The uses of Holography: from superconductivity to emergent symmetries

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Based on

PRL 107, 181601 (arXiv:1105.5392)
JHEP 1204, 135 (arXiv:1202.0006)
arXiv:1305.0011

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Plan

- Holography basics
- Application 1) Superconductivity
- Application 2) Emergent symmetries
- Conclusions

- Holography basics

Gauge/Gravity duality

Most optimistic definition: QFTs with a gravity dual

e.g., SU(N) YM — large N

in the double limit — large `t Hooft coupling

$$\lambda = g_{YM}^2 N_c$$

QFT geometrizes:

- μ -> (local) extra dimension
- spacetime symmetries -> isometry group
- fixed point -> scale invariant geometry
- Quantum fields -> classical fields

Heemskerk, Penedones, Polchinski Sully'09

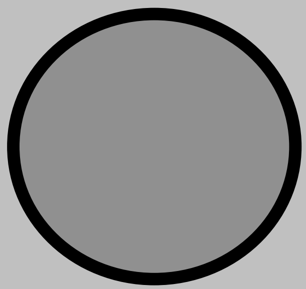
Sundrum '11

Fitzpatrick Kaplan '12

El-Showk Papadodimas'12

Gauge/Gravity duality

Simplest example: AdS – CFT



$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 - dt^2 + dx_{d-1}^2\right]$$

Conformal boundary at $z = 0$

$$z = e^{-y/\ell}$$

$$\mu = 1/z$$

Boundary conditions:

either $\phi\big|_{\partial M}$ or $\partial_z \phi\big|_{\partial M}$ must be specified

Gauge/Gravity duality

Operators – fields

$$\phi = z^{\Delta_-} J + z^{\Delta_+} O$$

Dimensions – masses

$$\Delta(d-\Delta) = M^2 \ell^2$$

Global symmetries – local symmetries

conserved current J_μ : $A_\mu = a_\mu + z^{d-2} J_\mu$

$T_{\mu\nu}$: $g_{\mu\nu} = z^{-2}\gamma_{\mu\nu} + z^{d-2} T_{\mu\nu}$

Gauge/Gravity duality

Finite T $\quad g_{\mu\nu} = z^{\Delta_- -2}\gamma_{\mu\nu} + \ldots$ $\quad$ Compact euclidean time $\quad \Delta t_E = 2\pi / T$

Finite μ $\quad A_\nu = a_\nu + \ldots$ $\quad A_0 = \mu$

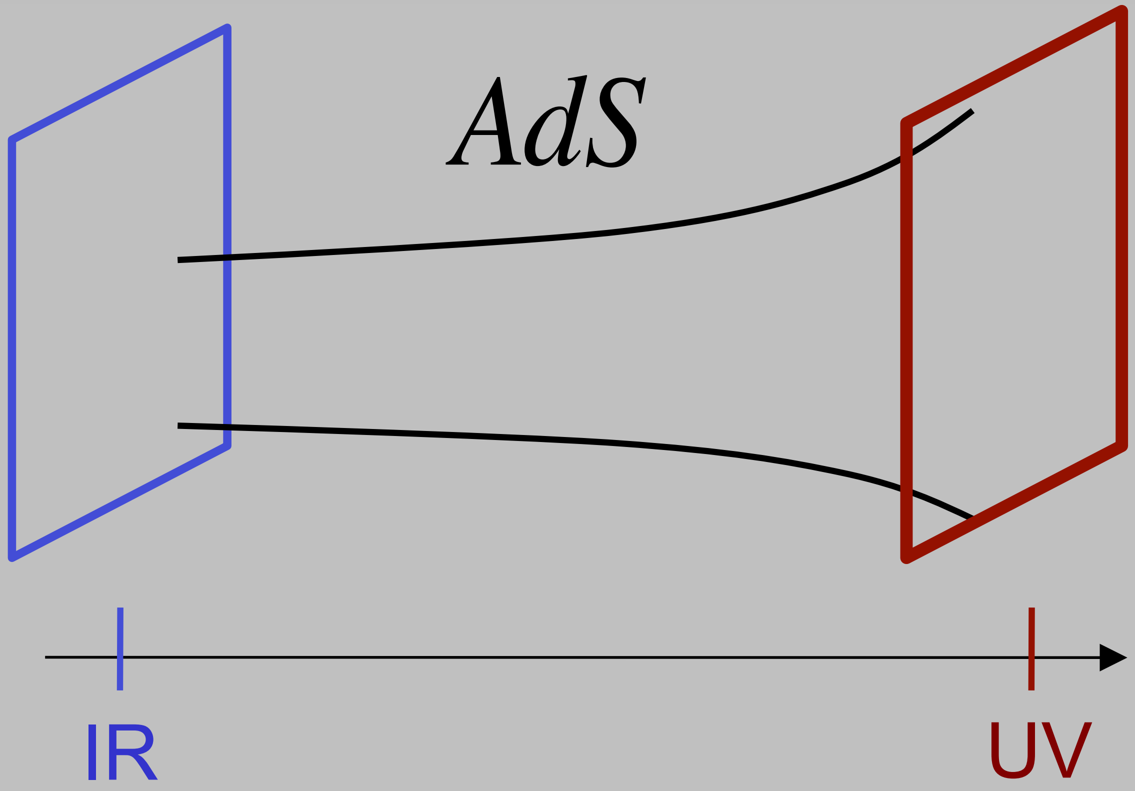
response

Partition function = On-shell Action

$$Z[J] = \int D\psi \exp\left(-S_D[\psi] - J\,O_\psi\right) = \exp\left(-S_{D+1}^{class}\left[\phi;\ \phi\big|_{\partial M} = J\right]\right)$$

correlators

Emergent phenomena



composite / emergent

elementary

e.g., mass gap = $1/z_{IR}$

Holographic *materials*

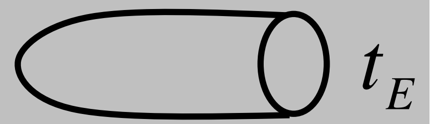
Holographic *materials*

de-confined plasma / fluid / conductor

no mass gap

Black brane

$$ds^2 = \frac{\ell^2}{z^2}\left[\frac{dz^2}{f(z)} - f(z)\,dt^2 + dx_{d-1}^2\right]$$



$$\Delta t_E = 2\pi / T$$

$$f(z) = 1 - \rho\, z^d$$

$$\rho \propto T^d$$

hydrodynamics: sound waves $c_S^2 = 1/3$

transport coefficients

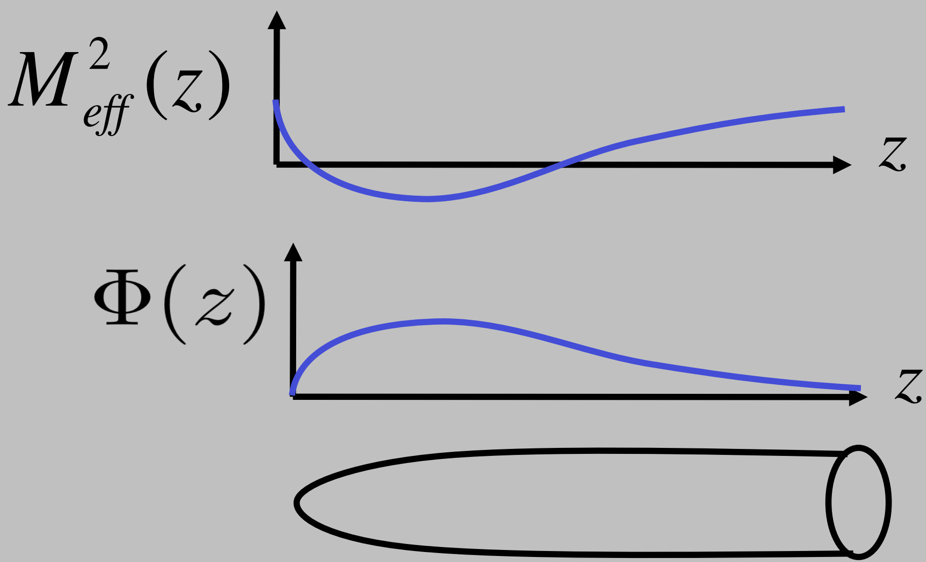
Holographic *materials*

Superconductor / conductor

Hartnoll Herzog Horowitz '08

charged condensate $\langle \mathcal{O} \rangle \neq 0$ $\quad \mathcal{O} \begin{cases} \text{dimension } \Delta \\ \text{charge } \quad n\,e \end{cases}$

