AdS/CFT & unbalanced superconductors

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Contents:

- Introduction: string/field theory correspondence.
- Unbalanced Superconductors.
- A Holographic Model.

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Some classes of (strongly coupled) quantum field theories are secretly theories of (classical) gravity.

Introduction: string/field theory correspondence

Holography: a quantum field theory in d dimensions may be modeled by gravity (strings) in $d + n$ dimensions.

Classical computations determine quantum field theories at strong coupling.

Example [Maldacena 1997]:

$\mathcal{N} = 4$ $SU(N_c)$ SYM in 4d equivalent to Type IIB on $AdS_5 \times S^5$.

Gravity regime: $N_c \gg 1$,
$\lambda = g_{YM}^2 N_c \gg 1$.

Is it true?

- No mathematical proof but infinite number of checks.

What is the gain?

- Can study systems at strong coupling (e.g. unconventional superconductors).
- Can study systems at finite charge density.
- Real-time physics readily accessible.

What is the price?

- Useful only for certain theories:
  - Large "number of degrees of freedom per site".
  - Few important operators: large gap in anomalous dimensions.

How does it work?

Every physical ingredient in quantum field theory (FT) is translated ($\Rightarrow$) in the dual gravity theory.

Dictionary:

- A ground state of the FT $\Rightarrow$ a background gravity solution.
- Each operator $\mathcal{O}$ of the FT $\Rightarrow$ a gravity field Φ.
- RG scale $\Rightarrow$ extra space-time dimension r.
- Temperature T, charge density ρ $\Rightarrow$ charged Black Hole with temperature T and gravity gauge field A_t.
- Response to external perturbations $\Rightarrow$ from perturbations of gravity fields.

How to compute:

[Witten, Gubser-Klebanov-Polyakov 1998]

$$\langle e^{-\int \Phi_0 \mathcal{O}} \rangle_{FT} = e^{-S_{gravity}(\Phi_0)}$$

- $\mathcal{O} \;\Rightarrow\; \Phi$ and $\Phi_0 = \lim_{r\to\infty} \Phi$.
- Φ_0 determines Φ via equations of motion.
- Plug the solution in the gravity action: $S_{gravity}(\Phi_0)$.
- LHS is generating functional: n-point functions from functional derivatives w.r.t. Φ_0 on the RHS.

Plan:

- Introduction: string/field theory correspondence.
- Unbalanced Superconductors.
- A Holographic Model.

In many systems in condensed matter physics (superconductors, cold atoms, ...) and in QCD (neutron stars) we deal with "unbalanced Fermi mixtures" at strong coupling.

Superconductors.

- Systems with infinite conductivity:

$$J = \sigma E \qquad \Rightarrow \qquad \sigma = \infty$$

- Directly from spontaneous breaking of $U(1)_{em}$ [Weinberg 1985].
- Condensation of electrically charged operator $\mathcal{O}$.
- Balanced BCS theory (weak coupling): vev of charged bosonic operator (Cooper pair)

$$\langle \mathcal{O} \rangle = \langle \psi^{\uparrow}(\vec{p}_1)\psi^{\downarrow}(\vec{p}_2)\rangle \sim \Delta\delta(\vec{p}_1 + \vec{p}_2)$$

Unbalanced Superconductors

There exist a very interesting class of high T_c “Unconventional superconductors” for which BCS does not work.

- $2\Delta/T_c \sim 8$ (cuprates) instead of $2\Delta/T_c \sim 3.5$.
- There is no completely satisfying theory.
- Probably strongly coupled (Non-Fermi Liquids).

Unbalanced Superconductors

Unbalanced superconductivity, macroscopically.

- Superconductivity: zero DC resistivity.
  Condensation of electrically charged operator $\mathcal{O}$.
- Unbalance: $\delta\mu$ chemical potential for $U(1)_s$ (decoupled from space-time symmetries in IR).
  Order parameter $\mathcal{O}$ uncharged under $U(1)_s$.

Essentially: $U(1)_{em} \times U(1)_s$ with only electrically charged $\mathcal{O}$.

Unbalanced Fermi mixtures.

- Fermions pile up (Pauli exclusion principle) and build up Fermi surfaces.
- Fermi surface: surface in momentum space where fermion 2-point function has poles.
- Different fermionic species in a system can have different Fermi surfaces: "imbalance".

Unbalanced Superconductors

Basic example: unbalanced superconductors.

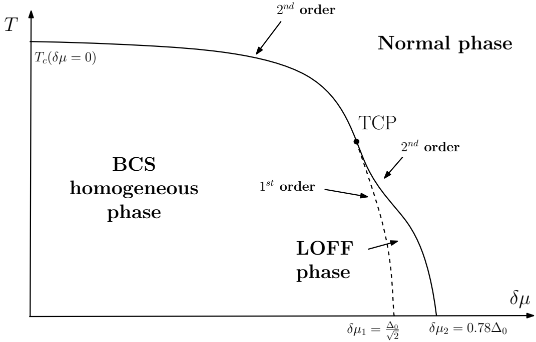
- Impurities, doping, external fields: different couplings with spins $\Rightarrow$ different Fermi surfaces for "up" and "down" electrons (generic situation).
- Example: superconductors with external magnetic field, interaction $\mathcal{H}_I = \bar{\Psi}\gamma^0 H \mu_B \sigma_3 \Psi$.
- Effective chemical potential mismatch $\delta\mu = H\mu_B$.

