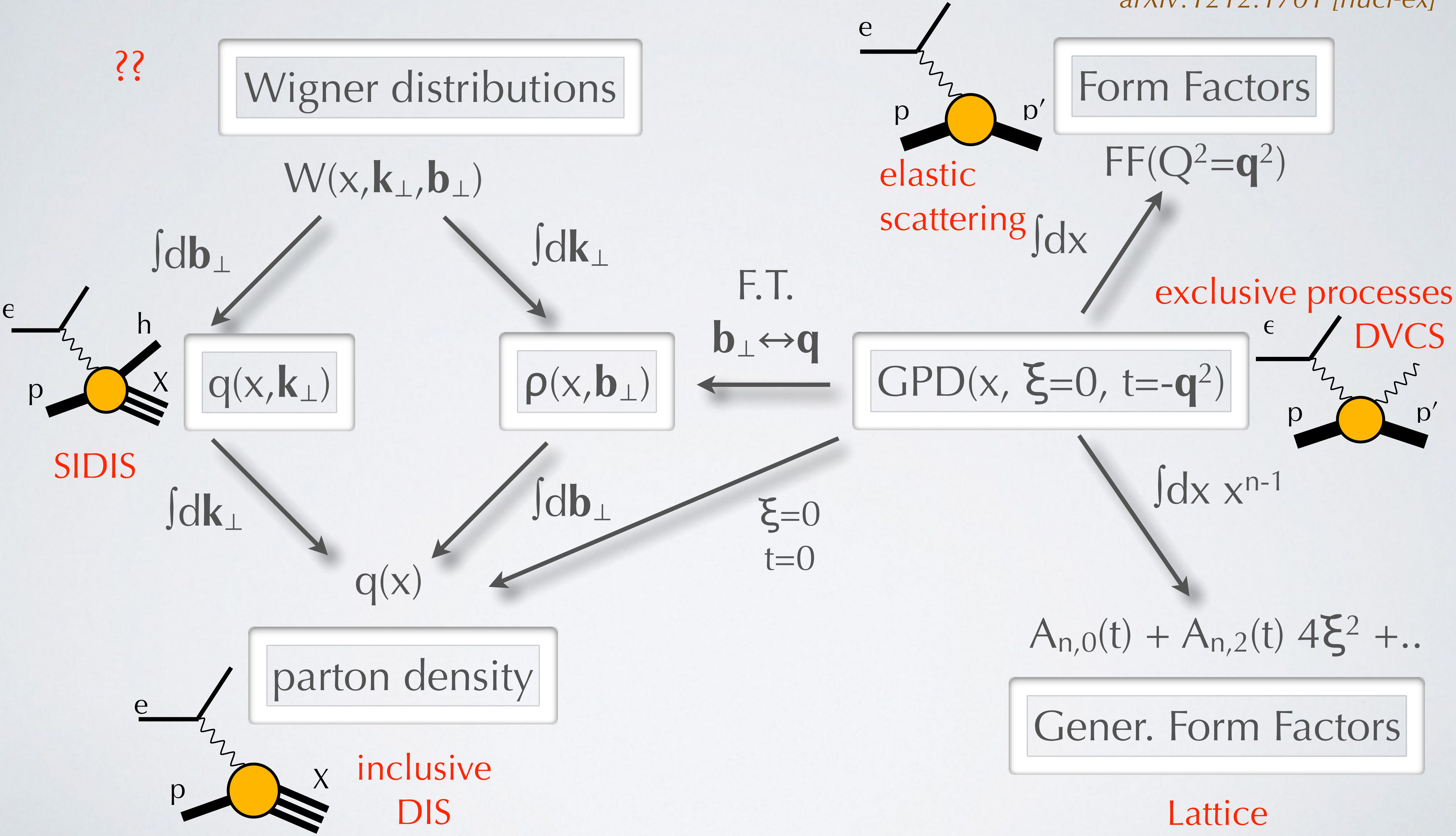


??

Wigner distributions

$W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

$\int d\mathbf{b}_\perp$

$\int d\mathbf{k}_\perp$

Form Factors

$FF(Q^2=\mathbf{q}^2)$

elastic scattering

$\int dx$

exclusive processes

DVCS

F.T.

