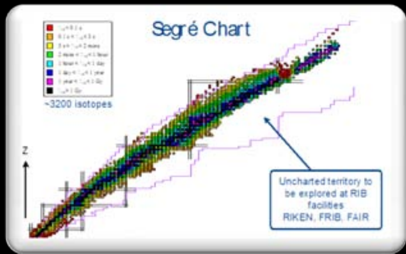


Equazione di stato di materia nucleare : dai nuclei alle stelle compatte

Fiorella Burgio (INFN Sezione di Catania)

Dipartimento di Fisica, Università di Genova, 27 Gennaio 2014

Collaborators :

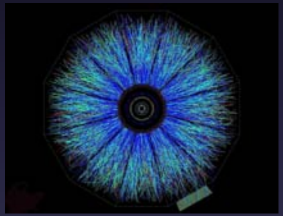
- M. Baldo, H.-J. Schulze, G. Taranto, D. Zappalà *(INFN Sezione di Catania)*
- V. Greco, U. Lombardo, S. Plumari *(INFN-LNS)*
- A. Bonanno *(INAF Catania)*
- V. Ferrari, L. Gualtieri (*Dip. Fisica "La Sapienza", Roma*)
- A. Li, X. R. Zhou (*Xiamen University, China*)
- M. Buballa *(TU Darmstadt, Germany)*
- H. Chen (*Wuhan University, China*)
- N. Yasutake (*Institute of Technology, Chiba, Japan*)
- V. Urpin (*IOFFE Institute, Saint-Petersburg, Russia*)
- F. Weber (*San Diego State University, USA*)
- W. Zuo *(Lanzhou University, China)*

What is an Equation of State (EoS) ?

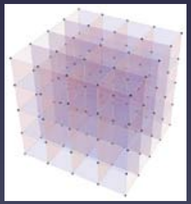
EoS : any non-trivial relation between thermodynamic variables characterizing the matter, e.g.

$$\longrightarrow \quad P(\rho, T) \ or \ \frac{E}{A}(\rho, T)$$

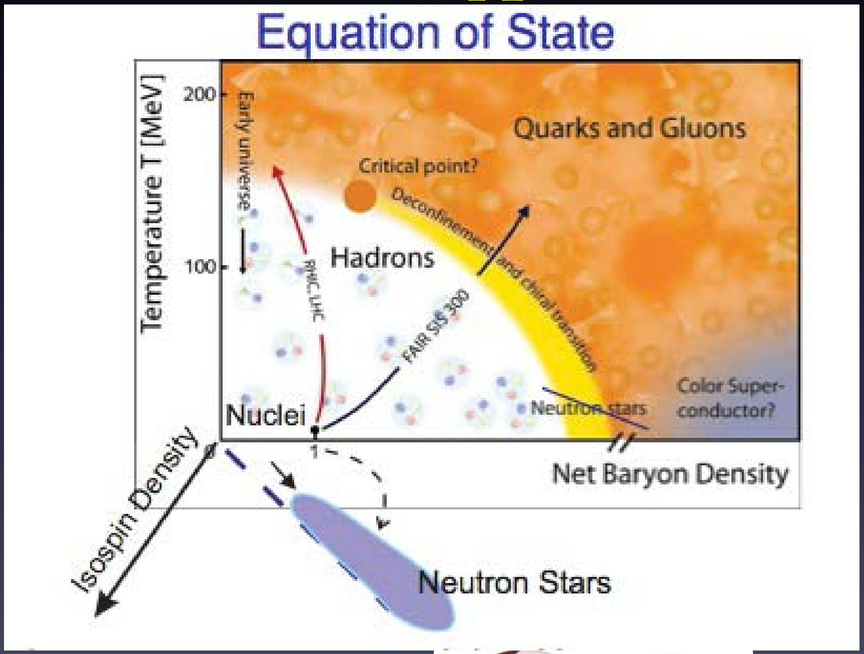
LHC

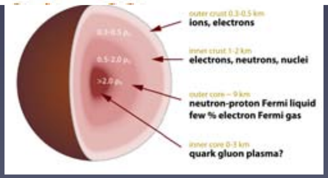


RHIC

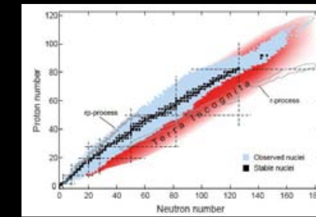


Lattice QCD

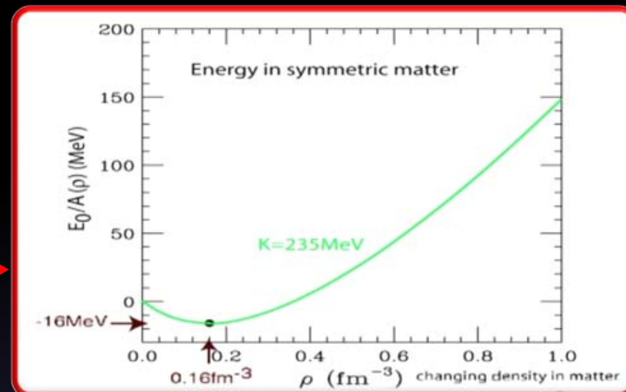




The EoS : where do we stand ?



<u>Droplet Model</u> : if A ->∞ and N=Z, neglecting Coulomb terms



By defining the asymmetry parameter I=(N-Z)/A , then in the vicinity of the saturation point ρ_o for symmetric matter, the binding energy is usually expanded as

Symmetry energy

Nuclear Incompressibility

$$\frac{E}{A}(\rho, I) = \frac{E}{A}(\rho, 0) + S_v(\rho) I^2$$

$$\epsilon = (\rho - \rho_0)/\rho_0 \qquad = \frac{E}{A}(\rho_0) + \frac{1}{18} K \epsilon^2 + \left[S_v(\rho_0) - \frac{1}{3} L \epsilon + \frac{1}{8} K_{sym} \epsilon^2 \right] I^2$$

$$S_v(\rho_0) = 30 \pm 2MeV$$

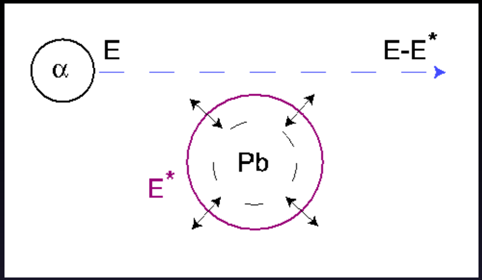
L and K_{sym} parameters characterizing the density dependence of the symmetry energy around the saturation point

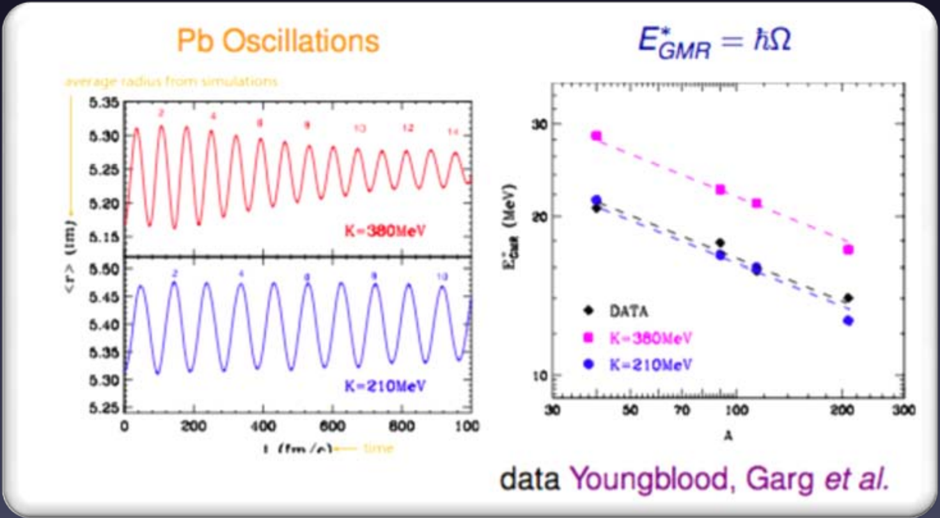
Nuclear Incompressibility for symmetric matter K

$$K = 9\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_o} = R^2 \frac{d^2}{dR^2}\left(\frac{E}{A}\right)_{\rho=\rho_o}$$

In radial oscillations induced by α-particles scattering :

$$E^* = \hbar\sqrt{\frac{K}{m_N < r^2 >_A}}$$





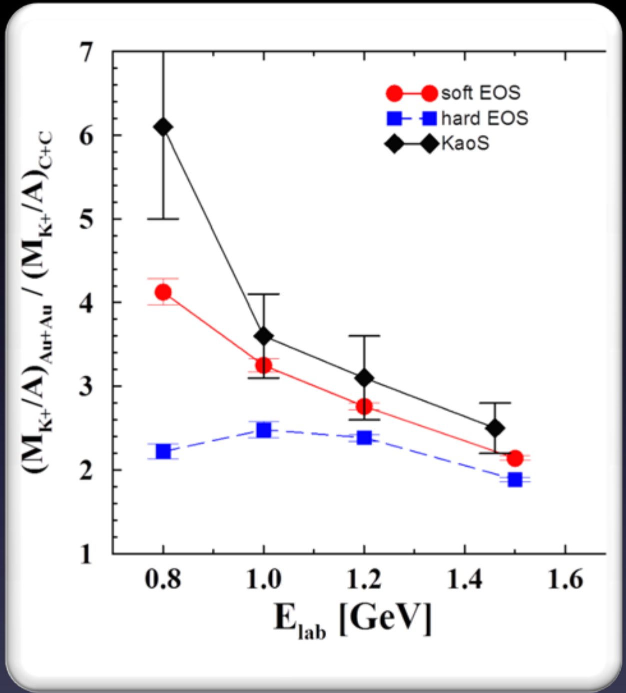
A soft EoS at density close to the saturation value is preferred.
Results are confirmed also by experiments on K^+ multiplicity in HIC

210 MeV < K< 250 MeV

Kaon production in heavy ion collisions

Kaons probe the dense nuclear medium

Sturm et al, PRL 2001



- K^-N interaction not well known
- K^+ produced by : $NN \to N\Delta \to N\Lambda K^+$
- Simulations must include nucleon excitations and must be relativistic.
- Production rate dependent on the maximal density ------> compressibility.

