Probing the thermal character of analogue Hawking radiation for shallow water waves?

Comparison with Vancouver experiment

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Based on:

arXiv:1404.7482

conversations with Germain Rousseaux

Analogue Gravity. I. Pioneering papers

- 1981. W. Unruh, PRL *"Experimental BH evaporation ?"*.
  Analogy btwn sound and light propagation in curved space
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  - the geometrical (IR) effects, and
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    are combined in a **single wave equation**.
  - Numerically showed the robustness of HR
    - **when** the dispersive UV scale $\Lambda \gg \kappa$,
    - **when** there is a horizon ($v = c$)

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Aims

Study and numerically solve the Unruh 2012 wave equation, in order to

- compare the solutions with their observations,
- better understand the role of the horizon $v = c$, and its absence $v < c$,
- make new predictions,
- guide new experiments.

Plan

- I. Wave equation, and turning point,
- II. Scattering in trans-critical flows,
- III. Scattering in sub-critical flows,
- IV. Comparison with Vancouver observations,
- V. Improved flow profile, future experiments ?

I. Wave equation

In an irrotational laminar flow of an inviscid, incompressible fluid, linear surface waves obey

$$\left[(\partial_t + \partial_x v)(\partial_t + v\partial_x) - ig\partial_x \tanh(-ih\partial_x)\right]\phi = 0, \qquad (1)$$

- $v(x,t)$ is the horizontal component of the flow velocity,
- $h(x,t)$ the background fluid depth,
- g the gravitational acceleration.

The dispersion relation is

$$\Omega^2 \equiv (\omega - vk_\omega)^2 = gk\tanh(hk). \qquad (2)$$

The linear perturbation of the velocity potential ϕ is related to the linear variation of the water depth by

$$\delta h(t,x) = -\frac{1}{g}(\partial_t + v\partial_x)\phi. \qquad (3)$$

I. Wave equation, quartic dispersion relation

- In stationary flows, work with (complex) stationary waves $e^{-i\omega t}\phi_\omega(x)$ with fixed lab. frequency ω.

- We expand to lowest non-trivial order in $h\partial_x$, and work with

$$\left[(-i\omega + \partial_x v)(-i\omega + v\partial_x) - g\partial_x h\partial_x - \frac{g}{3}\partial_x (h\partial_x)^3\right]\phi_\omega = 0. \quad (4)$$

Notice that the ordering of $h(x)$ and ∂_x has been preserved.

- The quartic dispersion relation is

$$\Omega_\omega^2 = (\omega - vk_\omega)^2 = c^2 k_\omega^2 \left(1 - \frac{h^2 k_\omega^2}{3}\right), \quad (5)$$

$c^2(x) = gh(x)$: the local group velocity^2 for low $k_\omega(x)$ waves

I. Wave equation, hydrodynamics, and black (white) hole metric

- In the hydrodynamical approximation, one neglects $(hk)^2 \ll 1$
- Then the wave eq.

$$\left[(-i\omega + \partial_x v)(-i\omega + v\partial_x) - g\partial_x h\partial_x - \frac{g}{3}\partial_x (h\partial_x)^3\right]\phi_\omega = 0$$

is (essentially) a Klein-Gordon in a 2D space-time metric

$$ds^2 = -c(x)^2 dt^2 + (dx - v(x)dt)^2,$$

- There is an analogue event horizon when $v(x)$ crosses $c(x)$.
- if v increases (*decreases*) along v, one gets a black (*white*) horizon, i.e., a decrease (*increase*) of the wave number k_ω of counter-prop. waves

Flow profiles: monotonic and non-monotonic ones

- In experim., it is fixed by the obstacle on the flume bottom.
- mathem., the flow profile can be fixed by the **water depth** $h(x)$, because at fixed flux J, one has

$$v(x) = J/h(x), \quad c(x) = \sqrt{g\,h(x)}. \tag{6}$$

- **Monotonic flows** are parameterized by

$$h(x) = h_0 + D \tanh \frac{\sigma x}{D}. \tag{7}$$

- The maximum slope of $h = \sigma$ is located at $x = 0$,
- D fixes the asymptotic height change $\Delta h = 2D$,

- **Non-monotonic flows** are

$$h_{\rm non-m}(x) = h_0 + D \tanh(\frac{\sigma_1}{D}(x+L)) \tanh(\frac{\sigma_2}{D}(x-L)), \tag{8}$$

where $2L$ gives the spatial extension of the flat minimum of h.