Holographically → (charged-)scalar "hair"



$$M^2_{eff} = M^2 - g^2 A_\mu A_\nu g^{\mu\nu}$$

$$\Phi(z) \to \cancel{j\, z^{\Delta_-}} + \langle \mathcal{O} \rangle\, z^{\Delta_+}$$

Holographic *materials*

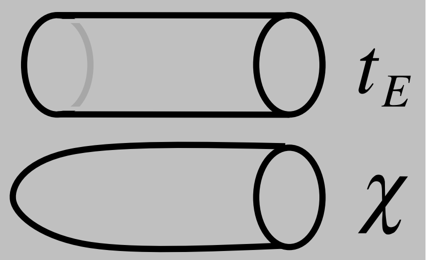
confining vacuum / solid / insulator

finite mass gap: $1/R$

AdS Soliton

$$ds^2 = \frac{\ell^2}{z^2}\left[\frac{dz^2}{f(z)} + f(z)\,d\chi^2 - dt^2 + dx_{d-2}^2\right]$$

$$f(z) = 1 - E_C\, z^d$$



$$\Delta\chi = 2\pi R$$

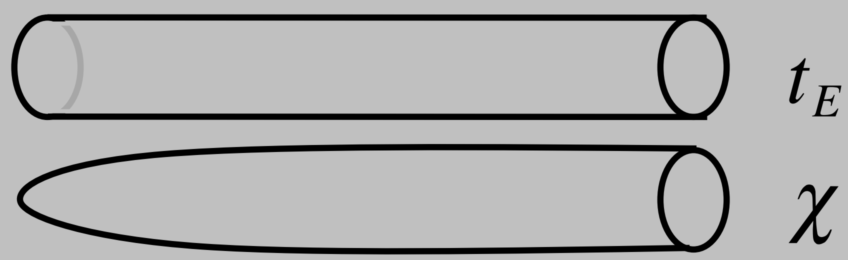
$$\Delta t_E = 2\pi / T$$

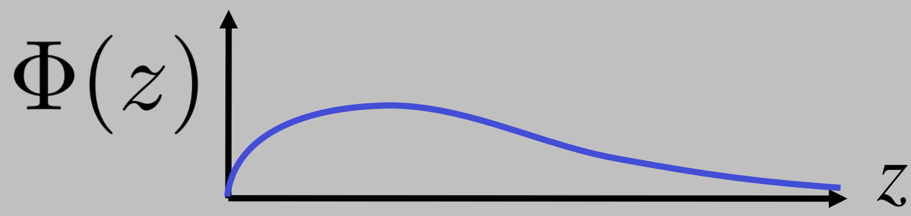
- groud state for $T < 1/R$
- Casimir Energy $E_C \propto 1/R^d$

Holographic *materials*

super-solid / super-insulator *(insulator-SC)*

AdS Soliton + scalar + large enough *μ*

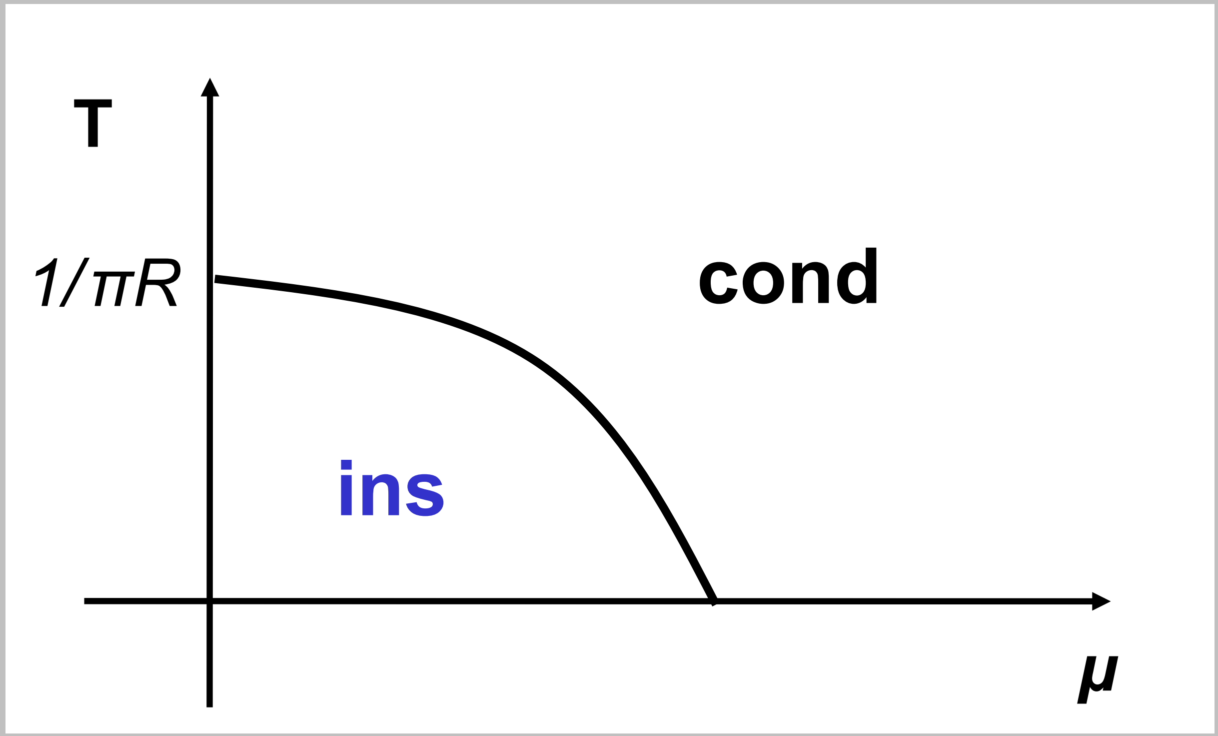




$\mu > \mu_c$

Holographic *materials*

conductor-insulator / de-confinement / Hawking-Page P.T.

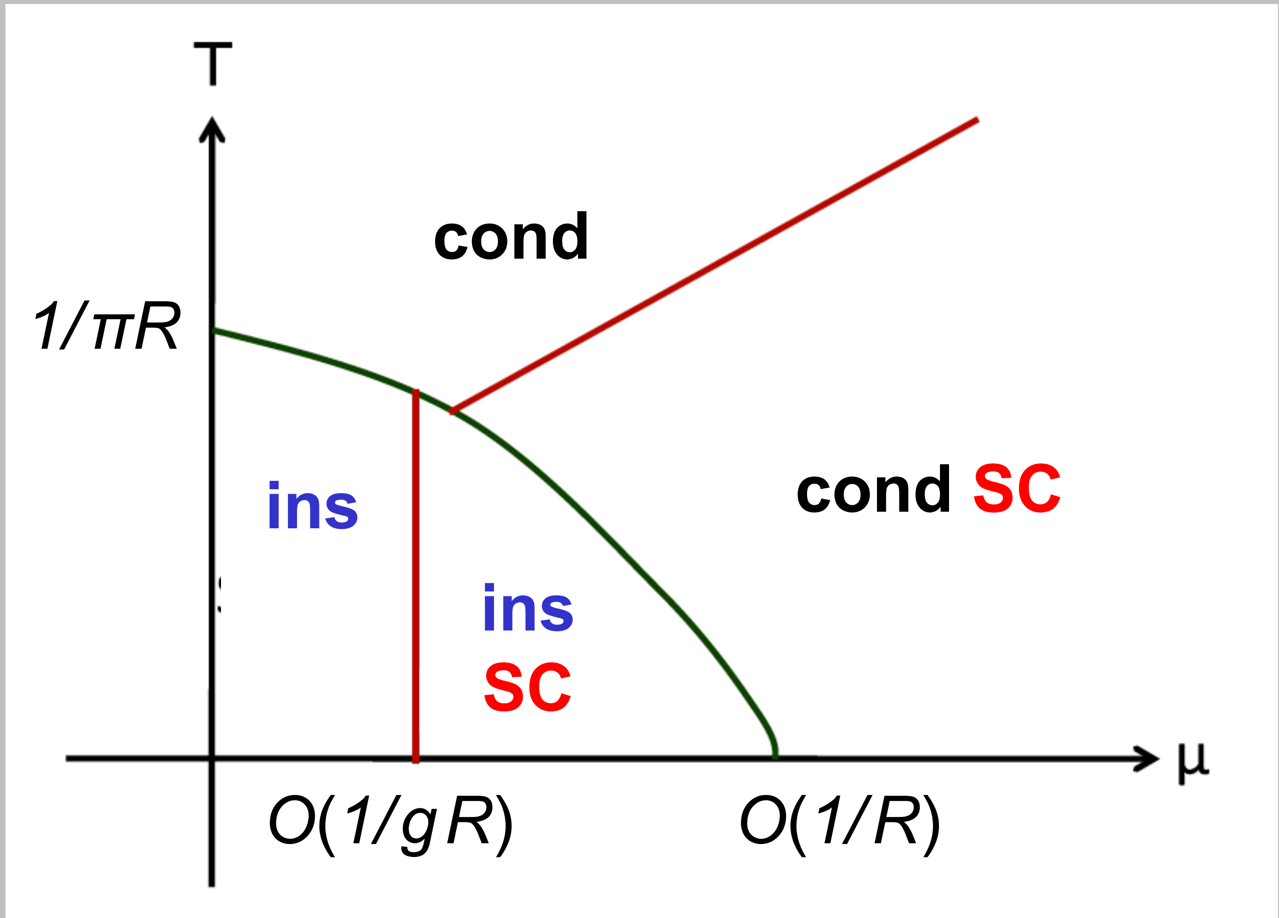


Free energy at μ=0 : $F^{(Sol)} = -N^2 / R^4$ $F^{(BH)} = -N^2 (T^4 + \mu^2)$

(Casimir energy)