- Experimental data by the KaoS and FOPI Collaborations :
- Double ratio : multiplicity per mass number for C+C collisions and Au+Au collisions at 0.8 AGeV and 1.0 AGeV .
- Largest density explored : $\rho \approx 2\text{-}3\ \rho_0$
- Only calculations with a compression $180 \le K_N \le 250$ MeV can describe the data (Fuchs, 2001)

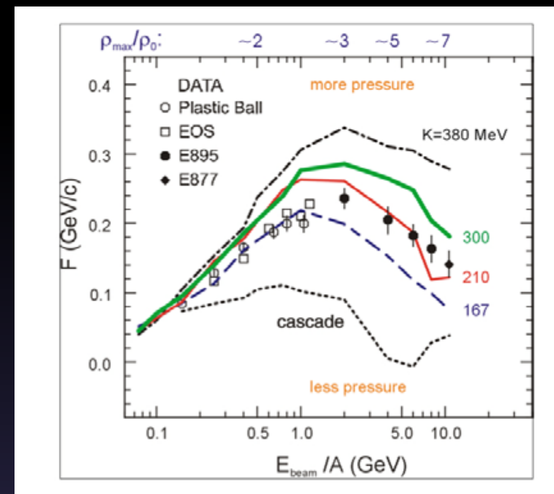
The nuclear equation of state up to $2\text{-}3\rho_0$ is **SOFT !**

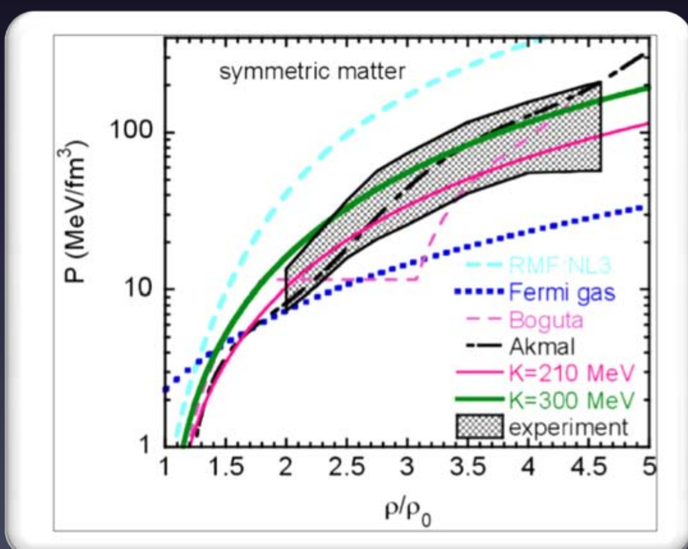
Determination of the Equation of State of Dense Matter

Pawel Danielewicz[1,2], Roy Lacey[3] & William G. Lynch[1,*]

Science **298**, 1592 (2002)

- Transverse flow measurements in Au + Au collisions at E/A=0.5 to 10 GeV
- Pressure determined from simulations based on the Boltzmann-Uehling Uhlenbeck transport theory





Flow data exclude

- very repulsive equations of state,
- very soft EoS (e.g. Fermi gas)

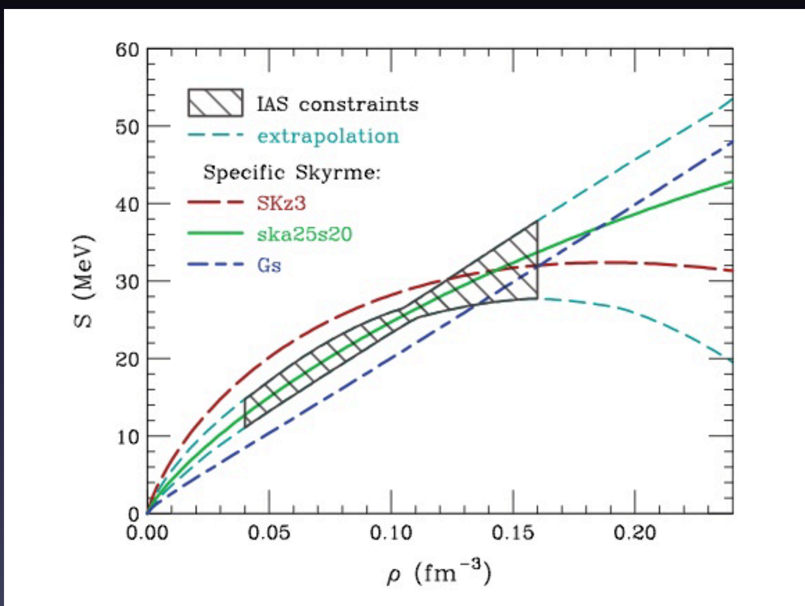
Importance of *S* and its slope *L*

R_{NS} ?

Lattimer & Prakash, ApJ(2001)

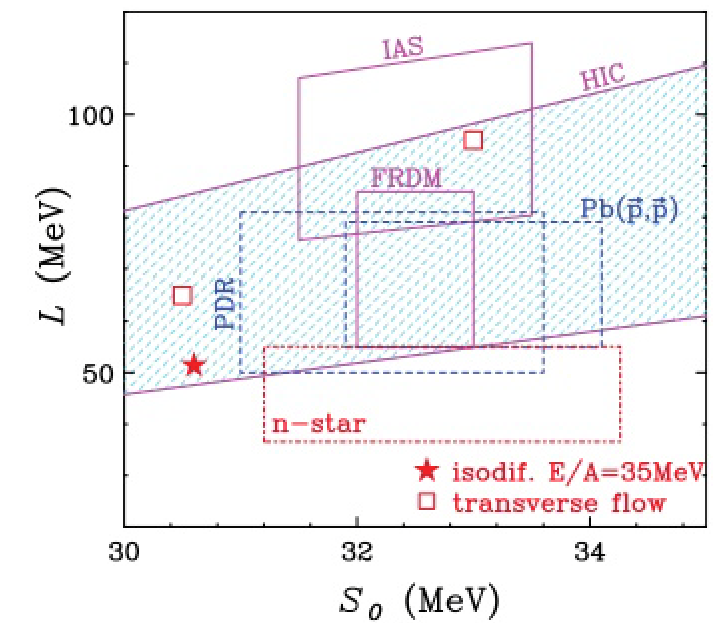
$$L = 3\rho_0 dS(\rho)/d\rho|_{\rho_0} = [3/\rho_0]P_0,$$

- Composition of neutron star matter.
- Neutrino DURCA processes responsible of cooling.



Danielewicz & Lee, arXiv:1307.4130

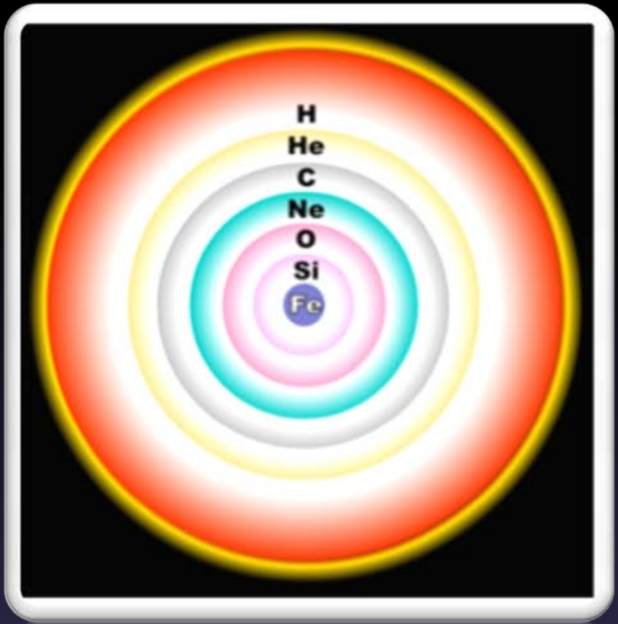
Tsang et al., PHYSICAL REVIEW C 86, 015803 (2012)



Evolution of a Star

- massive star => He fusion to carbon and oxygen
- if the star is heavy enough, this can go on to form neon,magnesium, silicon
- sequence of fusion reactions definitely ends with iron (the most bound nucleus)

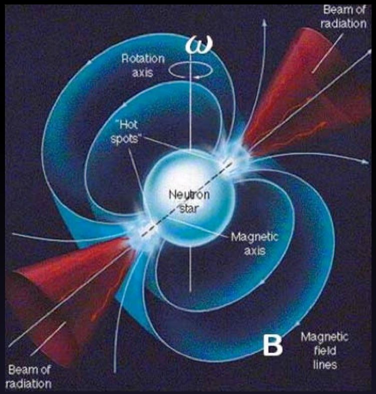
- after all possible fusion reactions are ceased
  - pressure goes down => star cannot withstand gravitational collapse any longer
  - supernova explosion
  - remnant becomes a white dwarf or a neutron star or a black hole.



Onion-like structure

NS observed as rotating pulsars (1967)

Jocelyn Bell



$$
\begin{aligned}
M_{NS} &= 1.35 - 2\ M_{\odot}, R = 10 - 12\ km \\
\rho_{NS} &\cong 10^{15} g\ cm^{-3}\ (\rho_0 = 2 \times 10^{14} g\ cm^{-3}) \\
P_{rot} &\geq 1.4\ ms \\
B &\approx 10^{10} - 10^{12}\ Gauss
\end{aligned}
$$

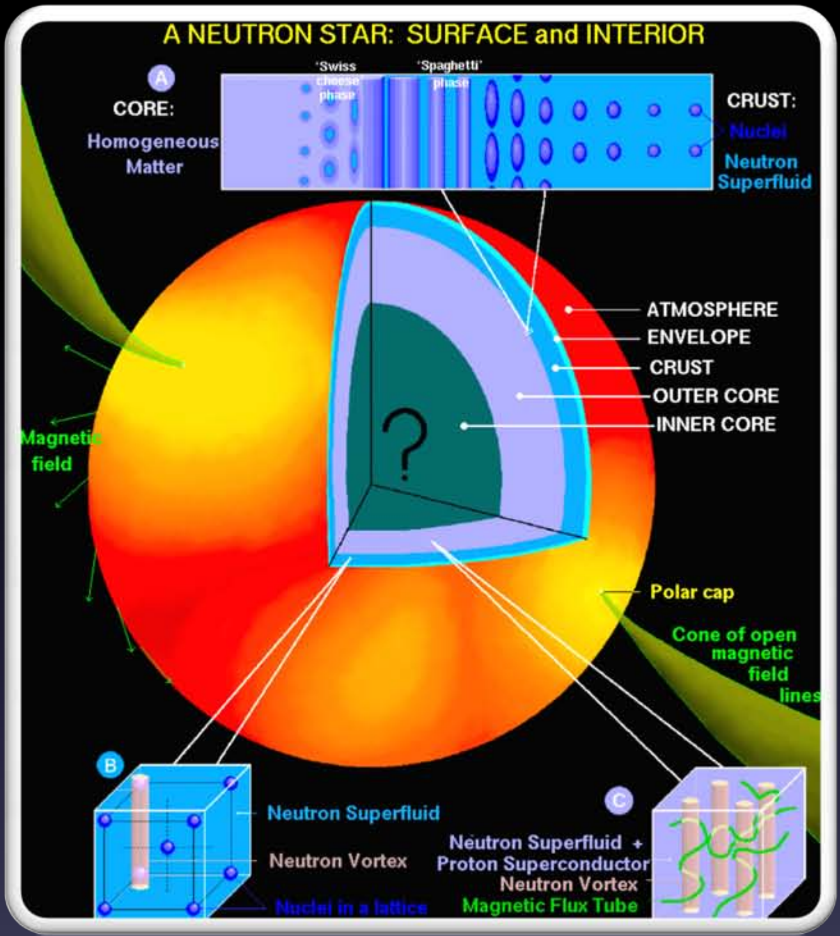
Four main layers

Outer crust ^{56}Fe ions immersed in an electron gas, as in normal metals.

