Quantum field theory on curved spacetime and backreaction effects

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Towards quantum gravity?

- **General relativity** and **quantum field theories** needs to be **combined** in some physical systems, (cosmology).
- We need: Quantum theory of gravity and matter

No satisfactory description.

- We can understand how that theory looks like analyzing *some particular regimes* *[Hawking]*.
  - Quantum Fields on **fixed** curved spacetimes
    (Hawking Radiation, Particle Creation)
    good for the description of the metric fluctuations.
  - Backreaction in a semiclassical fashion

$$G_{ab} = 8\pi \langle T_{ab} \rangle$$

good for the description of "evolution" in cosmological models.

- It should work: when fluctuations of $\langle T_{ab} \rangle$ are negligible.

Plan of the talk

- QFT on curved spacetime: the algebraic approach.
- States and asymptotic properties.
- Semiclassical gravity and backreaction.
- Extended semiclassical gravity and CMB fluctuations.

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Free QFT on flat spacetime

- Free relativistic massive **Klein Gordon** field on Minkowski space

$$-\Box\psi + m^2\psi = 0.$$

- $\mathcal{H}$ **one-particle Hilbert space**: positive frequency part of real sol.
- **Fock space** and the **vacuum** $|0\rangle$

$$\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus \ (\mathcal{H} \otimes_s \mathcal{H}) \ \oplus \ldots$$

- **Creation** $a^\dagger_{\vec{k}}$ and **annihilation** operators $a_{\vec{k}}$ and vacuum $|0\rangle$

$$\left[a_{\vec{k}_1}, a^\dagger_{\vec{k}_2}\right] = i\delta\left(\vec{k}_1 - \vec{k}_2\right) \ , \qquad a_{\vec{k}}|0\rangle = 0 \ .$$

- **Quantum field** (operator valued distribution)

$$\hat{\phi}(t,\vec{x}) := \int_{\mathbb{R}^3} \left[\frac{e^{i\omega(\vec{k})t + i\vec{k}\cdot\vec{x}}}{\sqrt{2\omega(\vec{k})}} a^\dagger_{\vec{k}} + \frac{e^{-i\omega(\vec{k})t - i\vec{k}\cdot\vec{x}}}{\sqrt{2\omega(\vec{k})}} a_{\vec{k}} \right] d^3\vec{k}$$

$$P\hat{\phi}(x) = 0 \ , \qquad \hat{\phi}^*(x) = \hat{\phi}(x) \ , \qquad [\hat{\phi}(x), \hat{\phi}(y)] = i\hbar\Delta(x,y) \ .$$

The algebraic approach

- Why we need an **algebraic approach**?
  - We don't have a preferred **time**!
  - We don't have **symmetries** used to construct a vacuum state.
  - Inequivalent representations arise very often.
- Base our theory on the **choice** of observables and on **relations** among them.
- The **algebraic approach** is what we need, permits to formulate QFT **without** have recourse to particular state or Hilbert space representation.

QFT: linear scalar field

- Scalar field coupled to gravity on (M, g) **globally hyperbolic** (GH)

$$-\Box\varphi + \xi R\varphi + m^2\varphi = P\varphi = 0$$

- Exist unique **advanced** and **retarded fundamental solutions** Δ_A Δ_R on GH spaces. *[Bär, Ginoux, Pfäffle]*.

  An **unique** causal propagator $\Delta = \Delta_A - \Delta_R$ **exists**.

- **Quantization**: $\mathcal{A}(M)$ $*-$algebra generated by smeared linear fields:

$$\varphi(Pf) = 0 \,, \qquad \varphi(f)^* = \varphi(\overline{f}) \,, \qquad [\varphi(f), \varphi(g)] = i\Delta(f, g)$$

[Borchers Uhlmann] , C^* version given by *[Dimock]*

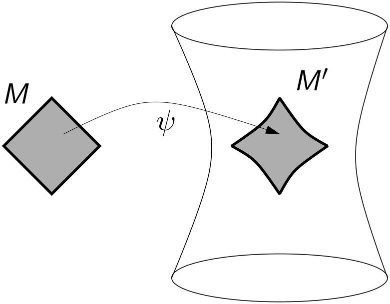
Local Covariant QFT

[Hollands, Wald, Bruentti, Fredenhagen, Verch]

$$\varphi(Pf) = 0\,, \qquad \varphi(f)^* = \varphi(\overline{f})\,, \qquad [\varphi(f), \varphi(g)] = i\Delta(f,g)$$