Trans- and sub-critical flow profiles, I.

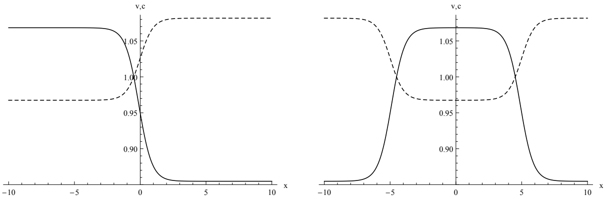
- the trans-critical character fixed by **"Froude number"** $F \equiv v/c$
- In units $g = J = 1$, one has

$$F(x) = \frac{1}{h(x)^{3/2}}, \tag{9}$$

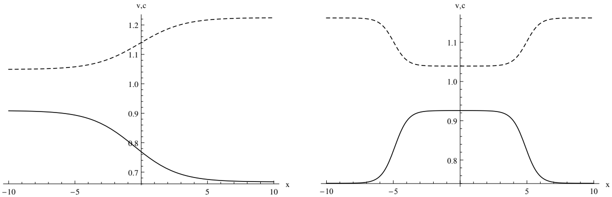
- For $h(x) < 1$, $F(x) > 1$ and the flow is transcritical.
- A **"phase velocity horizon"** corresponds to $v = c = h = F = 1$.
- The **surface gravity** $\kappa_G = |\partial_x(c - v)|_{v=c}$ reads

$$\kappa_G = |\partial_x F|_{F=1} \propto \sigma. \tag{10}$$

Trans- and sub-critical flow profiles, II.



trans-critical monotonic, and trans-critical non-monotonic flows.
velocity $v(x)$ (plain), and speed $c(x)$ (dashed)



sub-critical monotonic, and sub-critical non-monotonic flows.

Turning point and characteristics. I

- The properties of the scattering matrix is highly sensitive to the **presence/absence** of **turning points**.
- Turning points are double roots of the dispersion relation

$$(\omega - vk)^2 = c^2 k^2 (1 - k^2 h^2 / 3).$$

- the corresponding ω obeys

$$\omega_{\rm tp}(x) = \frac{c}{h} \sqrt{\frac{6\,(1-F^2)^3 \left(|F| + \sqrt{F^2+8}\right)}{\left(3\,|F| + \sqrt{F^2+8}\right)^3}}, \qquad (11)$$

where $\omega_{\rm tp}$, $c = (gh)^{1/2}$ and $F = J/(gh^3)^{1/2}$ are functions of $h(x)$.
NB. c/h is the dispersive "UV" frequency.

- All turning points are located in the **sub-critical domain** $F < 1$.

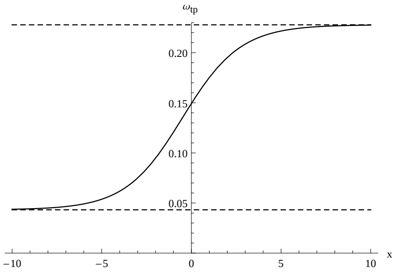
Turning point and characteristics, II.

- Given ω, Eq.(11) gives the location of $x_{tp}(\omega)$ through

$$\omega_{tp}(x_{tp}) = \omega. \tag{12}$$

- For monotonic flows, the max. and min. values of ω_{tp} are

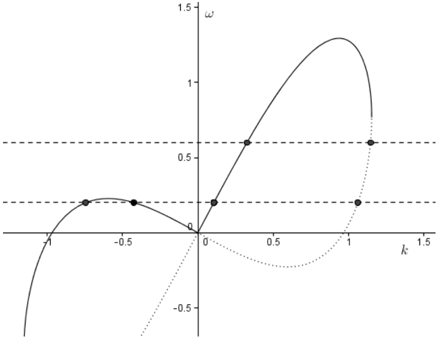
$$\omega_{max} = \omega_{tp}(x = \infty), \quad \omega_{min} = \omega_{tp}(x = -\infty), \tag{13}$$



Locii of $x_{tp}(\omega)$ for a monotonic, sub-critical flow.

Turning point and characteristics, III.

The **number** of real roots in the **left (high v) subsonic region** is **4 for** $0 < \omega < \omega_{\min}$, but only **2 for** $\omega_{\min} < \omega < \omega_{\max}$.



