Testing the Standard Model with the lepton g-2

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Preamble: today's values

$$a_e = 115965218 07.3\ (2.8) \times 10^{-13}$$

0.24 parts per billion !! (Hanneke et al., PRL100 (2008) 120801)

$$a_\mu = 116592089\ (63) \times 10^{-11}$$

0.5 parts per million !! (E821 – Final Report: PRD73 (2006) 072003)

$$a_\tau = -0.018\ (17)$$

Well, not much yet.... (PDG 2013)

Outline

- 1. Lepton magnetic moments: the basics
- 2. μ: The muon g-2: a quick update
- 3. e: Testing new physics with the electron g-2
- 4. τ: The tau g-2: opportunities & challenges (fantasies?)

1. Lepton magnetic moments: the basics

The beginning: g = 2

- **Uhlenbeck and Goudsmit in 1925 proposed:**

$$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$
$$g = 2 \quad (\text{not } 1!)$$

- **Dirac 1928:**

$$(i\partial_\mu - eA_\mu)\gamma^\mu \psi = m\psi$$

- **A Pauli term in Dirac's eq would give a deviation…**

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1+a)$$

…but there was no need for it! g=2 stood for ~20 yrs.

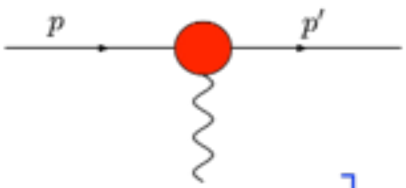
- **Kusch and Foley 1948:**

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc}\left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**



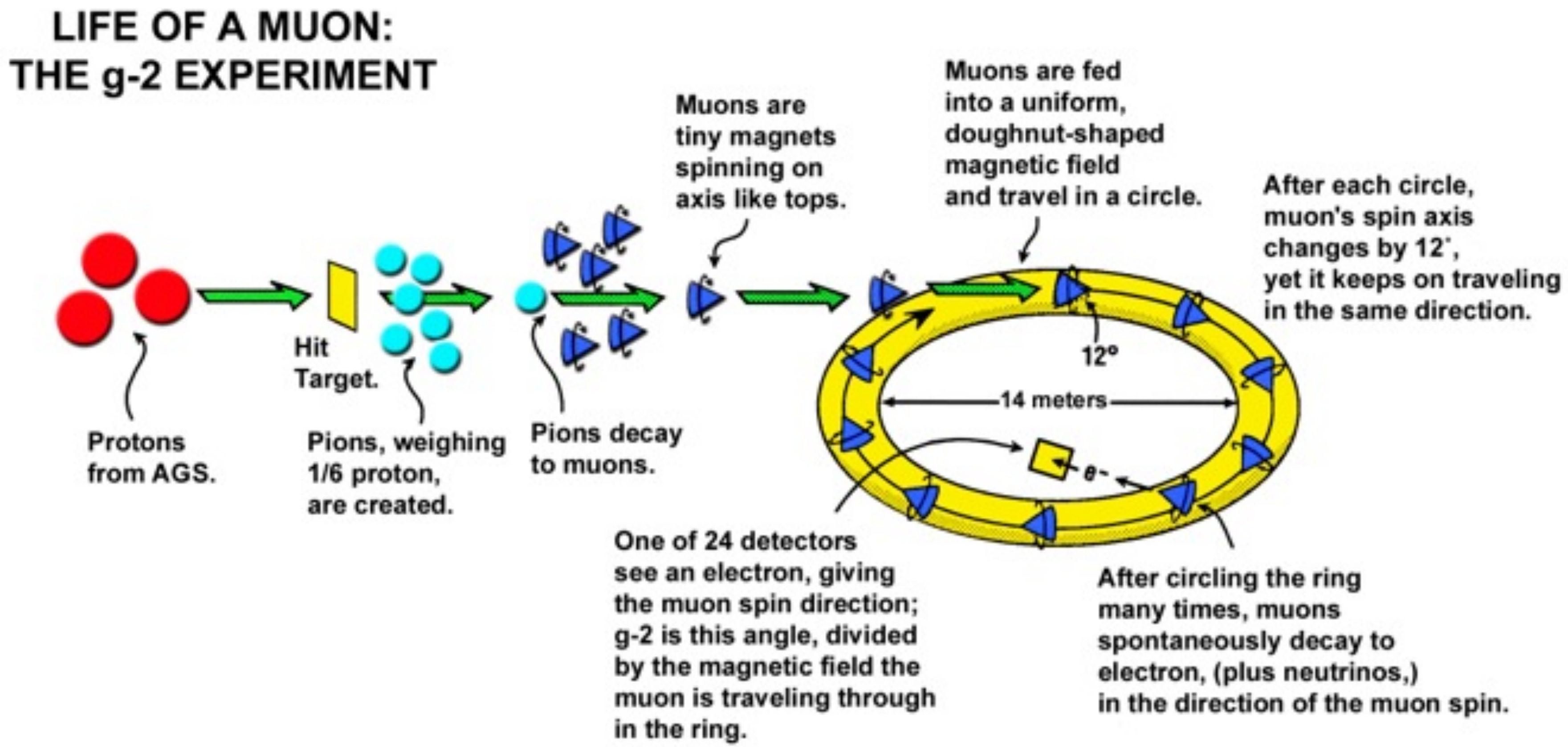
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

A pure “quantum correction” effect!

