Genova, 20/03/2013

New Confinement Phases from Singular SQCD Vacua

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Plan:

I. Confinement and XSB in QCD, Lessons from SQCD

- singular SCFT and confinement -

II. Monopoles and dyons in NA gauge theories

- A brief review: from Dirac, ‘t Hooft-Polyakov to Seiberg-Witten

III. Recent developments

- Argyres-Seiberg, Gaiotto-Seiberg-Tachikawa, Giacomelli -

IV. Singular SCFT and confinement

- New confinement picture -

I. Quark confinement vs Chiral Symmetry Breaking

Basic theme :

Conformal invariance (CFT) and confinement

UV CFT - - - - - - - - - - → Infrared-fixed point CFT

QCD:

quarks and gluons, AF - - - - - → collective behavior of color (confinement, XSB) ?

If confinement ~ deformation of an IR f.p. CFT

the understanding of the IR degrees of freedom in CFT

is the key to see the working of confinement / XSB

Quark confinement mechanism

- Abelian dual superconductor ? (dynamical Abelianization)

$$SU(3) \to U(1)^2 \to \mathbf{1}$$
$$\langle M \rangle \neq 0$$

't Hooft, Nambu, Mandelstam

☞ Doubling of the spectrum (*)

$$\Pi_1(U(1)^2) = \mathbf{Z} \times \mathbf{Z}$$

If confinement and XSB both induced by $\langle M^a_b \rangle = \delta^a_b \, \Lambda$

$$SU_L(N_F) \times SU_R(N_F) \to SU_V(N_F)$$

☞ Accidental SU(N_F^2) : too many NG bosons

- Non-Abelian monopole condensation

$$SU(3) \to SU(2) \times U(1) \to \mathbf{1}$$

$$\Pi_1(SU(2) \times U(1)) = \mathbf{Z}$$

☞ Problems (*) avoided but

Non-Abelian monopole are probably strongly coupled (sign flip of b_0 unlikely)

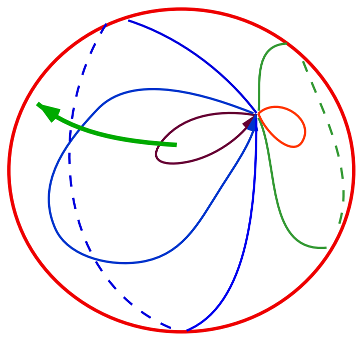
II. Monopoles and dyons in nonAbelian gauge theories

-- a 15 min. review --

Topology (Mapping: Space $\to G$)

- Charged particle $\psi(x)$ in a monopole field

$$\exp i g_e \oint_{\partial\Omega} A_i\, dx^i \to \exp i g_e \int_{S^2} d\mathbf{S}\cdot\mathbf{H} = \exp 4\pi i g_e\, g_m \qquad (\mathbf{H} = \nabla\frac{g_m}{r}).$$



$$2\, g_e\, g_m = n, \qquad n \in \mathbb{Z}, \qquad \Pi_1(U(1)) = \mathbb{Z}$$

Dirac ~1930

- $U(1)$ in a nonabelian theory G (Wu, Yang): monopole $\sim\ \Pi_1(G) \neq \emptyset$. Fiber bundle
- 't Hooft-Polyakov monopoles: $\Pi_2(SU(2)/U(1)) = \Pi_1(U(1)) = \mathbb{Z}$; Regular monopoles
- $G \xrightarrow{\langle\phi\rangle\neq 0} H$: similar $\to$ **monopoles with nonabelian charges** if $\Pi_2(G/H) \neq \emptyset$

Non-Abelian monopoles

$$G \xrightarrow{\langle\phi\rangle\neq 0} H$$

$$SU(3) \xrightarrow{\langle\phi\rangle} \frac{SU(2)\times U(1)}{\mathbb{Z}_2}, \qquad \langle\phi\rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -2v \end{pmatrix}$$

H: non-Abelian

Goddard-Nuyts-Olive, E.Weinberg, Lee,Yi,
Bais, Schroer, ... '77-80

$$F_{ij} = \epsilon_{ijk} \frac{r_k}{r^3} (\beta \cdot \mathrm{T}), \qquad 2\,\beta \cdot \alpha \in \mathbf{Z}$$

cfr. (Dirac)

$2\ m \cdot e \in Z$

"Monopoles are multiplets of $\tilde{\mathrm{H}}$ (GNOW)"

$\tilde{\mathrm{H}}$ generated by $\alpha^* \equiv \frac{\alpha}{\alpha \cdot \alpha}.$

<Φ> = v_I= h ·T

H	$\tilde{\mathrm{H}}$
U(N)	U(N)
SU(N)	SU(N)/Z_N
SO(2N)	$\widetilde{\mathrm{SO}(2N)}$
SO(2N+1)	USp(2N)

$$A_i(\mathrm{r}) = A_i^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\, S_a; \quad \phi(\mathrm{r}) = \chi^a(\mathrm{r}, \mathrm{h}\cdot\alpha)\, S_a + [\,\mathrm{h} - (\mathrm{h}\cdot\alpha)\,\alpha^*] \cdot \mathrm{T},$$

$$S_1 = \frac{1}{\sqrt{2\alpha^2}}(E_\alpha + E_{-\alpha}); \quad S_2 = -\frac{i}{\sqrt{2\alpha^2}}(E_\alpha - E_{-\alpha}); \quad S_3 = \alpha^* \cdot \mathrm{T},$$