Dispersion relation ω vs k, in a 'high' v subsonic region.
$\Omega^2 = (\omega - vk)^2 = c^2k^2(1 - k^2h^2/3)$, Plain $\Omega > 0$, **dashed** $\Omega < 0$

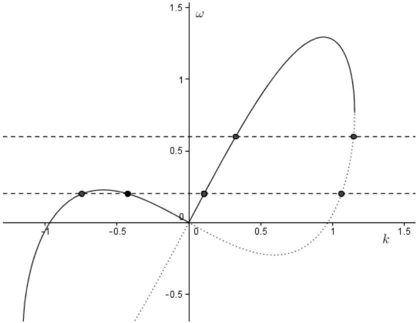
NB. Only the fourth highest k root lives on the **negative Ω branch**.

Asymptotic modes

For $0 < \omega < \omega_{\min}$, **the 4 k_ω are real**, and the asympt. 4 modes are

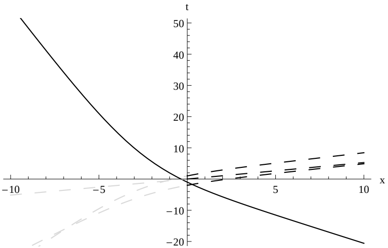
- $\phi_\omega^{\rightarrow,d}$ is dispersive and right-moving in the lab frame;
- $\phi_\omega^{\leftarrow}$ is hydrodynamic, and left-moving;
- $\phi_\omega^{\rightarrow}$ is hydrodynamic, and right-moving;
- $(\phi_{-\omega}^{\rightarrow,d})^*$ is dispersive, and right-moving.

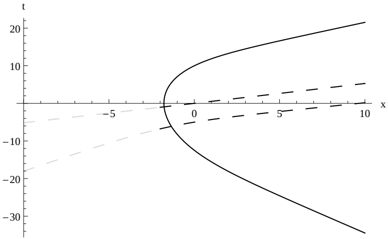
The last one has a **negative (Klein-Gordon) norm.**
(*Its corresponding root lives on the* **negative** Ω *branch.*)



Turning point and characteristics, IV.

The characteristics are solutions of Hamilton's eqs.:
$dx/dt = 1/\partial_\omega k_\omega$, $dk/dt = -1/\partial_\omega X_\omega$. (They give the locus of constructive interferences for WKB wave packets)





Left panel, for $0 < \omega < \omega_{\rm min}$, **there is no turning point,**
Right panel, for $\omega_{\rm min} < \omega < \omega_{\rm max}$, there is one.

Hence, for $0 < \omega < \omega_{\rm min}$, there are **four** (bounded) modes ϕ_ω whereas for $\omega_{\rm min} < \omega < \omega_{\rm max}$ there are only **3** (bounded) modes.

Mode mixing: 4X4 modes and 3X3

- For $0 < \omega < \omega_{\min}$, **four modes**, hence an *in* mode (from the left)

$$\phi_\omega^{\leftarrow,in} \to \alpha_\omega\, \phi_\omega^{\to,d,out} + \beta_\omega\, (\phi_{-\omega}^{\to,d,out})^* + \tilde{A}_\omega\, \phi_\omega^{\leftarrow,out} + A_\omega\, \phi_\omega^{\to,out}, \quad (14)$$

where "unitarity" gives

$$|\alpha_\omega|^2 - |\beta_\omega|^2 + |A_\omega|^2 + \left|\tilde{A}_\omega\right|^2 = 1. \quad (15)$$

- For $\omega_{\min} < \omega < \omega_{\max}$, **three modes**, :

$$\phi_\omega^{\leftarrow,in} \to \alpha_\omega \phi_\omega^{\to,d,out} + \beta_\omega (\phi_{-\omega}^{\to,d,out})^* + A_\omega \phi_\omega^{\to,out}, \quad (16)$$

no transmitted wave $\phi_\omega^{\leftarrow,out}$ and norm conservation now gives

$$|\alpha_\omega|^2 - |\beta_\omega|^2 + |A_\omega|^2 = 1. \quad (17)$$

- $-|\beta_\omega|^2$, **minus sign**, hence anomalous mode mixing, super-radiance, over-reflection, pair creation, ...