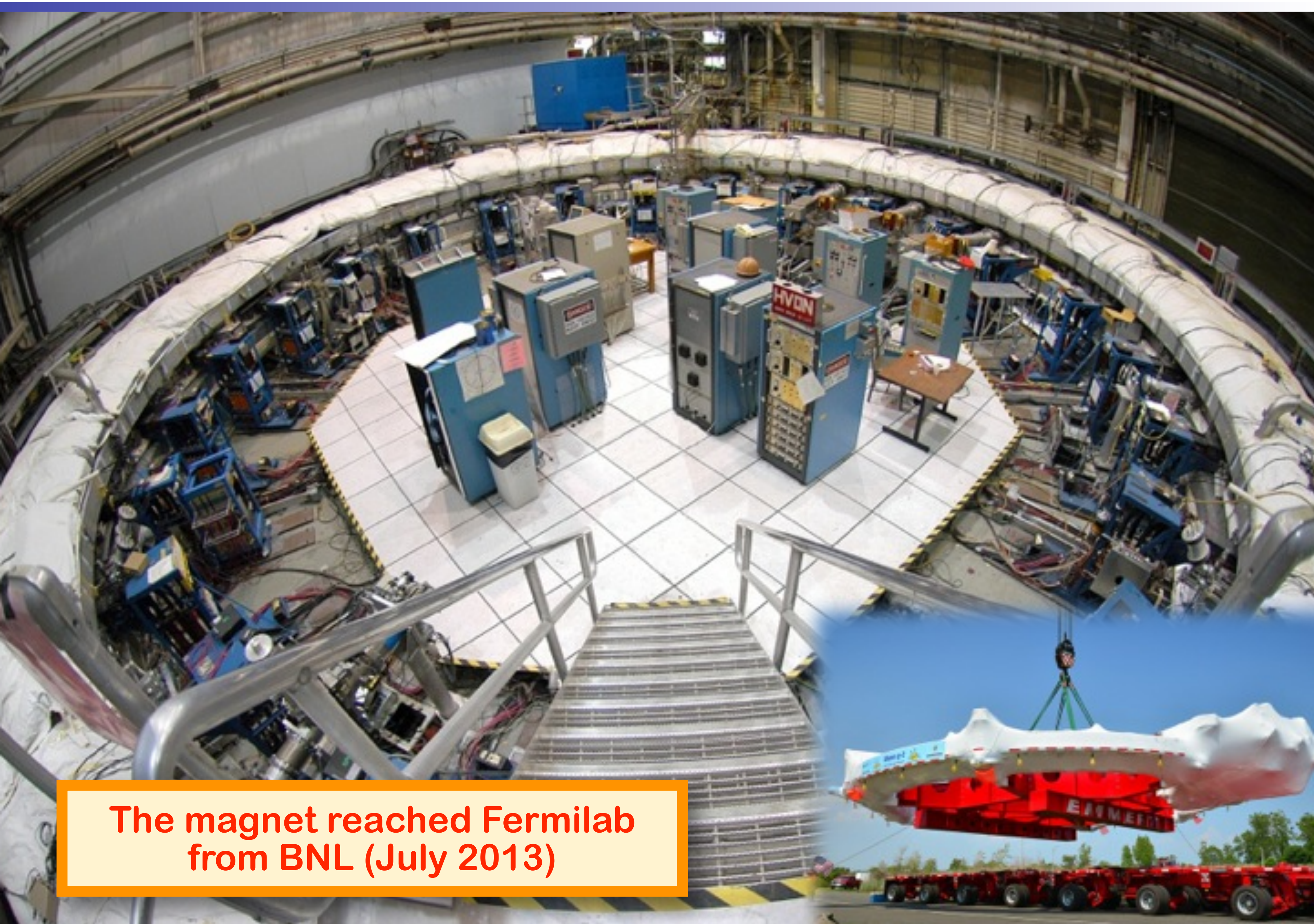
2. The muon g-2: theory update

The old experiment E821



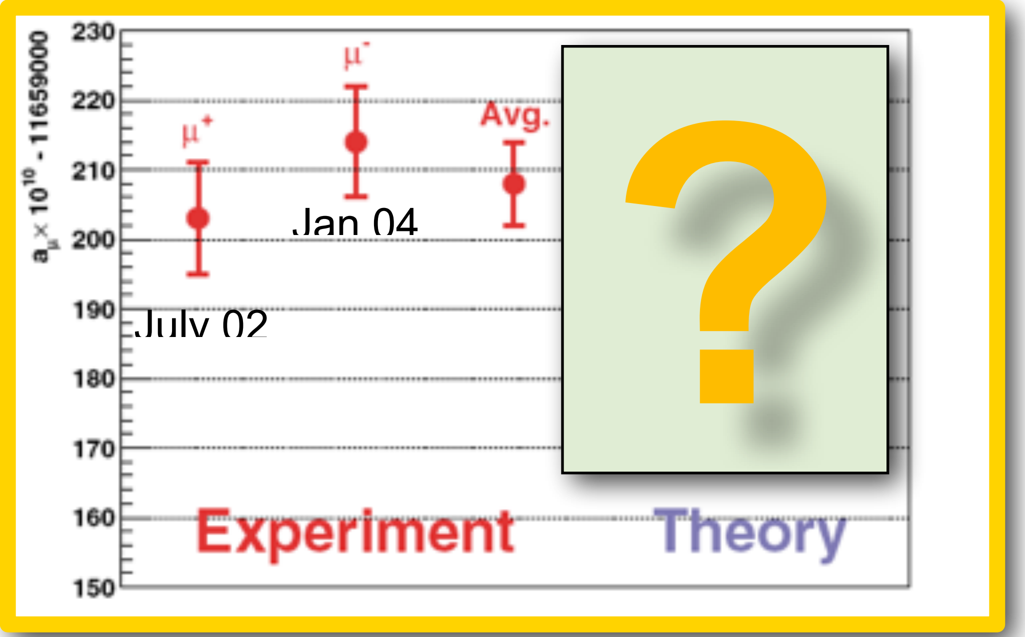
E821 @ BNL

The old experiment E821 (2)



The magnet reached Fermilab from BNL (July 2013)

The muon g-2: the experimental result



- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys})\times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments proposed at:
  - Fermilab E989, aiming at $\pm 16\times 10^{-11}$, ie 0.14ppm
  - J-PARC aiming at 0.1 ppm

  See B. Lee Roberts & T. Mibe @ Tau2012, September 2012

Sep 2012:
CD0 approval!
Data in
2016-17?