Unbalanced Superconductors

- Forming Cooper pairs costs more energy (excite one electron above its Fermi surface).
- Intuition: if imbalance $\delta\mu$ large $\Rightarrow$ loose superconductivity.
- In fact:
  - Homogeneous condensate disfavored beyond $\delta\mu_{max}$ (Chandrasekhar-Clogston bound [Chandrasekhar 1962, Clogston 1962]).
  - Alternatively: finite momentum condensate, inhomogeneous phase, LOFF [Larkin-Ovchinnikov 1964, Fulde-Ferrel 1964], e.g.

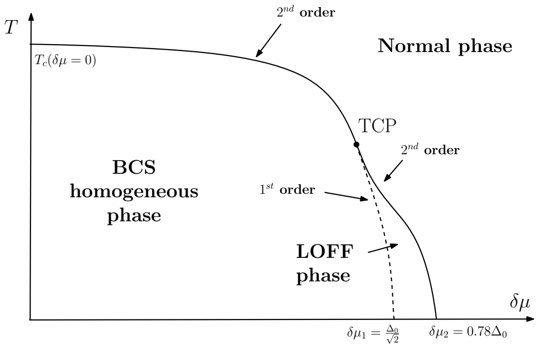
$$\langle \mathcal{O} \rangle \sim \Delta e^{i(\vec{p}^{\uparrow}+\vec{p}^{\downarrow})\vec{x}} \ , \qquad |\vec{p}^{\uparrow} + \vec{p}^{\downarrow}| \sim \delta\mu \ .$$

Phase diagram (weak coupling):



Unbalanced Superconductors

Phase diagram (weak coupling):



Strong coupling problem: phase diagram in high T_c superconductors?

Unbalanced Superconductors

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Essentially: $U(1)_{em} \times U(1)_s$ with only electrically charged $\mathcal{O}$.

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We build the simplest holographic model of s-wave unbalanced (layered) “superconductor”. [Bigazzi-Cotrone-Musso-Pinzani-Fokeeva-Seminara 2011]

Unbalanced holographic superconductors.

Dual ingredients for a superconductor [Gubser 2008, Hartnoll-Herzog-Horowitz 2008]:

- A charged Black Hole with a gravity gauge field A: normal phase of superconductor, $U(1)_{em}$.
- A charged scalar field ψ: the condensate $\langle \mathcal{O} \rangle$.

Extra dual ingredient for the imbalance:

- Another gravity gauge field B: the $U(1)_s$.

  [Bigazzi-Cotrone-Musso-Pinzani-Fokeeva-Seminara 2011]

Action:

$$S = \frac{1}{2\kappa_4^2}\int dx^4\sqrt{-g}\Big[\mathcal{R}+\frac{6}{L^2}-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{4}Y_{ab}Y^{ab} -V(|\psi|)-|\partial\psi-iqA\psi|^2\Big]$$

with:

$$F = dA\,,\qquad Y = dB\,,\qquad V(|\psi|) = -\frac{2}{L^2}\psi^\dagger\psi$$

Note:

- Scalar field ψ uncharged under $U(1)_s$.
- Dual operator of dimension 2 ($\Delta(\Delta-3)=m^2L^2$).

A Holographic Model

Ansatz for the ground states:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{r^2}{L^2}(dx^2 + dy^2) + \frac{dr^2}{g(r)}$$

$$\psi = \psi(r), \quad A_a dx^a = \phi(r)dt, \quad B_a dx^a = v(r)dt$$

Asymptotics:

$$IR: \quad g(r_H) = \phi(r_H) = v(r_H) = 0\,, \qquad \psi(r_H), \chi(r_H) \text{ constants}$$

Note: horizon at r_H, thermal systems.

$$UV: \quad \psi(r) = \frac{0}{r} + \frac{C_2}{r^2} + \ldots, \quad \phi(r) = \mu - \frac{\rho}{r} + \ldots, \quad v(r) = \delta\mu - \frac{\delta\rho}{r} + \ldots$$

$$g(r) = r^2 - \frac{\epsilon}{2r} + \ldots, \quad \chi(r) = 0 + \ldots$$

Note: asymptotically AdS.