- It is a **functorial procedure**
- $\mathcal{A} : M \to \mathcal{A}(M)\,, \qquad \psi \to \alpha_\psi$

$$\begin{tikzcd} M \arrow[r, "\psi"] \arrow[d, "\mathcal{A}"'] & M' \arrow[d, "\mathcal{A}"] \\ \mathcal{A}(M) \arrow[r, "\alpha_\psi"] & \mathcal{A}(M') \end{tikzcd}$$



and

$$\alpha_\psi \circ \alpha_{\psi'} = \alpha_{\psi \circ \psi'}\,, \quad \alpha_{\mathbb{I}_M} = \mathbb{I}_{\mathcal{A}(M)}\,.$$

- **Identify** the algebra of observables, and **local fields** over different space-times.

States

- **States** are positive linear functionals over the algebra of fields

$$\omega : \mathcal{A}(M) \to \mathbb{C}$$

- Described by the expectation values on their $n-$point functions (correlation functions, distributions on M^n)

$$\omega(\varphi(f_1), \ldots, \varphi(f_n)) := \omega_n(f_1 \otimes \cdots \otimes f_n)$$

- For simplicity we consider **Gaussian states**, those described by ω_2.
- Once a state is chosen, by **GNS theorem**, we recover the traditional picture. $(\mathfrak{H}_\omega, \pi_\omega, \Psi_\omega)$.
- There are **no generally covariant** states.

Extended algebra of fields

- We need to include observables like **energy density** in the algebra.

$$T_{ab} = \partial_a\varphi\partial_b\varphi - g_{ab}\left(\partial_\mu\varphi^\mu\partial\varphi + \xi R\varphi^2 + m^2\varphi^2\right)$$

- **Coinciding point limits** of fields. Their expectation values are not well defined.
- We have to **regularize** before computing expectation values.
- In flat spacetime it is done by **normal ordering**

$$:\varphi^2 := \varphi^2 - \langle 0|\varphi^2|0\rangle$$

- Restrict attention to a certain **class of states**, those whose 2−point functions have certain singular structure.
  - $: T_{ab} :$ is **local covariant**
  - **Conservation** $\nabla_a\omega(: T^a{}_b :) = 0$
  - Coincides with **normal ordering** w.r.t. the vacuum on flat space

- **Hadamard states** are states over which the point splitting regularization works.

$$\omega_2(x_1,x_2) = \frac{U(x_1,x_2)}{\sigma_\epsilon(x_1,x_2)} + V(x_1,x_2)\log\left(\frac{\sigma_\epsilon(x_1,x_2)}{\mu^2}\right) + W(x_1,x_2)$$

$$\omega_2 = \mathcal{H} + W$$

  - U, V depends on the **local geometry**.
  - σ is the squared geodesic.
  - W is the state dependent part.

  [de Witt, Brehme, Fulling, Kay, Wald...]
  For this kind of states the **point splitting** procedure works.

- They can be used for example to obtain Hawking effect in a natural way. *[Fredenhagen Haag]*

Microlocal spectrum condition

- A nicer characterization was given by Radzikowski

  microlocal spectrum condition

  It is a condition on the singular structure of ω_2 given in terms of the wave front set of the distribution

Definition

$\omega_2 \in \mathcal{D}'(M^2)$ satisfies the **microlocal spectrum condition** (μSC) if

$$\mathrm{WF}(\omega_2) = \left\{(x_1, x_2, k_1, k_2) \in T^*M^2 \setminus \{0\} \mid (x_1, k_1) \sim (x_2, -k_2), k_1 \triangleright 0\right\}.$$

It is a local (covariant) remnant of the **spectrum condition** (with respect to the first entrance, the singular directions are constrained on the forward light cone)

- A two point function has Hadamard form if and only if it satisfies the micro local spectrum condition. *[Radzikowski, Köhler, Brunetti, Fredenhagen]*.

Extended algebra of fields and Regularization Freedom

- Wick polynomials can be incorporated in the algebra of observables. **Extended algebra of fields** *[Hollands Wald]*.
- Other choices of $\mathcal{H}$ produce equivalent algebras.
- **Local fields** are determined up to some **regularization freedom**.
- Assuming some reasonable hypotheses the freedom is finite. *[Hollands Wald]*

$$\tilde{\varphi}^2 = \varphi^2 + \alpha R + \beta m^2$$

$$\tilde{T} = T + \alpha \, \Box R + \beta \, m^4 + \gamma \, m^2 R \, .$$

- Over this extended algebra it is possible to define the **time ordered products**.
- The **perturbative construction of interacting fields** can be understood. *[Kay, Hollands, Wald, Brunetti, Fredenhagen, Dütsch]*