Hawking radiation

- In 1974, Hawking found

$$n_\omega^{out} = \left|\beta_\omega^{\text{Black Hole}}\right|^2,$$

neglecting gray body factors, Planck spectrum:

$$|\beta_\omega^{\text{Black Hole}}|^2 = (e^{\omega/T_H} - 1)^{-1},$$

governed by the Hawking temperature,

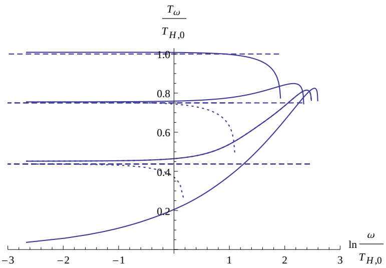
$$k_B T_H = \hbar\kappa/2\pi.$$

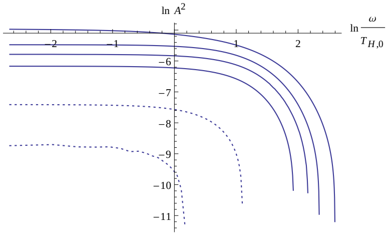
- We introduce the **effective temperature T_ω**

$$|\beta_\omega|^2 = (e^{\omega/T_\omega} - 1)^{-1}, \tag{18}$$

- **to compare**
  - T_ω numerically computed from the Unruh 2012 wave eq. to
  - $T_H = \kappa/2\pi$, where $\kappa = \partial_x F$ evaluated at $F = 1$

Monotonic trans-critical flows





- On the left,
  - in plain $T_\omega / T_{H,0}$ for 3 transcritical flows, $h_0 = 1, 1.1, 1.15$, and
  - a critical one with $F_{\max} = 1$.
  - the three horizontal dashed lines give T_H,
  - the two dotted curves, for $h_0 = 0.9$, and 0.85, i.e., higher $F_{\max}$.
- Right: logarithm of the reflexion coefficient $|A_\omega|^2$.
- Lessons →

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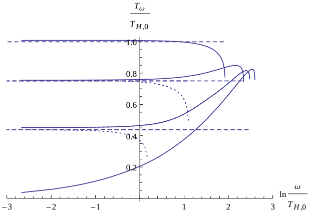

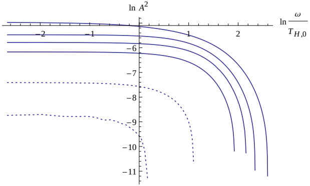

Lessons:

- when $F_{\rm max} - 1 \geq 0.1$, to a **good approx.**, the spectrum **is**
  - Planckian up to $\sim \omega_{\rm max}$, and
  - at the Hawking temperature $\kappa/2\pi$.
- in addition, $|A_\omega|^2 < e^{-5}$, hence $|\alpha_\omega|^2 - |\beta_\omega|^2 \simeq 1$.
- Hence, **in these conditions**, **the scat. of shallow water waves do follow Hawking's prediction**.
- Instead, when $F_{\rm max} - 1 \to 0$, **Planckianity is lost**.

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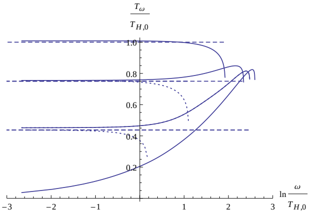

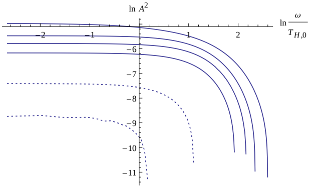

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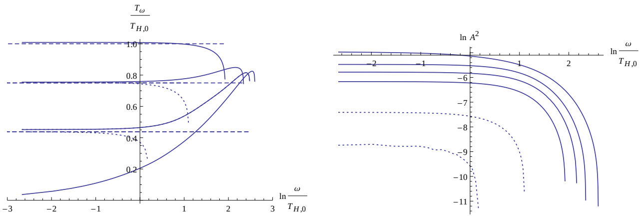

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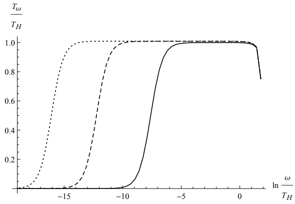
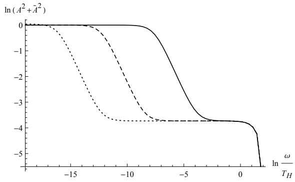
Non-monotonic trans-critical flows