$\mathbf{b}_\perp \leftrightarrow \mathbf{q}$

$q(x,\mathbf{k}_\perp)$

$\rho(x,\mathbf{b}_\perp)$

$GPD(x,\ \xi=0,\ t=-\mathbf{q}^2)$

SIDIS

$\int d\mathbf{k}_\perp$

$\int d\mathbf{b}_\perp$

$\xi=0$
$t=0$

$\int dx\ x^{n-1}$

$q(x)$

$A_{n,0}(t) + A_{n,2}(t)\ 4\xi^2 + ..$

parton density

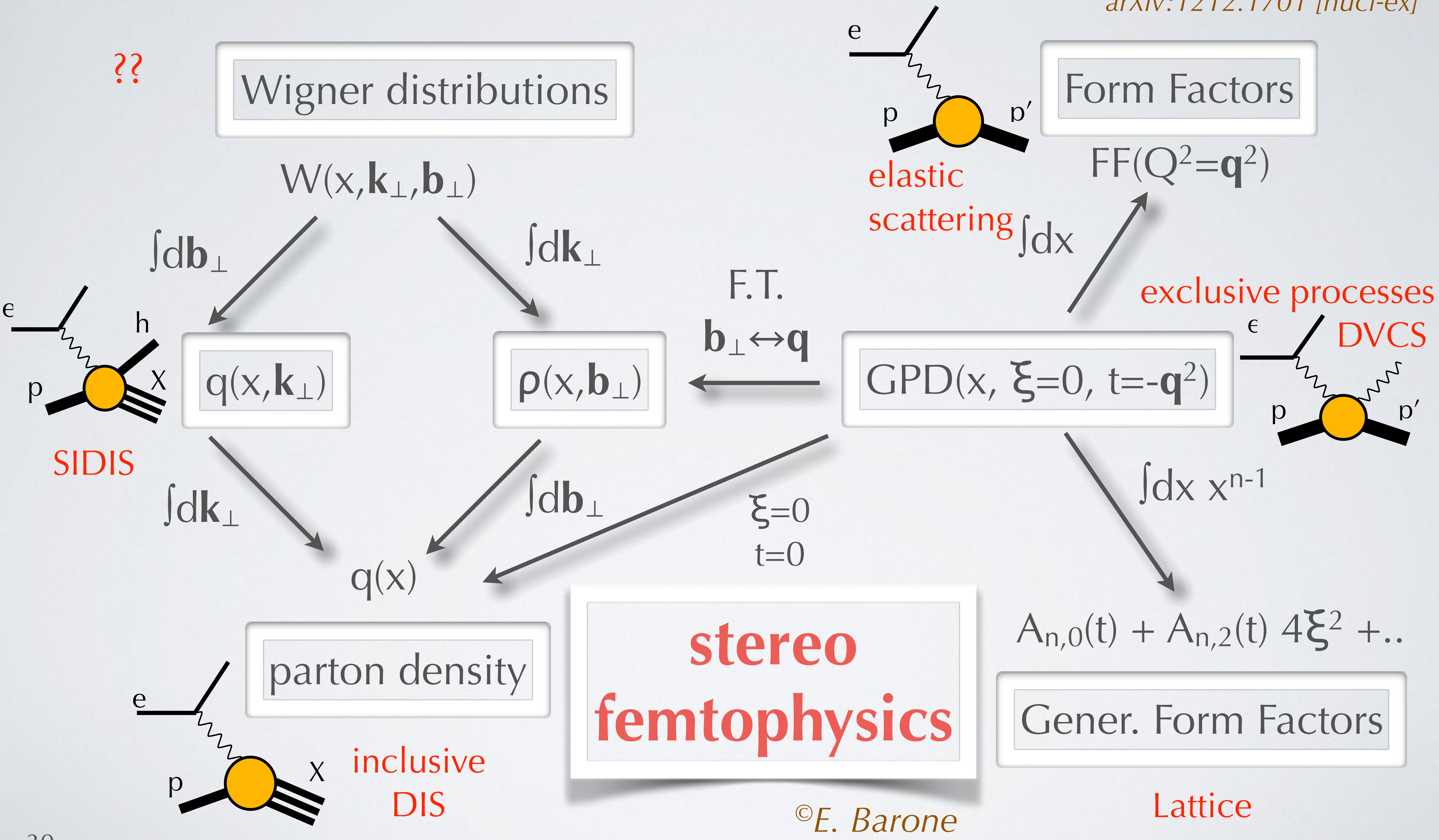
Gener. Form Factors

inclusive DIS

Lattice

“roadmap” to a multi-dim. picture of N

“Electron Ion Collider: the Next QCD Frontier”
arXiv:1212.1701 [nucl-ex]



??