Holographic *materials*

Holographic SuperConductors



Nishioka Ryu Takayanagi '10

- Application 1) SC – Flux Periodicities

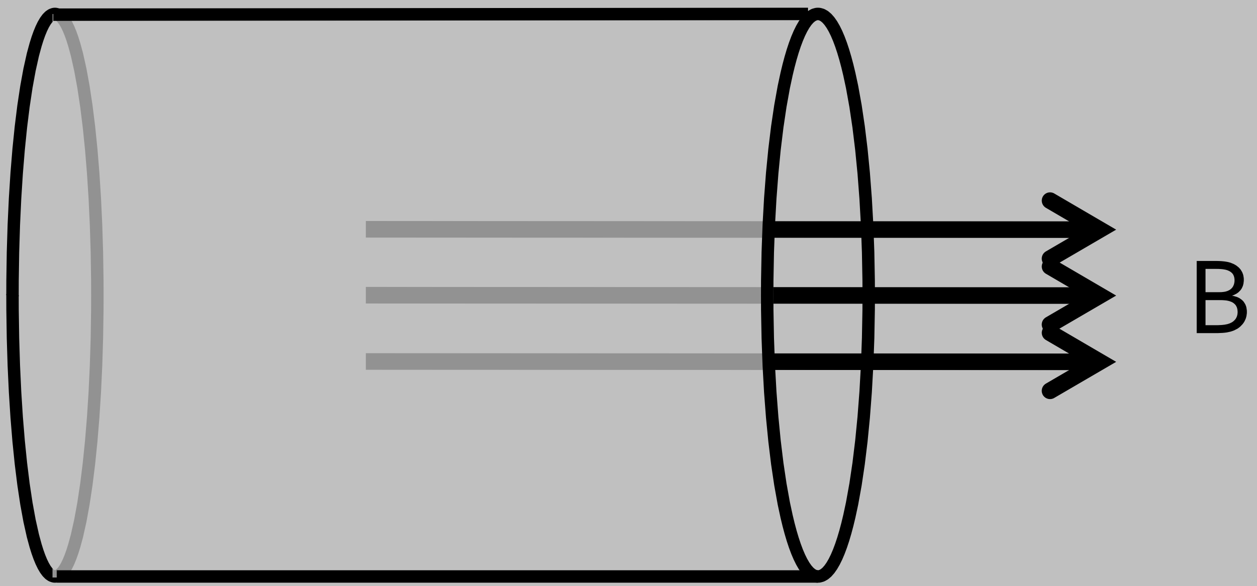
Flux Periodicities

Cylindrical material threaded by 'axial' B-field

(<-> Scherk-Schwartz compactification)

$$W \equiv \exp\left(ie\oint A_\mu dx^\mu\right) = \exp\left(ie\int d\vec{S}\cdot\vec{B}\right)$$

$$A_\mu \to \left(\mu, \vec{0}, A_\chi\right)$$



two different and competing effects:

Flux Periodicities

effect 1) ***Little-Parks*** – SC jumps between fluxoids

fluxoid configurations:

$$\Phi = \rho \exp\left(i \frac{m}{R} \chi \right)$$

$$A_\mu = \left(\mu, \vec{0}, A_\chi\right)$$

$$m = \frac{1}{2\pi} \oint \partial_\mu \theta \, dx^\mu$$

gauge-inv
(and quantized)

Flux Periodicities

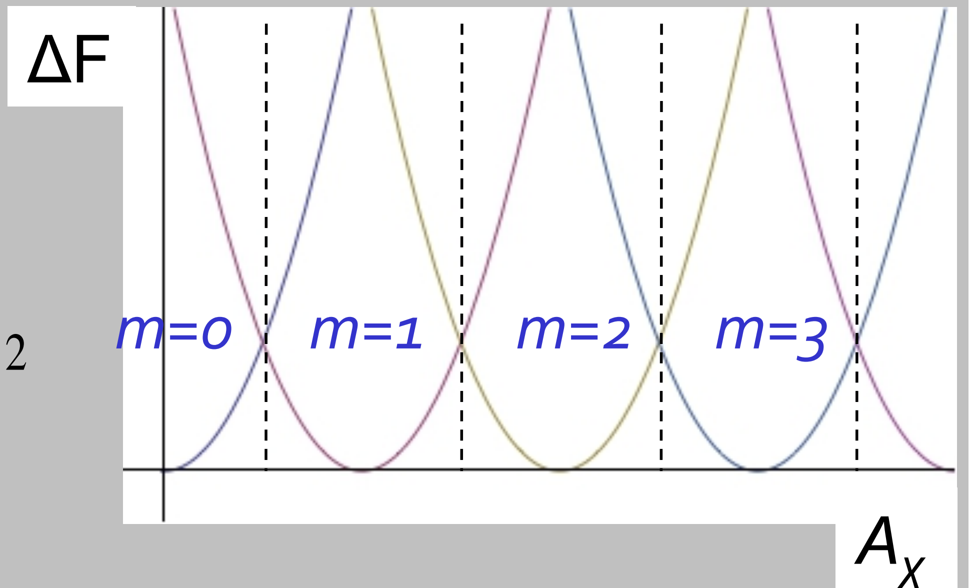
effect 1) ***Little-Parks*** – SC jumps between fluxoids

Free energy of ***fluxoids*** $\approx |D\Phi|^2 - M^2|\Phi|^2 - \lambda|\Phi|^4$

$$\Phi = \rho \exp\left(i \frac{m}{R} \chi \right)$$

$$A_\mu = \left(\mu, \vec{0}, A_\chi\right)$$

$$M_{eff}^2 = M^2 - g^2\mu^2 + \left(\frac{m}{R} - 2eA_\chi \right)^2$$



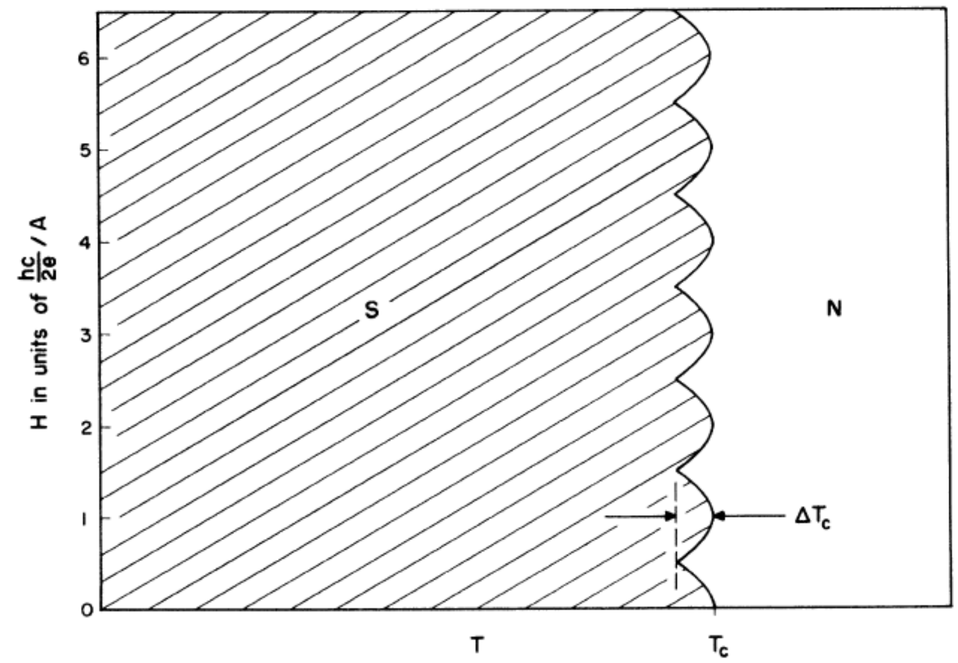
=> transitions between fluxoids

Periodicity: $\Delta A_\chi = 1/2eR$

Flux Periodicities

effect 1) ***Little-Parks*** – SC jumps between fluxoids

VOLUME 9, NUMBER 1 PHYSICAL REVIEW LETTERS JULY 1, 1962



$$\Delta T_c = \frac{\hbar^2}{16\, m^* R_0^2}\left(\frac{2e}{hc}\phi + n\right)^2$$

FIG. 1. Phase diagram for a thin cylindrical superconductor in an axial magnetic field. The scalloped edge of the superconducting phase results from the periodicity in the free energy of the bound pairs, in the magnetic flux through the cylinder.