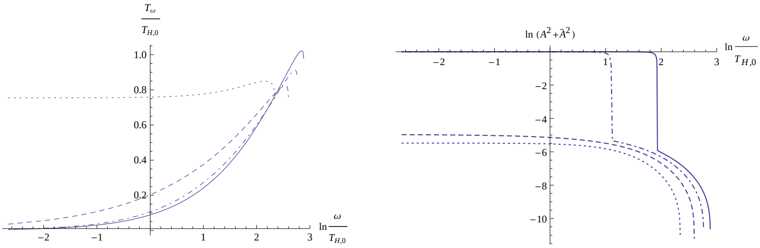
- Because the flow is asympt. sub-crit. on both sides, **four mode mixing**, the transmission coef. $\tilde{A}_\omega \neq 0$.
- Spectra for three different $L = 5$ (solid), 7 (dashed), and 10 (dotted)





- Except at ultra-low freq., no significant changes wrt to monot. flows.
- the critical low freq. $\omega_c \ll T_H$, hence **not relevant**.
- **Lessons:**
  - these can be used to test Hawking's prediction, with accuracy.
  - for ultra low freq., $|\beta_\omega|^2 \to 0$ and $\tilde{A}_\omega \to 1$, **total transmission**.

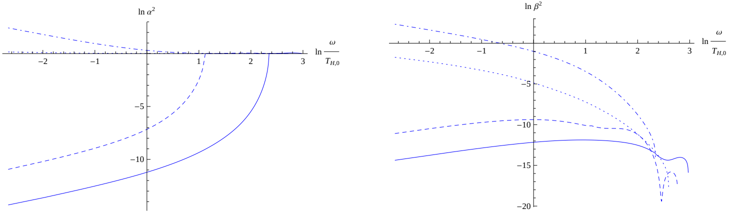
Monotonic sub-critical flows



Four flows with $F_{\max}$: 0.75 (solid), 0.87 (dot-dashed): both **subcritical**; $F_{\max} = 1.0$ critical (dashed); and $F_{\max} = 1.17$ transcritical (dotted).

For the two **subcritical ones**:

- four modes mixing for $0 < \omega < \omega_{\min}$, three for $\omega_{\min} < \omega < \omega_{\max}$.
- **above** $\omega_{\min}$, same behavior as for *slightly* trans-critical,
- **below** $\omega_{\min}$, $T_\omega \to 0$, and $|\tilde{A}_\omega|^2 \to 1$, $\to$ **total transmission**



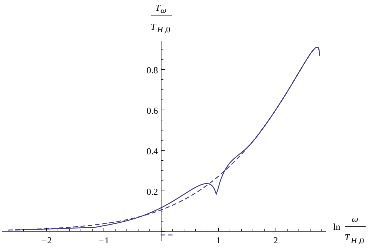
Same flows with $F_{\rm max} := 0.75$ (solid), and 0.87 (dot-dashed): both **subcritical**;
$F_{\rm max} = 1.0$ critical (dashed); and $F_{\rm max} = 1.17$ transcritical (dotted).

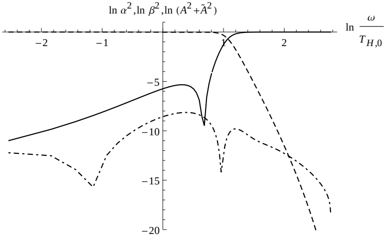
For the two subcritical ones: **main lessons**

- for $\omega < \omega_{\rm min}$, $|\alpha_\omega|^2$ **drop below 1**, because **no t.p.**
- for all ω, $|\beta_\omega|^2 \lesssim e^{-9}$, $\rightarrow$ **no** signif. mode amplification.
- for $\omega \rightarrow 0$, they **both** obey

$$|\beta_\omega|^2 \sim |\alpha_\omega|^2 \sim \omega \times e^{-(\frac{D}{\sigma h_0} F_{\rm max}^{-1/3})^2}.$$

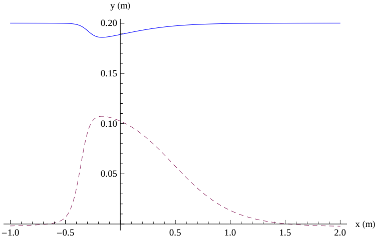
Non-monotonic sub-critical flows

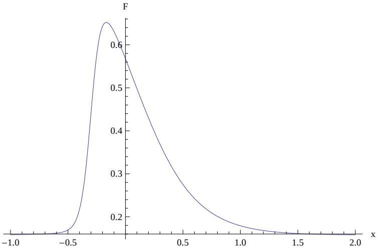




Main lesson:
NO significant difference wrt to monotonic subcritical flows.
They might be resonances, since there is a "cavity".