- **Are theorists ready for this (amazing) precision? No(t yet)**

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

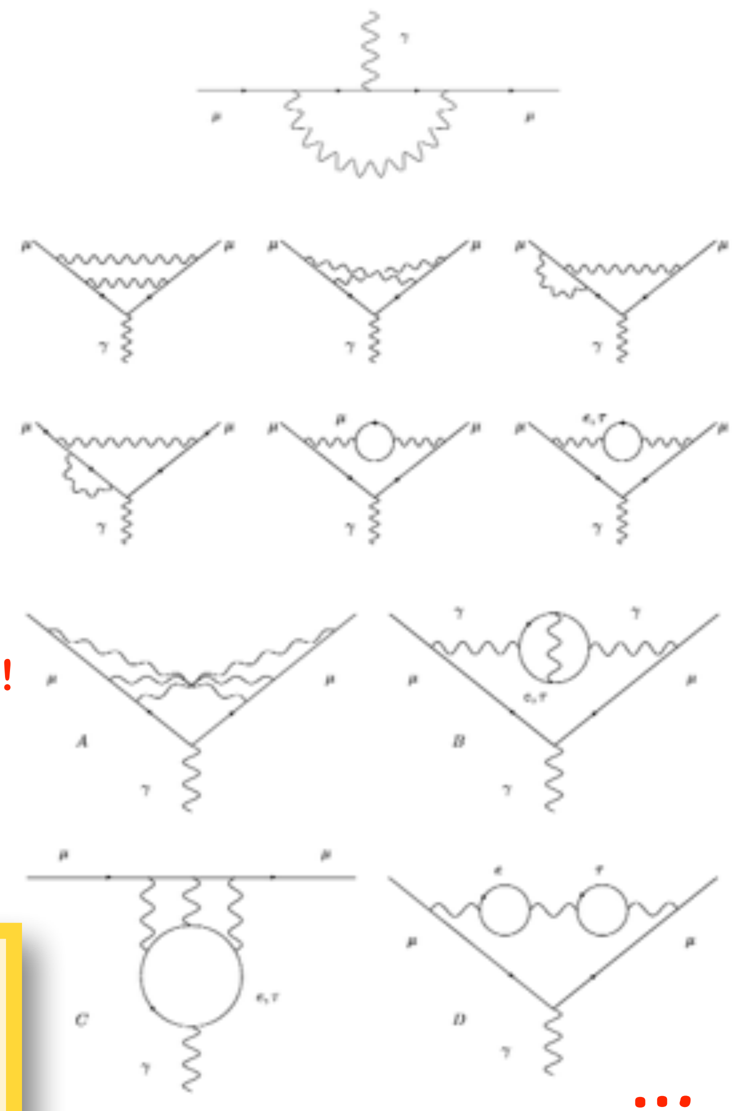
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8796\,(63)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,
Steinhauser et al. 2013 (analytic, in progress).

$+ \; 753.29\,(1.04)\,(\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012



Adding up, we get:

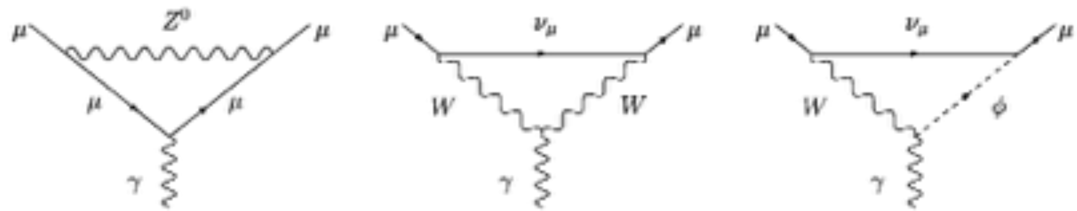
$a_\mu^{QED} = 116584718.951\,(22)(77) \times 10^{-11}$

from coeffs, mainly from 4-loop unc ⏎ ⮡ from $\delta\alpha$(Rb)

with $\alpha = 1/137.035999049(90)$ [0.66 ppb]