Inner crust Neutron gas in chemical equilibrium with electrons and ions. Ions get very neutron rich, until the neutrons drip from nuclei. Nuclear matter from drip point (4×10^{11} g/cc) up to about half the saturation density.

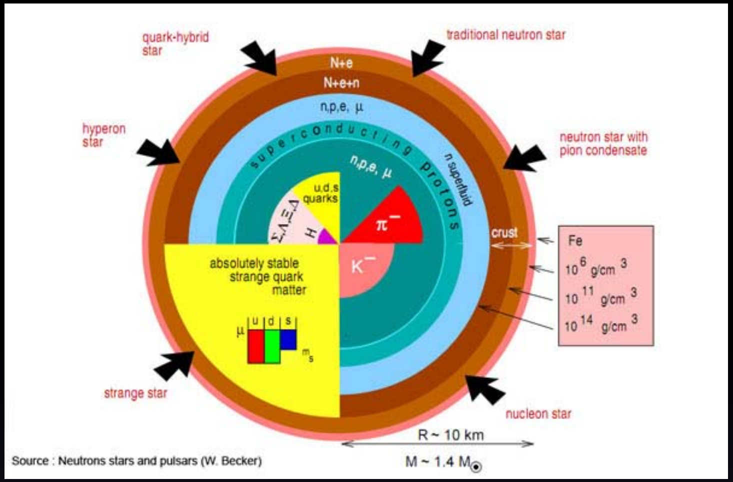
Outer core Asymmetric nuclear matter above saturation. Mainly composed by neutrons, protons, electrons and muons. Its exact composition depends on the nuclear matter Equation of State (EoS).

Inner core The most unknown region. “Exotic matter” . Hyperons. Kaons? Pions? Quarks ?



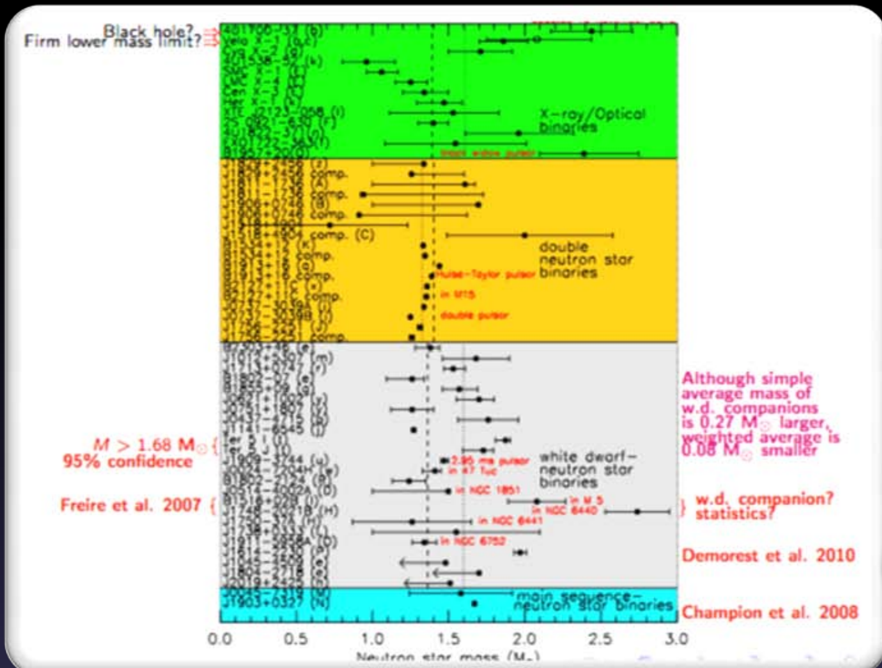
by Dany Page, UNAM Mexico City

- Properties like
  - maximum possible mass (Oppenheimer - Volkoff limit)
  - detailed composition of matter
  - maximum rotational frequencies
  - (non)-radial oscillation frequencies
  - temperature evolution



<u>depend crucially on the Equation of State of nuclear matter !</u>

EoS from astrophysical observations of NS binaries



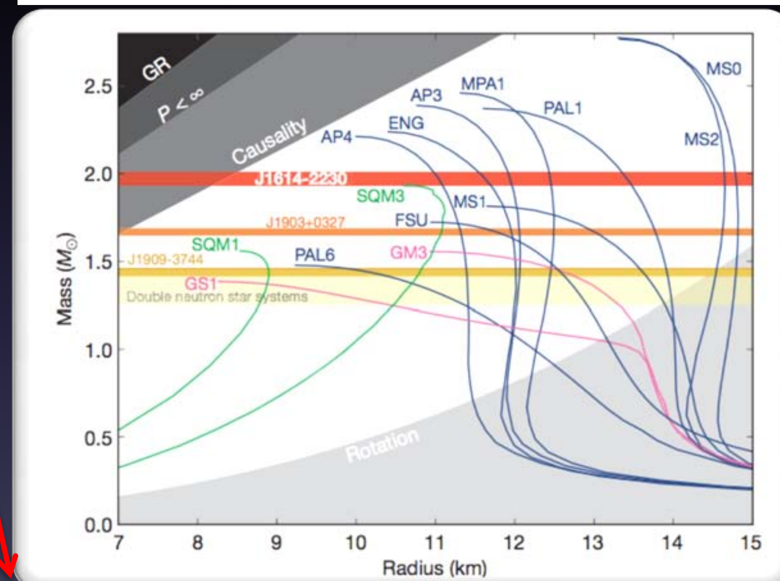
Compilation by Jim Lattimer

LETTER

doi:10.1038/nature09466

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

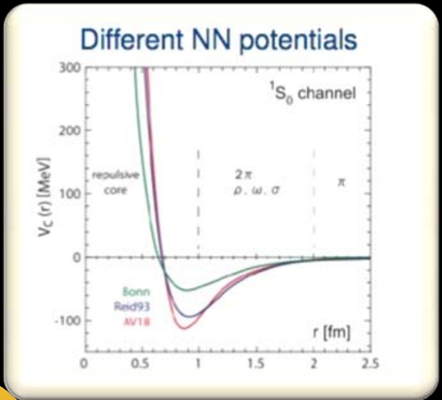


Several EoS are EXCLUDED !

Complication of the nuclear many-body problem

At short distance the NN interaction is too strong

Divergency problems in many-body calculations.



The construction of the EoS :
two possible philosophies

Phenomenological

- Effective NN interaction
- Density functional theory (DFT)
  Relativistic mean field models

Several free parameters. **FAST**

Ab initio

- Microscopic NN interaction
- Many-body technique
  (BHF, DBHF, Variational, Green's Functions Theory, Quantum Monte Carlo)

No free parameters. **SAFE**

M. Baldo, and F. Burgio, Reports on Progress in Physics **75** (2012)

Two different many-body techniques : BHF vs. Variational

- Bethe-Goldstone equation for the G-matrix

$$
\begin{aligned}
G(\rho;\omega) &= V + \sum_{k_a,k_b} V \frac{|k_a k_b > Q < k_a k_b|}{\omega - e(k_a) - e(k_b)} G(\rho;\omega) \\
e(k;\rho) &= \frac{k^2}{2m} + U(k;\rho) \\
U(k;\rho) &= Re \sum_{k' \leq k_F} < kk'|G(\rho:\omega)|kk' >
\end{aligned}
$$

$$
\frac{E}{A}(\rho) = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho} \sum_{k,k' \leq k_F} < kk'|G(\rho;\omega)|kk' >_a
$$

- Variational method

Ansatz : the ground state wave function Ψ can be written as

$$
\Psi(r_1, r_2, \ldots) = \prod_{i<j} f(r_{ij}) \Phi(r_1, r_2, \ldots)
$$

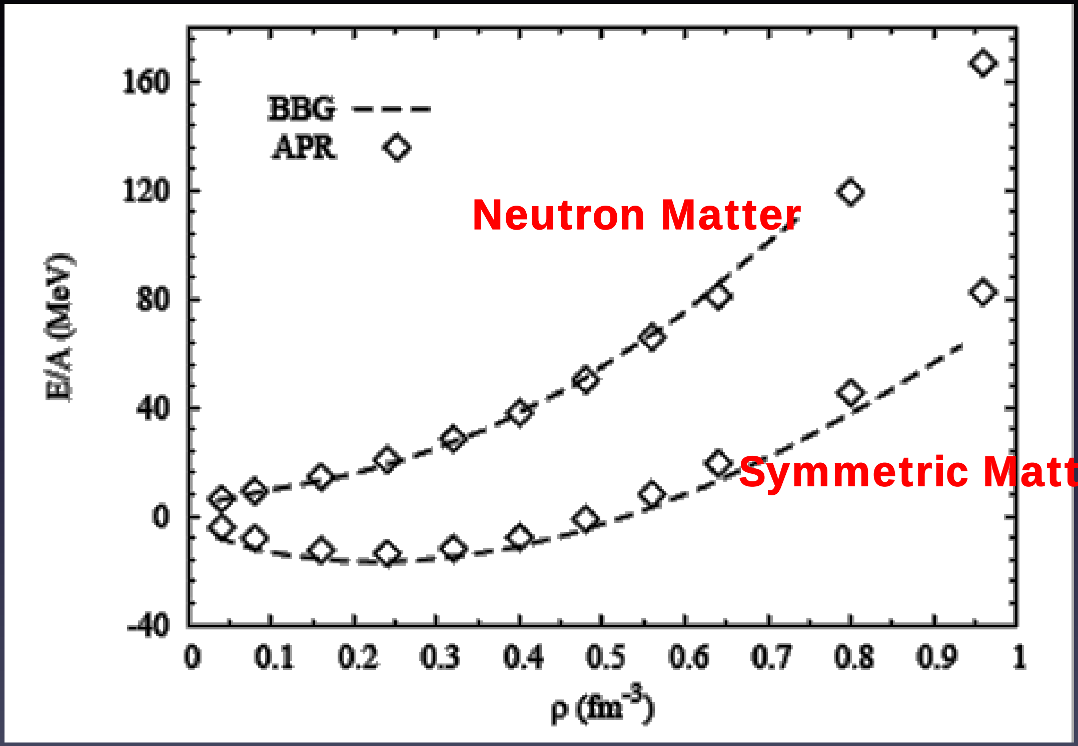
where Φ is the unperturbed ground state wave function and the correlation factor $f(r_{ij})$ is determined by the variational principle, i.e. by assuming that the mean value of the Hamiltonian gets a minimum

$$
\frac{\delta}{\delta f} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0
$$