Backreaction: choice of a state as initial conditions

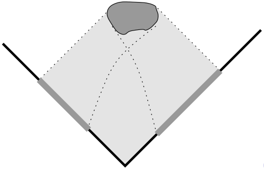
- Influence of quantum fields on the metric by

$$G_{ab} = 8\pi \langle T_{ab} \rangle_\omega$$

- We would like to consider an **initial value problem**.
- We need a **reference state**, we should take a generic Hadamard state but how can we control it?
- Existence proofs of Hadamard states are based on **deformation techniques**.
- Adiabatic states. *[Parker, Lüder Roberts, Junker Schrohe]*
- States of low energy. *[Olberman]*
- We need to prescribe a state in a space-time independent way.

States out of asymptotic properties of the metric.

- It is difficult to prescribe Hadamard states just analyzing them on a Cauchy surface (Cauchy surfaces are acausal)
- If the initial surface is a **null cone**, we could prescribe initial values and construct states inside of the cone. *[Christodoulou]*
- Hörmander **propagation of singularity theorem** can be used to control the regularity of the obtained states.
- Usually the boundary has a larger symmetry, it is thus possible to select some preferred state.
- This is the asymptotic form of many nice space times



Details

Lareger symmetry?

If the boundary has a larger symmetry it is possible to select states invariant under this symmetry.

States that are **"asymptotic vacuum"** can be constructed *[Sewell, Kay, Wald, Moretti, Dappiaggi ...]*

The described procedure works in many situations:

- Asymptotically flat space-times $\Longrightarrow$ **asymptotic vacuum**.
- Schwarzschild spacetiems $\Longrightarrow$ **Unruh states**.
- Cones in regular regions of spacetime
- Asymptotically de Sitter spaces $\Longrightarrow$ **asymptotic Bunch Davies states**.
- Flat Friedmann Robertson Walker Universes with null Big Bang structure $\Longrightarrow$ **asymptotic conformal vacuum**.

Application: Semiclassical equations in cosmology

- In first approximation the universe is **homogeneous** and **isotropic**.
- a spacetime $M = (I \times S, g)$
  - I is the interval of the *"cosmological time"*
  - S is a 3d manifold: the *"space"*, it has an high symmetry.
- Friedmann Robertson Walker metric

$$g = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\mathbb{S}^2(\theta,\varphi)\right].$$

- Knowing $a(t)$ is like knowing the "story" of the universe.
- Recent observations:
  - $\kappa \simeq 0 \Longrightarrow$ Conformally Flat.
  - $\frac{\dot{a}}{a} = H$ the Hubble parameter (very small positive almost constant) *(de Sitter Universe)*.

Standard Model of the Universe: Matter

- It takes the simple form $T_a{}^b = diag(-\rho, P, P, P)$
- Like a classical fluid (but $\frac{P}{\rho}$ is non constant).
- Einstein's equations become **Friedmann** equations $H = \frac{\dot{a}}{a}$

$$3H^2 = 8\pi\rho - \frac{3\kappa}{a^2}, \qquad 3\dot{H} + 3H^2 = -4\pi\,(\rho + 3P)$$

- Once an initial condition is chosen for a, FRW eq. is equivalent to

$$-R = 8\pi T, \qquad \nabla^a T_{ab} = 0\ .$$

- We shall model quantum matter by φ and discuss

$$-R = 8\pi\langle T\rangle_\omega$$

Obs. Prob.

Conservation equations for T_{ab} are satisfied: $\nabla_a \langle T^a{}_b \rangle_\omega = 0$ but (un)-fortunately the **trace** is different from the classical one.

$$\langle T \rangle_\omega := \frac{2[v_1]}{8\pi^2} + \left(-3 \left(\frac{1}{6} - \xi \right) \Box - m^2 \right) \langle \varphi^2 \rangle_\omega .$$

More precisely ($\xi = 1/6$) *[Wald 1978]*

$$2[v_1] = \frac{1}{360} \left(C_{ijkl} C^{ijkl} + R_{ij} R^{ij} - \frac{R^2}{3} + \Box R \right) + \frac{m^4}{4} .$$

The renormalization freedom for T is

$$\langle T' \rangle_\omega = \langle T \rangle_\omega + \alpha \, \Box R + \beta \, m^4 + \gamma \, m^2 R \, .$$