The muon g-2: the electroweak contribution

- **One-loop term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. ’80s

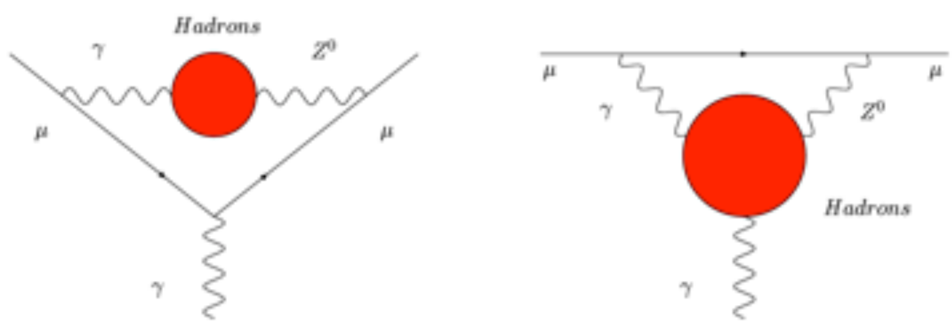
- **One-loop plus higher-order terms:**

$a_\mu^{EW} = 153.6\ (1) \times 10^{-11}$

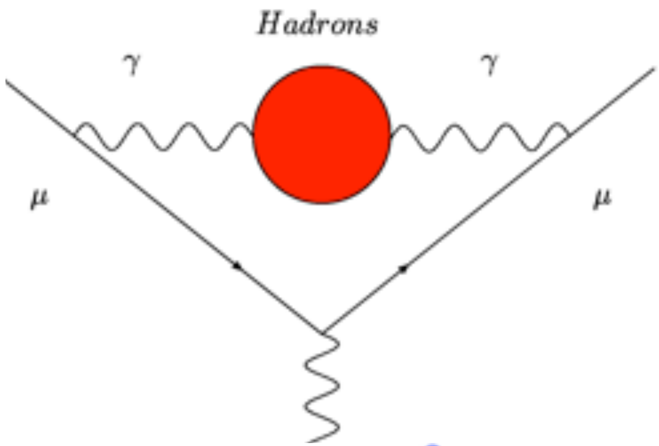
with M_{Higgs} = 125.6 (1.5) GeV

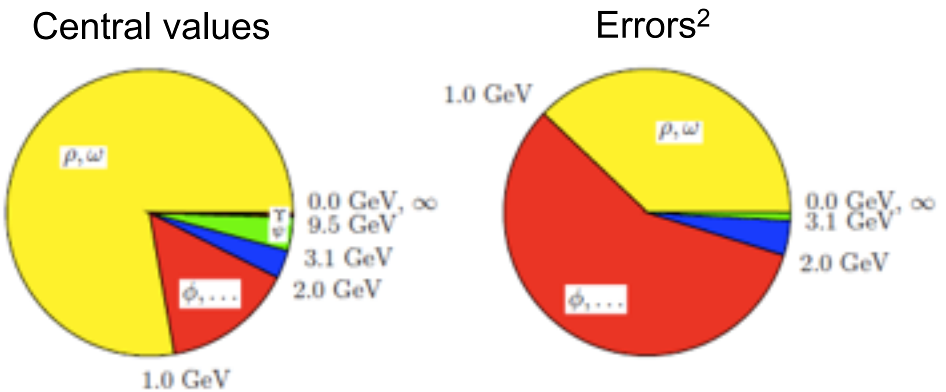
Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. ’92; Czarnecki, Krause, Marciano ’95; Knecht, Peris, Perrottet, de Rafael ’02; Czarnecki, Marciano and Vainshtein ’02; Degrassi and Giudice ’98; Heinemeyer, Stockinger, Weiglein ’04; Gribouk and Czarnecki ’05; Vainshtein ’03; Gnendiger, Stockinger, Stockinger-Kim 2013.



The muon g-2: the hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

$a_\mu^{HLO} = 6903\ (53)_{tot} \times 10^{-11}$ — F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

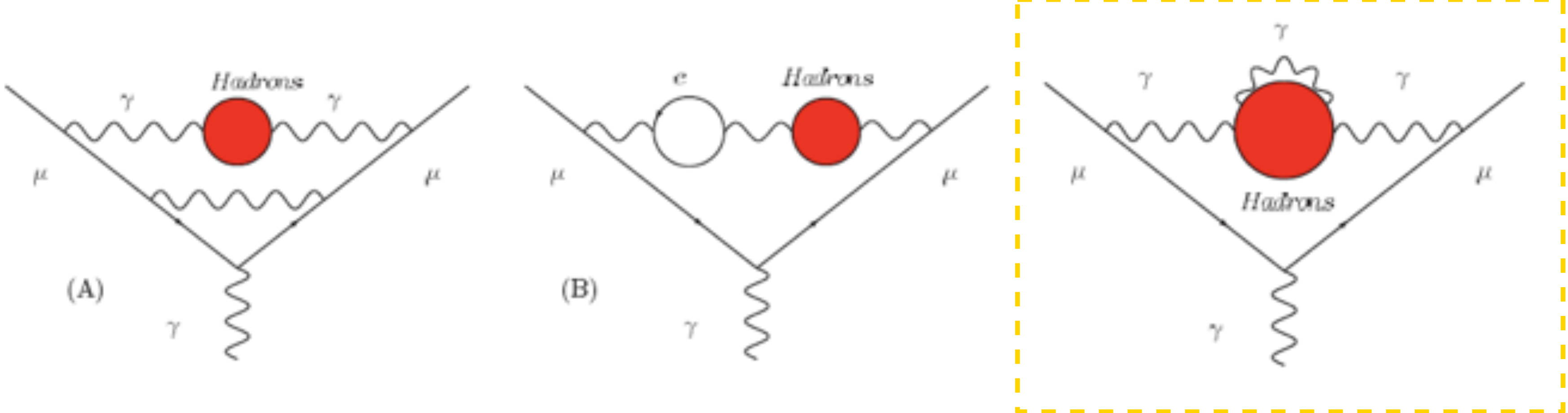
$= 6923\ (42)_{tot} \times 10^{-11}$ — Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$= 6949\ (37)_{exp}\ (21)_{rad} \times 10^{-11}$ — Hagiwara et al, JPG 38 (2011) 085003