Akmal, Pandharipande & Ravenhall, PRC **58**, 1804 (1998)

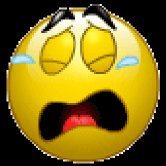
Dependence on the many-body scheme

- For the Av18 interaction, good agreement between var. and BBG up to 0.6 fm-3 (symmetric and neutron matter).
- The many-body treatment of nuclear matter EOS can be considered well understood. up to density below 0.6 fm$^{-3.}$

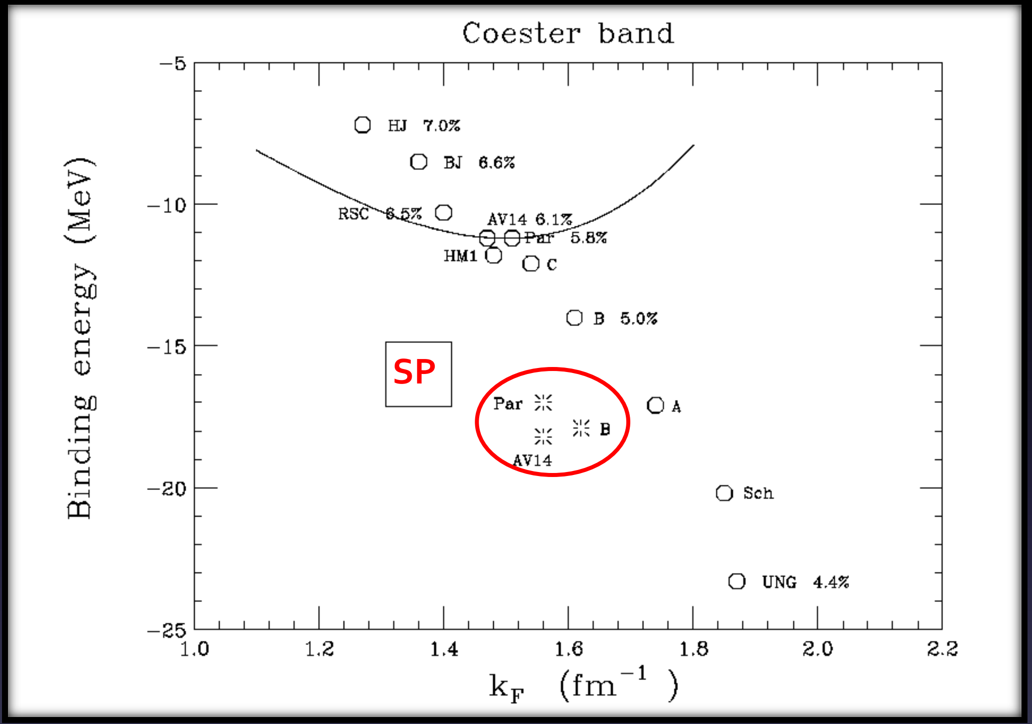


At two-body level, both methods give quite similar results.
Both methods give the wrong saturation point !

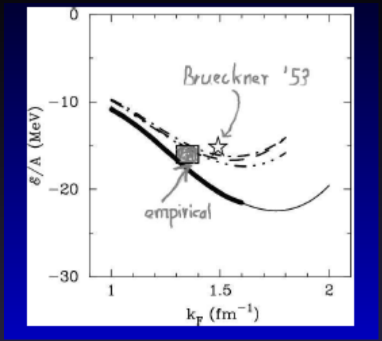
Coester et al., Phys. Rev. **C1**, 769 (1970)



Missing the saturation point …….



Just like in early calculations



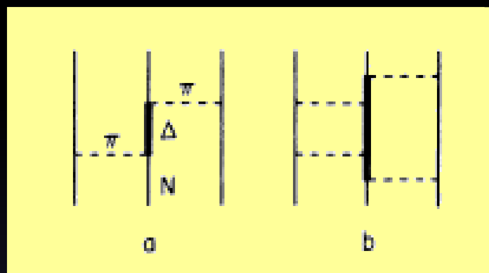
When three hole-line diagrams are included and modern NN interactions are used the Coester band reduces to a Coester "island"

The saturation "point" is still missed.

Three-nucleon forces

(No complete theory available yet !)

Urbana IX phenomenological model :

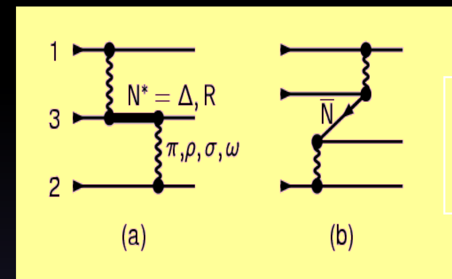


Carlson et al.,
NP A401,(1983) 59

(a) : 2π exchange (attractive)
(b) : Roper R resonance (repulsive)

Parameters fitted to saturation point !

Microscopic model :

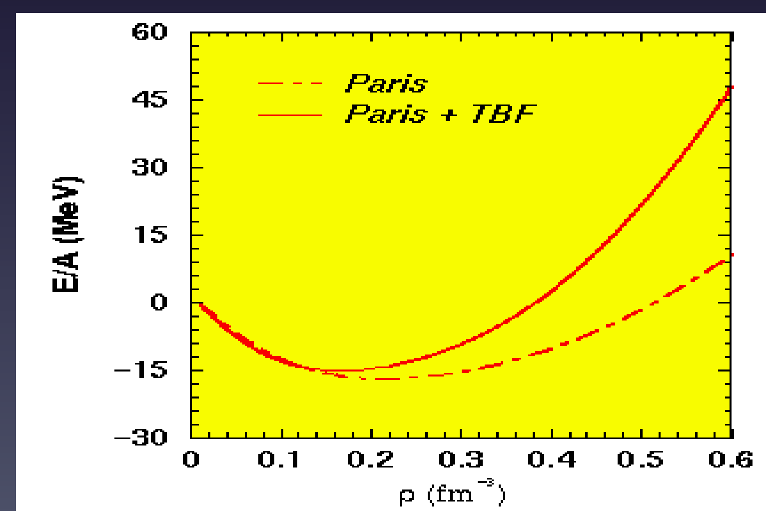


P. Grange' et al,
PR C40, (1989)
1040

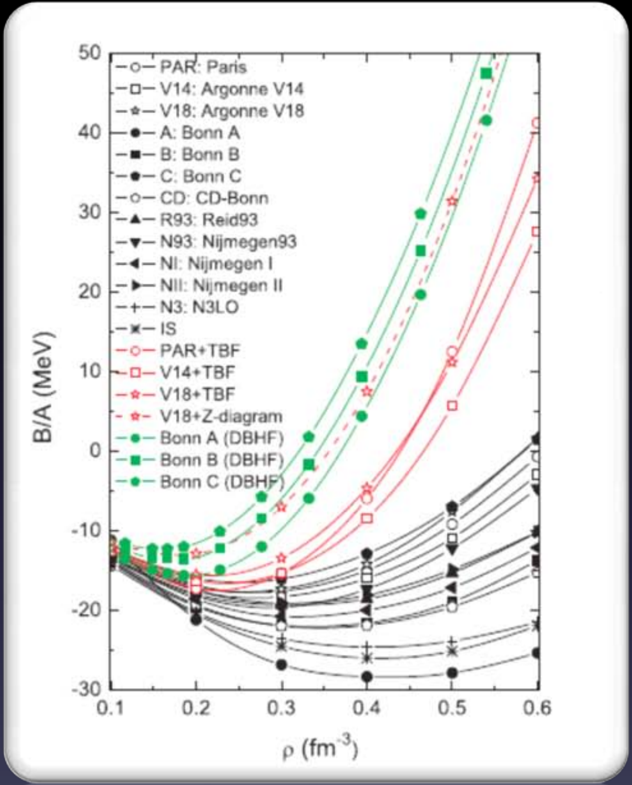
Exchange of π, ρ, σ, ω via Δ(1232), R(1440), NN
Parameters compatibile with two-nucleon potential

TBF reduced to an effective two-body force by averaging over the position of the third particle :

EoS

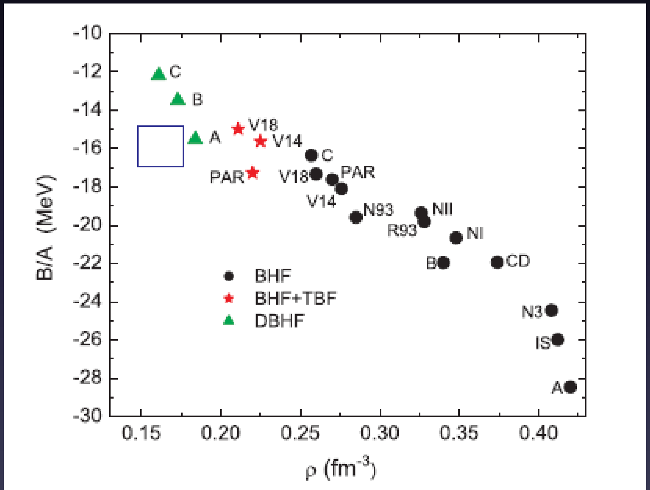


The problem is common to all non-relativistic many-body methods.



Z.H. Li, U. Lombardo, H.-J. Schulze, W. Zuo,
PRC 74, 047304 (2006)

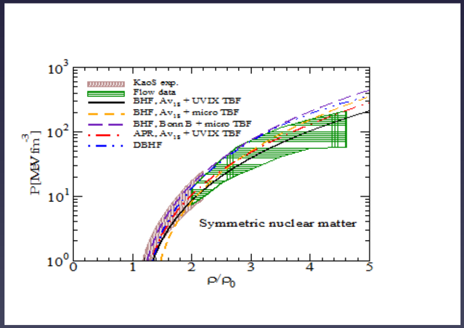
New Coester band

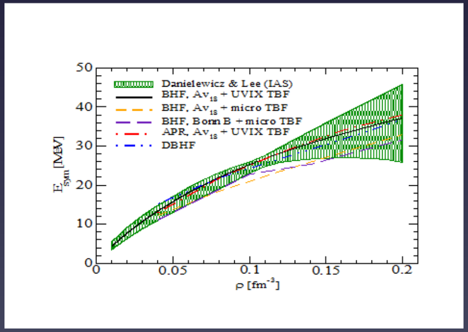


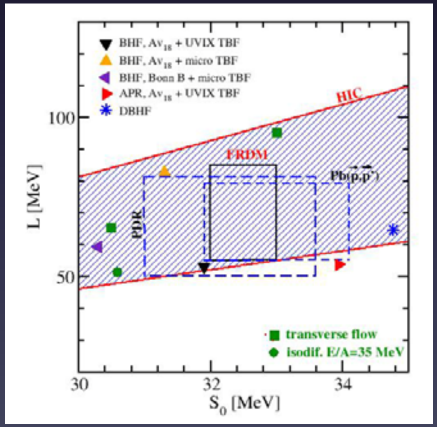
Microscopic EoS's reproduce well the data from phenomenology

TABLE I. Calculated properties of symmetric nuclear matter.