- In $\langle T \rangle_\omega$, three contributions: $T_{anomalies} + T_{ren.freedom} + T_{state}$.
- $\alpha \Longrightarrow$ cancel $\Box R$ from the trace $\Longrightarrow$ Wald's prescription.
- $\beta \Longrightarrow$ like a cosmological constant.
- $\gamma \Longrightarrow$ ren. of Newton Constant.
- We can **not** cancel $T_{anomalies}$ completely.
- $T_{anomalies}$ is **not** a mixture of perfect fluids: $\rho = H^4$

Massive model

With $\kappa = 0$ and $\xi = 1/6$, the equation $-R = 8\pi\langle T\rangle$ becomes

$$-6\left(\dot{H} + 2H^2\right) = -8\pi m^2\langle\varphi^2\rangle_\omega - \frac{1}{30\pi}\left(\dot{H}H^2 + H^4\right) + \frac{m^4}{4\pi}$$

Physical input: We would like to use **"vacuum states"** i.e. $\langle\varphi^2\rangle_\omega = 0$

Impossible: Adiabatic states, have similar properties

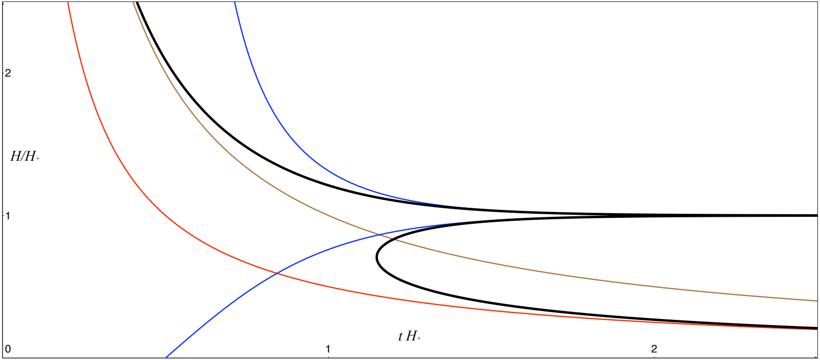
[Parker, Parker Fulling, Lüders Roberts, Junker Schrohe, Olbermann]

Assume (for the moment) $T_{state} = 0$

We have only $T_{anomalies}$ and $T_{ren.freedom} = \beta m^2 + \gamma R$

The differential equation is an ordinary one $\Longrightarrow$ it can be solved

With some choice of γ and β $H = 0$ and $H = H_+$ are stable solutions.



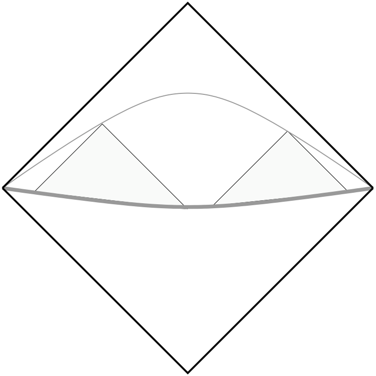
- $(m = 0)$ a length scale is introduced (proportional to G). **Two** fixed points instead of one. *[Wald 80, Starobinsky 80, Vilenkin 85]*
- Quantum effects are **not negligible** at least in the past.
- $(m \neq 0)$ H_+ is a renormalization constant. (It correspond to ren. of Newton constant)

Form of the initial singularity

Question

Where is the singularity t_0 in the Penrose diagram?

$$ds^2 = a^2\left(-d\tau^2 + d\mathbf{x}^2\right).$$



- **Classical solution**
  Radiation dominated:
  $\tau = \tau_0 + A(t - t_0)^{1/2} \to \tau_0$
  for $t \to t_0$
  Horizon problem.
- **Quantum Corrections**
  $\rho = 1/a(t)^2 \quad :$
  $\tau = \tau_0 + \log(t - t_0) \to -\infty$
  for $t \to t_0$
  Singularity is light like.

Power law inflation with
Null Big Bang (NBB) $\Im^-$

Existence and uniqueness in the early universe

- Consider an **asymptotic vacuum** ω . (Initial cond. of the problem) ▸ Def.
- We search for solutions of $-R = 8\pi\langle T\rangle_\omega$ near **NBB** $\Im^-$.
- Indicating by $X := H^{-1}$, we rewrite the equation as:

$$\frac{dX}{dt} = 1 - \frac{X^2}{X_c^2 - X^2} + m^2 \frac{C\, X^4}{X_c^2 - X^2} \langle \varphi^2 \rangle_\omega \, .$$

- It is **not** an ordinary differential equation.
- $\langle\varphi^2\rangle_\omega$ is a functional of $X = H^{-1}$.
- Let us rewrite the equation as $X = \mathcal{T}(X)$.