Wigner distributions

$W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$

$\int d\mathbf{b}_\perp$

$\int d\mathbf{k}_\perp$

SIDIS

$q(x,\mathbf{k}_\perp)$

$\rho(x,\mathbf{b}_\perp)$

F.T.

$\mathbf{b}_\perp \leftrightarrow \mathbf{q}$

$\mathrm{GPD}(x,\ \xi{=}0,\ t{=}{-}\mathbf{q}^2)$

elastic scattering

Form Factors

$\mathrm{FF}(Q^2{=}\mathbf{q}^2)$

$\int dx$

exclusive processes

DVCS

$\int d\mathbf{k}_\perp$

$\int d\mathbf{b}_\perp$

$\xi{=}0$
$t{=}0$

$\int dx\ x^{n-1}$

$q(x)$

parton density

inclusive DIS

stereo femtophysics

$A_{n,0}(t) + A_{n,2}(t)\ 4\xi^2 + ..$

Gener. Form Factors

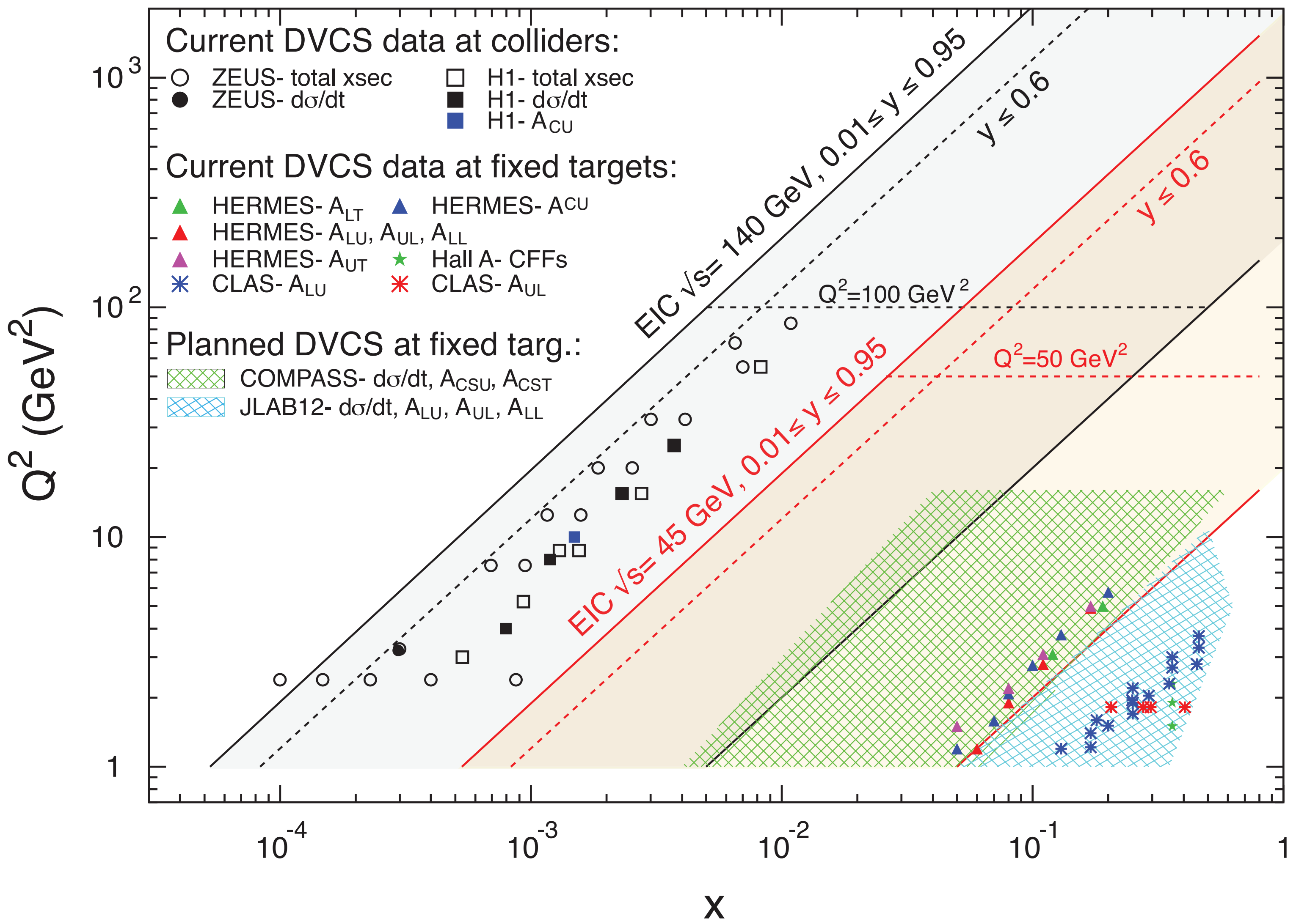
Lattice

©*E. Barone*

future directions ?