EoS	ρ_0 (fm^{-3})	$\frac{E}{A}$ (MeV)	K_0 (MeV)	S_0 (MeV)	L (MeV)
BHF, Av$_{18}$ + UVIX TBF	0.16	−15.98	212.4	31.9	52.9
BHF, Av$_{18}$ + micro TBF	0.2	−15.5	236	31.3	82.7
BHF, Bonn B + micro TBF	0.17	−16.	254	30.3	59.2
APR, Av$_{18}$ + UVIX TBF	0.16	−16.	247.3	33.9	53.8
DBHF, Bonn A	0.18	−16.15	230	34.4	69.4







G. Taranto et al., Phys. Rev. C87, 045803 (2013)

In stellar matter, beta-stable & neutrally charged matter

Assuming a conventional description of stellar matter as composed by n, p, e, μ

$$n \rightarrow p + e^- \quad , \quad p + e^- \rightarrow n$$

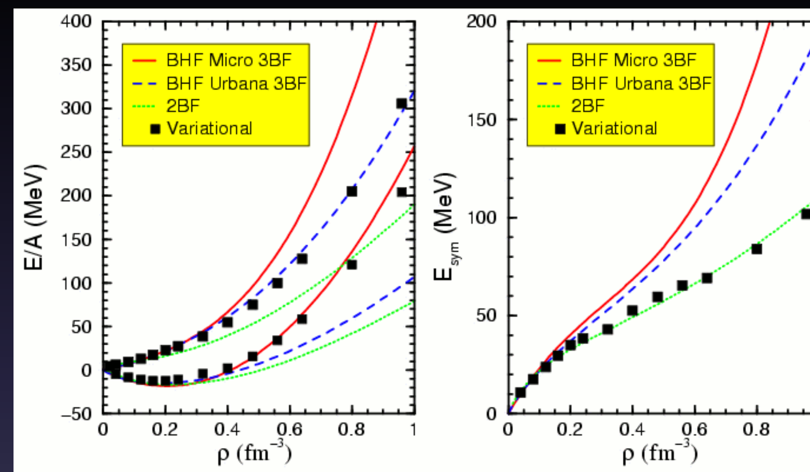
$$\mu_n - \mu_p = \mu_e$$
$$\mu_\mu = \mu_e$$
$$x_p = x_e + x_\mu$$
$$x_n + x_p = 1$$

Parabolic approximation

Asymmetry parameter $\beta = 1 - 2x_p$

$$\frac{B}{A}(\rho,\beta) \approx \frac{B}{A}(\rho,\beta=0) + \beta^2 E_{sym}(\rho)$$

$$E_{sym}(\rho) = -\frac{1}{4}\frac{\partial}{\partial x_p}\frac{B}{A}(\rho, x_p = 0) \approx \frac{B}{A}(\rho,\beta=1) - \frac{B}{A}(\rho,\beta=0)$$



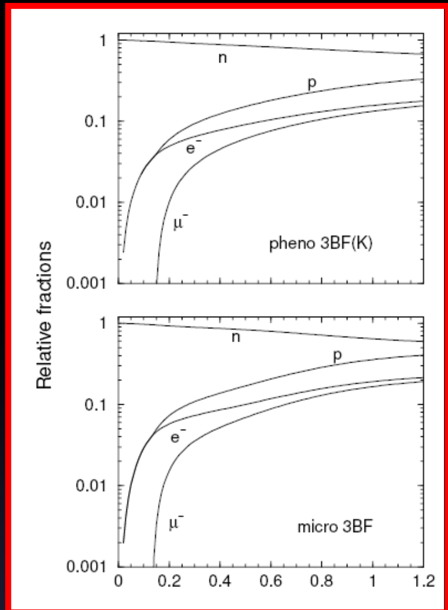
$$\mu_{p,n}(\rho,\beta) = \mu_N(\rho,0) + \beta^2 \rho \frac{\partial}{\partial \rho} E_{sym}(\rho) - (\beta^2 \pm 2\beta) E_{sym}(\rho)$$

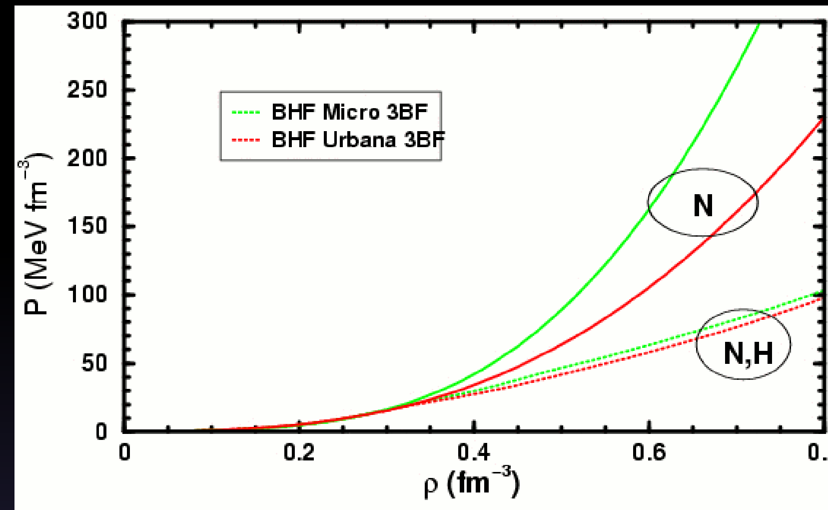
(+ for p, − for n), where $\mu_N(\rho,0)$ is the chemical potential of a nucleon in symmetric matter.

One obtains

$$\left[\mu_n - \mu_p\right](\rho,\beta) = 4\beta E_{sym}(\rho)$$

Particles' population → Equation of state



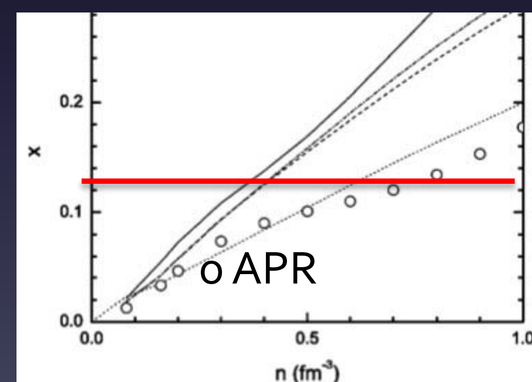


VOLUME 66, NUMBER 21 PHYSICAL REVIEW LETTERS 27 MAY 1991

Direct URCA Process in Neutron Stars

James M. Lattimer
Earth and Space Sciences Department, State University of New York at Stony Brook, Stony Brook, New York 11794
C. J. Pethick
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
and Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
Madappa Prakash
Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794
Pawel Haensel
Copernicus Astronomical Center, 00-716 Warszawa, Poland
(Received 29 January 1991)

We show that the direct URCA process can occur in neutron stars if the proton concentration exceeds some critical value in the range (11–15)%. The proton concentration, which is determined by the poorly known symmetry energy of matter above nuclear density, exceeds the critical value in many current calculations. If it occurs, the direct URCA process enhances neutrino emission and neutron star cooling rates by a large factor compared to any process considered previously.



$$n \rightarrow p + e^- + \bar{\nu}_e \quad , \quad p + e^- \rightarrow n + \nu_e$$

This is the main mechanism of NS cooling which dominates all other processes when the proton fra
is larger than about 12%.

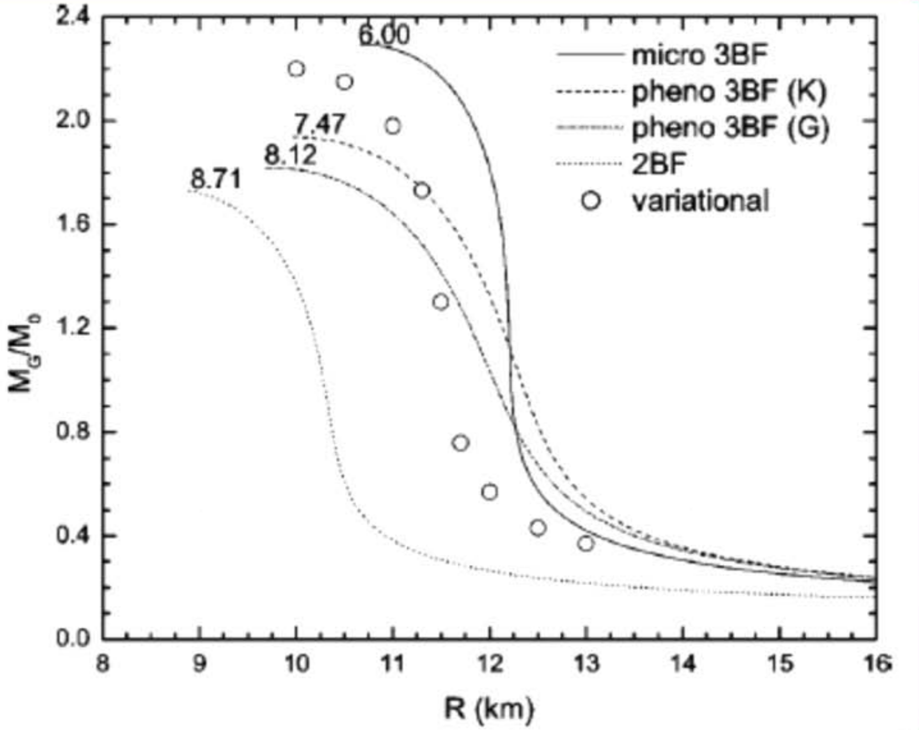
"Recipe" for neutron star structure calculation :