Theorem

$\mathcal{T}$ *is a* **contraction map**. *An unique solution exits thanks to* **Banach fixed point theorem**.

Weaker assumptions

- The upper branch is **not physical** (we need to add radiation with negative energy density). $\Longrightarrow$ We look for solution close to the **lower branch**.
- If we prescribe ω_2 to be a vacuum exactly on a Cauchy surface, we are defining an **adiabatic vacuum** of order 0.
- For this kind of states $\langle\varphi^2\rangle_\omega$ and its first functional derivative, as a functional of a and H can be **controlled** by a and H.
- Local existence can be obtained again by means of Banach fixed point theorem. *[np D. Siemssen, work in progress]*

▸ ΛCDM

Semiclassical Einstein equations and stochastic gravity

- Let's look again at

$$G_{ab} = 8\pi \langle T_{ab} \rangle_\omega$$

- The right hand side is a **probabilistic quantity**.
- If the variance of $\langle T_{ab} \rangle_\omega$ is not negligible, like for the **Brownian motion** $\Longrightarrow$ the equation can only make sense as a stochastic one.
  Fluct.
- The probabilistic distribution of φ^2 has been recently discussed by *[Fewster Ford Roman]*

Some comments

Question

What is the impact of fluctuations?

- It is believed that quantum fluctuations seeds structure formation in the universe.
- *[Verdaguer]* obtains a **scale free spectrum** of the **metric fluctuations** (Bardeen potentials) considering a "linearized version" of ω_2^2 as a source.
- The CMB anisotropies should give constraints on this point *[np, D. Siemssen]*

Stochastic approach

- Einstein-Langevin equation *[Verdaguer]*

$$G_{ab}(x) = \omega(T_{ab}(x))$$

- We could interpret it as a **stochastic equation**.
- It is not easy to compute the **probability distribution** for $\omega(T_{ab})$.
- The correlations of $\omega(T_{ab}(x))$ are **more complicated** than in *Wiener processes* or *Brownian motions*.
- We can equate their moments:

$$\langle G_{ab}(x)\rangle = \omega(T_{ab}(x))$$
$$\langle \delta G_{ab}(x_1)\delta G_{cd}(x_2)\rangle = Sym\left[\omega(\delta T_{ab}(x_1)\delta T_{cd}(x_2))\right]$$
$$\dots$$
$$\langle \delta G^n(x_1,\dots,x_n)\rangle = Sym\left[\omega(\delta T^n(x_1,\dots,x_n)))\right]$$

$$\delta G_{ab} = G_{ab} - \langle G_{ab}\rangle\ , \qquad \delta T_{ab} = T_{ab} - \langle T_{ab}\rangle$$

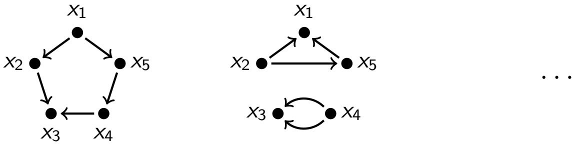
Graphical representation for $\delta\varphi^2$ in a Gaussian state

$$\langle \delta G(x_1)\delta G(x_2)\rangle = m^4\,\omega(\delta\varphi^2(x_1)\delta\varphi^2(x_2)) = \frac{m^4}{2}\omega_2^2(x_1,x_2) + \frac{m^4}{2}\omega_2^2(x_2,x_1)$$

$n = 2$:

$n = 3$:

$n = 5$:



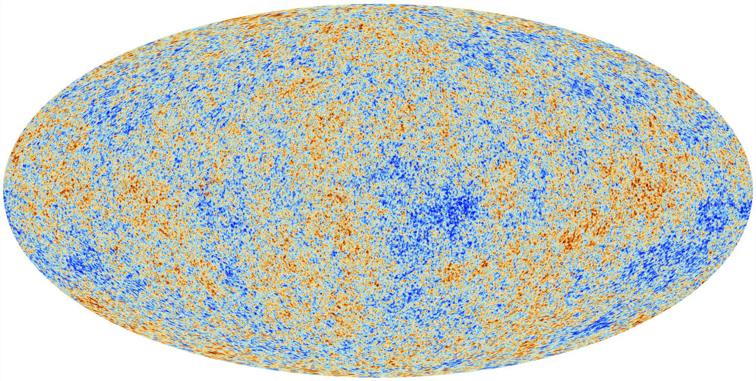
All these graphs are well defined distributions:

$$\omega_2^2(x,y) \qquad \omega_2(x,y)\omega_2(y,z) \qquad \ldots.$$

Comments

- Semiclassical Einstein equations **link** quantum matter fluctuations with curvature fluctuations
- We can compare this model with observations!
- δG is not Gaussian (3-point function do not vanish ...)
- The non Gaussianity seems to be detected in CMB anisotropies.