*"Electron Ion Collider:
the Next QCD Frontier"
arXiv:1212.1701 [nucl-ex]*

example : DVCS data



future directions ?

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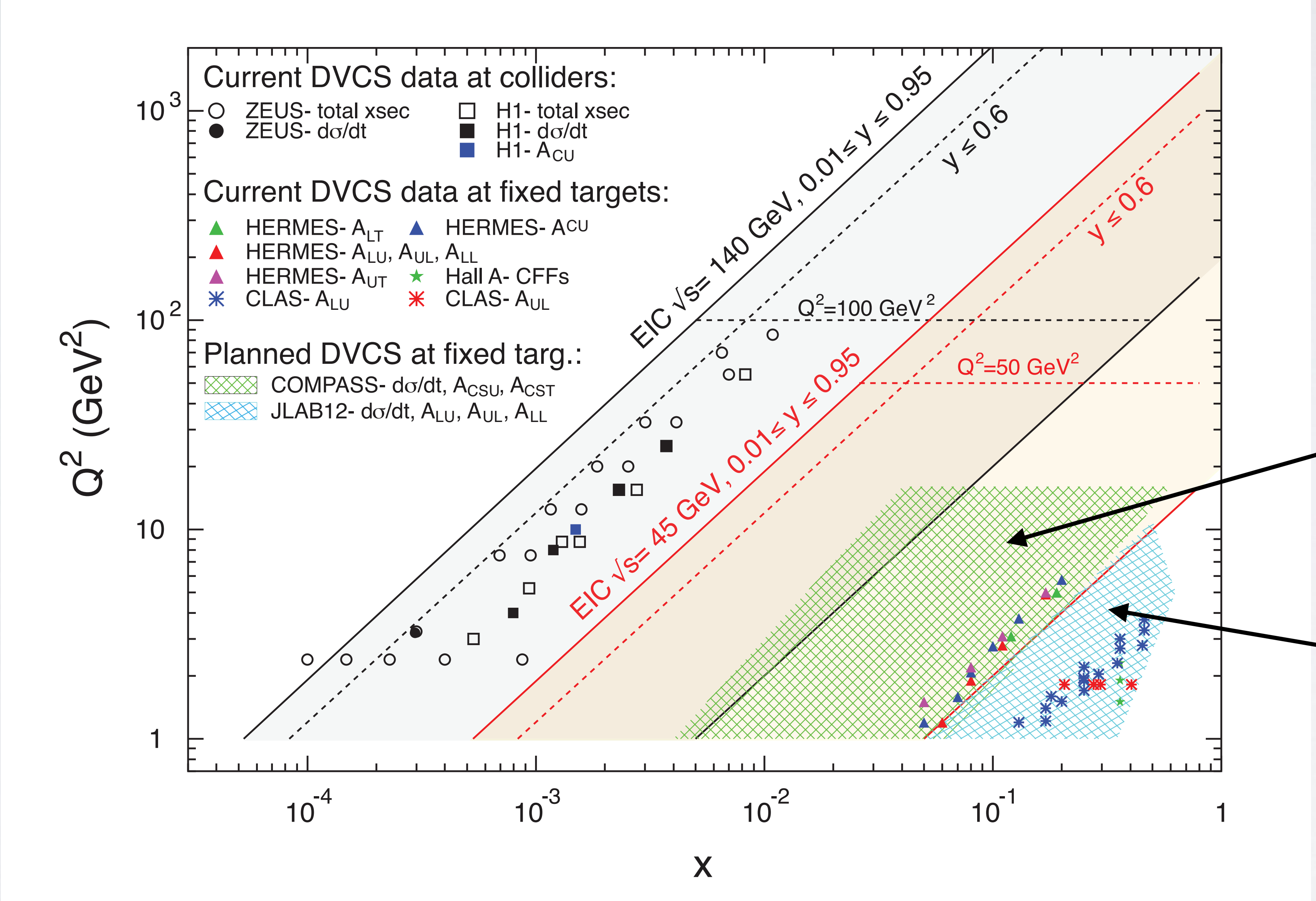
arXiv:1212.1701 [nucl-ex]