Non-Abelian monopoles: difficulties

- Non normalizable gauge zeromodes ⇒ NOT a H-multiplet
  N. Dorey et al '96
- Topological obstruction
  "Unbroken" $H \subset G$ not defined globally
  Coleman: "Colored dyons do not exist"
  Abouelsaad et al '83
  Coleman,
  Balachandran et al
  $SU(5) \to SU(3) \times SU(2) \times U(1)$
- GNO quantization $2\beta \cdot \alpha \in \mathbb{Z}$
  (β = weight vector of $\tilde{H}$ ("EM" dual of H))
  monopoles transform under $\tilde{H}$ (non locally under H)
  Goddard-Nuyts-Olive '73
- Phase: $\tilde{H}$ in confinement phase
  ⇐ H in Higgs phase
- NA monopoles require flavor ⇐ NA Vortices
  2003 –
  Pisa
  Cambridge
  Minnesota
  Tokyo

Seiberg-Witten '94 $\mathcal{N}=2$ SUSY G. Th.

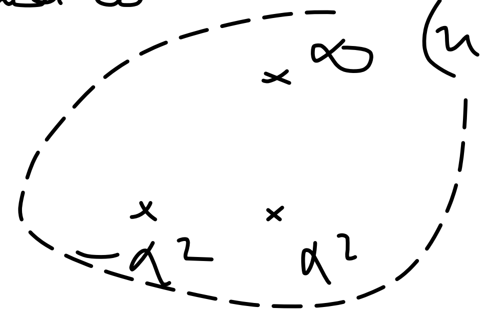
- Exact $\mathcal{L}_{eff}$; exact BPS spectra of monopoles
- $SU(2)$ $W = (\lambda, F_{\mu\nu})$; $\Phi = (\phi, \psi)$ $\langle \phi \rangle = \begin{pmatrix} a & \\ & -a \end{pmatrix} \Rightarrow$ Vac deg. (moduli)

$a \neq 0$ $SU(2)/U(1) \Rightarrow$

$$S = \int \frac{\partial F(A)}{\partial A} \bar{A} + \frac{\partial^2 F}{\partial A^2} W W_\alpha$$

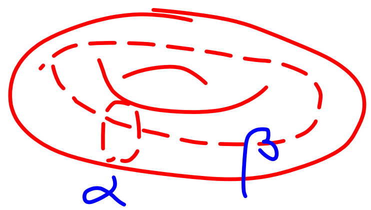
inv. under EM transformation

at $u = \pm \Lambda^2$ $\exists$ massless monopole



- Soln: auxil. curve

$$y^2 = (x - \Lambda^2)^2 (x - u)$$



$$a_D = \frac{\partial F(a)}{\partial a} = \oint_\alpha \lambda \; ; \quad \frac{d a_D}{du} = \oint_\alpha \omega \; ; \quad \omega = \frac{dx}{y} \text{ . holomorphic differ.}$$

$$a = \oint_\beta \lambda \; ; \quad da/du = \oint_\beta \omega \; ; \quad \tau = \frac{d a_D}{da} = \frac{\theta}{2\pi} + 4\pi i / g^2$$

- $\lambda =$ SW diff. $= dx\, x/y$

$$M_{n_m, n_e} = \sqrt{2} \, | n_m a_D + n_e a |$$

$\longleftarrow M_{mon} = \langle \phi \rangle / g$: $M_W = g \langle \phi \rangle$

Tuesday, March 19, 13

What $\mathcal{N}$=2 SQCD (softly broken) teaches us

- Abelian dual superconductivity ✔

Seiberg, Witten

SU(2) with N_F = 0, 1,2,3
monopole condensation ⇒ confinement & dyn symm. breaking

SU(N) $\mathcal{N}$ =2 SYM : SU(N) ⇒ $U(1)^{N-1}$

Beautiful, but don't look like QCD

- Non-Abelian monopole condensation for SQCD ✔

Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00

SU(N), N_F quarks
SU(N) ⇒ SU(r) x U(1) x U(1) x .... $r \le N_F/2$
r vacua are local, IR free theories

Beautiful, but don't look like QCD

- Non Abelian monopoles interacting very strongly

SCFT of higher criticalities, EHIY points

Beautiful, interesting but difficult

Argyres,Plesser,Seiberg,Witten,
Eguchi-Hori-Ito-Yang, '96

Effective degrees of freedom in the quantum r vacua of softly broken N=2 SQCD

$(r \leq N_f\ / 2\ )$

Seiberg-Witten '94
Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00

	$SU(r)$	$U(1)_0$	$U(1)_1$	$\ldots$	$U(1)_{N-r-1}$	$U(1)_B$
$N_F \times \mathcal{M}$	$\underline{\mathbf{r}}$	1	0	$\ldots$	0	0
M_1	$\underline{\mathbf{1}}$	0	1	$\ldots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
M_{N-r-1}	$\underline{\mathbf{1}}$	0	0	$\ldots$	1	0