CMB Temperature fluctuations



- CMB anisotropies observed by the Planck space telescope.
- Produced at the time of matter/radiation decoupling.
- Usually explained by inflation.

CMB Temperature fluctuations

$$\Theta(\tau,\vec{x},\vec{e}) = \frac{\delta T(\tau,\vec{x},\vec{e})}{T(\tau)} = \sum_{\ell,m} \Theta_{\ell m}(\tau,\vec{x}) Y_{\ell m}(\vec{e})$$

$\Theta_{\ell m}(\tau_0,\vec{x}_0)$ are **statistically homogeneous random variables** with correlations

$$\langle \Theta_{\ell m}(\tau_0,\vec{x}_0)\Theta_{\ell' m'}(\tau_0,\vec{x}_0)^* \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell$$

where, in terms of **Newtonian perturbations** Ψ,

$$C_\ell = 4\pi \int_0^\infty T_\ell(k)^2 \, \langle \widehat{\Psi}(\tau_1,k)\widehat{\Psi}(\tau_1,k) \rangle \, k^2 \, dk$$

In order to be coherent with observations, for small k it should be

$$\langle \widehat{\Psi}(\tau_1,k)\widehat{\Psi}(\tau_1,k) \rangle \approx C \left(\frac{k_0}{k} \right)^{3-\epsilon}$$

we shall compare this with results obtained in an inflationary model.

Perturbations around an inflationary spacetime

- We start with a de Sitter spacetime

$$\overline{g} = \frac{1}{(H\tau)^2}\left(-d\tau^2 + d\vec{x}^2\right)$$

- Let's add Newtonian perturbations $\overline{g} \to g = \overline{g} + \epsilon\tilde{g}$:

$$g = \frac{1}{(H\tau)^2}\left(-(1+2\Psi)d\tau^2 + (1-2\Psi)d\vec{x}^2\right)$$

- Linear perturbation of **scalar curvature**

$$\delta G = \overline{g}^{ab}\left(G_{ab} - \langle G_{ab}\rangle\right) = 6(H\tau)^4\left(\frac{\partial^2}{\partial\tau^2} - \frac{1}{3}\vec{\nabla}^2\right)\frac{\Psi}{(H\tau)^2}$$

- Inverting (with retarded propagator) we get

$$\langle\Psi(x_1)\Psi(x_2)\rangle = m^4 \left(\quad \right)$$

Power spectrum of Ψ: $P(\tau, k)$

Is obtained computing the spatial Fourier transform of $\langle \Psi(x_1)\Psi(x_2)\rangle$

where the state is

$$\omega_2(x_1, x_2) = \frac{U}{\sigma_\epsilon} + \text{less singular term} = H^2 \tau_1 \tau_2\, \omega_{\mathbb{M}}(x_1, x_2) + \text{less singular term}$$

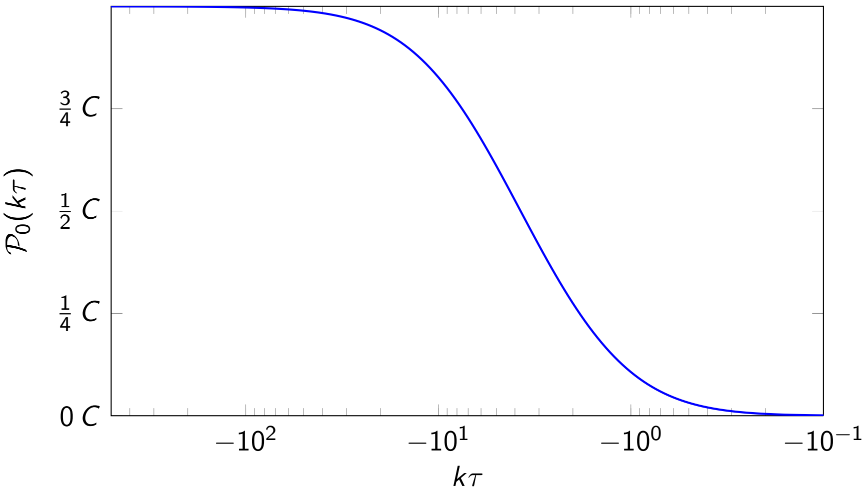
and the square of the two-point function

$$\widehat{\omega_2^2}(\tau_1, \tau_2, \vec{k}) = \frac{1}{16\pi^2} \int_k^\infty e^{-ip(\tau_1 - \tau_2)} dp$$

We can consider the tree contribution to the power spectrum separately.

$$\langle \widehat{\Psi}(\tau, k)\widehat{\Psi}(\tau, k)\rangle \approx \frac{1}{k^3} \mathcal{P}_0(k\tau)$$

The rescaled power spectrum



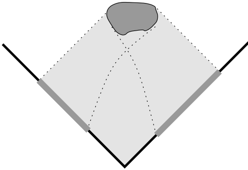
Which is consistent with observations.