Radiative Corrections are crucial! S.Actis et al, Eur. Phys. J. C66 (2010) 585

- ## HNLO: Vacuum Polarization

Already included in a_μ^{HLO}

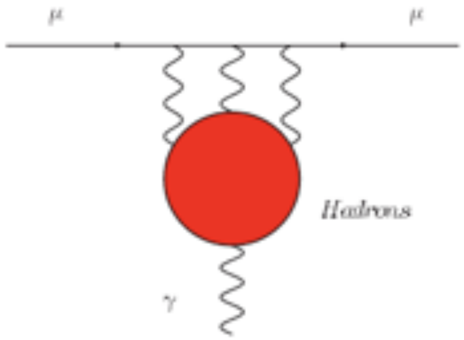


$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



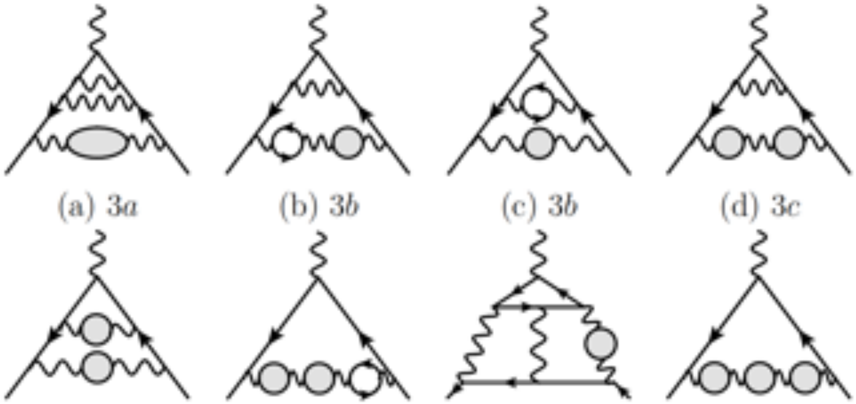
- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HHO}(lbl) = +116\ (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- "Bound" $a_\mu^{HNLO}(lbl) < \sim 160 \times 10^{-11}$ Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- Lattice? Very hard... but in progress. M. Golterman @ PhiPsi 2013; T. Blum @ Lattice 2012
- Pion exch. in holographic QCD agrees. D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- "By far not complete" calculation: 188×10^{-11} Fischer et al, PRD87(2013)034013
- Dispersive approach recently proposed Colangelo, Hoferichter, Procura, Stoffer 1402.7081

HNNLO: Vacuum Polarization



$O(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

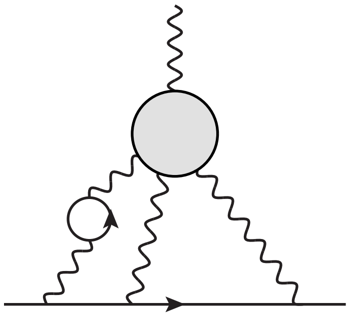
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

HNNLO: Light-by-light

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda=\mu_\mu/\mu_p$ from CODATA'06

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$	σ
116 591 809 (66)	$280\ (91) \times 10^{-11}$	3.1 [1]
116 591 829 (57)	$260\ (85) \times 10^{-11}$	3.1 [2]
116 591 855 (58)	$234\ (86) \times 10^{-11}$	2.7 [3]

with the "conservative" $a_\mu^{HNLO}(lbl)$ = 116 (39) x 10^{-11} and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

Note that the th. error is now about the same as the exp. one

The muon g-2: connection with the SM Higgs mass

- Δa_μ can be explained in many ways: errors in LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or, we hope, New Physics!
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

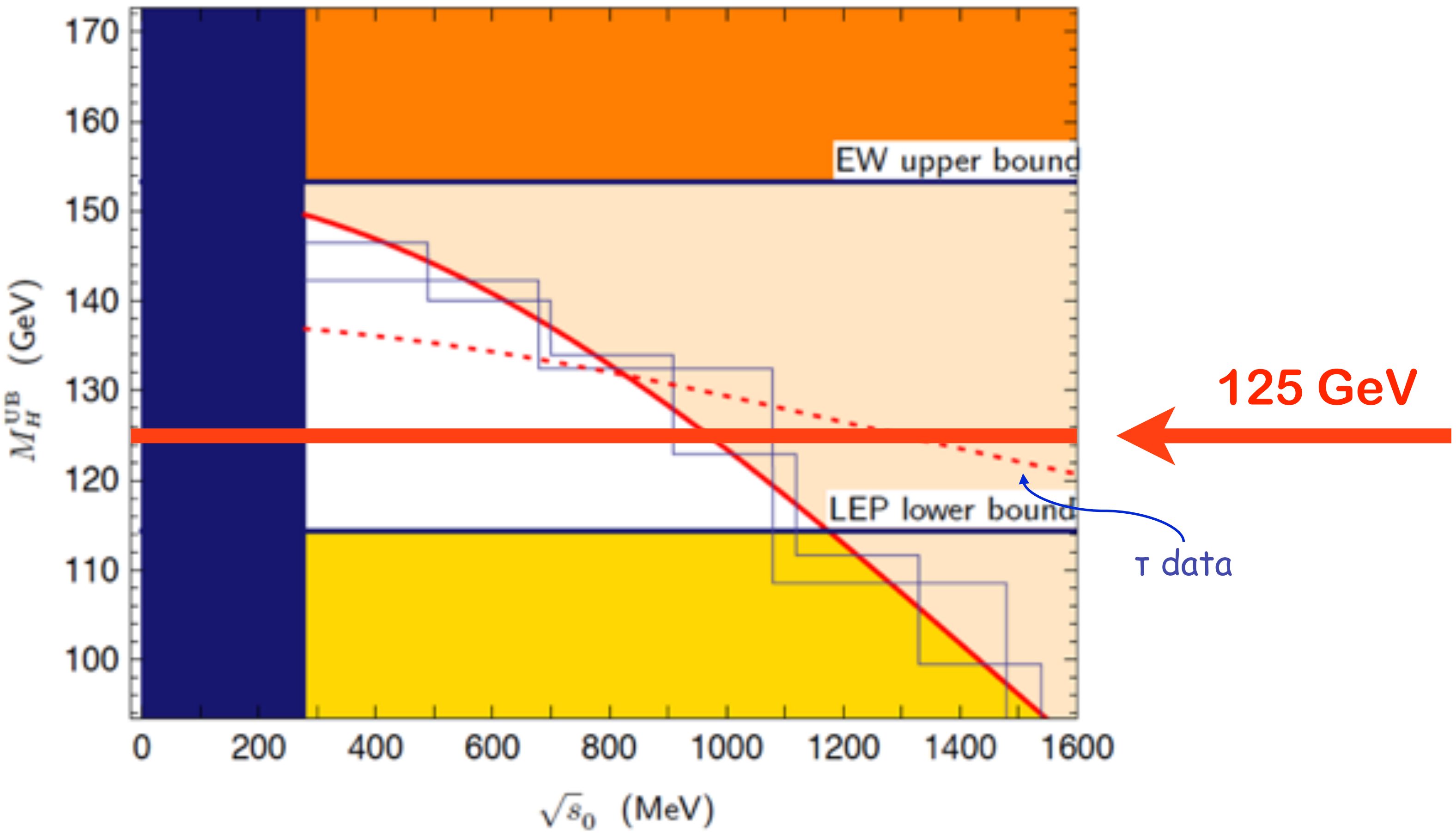
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