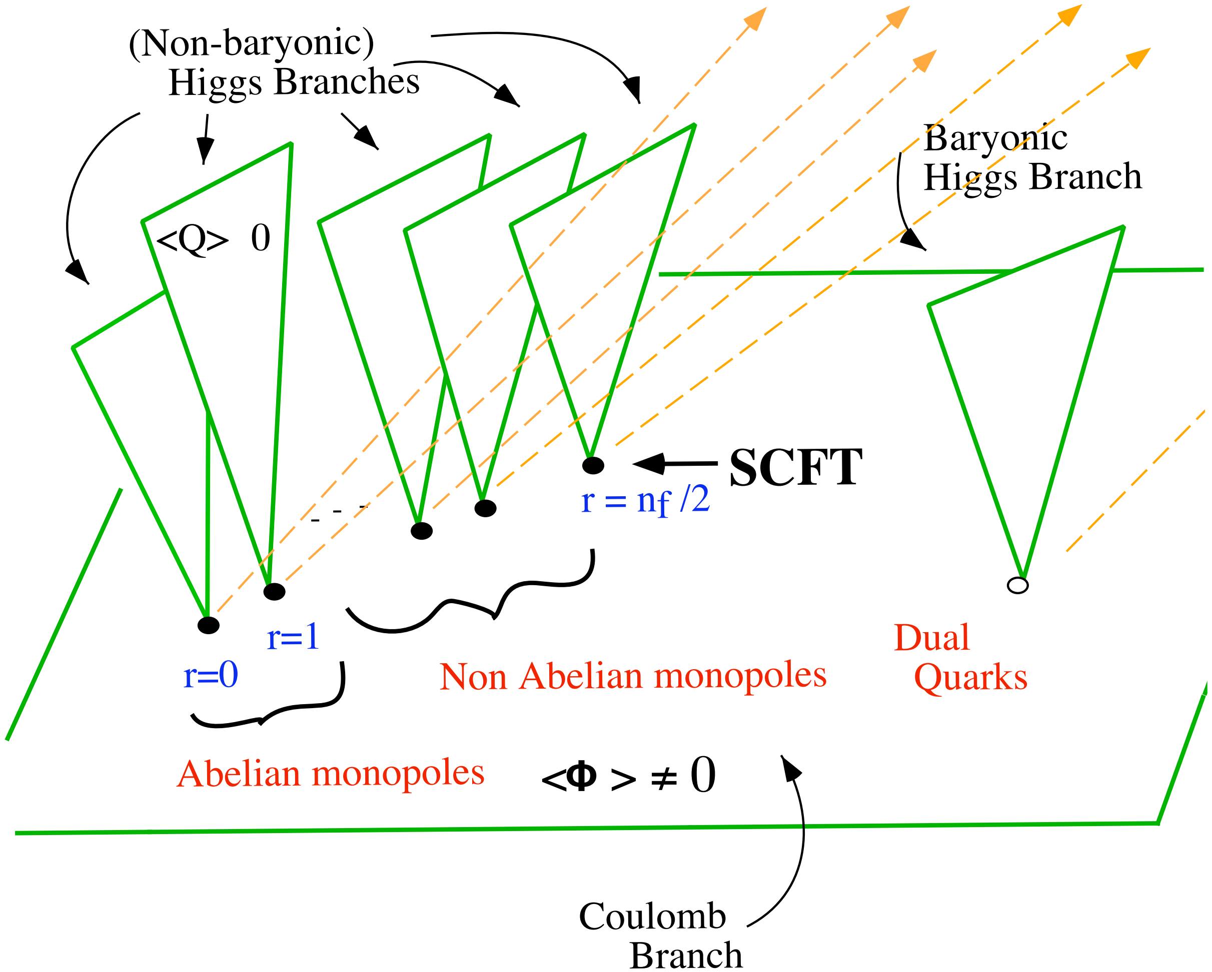
The massless non-Abelian and Abelian monopoles and their charges at the r vacua

- "Colored dyons" do exist !!!
- they carry flavor q.n.

$\mu\Phi^2$ perturbation

- $\langle q^i{}_\alpha \rangle = v\ \delta^i{}_\alpha \Rightarrow \quad U(N_f) \Rightarrow U(r) \times U(N_f - r)$

QMS of N=2 SQCD (SU(n) with n_f quarks)



● N=1 Confining vacua (with $\mu\Phi^2$ perturbation)