Summary

- Algebraic Quantum Field Theory over curved spacetime is a solid theory.
- Semiclassical Backreaction can also be analyzed.
- Fluctuations can be studied in the same framework.
- Comparison with observation in cosmology **can be made**.

Thanks a lot for your attention!

Let $\mathcal{B}$ be the past null boundary of some cone $\mathcal{C}$.
Project the algebra $\mathcal{A}(\mathcal{C})$ on $\mathcal{B}$ using the causal prop.



$$\iota : \mathcal{A}(\mathcal{C}) \mapsto \mathcal{A}(\mathcal{B})$$

$$\iota(\varphi(f)) = \varphi_{\mathcal{B}}(\Delta \restriction_{\mathcal{B}} (f))$$

The action on the symplectic structure is a **symplectomorphism**.

$$[\varphi(f), \varphi(h)] = i\Delta(f, h) = [\varphi_{\mathcal{B}}(\Delta \restriction_{\mathcal{B}} (f)), \varphi_{\mathcal{B}}(\Delta \restriction_{\mathcal{B}} (h))]$$

Pullback states (functionals) from the boundary

$$\iota^*(\omega_{\mathcal{B}}) = \omega_{\mathcal{C}}$$

We can analyze the singular structure of the two-point function

$$\omega^2_{\mathcal{C}} = (\Delta \restriction_{\mathcal{B}} \otimes \Delta \restriction_{\mathcal{B}}) \circ \omega^2_{\mathcal{B}} \,.$$

If $WF(\omega^2_{\mathcal{B}})$ does not contain "negative frequencies", **propagation of singularity** theorem implies that the pulled back state is Hadamard.

[Hollands, Moretti, Dappiaggi, Hack ...] Back

Cosmological scenario: Observation

- If we use **Radiation**, **Dust** and **cosmological constant** to model the present day observations:
  - Radiation is less important. $\rho_R \sim a(t)^{-4}$
  - We look for a mixture of $\rho_M \sim a(t)^{-3}$ and $\rho_\Lambda \sim C$

We have a problem

in modeling CMB and Supernovae red-shift observation:

Total **Energy density** is:
$\sim$ **74%** *Cosmological constant*, $\sim$ **26%** *Dust*.

Known matter: only $\sim$ **4%**.

Back

Asymptotic vacuum in cosmological spaces with NBB

The **pure**, **homogeneous** and **isotropic**

$$\omega_2(x,y) := \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} \frac{\overline{\chi_k}(x_0)}{a(x_0)}\frac{\chi_k(y_0)}{a(y_0)} e^{i\mathbf{k}\cdot(x-y)} d\mathbf{k} \,,$$

$\forall k \geq 0$, χ_k is a smooth function satisfying

$$\chi_k''(\tau) + (m^2 a(\tau)^2 + k^2)\chi_k(\tau) = 0,$$

$$\overline{\chi_k}\frac{d}{d\tau}\chi_k - \frac{d}{d\tau}\overline{\chi_k}\chi_k = i \,.$$

Condition for being an **asymptotic vacuum**

$$\lim_{\tau\to-\infty} \chi_k(\tau)e^{ik\tau} = \frac{1}{\sqrt{2k}} \,, \qquad \lim_{\tau\to-\infty} \chi_k'(\tau)e^{ik\tau} = -i\sqrt{\frac{k}{2}} \,.$$

It is an Hadamard state. Back

Comparison with the ΛCDM model

- The late time behavior is **not** under control $\Longrightarrow$ some assumptions

Before "local vacuum": $\langle\varphi^2\rangle_\omega \sim 0$ with certain α and β

Now "local thermal state": $\langle\varphi^2\rangle_\omega \sim \frac{T^3}{a^3} + O\left(\frac{1}{a^5}\right)$

(A minimal model with two fields a massive scalar field a massless one)