$\Delta A_\chi = 1/2eR$ hallmark of 'pairing'

LP = semiclassical effect

Flux Periodicities

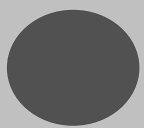
effect 2) ***Aharonov-Bohm effect***

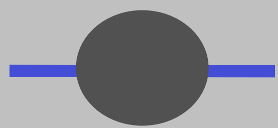
Montull OP
Salvio Silva '11

Periodicity $(\Delta A_\chi)^{AB} = 1/eR = 2(\Delta A_\chi)^{LP}$

$$V_{eff} = V_0(W) - M^2(W)\,|\Phi|^2 - \lambda(W)\,|\Phi|^4$$

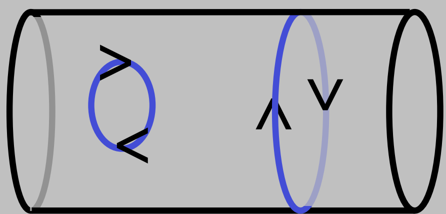
`Casimir – AB' effect





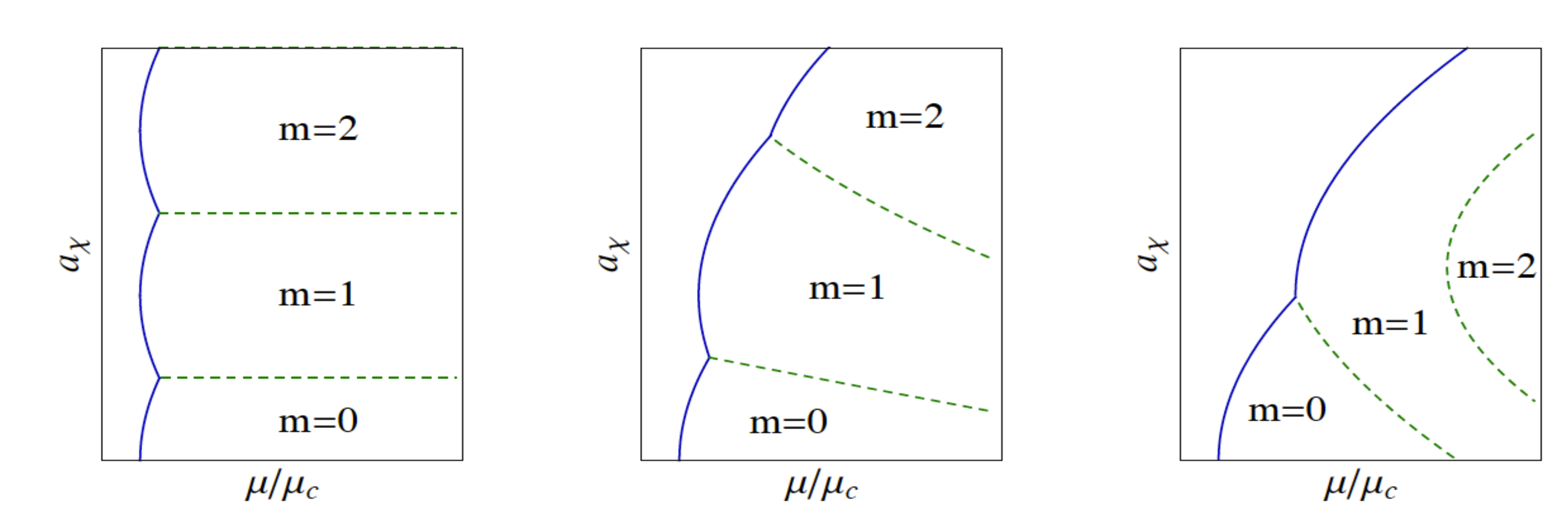
$$\delta M^2 \sim \frac{f(W)}{R^2}$$

f(W) depends on vacuum &
not small if strong coupling



$$W \equiv \exp\left(i\,2\pi\, eA_\chi R\right)$$

Flux Periodicities



$\delta M^{(AB)} = 0$ $\quad$ $\delta M^{(AB)} << M_{eff}^{(LP)}$ $\quad$ $\delta M^{(AB)} \sim M_{eff}^{(LP)}$

"LITTLE-PARKS degeneracy"

with $a_\chi \equiv m'/gR + \widetilde{a}_\chi$,

configurations:
$m = m' = k$
$k = 0, 1, ..., N-1$

define magnetic variety of QH

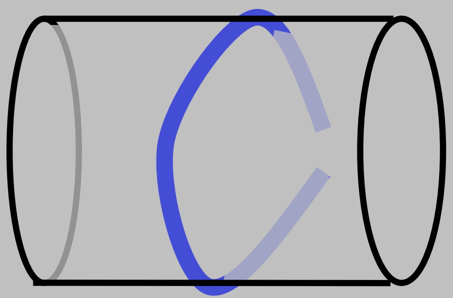
Flux Periodicities

Cond Matt literature disscussing Flux Periodicities

[2] V Vakaryuk, Phys. Rev. Lett. **101**, 167002 (2008).
[3] F Loder *et al.*, Nat. Phys. **4**, 112 (2008).
[4] TC Wei, PM Goldbart Phys. Rev. **B77**, 224512 (2008).

→ Doubling of flux-period occurs for $R \leq \xi_0$

As if AB-effect amongst the 2 electrons in the pair



Holography completely agrees:

BB: $R >> \xi_0 \approx 1/\mu$ => A-B suppressed

Sol: $R \approx \xi_0$ => A-B unsuppressed

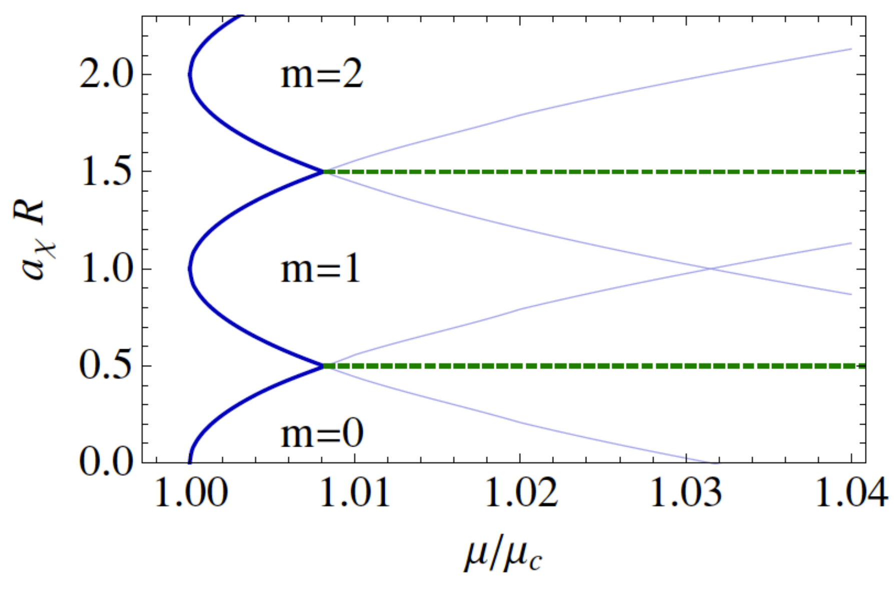
Flux Periodicities

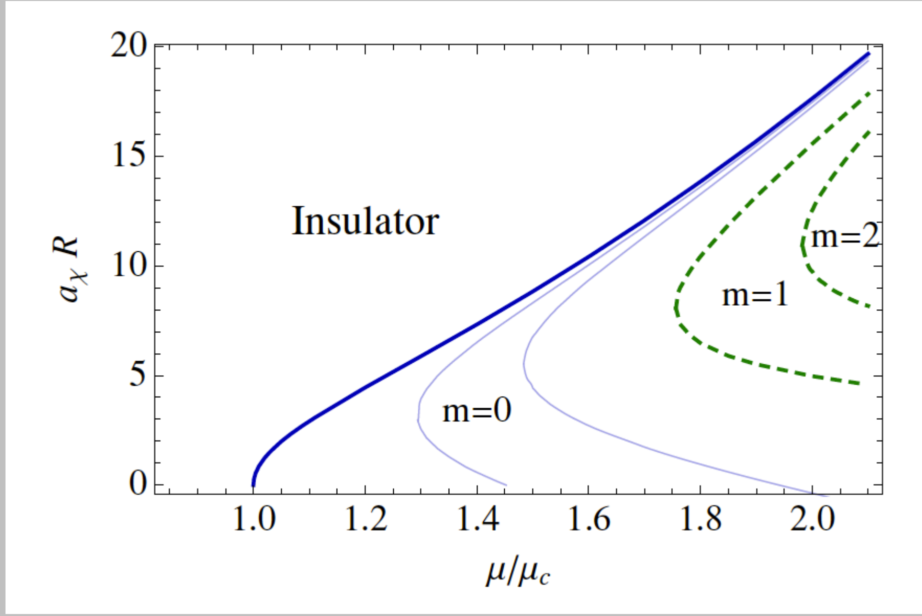
Montull OP Salvio Silva '11,12

In Holographic SCs:

Black Brane (cond/SC)

AdS Sol (ins/SC)





LP degeneracy

LP degeneracy uplifted

Generically, flux periodicity

- $\Delta\Phi_B = 1/2e$ for conductors
- $\Delta\Phi_B = 1/e$ for insulators

Not only at large N

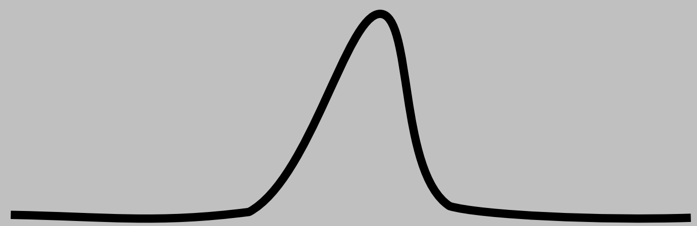
CFT Interpretation

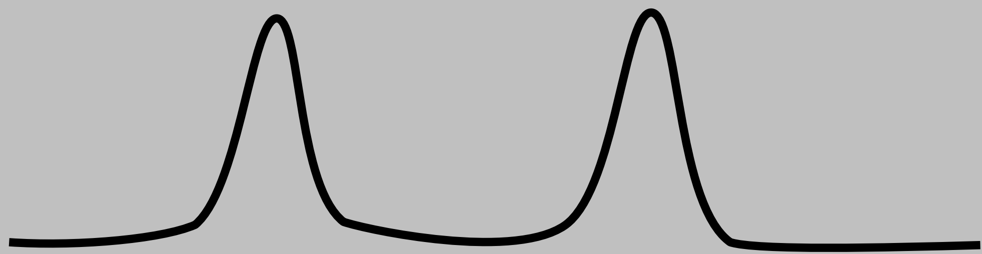
Large N is a classical limit $$\langle AB \rangle = \langle A \rangle \langle B \rangle + 1/N$$

Classical behaviour is recovered at large N
if quantum state admits classical counterpart

Yaffe '82

Ex: Gaussian wave-packet vs. superposition of 2 wavepackets





$\Delta x, \Delta p \to 0$ $\hbar \to 0$ $\Delta x, \Delta p \not\to 0$

classical point particle no classical state !

CFT Interpretation

Large N is a classical limit $\langle AB \rangle = \langle A \rangle \langle B \rangle + 1/N$

Classical behaviour is recovered at large N
if quantum state admits classical counterpart

BH/cond -> plasma -> has a classical counterpart

=> AB effects suppressed

Sol/ins -> confining vac. -> no classical counterpart

=> AB effects persists!

Montull OP
Salvio Silva '11

Conclusions – 1)

Rather generic prediction:

Flux-threaded cylindrical SCs exhibit

1/e flux-periodicity – if normal phase is insulator

1/2e flux periodicity – if conductor

We'll see...?

Large-N no-hair theorems?

- **Application 2) Emergent symmetries**

Emergence of Lorentz Invariance

can **Lorentz Invariance (LI)**
be *accidental* ??

Context: **LV** → QFT of Gravity (Hořava Gravity)

From phenomenological perspective, the recovery of LI at low energies is the most pressing issue

Emergence of Lorentz Invariance

can **Lorentz Invariance (LI)** be *accidental* ??

Context: **LV** → QFT of Gravity (Hořava Gravity)

From phenomenological perspective, the recovery of LI at low energies is the most pressing issue

1) Bounds on Lorentz Violation

Matter Sector

$\delta c_\psi \, \bar{\psi}\gamma^i D_i \psi$ $\qquad \delta c_\gamma^2 \, \vec{B}^2$ $\qquad \delta c_H^2 \; |D_i H|^2$ $\qquad$ CPT+ Dim4

Observational bounds:
$$\begin{cases} |c_e - c_\gamma| < 10^{-15} \; ! \\ |c_p - c_\gamma| < 10^{-20} \; !! \end{cases}$$

FINE TUNING

EFT expectation: $\delta c \sim 1 - 10^{-3} \; !!!$

Collins Perez Sudarsky Urrutia Vucetich '04

Iengo Russo Serone '09

Giudice Strumia Raidal '10

Anber Donoghue '11

Challenge: can one achieve naturally ~ 10^{-20} suppression?

2) LV & RG

RG-Emergence of Lorentz Invariance

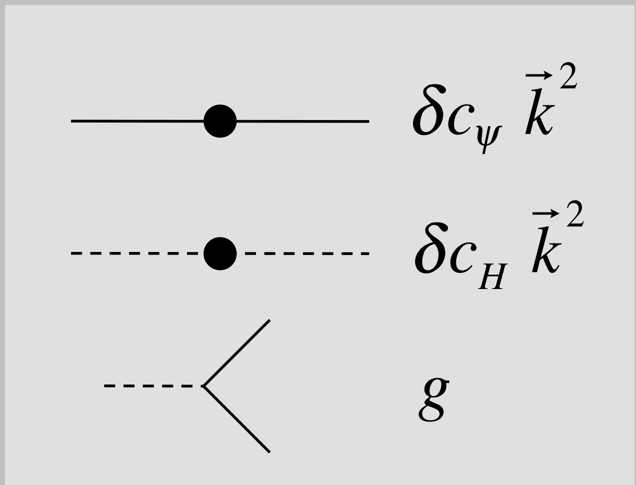
LI-fixed point is IR-attractive !!

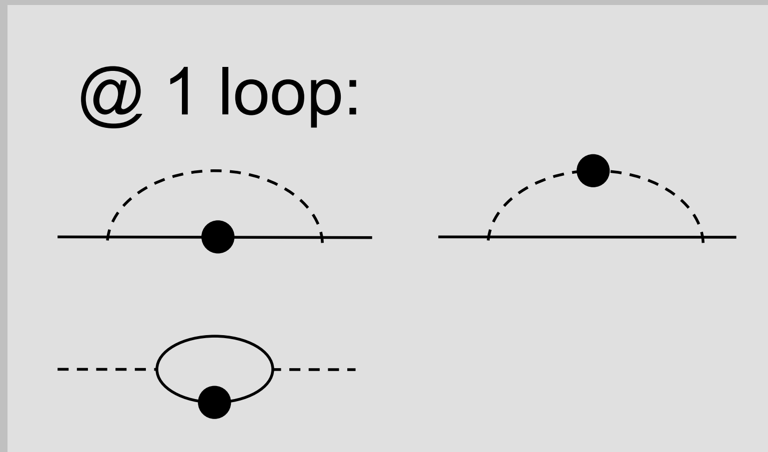
Chadha Nielsen' 83

RG-Emergence of Lorentz Invariance

LI-fixed point is IR-attractive !! Chadha Nielsen' 83

E.g., **LV – Yukawa** theory: $L = (\partial h)^2 + \overline{\psi}\,\gamma\cdot\partial\,\psi + g\,h\overline{\psi}\psi + \delta c's$





$$(4\pi)^2 \frac{d\ \delta c}{d\log\mu} = \beta_{\delta c}\, g^2\, \delta c$$

$$(4\pi)^2 \frac{d\ g}{d\log\mu} = \beta\, g^3$$

$$\delta c = \frac{\delta c_0}{\left[1 - \beta\, g_0^2 \log(\mu/M)\right]^{\frac{\beta_{\delta c}}{\beta}}}$$