The muon g-2: connection with the SM Higgs mass (2)

- **How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?**



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (3)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV).

- Vice versa, assuming we now have a SM Higgs with M_{Higgs} = 125 GeV, if we bridge the M_{Higgs} discrepancy in the EW fit via changes in the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010 (and work in progress)

3. Testing new physics with the electron g-2

G.F. Giudice, P. Paradisi & MP, arXiv:1208.6583

The QED prediction of the electron g-2

$$a_e^{QED} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$$

$$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$$

$$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$$

$$+ 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$$

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$

$$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$$

$$A_2^{(6)}\,(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$$

$$A_3^{(6)}\,(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$$

$$- 1.9097\,(20)\,(\alpha/\pi)^4$$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012

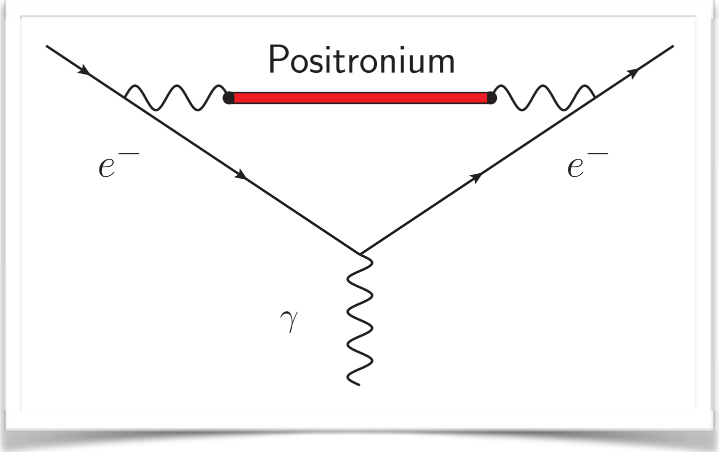
$$+ 9.16\,(58)\,(\alpha/\pi)^5$$ **COMPLETED! (12672 mass independent diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807.

Is there a positronium contribution to the electron g-2?

- **The leading contribution of positronium to a_e comes from:**

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690, Eides 1402.5860, Hayakawa 1403.0416



- **The electron-positron bound state appears as poles in $\Pi(q^2)$ below the $q^2 = (2m)^2$ branch-point. Their contribution is:**

$$a_e^{\mathrm{P}} = \frac{\alpha^5}{4\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right) = 8.94 \times 10^{-14} = 1.32\left(\frac{\alpha}{\pi}\right)^5$$

- **This result is of the same magnitude of the experimental uncertainty of a_e and of the same order of α as the 5-loop one…**

Is there a positronium contribution to the electron g-2? (II)

- …but it should not be added to the perturbative 5-loop result!

- Indeed, a recently determined nonperturbative contribution of the continuum right above threshold cancels one-half of it,

$$a_e(\mathrm{vp})^{\mathrm{cont,np}} = -\frac{|\alpha|^5}{8\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right)$$

- And it is argued that the remaining half is already included in the 5-loop perturbative QED result:

$$a_e^{(10)}(\mathrm{vp}) = \frac{a_e^{\mathrm{P}}}{2} + \cdots$$

The SM prediction of the electron g-2

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [Codata 2012]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- The Hadronic contribution is: Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97

$$a_e^{HAD} = 16.82\,(16) \times 10^{-13}$$

- Which value of α should we use to compute a_e^{SM} and compare it with a_e^{EXP} ?? Not the PDG/Codata one (obtained equating $a_e^{SM}(\alpha) = a_e^{EXP}$)! Use atomic-physics measurements of alpha.