○ N=1 vacua (with $\mu\Phi^2$ perturbation) in free magnetic pha

$m = m^{cr}$

next slide

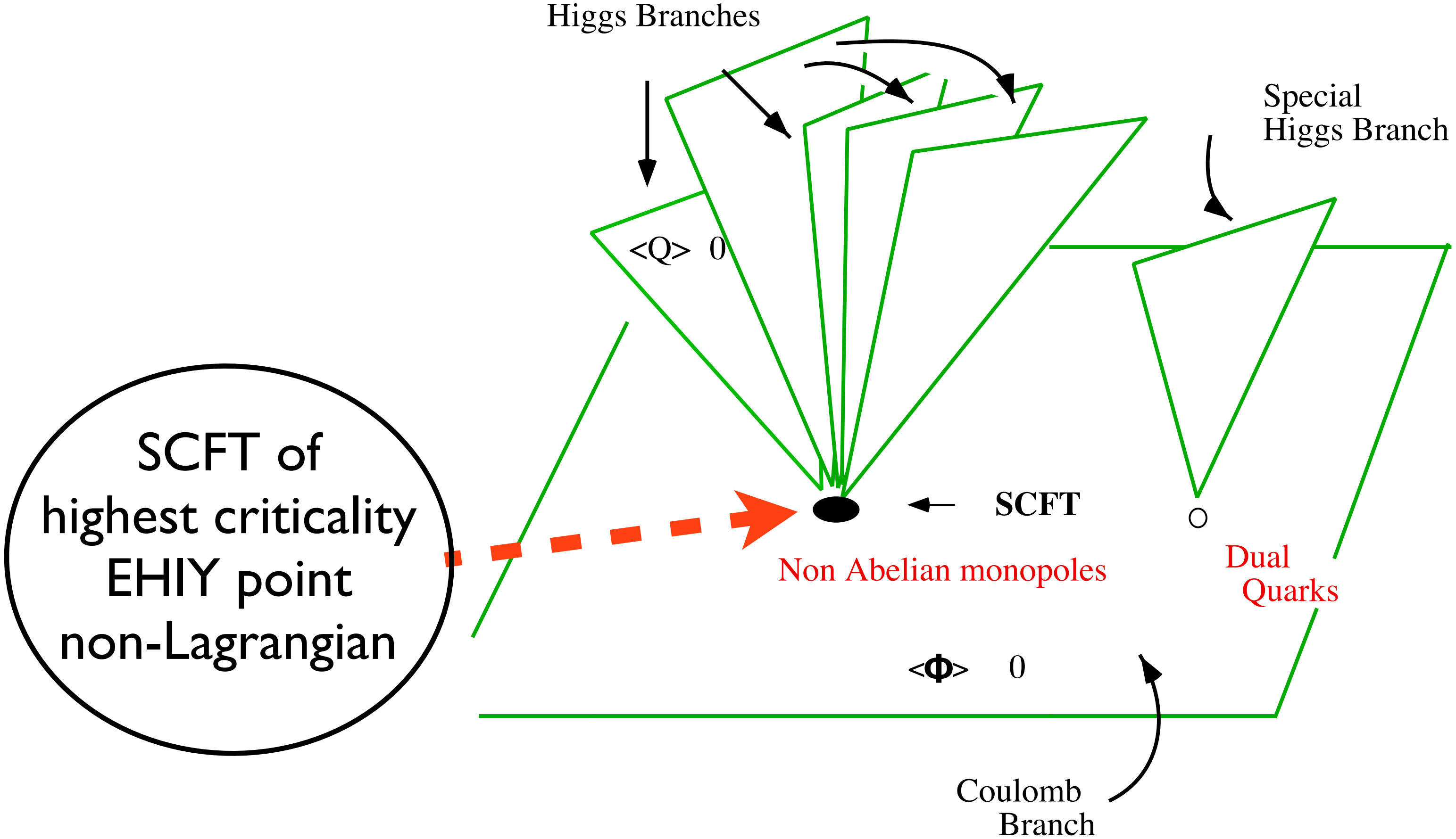
Di Pietro, Giacomelli '11

previous slide (Universality)

$m \neq 0$

QMS of N=2 USp(2n) Theory with n_f Quarks

(m = 0)



Carlino-Konishi-Murayama '00

• N=1 Confining vacua (with Φ^2 perturbation)

○ N=1 vacua (with Φ^2 perturbation) in free magnetic pha

III. Recent key developments

N=2 SCFT's

Gaiotto, Seiberg, Argyres, Tachikawa, Moore, Maruyoshi, ... '09 - '12

- S-duality in SCFT at $g = \infty$ e.g. SU(N) w/ $N_F = 2N$

Argyres-Seiberg '07

Gaiotto '09

- Argyres-Seiberg S-duality applied to SCFT (IR f.p.) of highest criticality (EHIY points)

Gaiotto-Seiberg-Tachikawa '11

- GST duality generalized to USp(2N), SO(N)

Giacomelli '12

- Colliding r-vacua and EHIY in SU(N)

Giacomelli, Di Pietro '11

- GST duality in USp(2N) and SU(3), N_F =4 and confinement

Giacomelli, Konishi '12 and '13

Argyres-Seiberg's S duality

- SU(3) with $N_F = 6$ hypermultiplets ($Q_i, \tilde{Q}_i$ ' s) at infinite coupling

Minahan-Nemeschansky '96

$$SU(3)\ w/\ (6\cdot\mathbf{3}\oplus\bar{\mathbf{3}}) \quad = \quad SU(2)\ w/\ (2\cdot\mathbf{2}\oplus\mathrm{SCFT}_{E_6})$$

g = ∞   g = 0

$$SU(2)\times SU(6)\subset E_6$$

Flavor symmetry ~ SU(6) x U(1)

- USp(4) with $N_F = 12$ Q' s at infinite coupling

$$USp(4)\ w/\ 12\cdot\mathbf{4} = SU(2)\ w/\ SCFT_{E_7}$$

$$SU(2)\times SO(12)\subset E_7$$

Gaiotto-Seiberg-Tachikawa (GST)

- Apply the basic idea of Argyres-Seiberg duality to the IR f.p. SCFT
- SU(N) with N_F = 2n :

$$y^2 = (x^N + u_1 x^{N-1} + u_2 x^{N-2} + \cdots + u_N)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x + m_i)$$