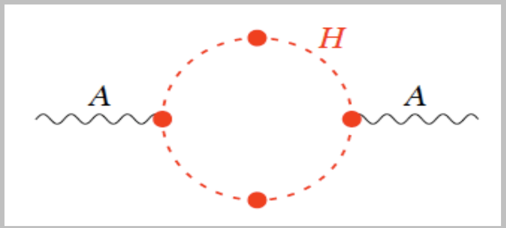
$$g^2 = \frac{g_0^2}{1 - \beta\, g_0^2 \log(\mu/M)}$$

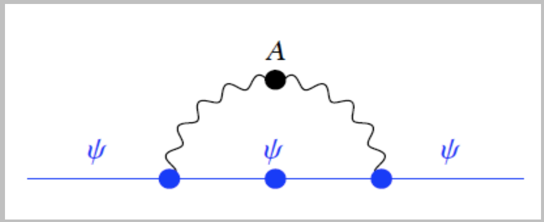
RG-Emergence of Lorentz Invariance

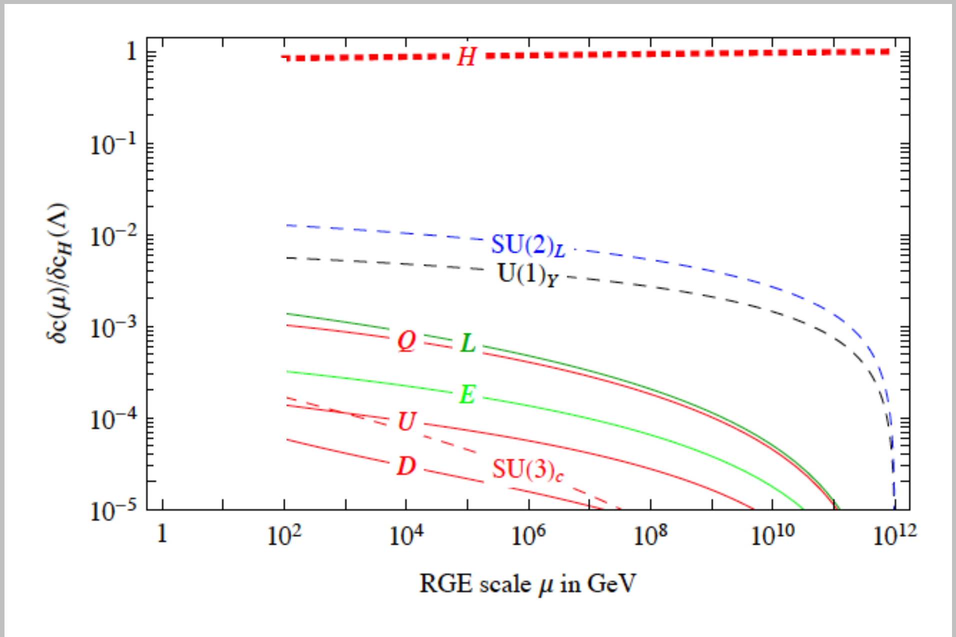
LI-fixed point is IR-attractive !!

Giudice Strumia Raidal' 10

E.g., **LV – Standar Model** (SME)







RG-Emergence of Lorentz Invariance

LI

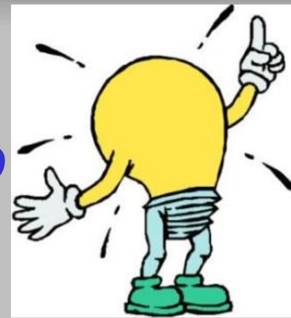
The RG flow by itself already provides a mechanism for the Emergence of LI

But (*in weakly coupled theories*) the emergence is too slow!

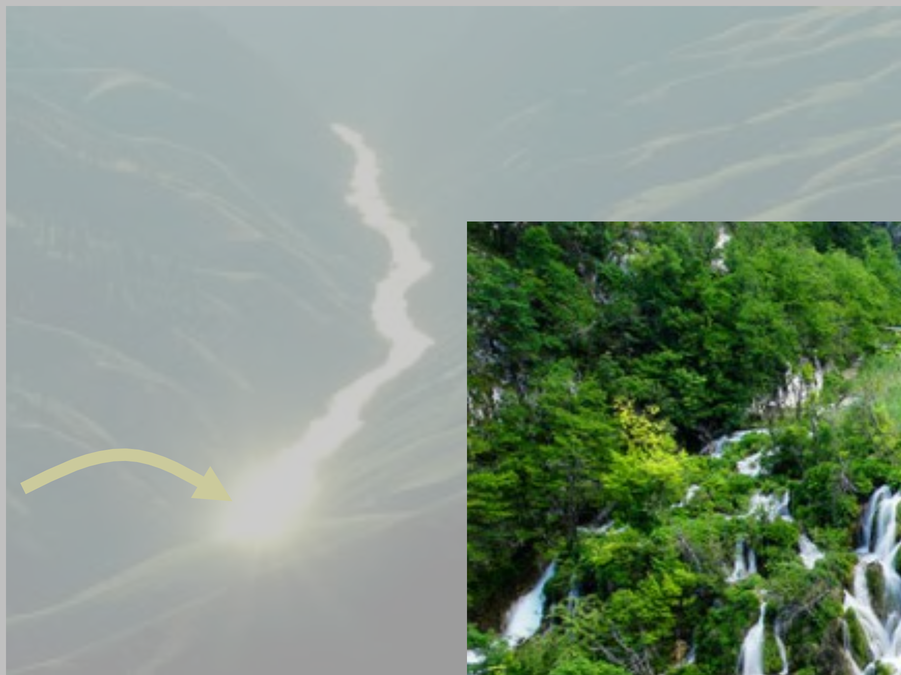
Suppression is only for a factor $Log\left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right) \sim 10$

RG-Emergence of Lorentz Invariance

let's accelerate the running by turning to strong coupling



LI



LI

RG-Emergence of Lorentz Invariance

Idea:

$$(4\pi)^2 \frac{d\,\delta c}{d\log\mu} = \beta_{\delta c}\, g^2\, \delta c$$

If strongly-coupled fixed point:

$$\delta c = \mu^{\frac{\beta_* g_*^2}{(4\pi)^2}} \delta c_0$$

accelerated running

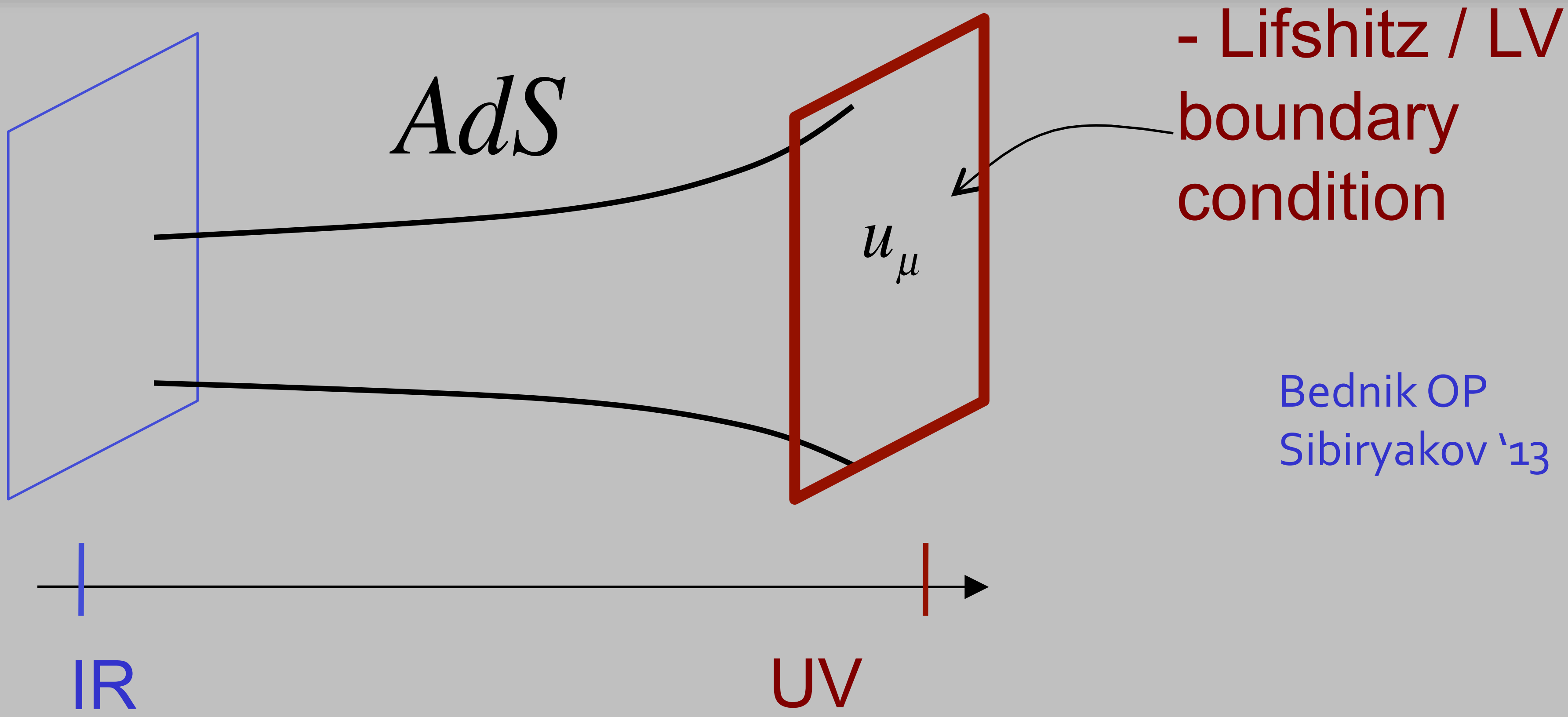
power > 0 granted $(\beta_{\delta c} > 0)$