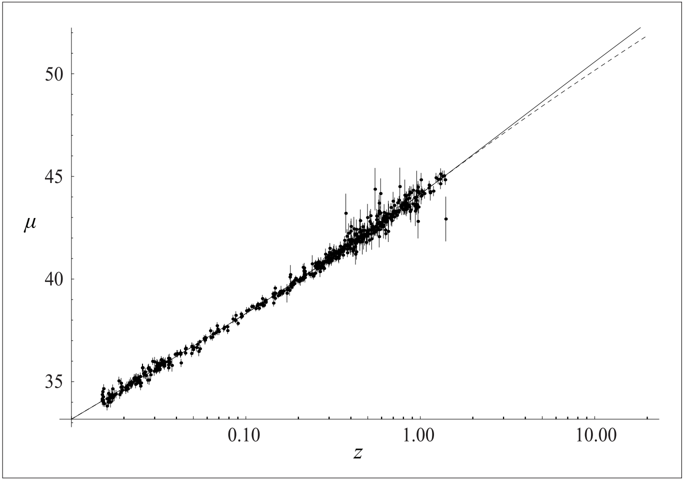
$$H^2 = H_*^2 \pm \sqrt{H_*^4 - \frac{C_1}{a^4} - C_2 - C_3\frac{T^3}{a^3}}$$

  - lower branch if H_*^4 is very large we get ΛCDM plus quantum correction

- Phenomenological law for $\mu(z)$ *(distance modulus, difference between absolute and apparent magnitude)* w.r.t. red-shift $z = \frac{1}{a} - 1$ *(temporal distance)* for the SN1a explosions.

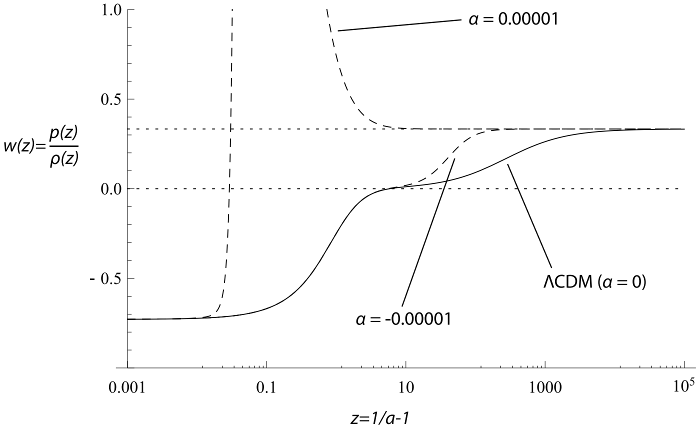
$$\mu(z) = 5\log\left((1+z)\int_0^z \frac{1}{H(z')}dz'\right) + K$$

- Compare it with observations: best fit is obtained by minimizing χ^2.



Union2 supernova compilation *[Amanullah et al. 2010]*

- Upper branch: **"gravity is repulsive"**, Newton constant is too large $\Longrightarrow$ **rule out**.
- Lower branch with relaxed conditions $T = \cdots + \alpha \Box R$,



[T-P. Hack, work in progress] Back.

Analysis of the fluctuations

The solution is meaningful provided the variance of $T_\mu{}^\mu$ is small

- The anomaly is a $C-$number
- The variance of $\langle \varphi^2 \rangle$

$$\Delta_\omega(\varphi^2) := \omega(\varphi^2 \star_{\mathcal{H}} \varphi^2) - \omega(\varphi^2)\omega(\varphi^2)$$

diverges: it is proportional to $\omega_2 \cdot \omega_2(x, x)$

When smeared the situation is better, consider the family centered in x_τ

$$f_{\delta t, \delta x}(\tau', \mathbf{x}) = \frac{1}{\delta t \delta x^3} f\left(\frac{(\tau' - \tau)}{\delta t} + \tau, \frac{\mathbf{x}}{\delta x}\right), f(x_\tau) = 1, \qquad \int_M f \, d\mu(g) = 1$$

We study the limit

$$\lim_{\delta_t \to 0} \lim_{\delta_x \to \infty} [R(f_{\delta_t,\delta_x}) + 8\pi \langle T \rangle_\omega (f_{\delta_t,\delta_x})] = R(x_\tau) + 8\pi \langle T \rangle_\omega (x_\tau)$$

Theorem

We have

$$\lim_{\delta x \to \infty} \Delta_{\omega_{1,0}}(\varphi^2(f_{\delta_t,\delta_x})) = 0 \ .$$

- In a **weaker** sense, the solution we have found is meaningful also when H is very large.

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