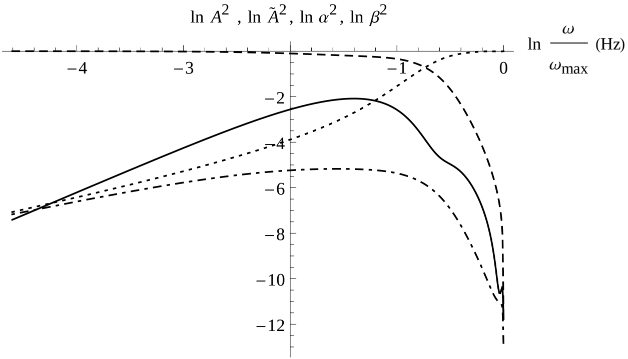
Vancouver experiment. I. the background flow





- On the left, the free surface (plain), and the obstacle (dashed).
- On the right, $F(x) = v(x)/c(x)$. The maxim. is $\simeq 0.7$, significantly less than 1, hence no horizon, no white hole.
- yet, observation of
  - wave blocking, **as if** no transmission, and
  - $R = |\beta_\omega|^2/|\alpha_\omega|^2 \sim e^{-\omega/\omega_V}$ **as if** Planck spectrum

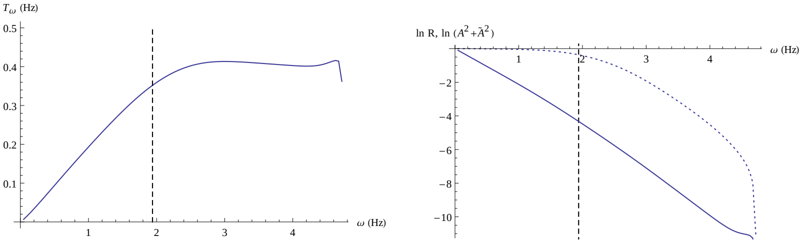
Vancouver experiment. II. Scattering coefficients



Log. of $|\alpha_\omega|^2$ (dotted), $|\beta_\omega|^2$ (dot-dashed), $|\tilde{A}_\omega|^2$ (dashed), and $|A_\omega|^2$ (solid), as functions of $\ln \omega/\omega_{\max}$ for the V. flow

- NB. $\omega_{\min}/\omega_{\max} \simeq 2/5$, $\omega_{\max} \simeq 5Hz$.
- for $\omega < \omega_{\min}$ one recovers the severe drop of $|\alpha_\omega|^2$ below 1,
- one also recovers $|\beta_\omega|^2 \lesssim e^{-5} \ll 1$ for all ω.

Vancouver experiment. III. Spectra



- Left, effective temperature T_ω. It vanishes for $\omega \to 0$. (Vertical line $\omega = \omega_{\rm min}$) **Not reported in V.**.
- Right, solid, $\ln R_\omega \equiv \ln |\beta_\omega|^2/|\alpha_\omega|^2$. Essentially linear in ω, **as if** a thermal spectrum. **Observed in Vancouver, with a slope in agreement of** 30%**.**

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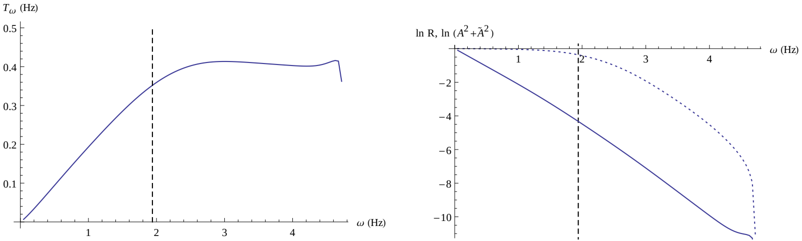

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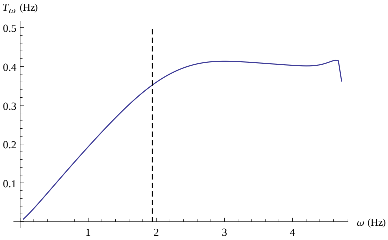

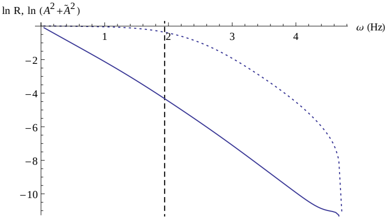

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  Essentially linear in ω, **as if** a thermal spectrum.
  Observed in Vancouver, with a slope in agreement of 30%.