Unitarity bound $Dim\left(\partial_\mu \phi^* \partial_\nu \phi\right) \geq 4$

LV deformation $\delta c\, u^\mu u^\nu\, \partial_\mu \phi^* \partial_\nu \phi$ => δc irrelevant coupl

Toy model #1: Randall-Sundrum holography

LV-Randall-Sundrum



Bednik OP
Sibiryakov '13

Dual to a **CFT** with UV cutoff (coupling to LV, gravity, $M_{Lifshitz}$, M_P)

and IR cutoff (Λ_{QCD}) ←→ RG scale

LV-Randall-Sundrum

$$L = L_{CFT}(O_\Delta) - \phi\left(w^2 - c^2 k^2\right)\phi + \lambda \phi O_\Delta$$

LV-Randall-Sundrum

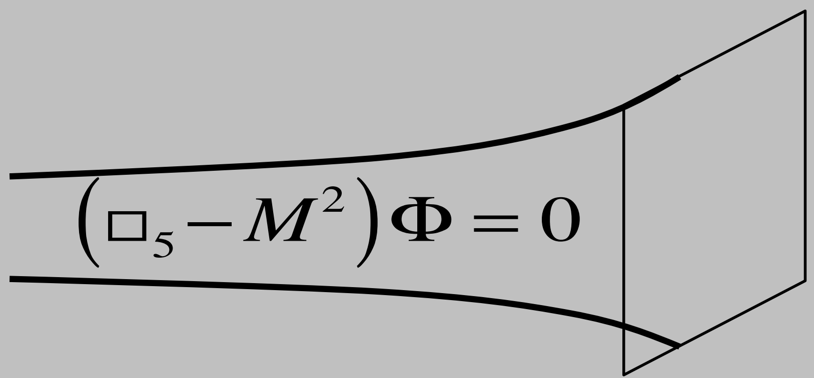
RS Realizes a CFT with an operator O_Δ and a LV source ϕ

$$L = L_{CFT}(O_\Delta) - \phi\left(w^2 - c^2k^2\right)\phi + \lambda\phi O_\Delta$$

(for ***Δ*>2** *– standard quantization)*

if λ ***relevant*** *(Δ< 3)*
=> Emergent LI

$$G_\phi(w,k)^{-1} \simeq w^2 - c^2k^2 + \lambda^2 (p^2)^{\Delta-2}$$



probe scalar with LV boundary

$$\partial_5\Phi = (w^2 - c^2k^2)\Phi$$

LV-Randall-Sundrum

Schematic form of the dispersion relations:

Bednik OP Sibiryakov '13

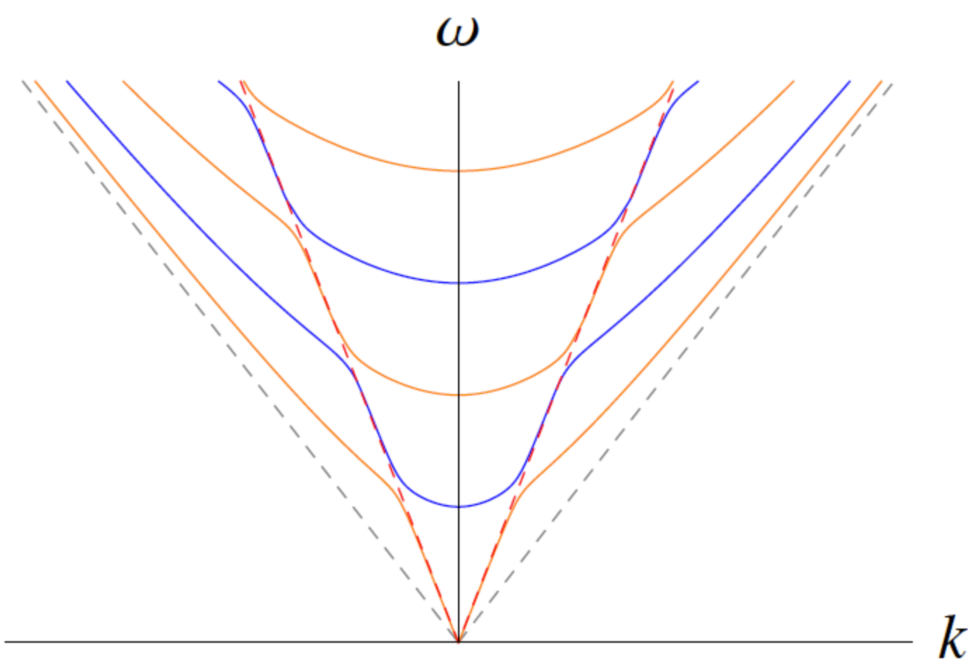
$$w_i^2(k^2) \simeq m_i^2 + \delta c_i^2 k^2 + \sum \frac{k^{2+2n}}{M_{(i,n)}^{2n}}$$

$$\delta c_i^2 \sim \frac{\delta c_{UV}^2}{\lambda^2}\left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^{2(3-\Delta)}$$

power-law suppressed!

for relevant couplings ($\Delta < 3$)

(Optimal case, Δ=2)