At u = m=0,* $y^2 \sim x^{N+n}$ (EHIY point)

relatively non-local massless monopoles and dyons

Note

* except for one u

Eguchi-Hori-Ito-Yang

- Straightforward treatment of fluctuations around u=m=0, gives an incorrect scaling laws for the masses

❀ To get the correct scale-invariant fluctuations, introduce two different scalings:

$$u_{N-n+2} \sim O(\epsilon_A^2), \quad u_{N-n+3} \sim O(\epsilon_A^3), \quad \ldots, \quad u_N \sim O(\epsilon_A^n).$$

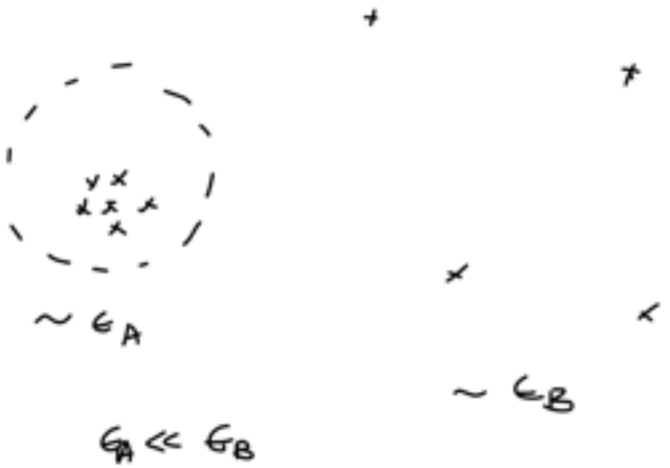
$$u_1 \sim O(\epsilon_B), \quad u_2 \sim O(\epsilon_B^2), \quad \ldots, \quad u_{N-n+2} \sim O(\epsilon_B^{N-n+2}).$$

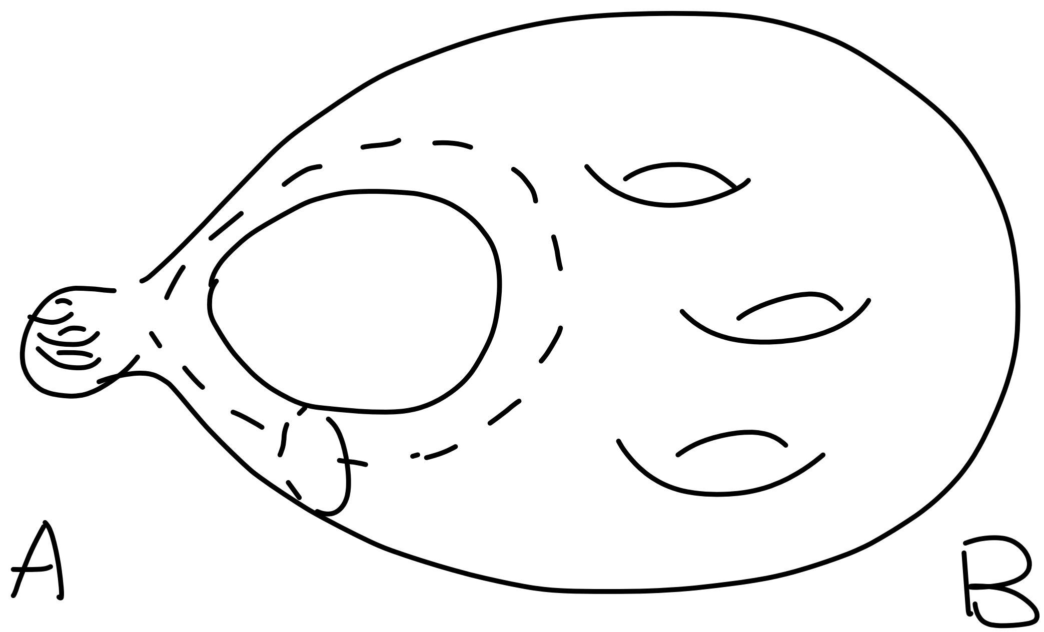
$$\epsilon_A^2 = \epsilon_B^{N-n+2} \qquad \Longrightarrow$$

$$a_i = \oint_{\alpha_i} \lambda, \quad a_{D\,i} = \oint_{\beta_i} \lambda$$

$$\lambda \sim dx\, y/x^n$$

$$m_{(n_m, n_e, n_i)} = \sqrt{2}\, |n_m a_D + n_e a + n_i m_i|$$


~ εA
~ εB
εA << εB


A
B

Gaiotto-Seiberg-Tachikawa '11

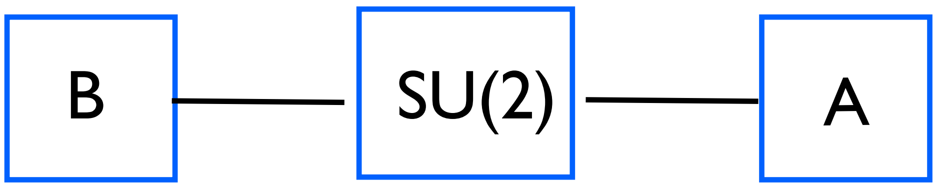
- $U(1)^{N-n-1}$ gauge multiplets

- SU(2) gauge multiplet (infrared free) coupled to the SU(2) flavor symmetry of the two SCFT's A & B

$$b_0 = \frac{N-n}{N-n+2}$$

- The A sector: the SCFT entering in the Argyres-Seiberg dual of SU(n), N_F =2 n, having SU(2)xSU(2n) flavor symmetry

- The B sector: the maximally singular SCFT of the SU(N-n+1) theory with two flavors



where

A: 3 free <u>2</u> 's (n=2); E_6 of Minahan-Nemechansky (n=3), etc.