Explanations, T_ω vs R_ω

- the effect. T_ω^n defined by $n_\omega = |\beta_\omega|^2 = 1/(e^{\omega/T_\omega^n} - 1)$, and
- that defined from $R_\omega \equiv |\beta_\omega|^2/|\alpha_\omega|^2 = e^{-\omega/T_\omega^R}$
- agree iff $|\alpha_\omega|^2 - |\beta_\omega|^2 = 1$ to a good precision,
  (which is **not** fulfilled in **sub-critical flows**)

- ln R_ω is always linear in ω for $\omega \to 0$.
  because $|\alpha_{-\omega}|^2 = |\beta_\omega|^2$ (i.e., crossing symmetry)
- as a result, ln R_ω cannot be used to establish thermality.

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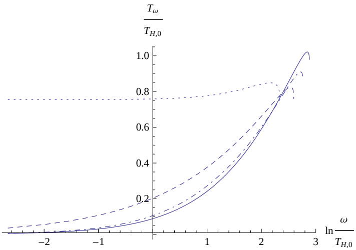
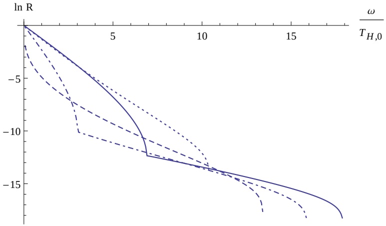
Explanations, T_ω vs R_ω

Compare the behavior of T_ω and R_ω for the above four flows

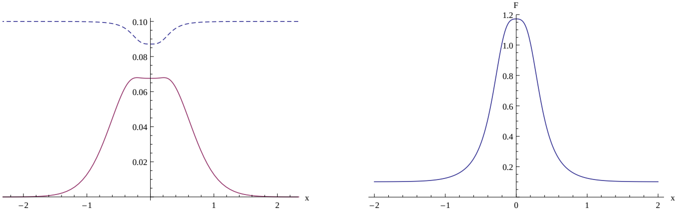




Four flows with $F_{\max}$: 0.75 (solid), 0.87 (dot-dashed): both **subcritical**; $F_{\max} = 1.0$ critical (dashed); and $F_{\max} = 1.17$ transcritical (dotted).

- On the left, T_ω shows that Planckianity is **lost for sub-crit. fl.**
- From the right plot, it is clear that R_ω is oblivious to this loss.
- Hence R_ω cannot be "trusted"

In reality, I do not know; but in principle, yes:

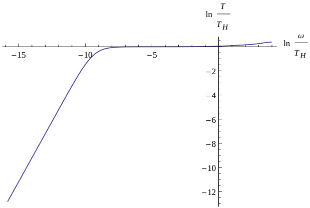


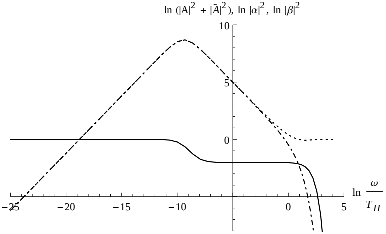
Free surface (dashed), obstacle (solid) (Left), and $F(x)$ (Right) obtained by **solving the non-linear hydrodynamical equations**. $F_{\max} = 1.17$.

Notice that there is no *undulation*.

Conclusions: can one improve the experiment ? II

The corresponding spectra:





Left, $\ln(T_\omega / T_H)$ as a function of $\ln(\omega / T_H)$. $T_H \simeq 0.164\ Hz$.
Very good agreement with Hawking's prediction.

Right, log of $|\alpha_\omega|^2$ (dashed), $|\beta_\omega|^2$ (dot-dashed), and $|A_\omega|^2 + |\tilde{A}_\omega|^2$ (solid).
$|\alpha_\omega|^2$ **(dashed),** $|\beta_\omega|^2$ **both dominate in a wide** ω **range.**

Can this type of flows be realized, and used to test the Hawking effect ?