

Some examples on probability and distributions

1) Histogram some uniform random numbers

2) A hitchhiker stands by the road where cars pass randomly (according to a Poisson distribution) at a rate of 1 per minute.

What is the probability they are still waiting (1) after 60 cars have passed (2) after 1 hour.

3) Why does the sum of 12 uniform $[0, 1]$ random numbers, minus 6, give (to an approximation) a random number distributed according to the unit Gaussian.

Try it and see.

4) In a search experiment you see 0 events. (Often happens!)

For what Poisson mean μ is $P(0; \mu) = 5\%$?

What can you say, using frequentist probability, with 95% confidence?

5) For the situation in Question 4, what can you say using Bayesian probability, if you assume a prior which is uniform in μ

What can you say if you assume a prior uniform in $\ln \mu$?

6) Use the inversion method to generate a probability distribution that generates numbers between 0 and 10, with a density that increases linearly from 0 at zero.

Do the same thing using the von Neumann technique.

7) You have several estimates x_i of the same quantity X , each with (in principle) different errors σ_i .

If these are Gaussian measurements, show that the Maximum Likelihood estimator \hat{X} is $(\sum x_i w_i) / \sum w_i$ with $w_i = \frac{1}{\sigma_i^2}$

You repeat a counting experiment 3 times, and get results 4,5, and 6 counts. What is the ML estimated mean?