B: the maximally singular SCFT of SU(2), N_F =2 (Seinberg-Witten) for N=3, n=2 , etc.

- Analogous results for USp(2N), SO(N)

Giacomelli '12

IV. GST duals and confinement

USp(2N) theory w/ $N_F = 2n$

Note

- Two types of Chebyshev* vacua ($\phi_1 = \phi_2 = \ldots = 0$; $\phi_n^2 = \pm \Lambda^2$; ϕ_m det'd by Cheb. polynom.)

$$xy^2 \sim \left[x^n(x-\phi_n^2)\right]^2 - 4\Lambda^4 x^{2n} = x^{2n}(x-\phi_n^2-2\Lambda^2)(x-\phi_n^2+2\Lambda^2).$$

$y^2 \sim x^{2n}$ singular SCFT (EHIY point);
strongly interacting, relatively non-local monopoles and dyons

- A strategy: resolve the vacuum by adding small $m_i \neq 0$ and determining the vacuum moduli (u_i's or ϕ_i's) requiring the SW curve to factorize in maximally Abelian factors (double factors) (i.e., Vacua in confinement phase surviving N=1, $\mu\, \Phi^2$ perturbation)

Carlino-Konishi-Murayama '00

$\Rightarrow$➔

$$\binom{N_f}{0} + \binom{N_f}{2} + \ldots \binom{N_f}{N_f} = 2^{N_f-1}$$

even r vacua, from one of the Cheb. vacua

$$\binom{N_f}{1} + \binom{N_f}{3} + \ldots \binom{N_f}{N_f-1} = 2^{N_f-1}$$

odd r vacua, from one of the Cheb. vacua

$$xy^2 = \left[x\prod_{a=1}^{n_c-1}(x-\phi_a^2)(x-2\Lambda^2-\beta)+2\Lambda^2 m_1\cdots m_{n_f}\right]^2 - 4\Lambda^4\prod_{i=1}^{n_f}(x+m_i^2)$$

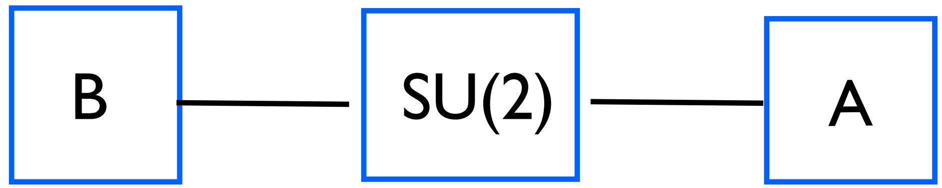
$\xrightarrow{\phi_i\text{'s}}$

$$xy^2 = x(x-4\Lambda^2-\gamma)\prod_{a=1}^{n_c}(x-\alpha_a)^2,$$

GST dual for the Chebyshev point of USp(2N) (also SO(N))

Giacomelli '12

$$y^2 \sim x^{2n} \qquad \approx$$



- $U(1)^{N-n}$ gauge multiplets

- The A sector: a (in general) non-Lagrangian SCFT having SU(2)xSO(4n) flavor symmetry

- The B sector: a free doublet (coupled to U(1) gauge boson)

For $N_F = 2n = 4$, A sector ~ 4 free doublets

But this allows a direct description of IR physics !!

For $USp(2N), \ N_f = 4$

the GST dual is (both the A and B sectors are free doublets) :

Giacomelli,Konishi '12,'13

$$\boxed{1} - SU(2) - \boxed{4}\ .$$

U(1)

the effects of m_i and $\mu\,\Phi^2$ perturbation can be studied from the superpotential:

$$\sqrt{2}\,Q_0(A_D+m_0)\tilde{Q}^0+\sqrt{2}\,Q_0\phi\tilde{Q}^0+\sum_{i=1}^{4}\sqrt{2}\,Q_i\phi\tilde{Q}^i+\mu A_D\Lambda+\mu\,\mathrm{Tr}\phi^2+\sum_{i=1}^{4}m_i\,Q_i\tilde{Q}^i\ .$$

cfr. UV Lagrangian:

$$W=\mu\mathrm{Tr}\Phi^2+\frac{1}{\sqrt{2}}Q_a^i\Phi_b^aQ_c^i\ J^{bc}+\frac{m_{ij}}{2}Q_a^iQ_b^j\ J^{ab}$$

$$m=-i\sigma_2\otimes\mathrm{diag}\,(m_1,m_2,\ldots,m_{n_f})\ .$$

Correct flavor symmetry for all {m}

- $m_i = m$: SU(4) x U(1) ;
- $m_i = 0$: SO(8) ; etc.,

vacuum equations

$$\sqrt{2}\,Q_0\tilde{Q}_0 + \mu\Lambda = 0\;;$$

$$(\sqrt{2}\,\phi + A_D + m_0)\tilde{Q}_0 = Q_0\,(\sqrt{2}\,\phi + A_D + m_0) = 0\;;$$