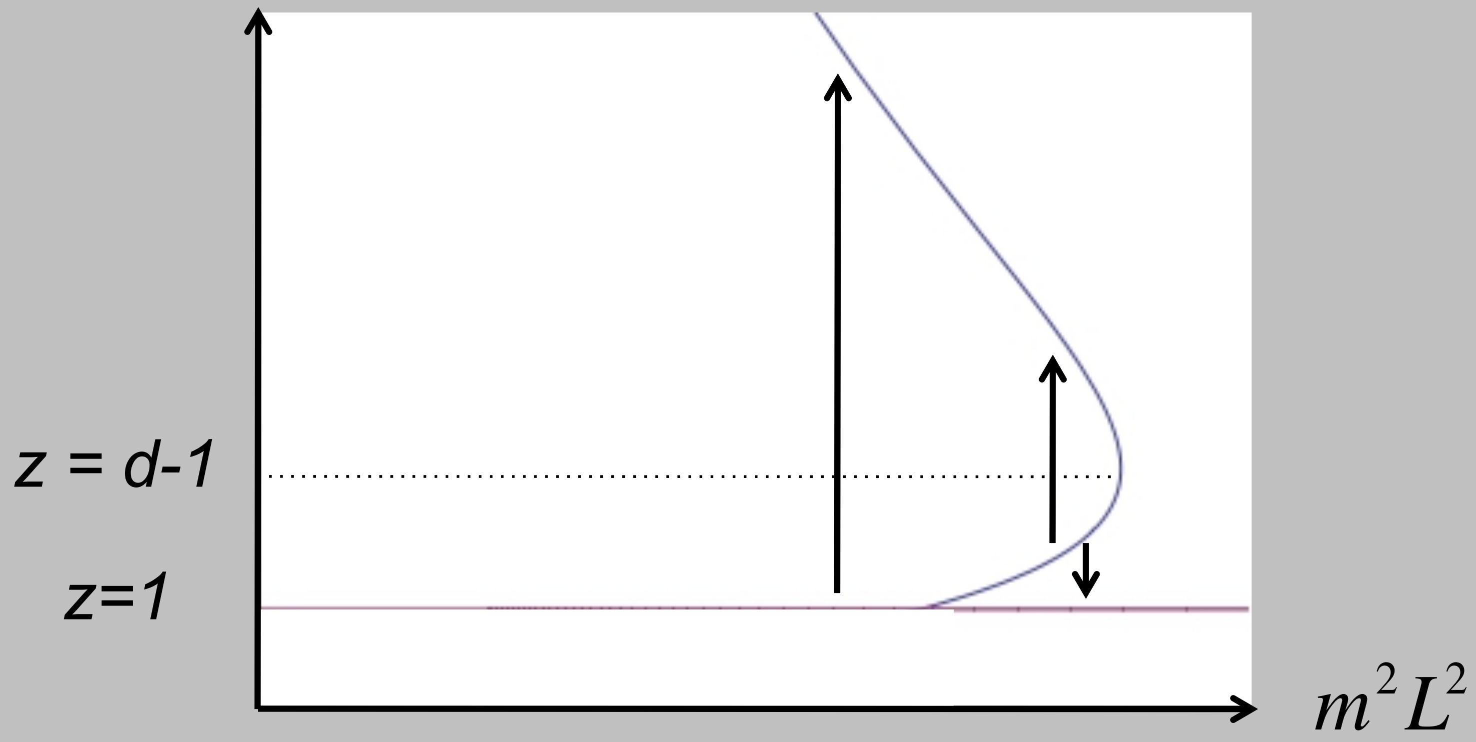
Toy model #2: Lifshitz flows

Lifshitz Holography

Kachru Liu Mulligan ‘08

$$ds^2 = \frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}d\vec{x}^2 - \frac{r^{2z}}{\ell^{2z}}dt^2 \qquad z > 1$$

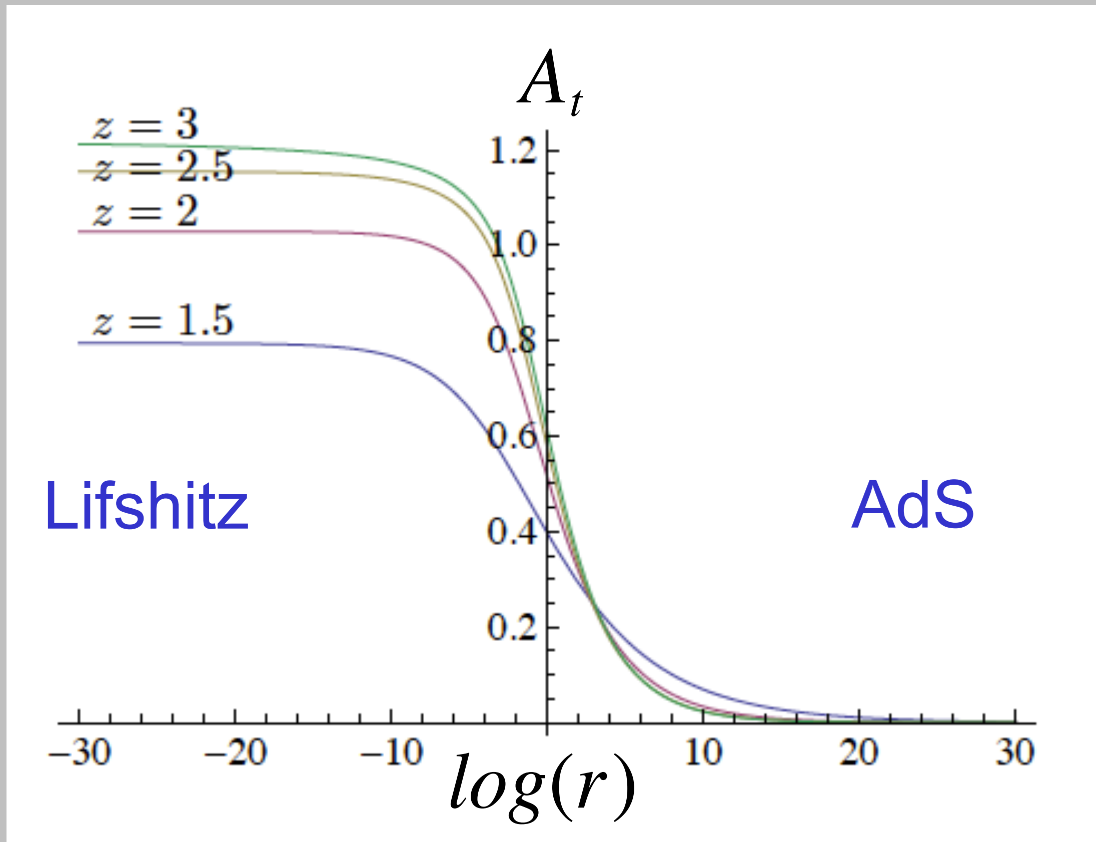
Lifshitz solutions in Einstein + Proca + Λ : $A_t \propto r^z$



Lifshitz Holography

Kachru Liu Mulligan '08

$$ds^2 = g(r)\frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}d\vec{x}^2 - f(r)\frac{r^{2z}}{\ell^{2z}}dt^2$$



Lifshitz flows

The flow imprints modified scaling into the scalar propagator

$$\delta G_\phi(w,k)^{-1} \simeq (p^2)^{\Delta-2}\left[1+w^2\left(\frac{(p^2)^{(\Delta_1-5)}}{\Lambda_*^{2(\Delta_1-4)}}+\frac{(p^2)^{(\Delta-2)}}{\Lambda_*^{2(\Delta-1)}}+\dots\right)+\dots\right]$$

... and into the dispersion relations of bound states

$$\delta c^2 \simeq \begin{cases} (\Lambda_{IR}L_{UV})^{2(\Delta_1-4)} \\ (\Lambda_{IR}L_{UV})^{2(3-\Delta)} \end{cases}$$

Bednik OP
Sibiryakov '13

$\Delta_1 \leq 4.35$ in the simplest model – **not very large suppression**

(model dependent)

Conclusions – 2)

QFT (RG engine)

Strong coupling

Conclusions – 2)

RG + Strong Dynamics => fast Emergence of LI is possible

Emergent LI may not be an exceptional phenomenon

The leading LV corrections are characterized by an exponent

determined by the ***LILVO – least irrelevant LV operator***

$$\delta c \sim \left(\frac{\Lambda_{IR}}{\Lambda_{UV}} \right)^{\Delta_{LILVO} - 4}$$

-> RG scale = compositeness scale

-> how large can Δ_{LILVO} *be ??*

Discussion

Applications to Condensed Matter

Is ELI already at work in some material? Graphene…??

– QED3 has been argued to display ELI since long ago

Related phenomenon: emergence of isotropy

Discussion

Implications in Particle Physics / Non-Relativistic Gravity

compositeness – at low Energies ~ 100 TeV

more strongly tested species -> more composite

Limits on compositeness in SM? $\Lambda \geq few\, 10\, TeV$

Realizing a composite SM? – no problem with fermions & H

Discussion

Several QFT-mechanisms for Emergence of LI exist

- Non Relativistic SUSY – Groot-Nibelink Pospelov '04
- Large N species (extra dim) – Anber & Donoghue '11
- Separation of scales – Pospelov & Shang '10

Via naturalness, NRQG becomes very predictive: new physics at *much lower* energies

Thank you!

Quantum Hair

Krauss Wilczek '89

QH Example) discrete Z_n gauge charge

$U(1) \rightarrow Z_n$ by charge-n condensate

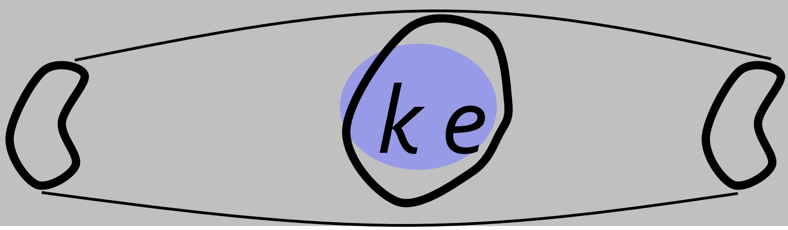
$$\begin{array}{lll} \psi & e & \\ \phi & g \equiv n e & \langle \phi \rangle \neq 0 \end{array}$$

(In SCs, $n = 2$)

U(1) Higgsed => electric charge is short-range

but (q mod n) measurable at infinity by CS-AB

$$\Phi_B^{CS} = \frac{2\pi}{n e}$$



$$W = \exp\left(\frac{2\pi i k}{n}\right)$$