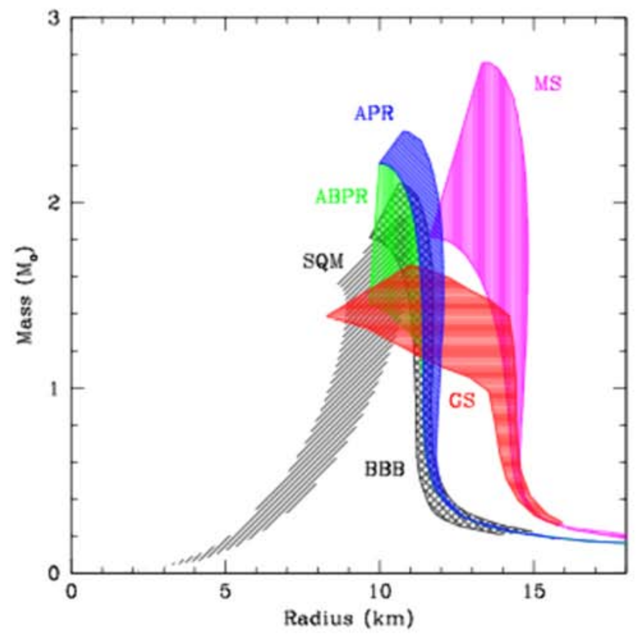
- Brueckner calculation : $\epsilon(\rho, x_i); x_i = \rho_i/\rho$
- Chemical potentials : $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$
- Beta-equilibrium : $\mu_i = b_i \mu_n - q_i \mu_e$
- Charge neutrality : $\sum_i x_i q_i = 0$
- Composition : $x_i(\rho)$
- Equation of state : $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$
- TOV equations :

$$\begin{cases} \frac{dP}{dr} = -\frac{Gm}{r^2}\frac{(\epsilon + P)(1 + 4\pi r^3 P/m)}{1 - \frac{2Gm}{r}} \\ \frac{dm}{dr} = 4\pi r^2 \epsilon(r) \end{cases}$$

- Structure of the star : $\rho(r), M(R), etc.$

Neutron star Mass M and Radius R

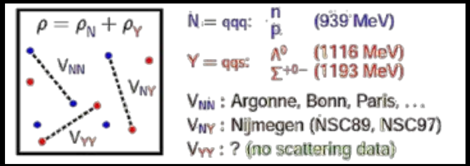




from F. Ozel, “Surface emission from Neutron Stars and Implications for the Physics of their interiors”, arXiv:1210.0916

Different many-body techniques and matter compositions predict different results for the M-R relation.

INCLUDING HYPERONS



- Extension of the BBG theory.

  Several reaction channels involved, more time consuming calculations.

- Few experimental data of hypernuclei on nucleon-hyperon interaction.

  Nijmegen parametrization, Phys. Rev. C40, 2226 (1989) (NSC89)

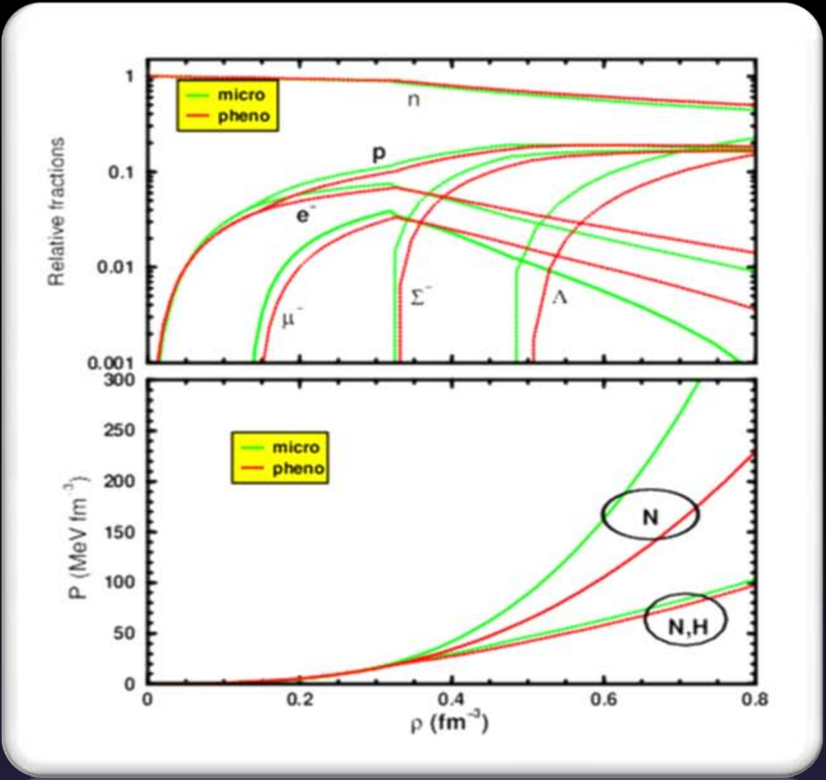
- Unknown HH interaction.

  Use of an extrapolation of NSC89, called NSC97

- Strong consequences for NS structure.

  Softening of the EoS

$$\mu_n = \mu_p + \mu_e$$
$$\mu_e = \mu_\mu$$
$$2\mu_n = \mu_p + \mu_{\Sigma^-}$$
$$\mu_n = \mu_\Lambda$$
$$\rho_p = \rho_e + \rho_\mu + \rho_{\Sigma^-}$$
$$\rho = \rho_p + \rho_n + \rho_\Sigma + \rho_\Lambda$$

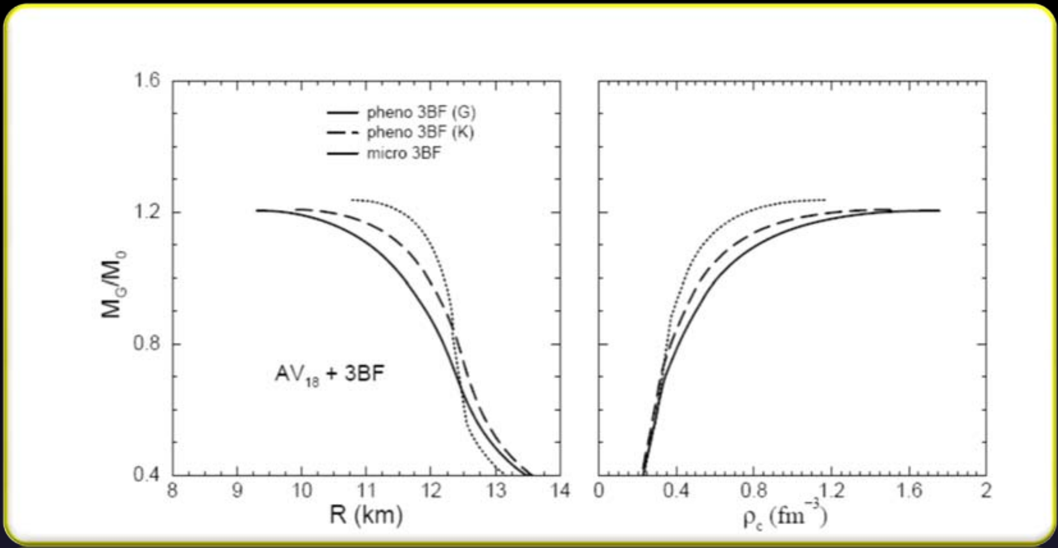


Hyperon onset at density close to 2-3 times the saturation value.

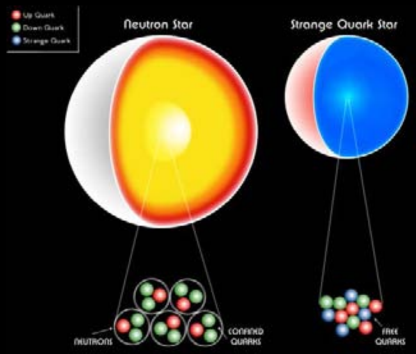
In asymmetric beta-stable matter, nucleons and hyperons are equally present at high densities.

Strong softening of the EoS, no matter the nucleonic TBF's.

- Small limiting masses.



- New experimental data from hypernuclei are needed.



Quark matter in NS (hybrid stars)

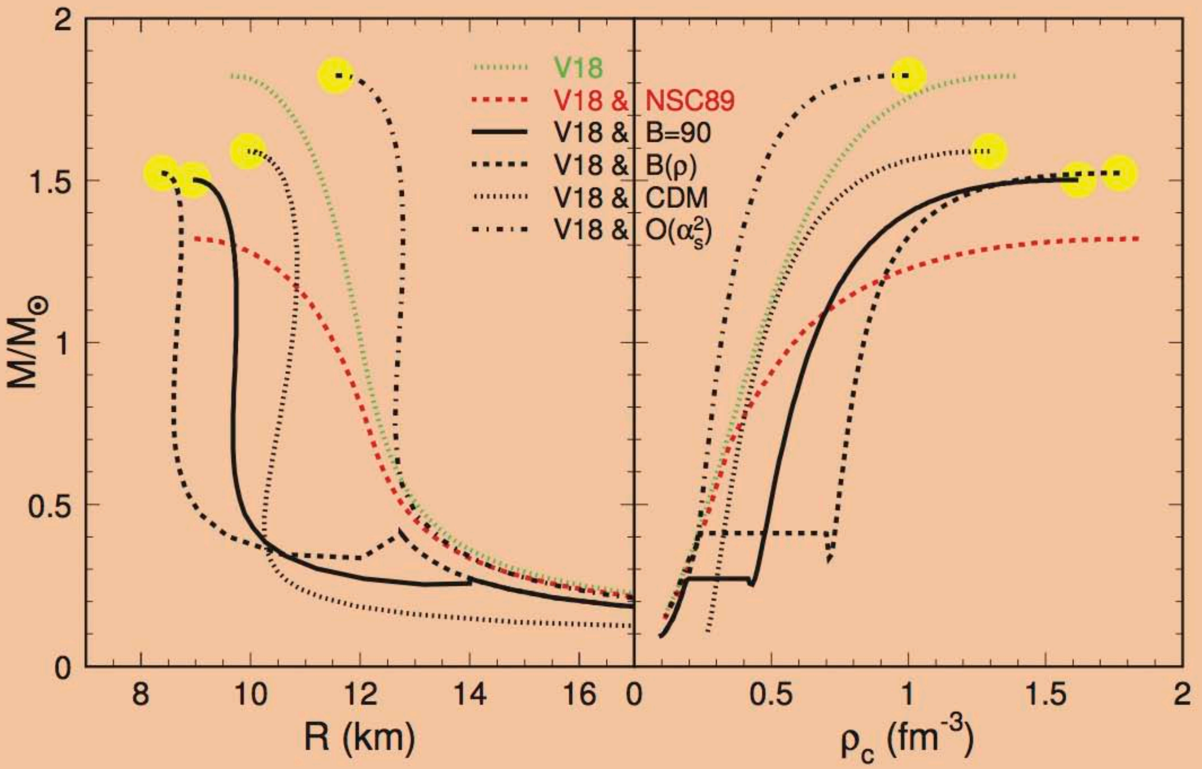
- Since we have no theory which describes both confined and deconfined phases, we use two separate EOS for baryon and quark matter and look at the crossing in the P-μ plane .
- Constraints from nuclear phenomenology on the general quark EOS : In symmetric nuclear matter one can expect a transition to quark matter at some density, but it must be larger than at least normal nuclear matter density.
- Maximum NS mass at least equal to 1.97 M_0

Several models of quark matter

- MIT bag model
- Nambu-Jona—Lasinio model
- Color Dielectric model
- Dyson-Schwinger model

They all give different hybrid star structure and mass limits.