$$\sqrt{2}\;\left[\frac{1}{2}\sum_{i=1}^{4} Q_i^a\tilde{Q}_b^i - \frac{1}{4}\delta_b^a Q_i\tilde{Q}^i + \frac{1}{2}Q_0^a\tilde{Q}_b^0 - \frac{1}{4}\delta_b^a Q_0\tilde{Q}^0\right] + \mu\,\phi_b^a = 0\;;$$

$$(\sqrt{2}\,\phi + m_i)\,\tilde{Q}^i = Q_i\,(\sqrt{2}\,\phi + m_i) = 0, \quad \forall i\;.$$

0

Solutions

$$Q_0 = \tilde{Q}_0 = \begin{pmatrix} 2^{-1/4}\sqrt{-\mu\Lambda} \\ 0 \end{pmatrix}$$

four solutions

$$a = -\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} f_i \\ 0 \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

four more solutions

$$a = +\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} 0 \\ g_i \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

$$f_i^2 = \frac{\mu\Lambda - 4\,a}{\sqrt{2}} = \mu(\frac{\Lambda}{\sqrt{2}} + 2m_i).$$

$$g_i^2 = \frac{-\mu\Lambda + 4\,a}{\sqrt{2}} = -\mu(\frac{\Lambda}{\sqrt{2}} - 2m_i).$$

They are 4 + 4 , r=1 vacua ! (p. -2)

But where are the even r-vacua (r=0,2) ???

Answer: in the second Chebyshev vacuum:

$$\sqrt{2}\,Q_0(A_D+m_0)\tilde{Q}^0+\sqrt{2}\,Q_0\phi\tilde{Q}^0+\sum_{i=1}^{4}\sqrt{2}\,Q_i\phi\tilde{Q}^i+\mu A_D\Lambda+\mu\,\mathrm{Tr}\phi^2+\sum_{i=1}^{4}\tilde{m}_i\,Q_i\tilde{Q}^i$$

with

$$\tilde{m}_1=\frac{1}{4}(m_1+m_2-m_3-m_4)\;;$$
$$\tilde{m}_2=\frac{1}{4}(m_1-m_2+m_3-m_4)\;;$$
$$\tilde{m}_3=\frac{1}{4}(m_1-m_2-m_3+m_4)\;;$$
$$\tilde{m}_4=m_0=\frac{1}{4}(m_1+m_2+m_3+m_4)\;;$$

--------→

Spinor representation of SO(2N_F) ...

Magnetic Monopoles!

GST duality is not electromagnetic duality

Flavor symmetry OK in all cases:

m_i	$\tilde{m}_i$	Symmetry in UV	Symmetry in IR
$m_i=0$	$\tilde{m}_i=0$	$SO(8)$	$SO(8)$
$m_i=m\neq 0$	$\tilde{m}_4,\ \tilde{m}_1=\tilde{m}_2=\tilde{m}_3=0$	$U(1)\times SU(4)$	$U(1)\times SO(6)$
$m_1=m_2,\ m_3,\ m_4,$ generic	$\tilde{m}_2=-\tilde{m}_3,\ \tilde{m}_4,\tilde{m}_1$ generic	$U(1)\times U(1)\times U(2)$	$U(1)\times U(1)\times U(2)$
$m_1=m_2,\ m_3=m_4,\ m_1\neq m_3$	$\tilde{m}_2=\tilde{m}_3=0,\ \tilde{m}_4,\ \tilde{m}_1,$ generic	$SU(2)\times U(1)\times SU(2)\times U(1)$	$SO(4)\times U(1)\times U(1)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

Solutions similar to the previous case but: 1 + 1 + 6 in the m_i → m

To recapitulate:

- Mass perturbation of the EHIY (SCFT) singularity :
  the resolution of the Chebyshev vacua into the sum of the r-vacua:
  (local Lagrangian theories with $SU(r)xU(1)^{N-r}$ gauge symmetry)

  - Correct identification of the N=1 vacua surviving $\mu\ \Phi^2$ perturbation

  - But physics was unclear (strongly-coupled monopoles and dyons) in $m \to 0$ limit

- But **we have now checked** that these features of the singular EHIY (SCFT) theory is correctly reproduced by GST duals after $\mu\ \Phi^2$ perturbation.

☞

- The limit $m \to 0$ can be taken smoothly in the GST description
  (cfr. the usual monopole picture)

Physics of USp(2N), N_F = 4 theory at m=0

$$\sqrt{2}\,Q_0(A_D+m_0)\tilde{Q}^0+\sqrt{2}\,Q_0\phi\tilde{Q}^0+\sum_{i=1}^{4}\sqrt{2}\,Q_i\phi\tilde{Q}^i+\mu A_D\Lambda+\mu\,\mathrm{Tr}\phi^2\ .$$