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → best determination of alpha (2014):

$$\alpha^{-1} = 137.035\,999\,184\ (35) \qquad [0.25\ \text{ppb}]$$

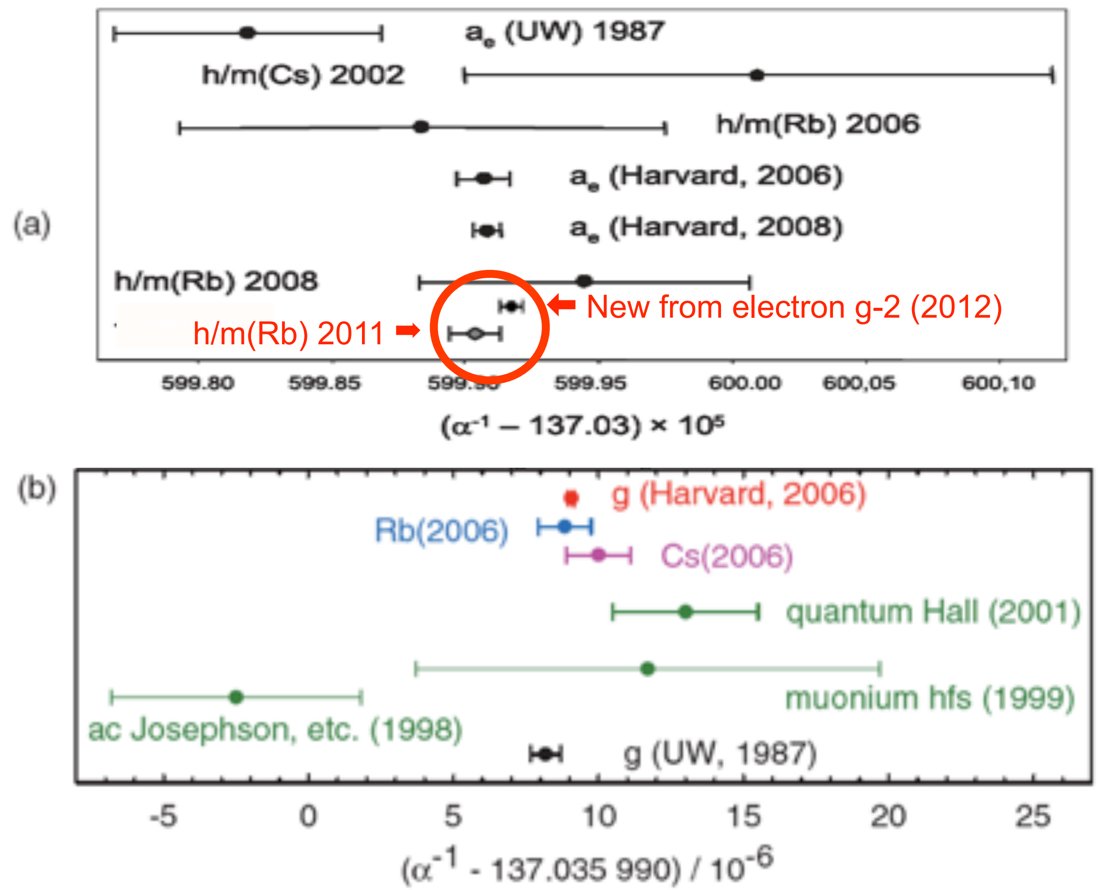
- Compare it with other determinations (independent of a_e):

$\alpha^{-1} = 137.036\,000\,0\ (11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

$\alpha^{-1} = 137.035\,999\,049\ (90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
Bouchendira et al, PRL106 (2011) 080801

The electron g-2: SM vs. Experiment

- Using α = 1/137.035 999 049 (90) [^{87}Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,17.8\,(0.6)\,(0.4)\,(0.2)\,(7.6) \times 10^{-13}$$

(0.6) → δC_4^{qed}; (0.4) → δC_5^{qed}; (0.2) → δa_e^{had}; (7.6) → from $\delta\alpha$

- The EXP-SM difference is:

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.5\,(8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).
NB: The 4-loop contrib. to a_e^{QED} is $-5.56 \times 10^{-11} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.6)_{\rm QED4}, \quad (0.4)_{\rm QED5}, \quad (0.2)_{\rm HAD},}_{(0.7)_{\rm TH}} \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\rm EXP}}$$

⬅ may drop to 0.2 or 0.3

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

F. Terranova & G.M. Tino, arXiv:1312.2346

- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (II)

- The experimental sensitivity in Δa_e is not far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- BSM scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e (& Δa_τ) and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

4. The tau g-2: opportunities & challenges

Work in progress in collaboration with
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302
arXiv:1310.1081

The SM prediction of the tau g-2

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM}	=	117324	(2)	x 10^{-8}	QED
	+	47.4	(0.5)	x 10^{-8}	EW
	+	337.5	(3.7)	x 10^{-8}	HLO
	+	7.6	(0.2)	x 10^{-8}	HHO (vac)
	+	5	(3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

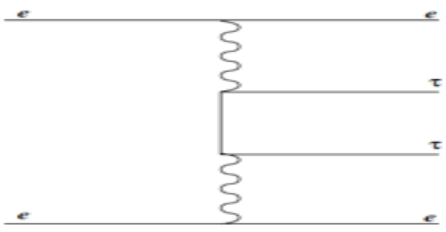
	Muon	Tau
a^{EW}	1/45	1/7
a^{EW}	3	10

… if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\ (17)$$

PDG 2012

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.004 < a_\tau^{NP} < 0.006 \quad (95\%\ CL)$$