- Different quark EOS: bag models, color dielectric model:



NJL, Dyson-Schwinger models: hyperons prevent phase transition

→ Maximum masses: 1.5…1.9 $M_\odot$, Radii are different !

Maximum mass of neutron stars

- The inclusion of the deconfined phase seems to stabilize the value of the maximum mass, which lies in a relatively narrow range, between 1.5 and 1.9 solar mass.
  However the structure of neutron star is quite different in the different models.

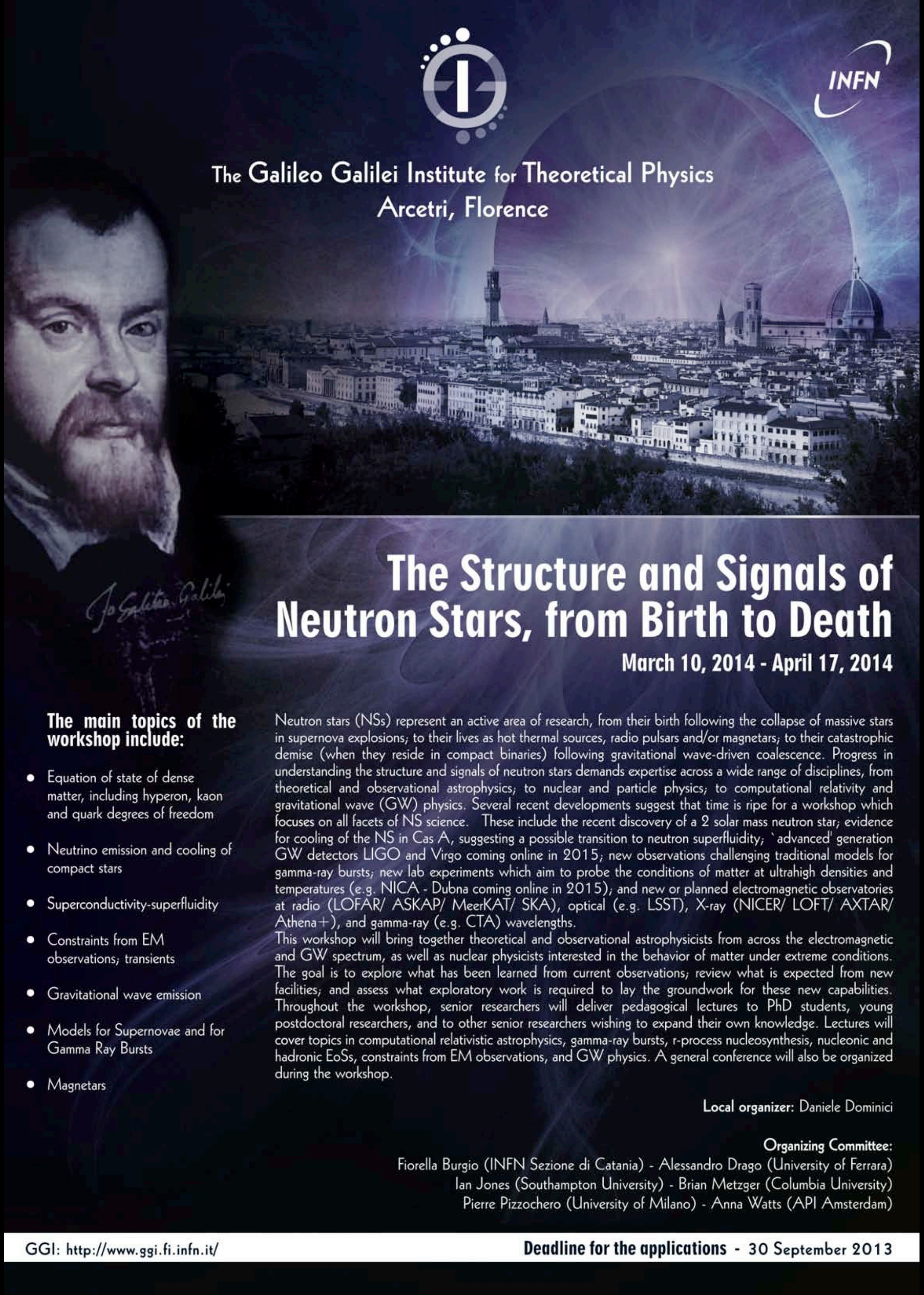
- The observation of ***PSR*** J0348+0432, **M=2.01± 0.04 Mo** implies that additional repulsion is required in the currently adopted quark models.

We learn about strong force just by looking at the sky !

Final considerations

- The EoS of nucleonic matter is much under control up to 4-5 times normal density.
- Excellent agreement with experimental data on nuclei from HIC.
- Large uncertainty in the TBF's.

- Hyperons appearance : open question
- Phase transition to quark matter : open question

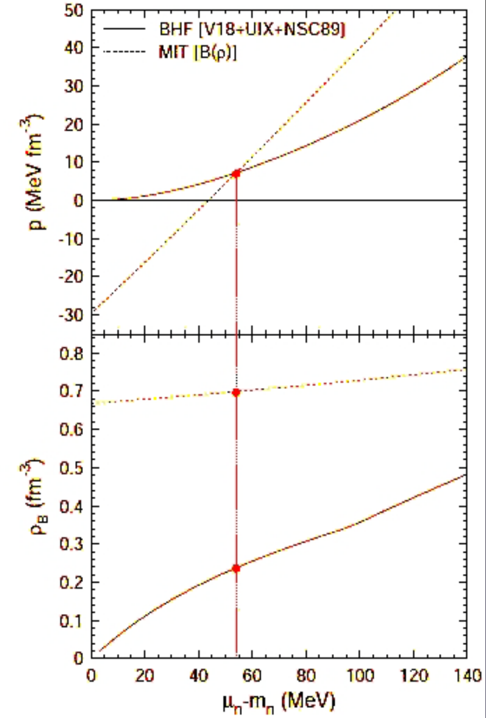


This conference is supported in part by the European network CompStar (MPNS COST Action MP1304 - Exploring fundamental physics with compact stars)

Hadron-Quark Phase Transitions:

- Phase coexistence under mech. and chem. equilibrium:

$$p_H = p_Q$$
$$\mu_i = B_i\mu_B + Q_i\mu_Q$$
$$(i = n, p, \Lambda, \Sigma, u, d, s, e, \mu)$$

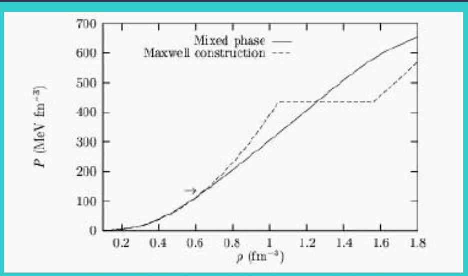


- Maxwell construction:

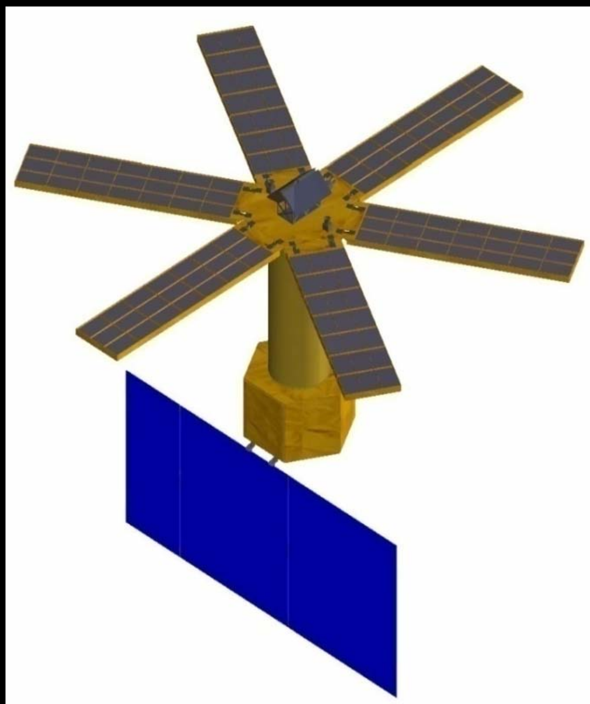
Local charge neutrality in Q and H phases

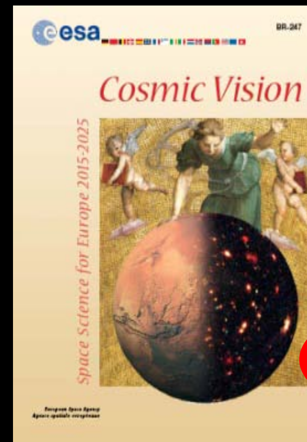
↪ Discontinuous H-Q transition inside the star: $\rho_Q \gg \rho_H$

Imposing the global charge neutrality ⟹ Gibbs construction



LOFT: the Large Observatory For x-ray Timing





3. What are the fundamental physical laws of the Universe?

3.1 Explore the limits of contemporary physics

Use stable and weightless environment of space to search for tiny deviations from the standard model of fundamental interactions

3.2 The gravitational wave Universe

Make a key step toward detecting the gravitational radiation background generated at the Big Bang

3.3 Matter under extreme conditions

Probe gravity theory in the very strong field environment of black holes and other compact objects, and the state of matter at supra-nuclear energies in neutron stars

LOFT Science Team composed of scientists from:

*Australia, Brazil, Canada, CzechRepublic, Denmark,
Finland, France, Germany, Greece, Ireland, Israel, Italy,
Japan, theNetherlands, Poland, Spain, Sweden,
Switzerland, Turkey, United Kingdom, USA*