➜ $$Q_0=\tilde{Q}_0=\begin{pmatrix}2^{-1/4}\sqrt{-\mu\Lambda}\\ 0\end{pmatrix}$$

$$(Q_1)^1=(\tilde{Q}^1)_1=2^{-1/4}\sqrt{\mu\Lambda}\,,\qquad Q_i=\tilde{Q}_i=0,\quad i=2,3,4.$$

$$\phi=0,\quad A_D=0\ .$$

➜ XSB

$$SO(8)\to U(1)\times SO(6)=U(1)\times SU(4)=U(4),$$

OK with results at $\mu, m \gg \Lambda$

➜ Confinement

UV: $\Pi_1(USp(2N))=\mathbf{1}$

IR: $\mathcal{L}_{GST}=SU(2)\times U(1),\qquad \Pi_1(SU(2)\times U(1))=\mathbf{Z}$

Higgsed at low energies; the vortex = the unique (N=n) confining string

Cfr. Abelianization implies $U(1)^N$ low energy theory

- The confining string is Abelian cfr. non-Abelian vortex of r-vacua → multiplication of the meson spectrum
- Confining o.p. (Q_0) triggers XSB o.p. (Q_i)

Some other systems studied

Giacomelli, Konishi '13

- Colliding r-vacua of SU(3), N_f =4 : G_f = U(4) unbroken
- Singular r=2 -vacua of SU(4), N_f =4 : G_f = U(4) ⇒ U(2)xU(2)
- SO(2N+1), N_f =1 : G_f = USp(2) =SU(2) ⇒ U(1)
- SO(2N), N_f =2 : G_f = USp(4) ⇒ U(2)

In all cases the GST description gives the correct number of the vacua and the symmetry breaking pattern known from the large μ analysis

Conclusion

- Gaiotto-Seiberg-Tachikawa duals of singular, IRFP SCFT allows to describe (confining) systems which naively involve infinitely-strongly coupled nonAbelian monopoles and dyons

- More general cases involve nonlocal (non Lagrangian) systems

- Features different from the naive dual superconductor picture. Confinement order parameters triggers the order parameter of the global symmetry breaking

⇒ A new picture for Q C D ?

The End

Colliding r vacua of SU(3), N_F =4 theory

The GST dual is now:

$$D_3 - SU(2) - \boxed{3}$$

where D_3 is the most singular SCFT of the $\mathcal{N}=2$ SU(2), N_F =2, theory, and 3 is three free doublets of SU(2). D_3 is a nonlocal theory,* it not easy to analyze.

We replace the system by

$$SU(2) \overset{P}{-} SU(2) - \boxed{3}$$

* arises from the collision of a doublet vacuum and a singlet vacuum

where P is a bifundamental field. The superpotential is:

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} P \Phi \tilde{P} + \sqrt{2} \tilde{P} \chi P + \mu \, \chi^2 + m^{'} \, \tilde{P} P,$$

The first SU(2) is AF: its dynamics is not affected by the second SU(2). But to exract the D_3 point, need to keep $m' \simeq \pm \Lambda^{'}$, but not exactly equal. The system Abelianizes →

Doublet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} M \Phi \tilde{M} + \sqrt{2} \tilde{M} A_\chi M + \mu \, A_\chi \Lambda' ,$$

with m

$$\tilde{m}_1 = \frac{1}{4}(m_1 + m_2 - m_3 - m_4) \; ;$$
$$\tilde{m}_2 = \frac{1}{4}(m_1 - m_2 + m_3 - m_4) \; ;$$
$$\tilde{m}_3 = \frac{1}{4}(m_1 - m_2 - m_3 + m_4) \; ,$$

→ correct symmetry

for all m_i :

→ six solutions (r=2 vacua)

SU(3), N_F =4 theory has r=0,1,2 vacua: where are the r=0,1 vacua? Answer:

Singlet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} \tilde{N} A N + \mu \, A \Lambda' + m' \, \tilde{N} N .$$

AF: becomes strongly coupled. → 4 + 1 vacua of

SW SU(2) N_F =3 theory! → r=1 (4) and r=0 (1) vacua

Vacuum structure OK

↩

Tuesday, March 19, 13

* Remarks

Tachikawa, Wecht '09

- N=2 SCFT's in UV flow into N=1 SCFT, upon N=1, $\mu\,\Phi^2$ perturbation (27/32)

- Some of them survive and brought into confinement phase; r-vacua, (r=0,1,2,...) upon N=1, $\mu\,\Phi^2$ perturbation

- Not all singular N=2 SCFT's survives N=1, $\mu\,\Phi^2$ perturbation (e.g., Aygyres-Douglas point in pure SU(3))

- N=2 IRFP SCFT's can survive and brought into confinement phase; e.g., Colliding r-vacua of SU(N) theories, m=0, USp(2N) theory (Tchebyshev vacua); m=0, SO(N) theory

- They are strongly interacting, nonlocal theory of monopoles and dyons, in confinement phase: interesting system to understand!!

r-vacua

$$y^2 = \prod_{a}^{N}(x-\phi_a)^2 - \Lambda^{2N-2n}\prod_{i=1}^{2n}(x+m_i)$$

$$\phi_a = (-m_1, -m_2, \ldots, -m_r, \phi_{r+1}, \ldots)$$

$m_i \to m,$

$$y^2 = (x+m)^{2r} \prod_{b=1}^{N-r}(x-\alpha_b)^2 \, (x-\gamma)(x-\delta)$$

This describes SU(r) x U(1)$^{N-r}$ theory

↩