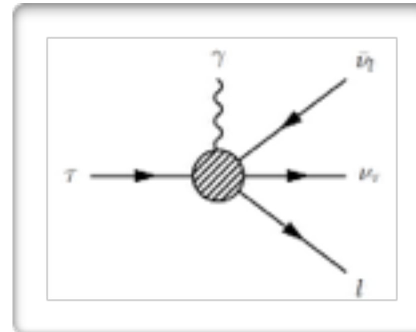
Escribano & Massó 1997

$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ CL)$$

Gonzáles-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \rightarrow \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

The tau g-2 via its radiative leptonic decays: a proposal



- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx\,dy\,d\cos\theta} = \frac{\alpha\, M_\tau^5\, G_F^2\, y\,\sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6}\, G_0(x, y, \cos\theta, r)$$

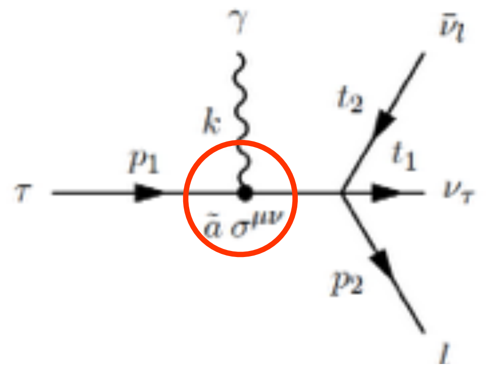
Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\left.\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\text{total}}}\right|_{E_\gamma > 10\text{MeV}} = 1.836\% \quad \text{vs} \quad 1.75(18)\%$$

CLEO 2000

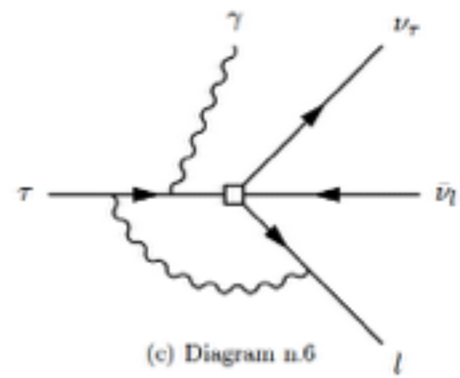
$$\left.\frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\text{total}}}\right|_{E_\gamma > 10\text{MeV}} = 0.367\% \quad \text{vs} \quad 0.361(38)\%$$

$$x = \frac{2E_l}{M_\tau},\; y = \frac{2E_\gamma}{M_\tau},\; r = \frac{m_l}{M_\tau}$$



- **Add the contribution of the effective coupling and the QED corrections:**

$$G_0 \to G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{\text{RC}}$$



(c) Diagram n.6

- **Measure $d^3\Gamma$ precisely and get $\tilde{a}_\tau$!**

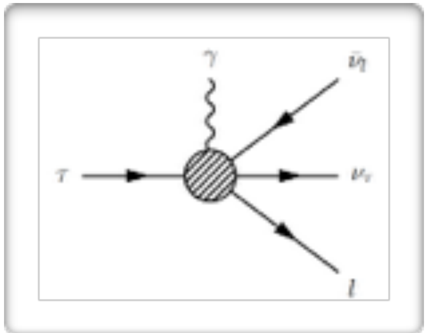
[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]

The tau g-2 via its radiative leptonic decays: a proposal

- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx\,dy\,d\cos\theta} = \frac{\alpha\, M_\tau^5\, G_F^2\, y\, \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6}\, G_0(x, y, \cos\theta, r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

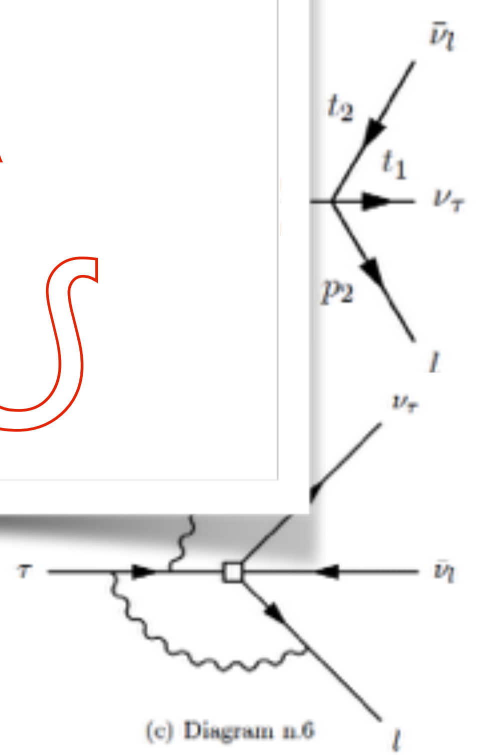


$\Gamma(\tau^-$ … $, \; r = \frac{m_l}{M_\tau}$

$\Gamma(\tau^-$ …

WORK IN PROGRESS

- **Add** …
  coup …



- **Measure d³Γ precisely and get ã$_\tau$!**
  [see also Laursen, Samuel, Sen, PRD29 (1984) 2652]

Conclusions

- The lepton g-2 provide beautiful examples of interplay between theory and experiment.
- The discrepancy is $\Delta a_\mu \sim 3 \div 3.5\ \sigma$. Is it NP? New g-2 experiment, ring now in Fermilab! QED & EW terms ready for the challenge; How about the hadronic one? Future of LBL??
- Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies (below ~ 1GeV).
- The sensitivity of the electron g-2 has improved. The positronium contribution should not be added. It may soon be possible to test if Δa_μ manifests itself also in the electron g-2! A robust and ambitious exp program is needed to improve α & a_e.
- The tau g-2 is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

The End