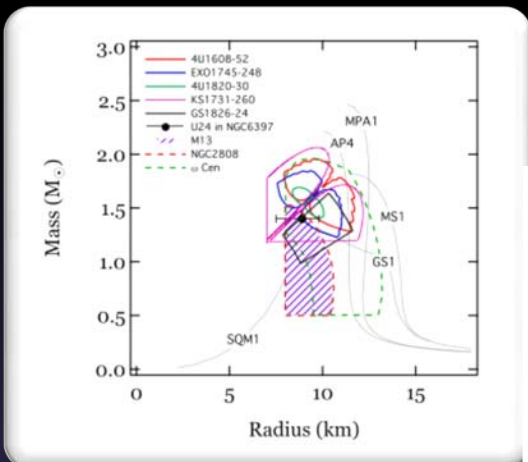
LOFT Consortium: national representatives:

Jan-Willem den Herder — SRON, the Netherlands
Marco Feroci — INAF/IAPS-Rome, Italy
Luigi Stella — INAF/OAR-Rome, Italy
Michiel van der Klis — Univ. Amsterdam, the Netherlands
Thierry Courvousier — ISDC, Switzerland
Silvia Zane — MSSL, United Kingdom
Margarita Hernanz — IEEC-CSIC, Spain
Søren Brandt — DTU, Copenhagen, Denmark
Andrea Santangelo — Univ. Tuebingen, Germany
Didier Barret — IRAP, Toulouse, France
Renè Hudec — CTU, Czech Republic
Andrzej Zdziarski — N. Copernicus Astron. Center, Poland
Juhani Huovelin — Univ. of Helsinki, Finland
Paul Ray — Naval Research Lab, USA
Joao Braga — INPE, Brazil
Tad Takahashi — ISAS, Japan

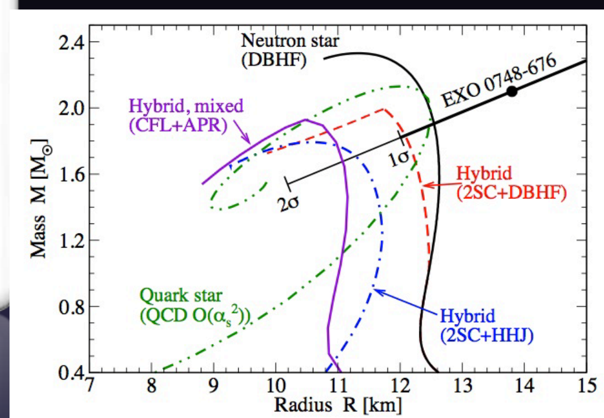
EoS from astrophysical observations from isolated NS

$$F_\infty = \sigma_B T^4_{eff,\infty}\left(\frac{R}{D}\right)^2\left(1-\frac{2GM}{Rc^2}\right)^{-1}$$

TOV inversion to get model-independent EoS

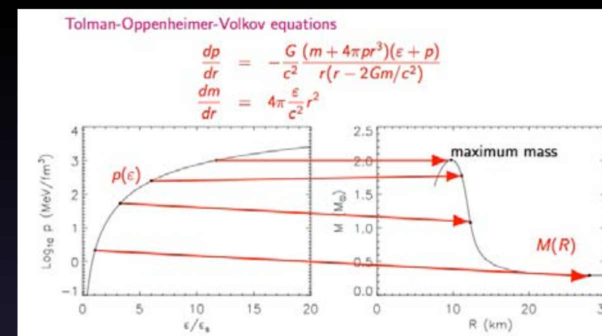


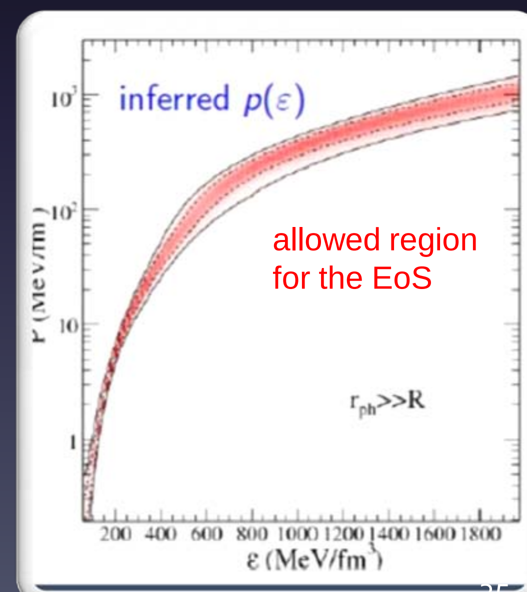
F. Ozel, arXiv:1210.0916.
Reports on Progress in Physics



Alford, Drago et al., Nature 2007

(Steiner, Lattimer, 2010)

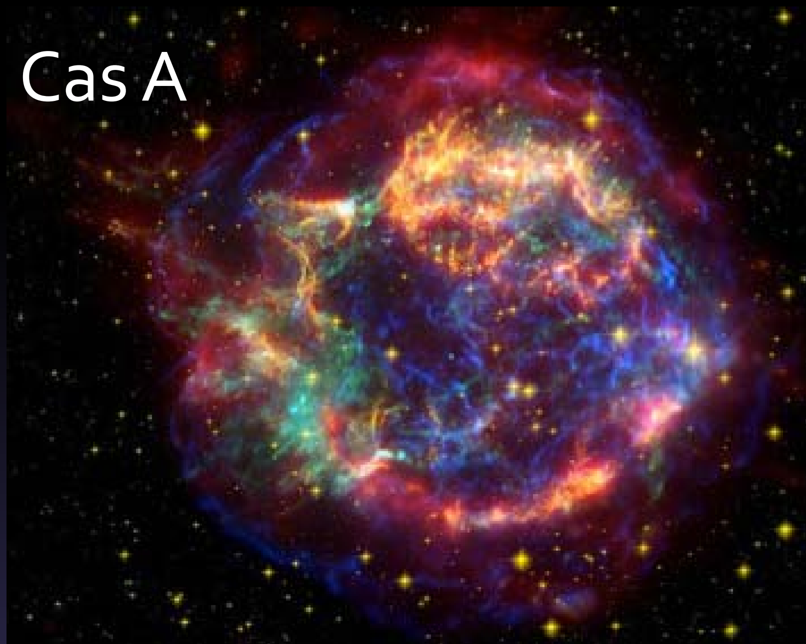




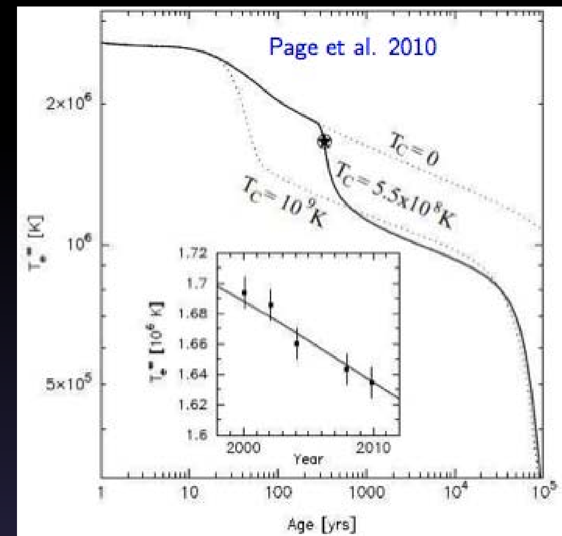
- All current measurements are consistent with radii in the range 8-12 km.
- Quark matter in the inner core cannot be excluded

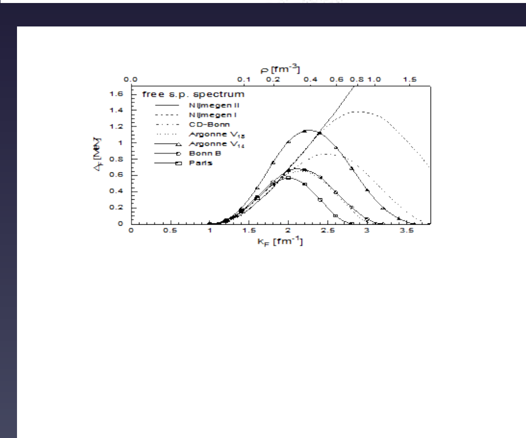
Lombardo & Schulze, Lect.Notes.Phys. 2001

Effects of superfluidity on cooling



P. Shternin, MNRAS 2011





Baldo et al., PRC 1998

Observations of CasA put constraints

- Superfluidity of neutrons in the $^3P_2$ channel
- Lower limit on the gap $\Delta \approx 1$ MeV

Superfluidity as open problem in NS physics

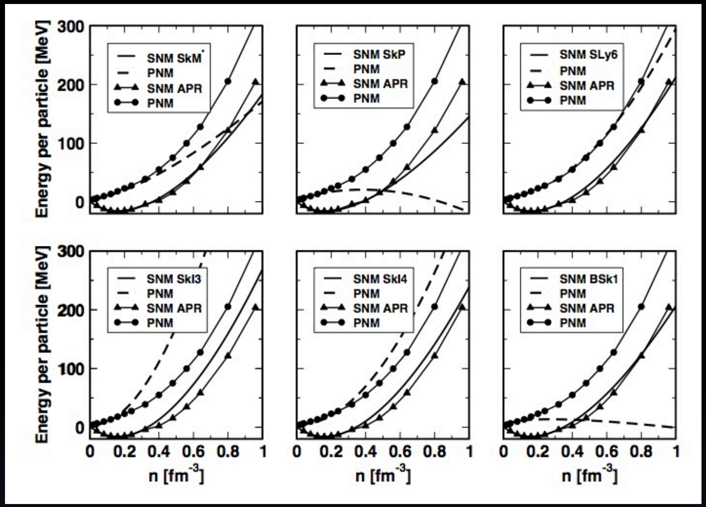
The method of energy density functional : mean field with effective interaction

$$
\begin{aligned}
E[\hat{\rho}] &= \int d\vec{r}\, \mathcal{H}(\vec{r}) \\
\mathcal{H}(\vec{r}) &= \frac{\hbar^2}{2m}\tau(\vec{r}) + \frac{3t_0}{8}\rho^2(\vec{r}) + \frac{t_3}{16}\rho^3(\vec{r}) \\
\tau(\vec{r}) &= \frac{3}{5}\left(\frac{6\pi^2}{g}\right)^{2/3} \rho^{5/3}(\vec{r})
\end{aligned}
$$

The parameters are adjusted to reproduce the empirical saturation point of symmetric nuclear matter.

In general one can introduce a more complex structure for the effective forces (gradient terms, etc.) and correspondingly more parameters. In this case it is possible to reproduce also the binding energy and radii of spherical nuclei with a high degree of accuracy and many spectroscopic data (giant resonances, and so on)

Binding energy of SNM and PNM



J. R. Stone, "Skyrme Interaction and Nuclear Matter Constraints", arXiv:1202.3902

The density dependence of E for SNM is rather similar for all the selected Skyrme forces and compares well with APR model. This doesn't hold true for PNM, which agrees with the APR result only for SLy6.

Symmetry energy