example : DVCS data



COMPASS

also
polarized
Drell-Yan

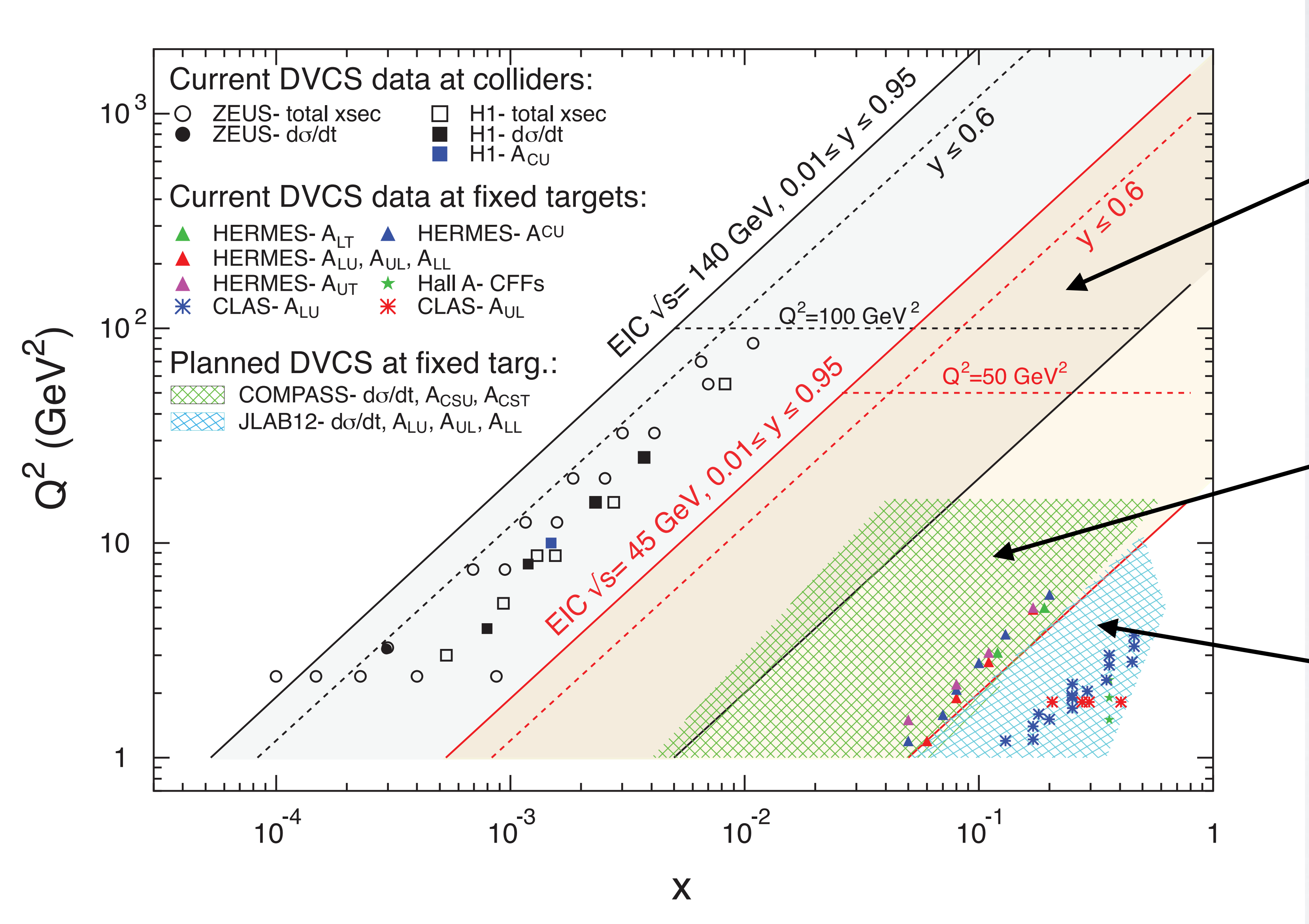
Jefferson Lab @12 GeV

valence x

future directions ?

"Electron Ion Collider:
the Next QCD Frontier"
arXiv:1212.1701 [nucl-ex]

example : DVCS data



EIC

small x

COMPASS

also polarized Drell-Yan

Jefferson Lab @12 GeV

valence x

LHeC even smaller x, but no polarization…

> “With 3D projections, we will be entering a new age. Something which was never technically possible before: a stunning visual experience which ‘turbocharges’ the viewing.”
>
> James Cameron