Equations of motion:

$$\psi'' + \psi'\left(\frac{g'}{g} + \frac{2}{r} - \frac{\chi'}{2}\right) - \frac{V'(\psi)}{2g} + \frac{e^{\chi} q^2 \phi^2 \psi}{g^2} = 0$$

$$\phi'' + \phi'\left(\frac{2}{r} + \frac{\chi'}{2}\right) - \frac{2q^2\psi^2}{g}\phi = 0$$

$$\frac{1}{2}\psi'^2 + \frac{e^{\chi}(\phi'^2 + v'^2)}{4g} + \frac{g'}{gr} + \frac{1}{r^2} - \frac{3}{gL^2} + \frac{V(\psi)}{2g} + \frac{e^{\chi} q^2 \psi^2 \phi^2}{2g^2} = 0$$

$$\chi' + r\psi'^2 + r\frac{e^{\chi} q^2 \phi^2 \psi^2}{g^2} = 0$$

$$v'' + v'\left(\frac{2}{r} + \frac{\chi'}{2}\right) = 0$$

A Holographic Model

First ground state, Normal phase $\langle \mathcal{O} \rangle = 0$:
$U(1)^2$-charged Reissner-Nordstrom-AdS_4 black hole ($\psi = 0$).

$$\begin{aligned} ds^2 &= -f(r)dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{f(r)} \\ f(r) &= r^2\left(1 - \frac{r_H^3}{r^3}\right) + \frac{\mu^2 r_H^2}{4r^2}\left(1 - \frac{r}{r_H}\right) + \frac{\delta\mu^2 r_H^2}{4r^2}\left(1 - \frac{r}{r_H}\right) \\ \phi(r) &= \mu\left(1 - \frac{r_H}{r}\right) = \mu - \frac{\rho}{r} \\ v(r) &= \delta\mu\left(1 - \frac{r_H}{r}\right) = \delta\mu - \frac{\delta\rho}{r} \end{aligned}$$

Note: "Forced" ferromagnet: $\delta\rho$ supported by non zero $\delta\mu$ (external magnetic field).

(In)stability:

- At $T = 0$, near-horizon $AdS_2 \times R^2$.
- Fluctuating scalar: $m^2_{eff} = m^2 - \frac{2q^2}{(1+\delta\mu^2/\mu^2)}$.
- AdS_2 BF-bound (scalar instability): $L^2_{(2)} m^2_{\mathrm{eff}} = \frac{1}{6} m^2_{\mathrm{eff}} < -\frac{1}{4}$.

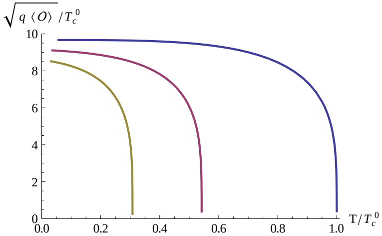
Thus:

$$\left(1 + \frac{\delta\mu^2}{\mu^2}\right)\left(m^2 + \frac{3}{2}\right) < 2q^2$$

- If $m^2 < -\frac{3}{2}$: instability for every $\frac{\delta\mu^2}{\mu^2}$.
- If $m^2 > -\frac{3}{2}$: instability for $\frac{\delta\mu^2}{\mu^2} < 2q^2 \frac{1}{\left(m^2+\frac{3}{2}\right)} - 1$ (CC-like).

Second ground state, Superconducting phase, $\langle \mathcal{O} \rangle \neq 0$:
$U(1)^2$-charged hairy-AdS_4 black hole ($\psi \neq 0$, numeric solution).

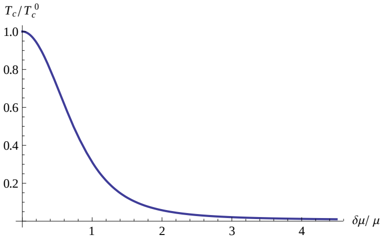
The condensate for $\delta\mu/\mu = 0$ (blue), 1 (red), 1.5 (green):



- $T > T_c$: no condensate.
- $T < T_c$: $\langle \mathcal{O} \rangle \neq 0$, breaks spontaneously $U(1)_{em}$.
- Around T_c, 2^{nd}-order phase transition: $\langle \mathcal{O} \rangle \sim \sqrt{T_c - T}$.
- $\delta\mu$ reduces T_c.

A Holographic Model

The phase diagram:



- At $T = 0$ always superconducting: no CC-like bound.
- Phase transition always second order (from free energy).
- LOFF phases unlikely (checked at $q \gg 1$ for plane-waves).
- Features depend on model parameters.

A Holographic Model

Real time physics: conductivities.

Vector fluctuations on background:

$$A_x''+\left(\frac{g'}{g}-\frac{\chi'}{2}\right)A_x'+\left(\frac{\omega^2}{g^2}e^{\chi}-\frac{2q^2\psi^2}{g}\right)A_x-\frac{\phi'}{g}e^{\chi}\left(B_x v'+A_x\phi'\right)=0$$

$$B_x''+\left(\frac{g'}{g}-\frac{\chi'}{2}\right)B_x'+\frac{\omega^2}{g^2}e^{\chi}B_x-\frac{v'}{g}e^{\chi}\left(B_x v'+A_x\phi'\right)=0$$

- A_x, B_x fluctuating fields, others background.
- $O(2)$ symmetry, picked x-direction. Time dependence: $e^{-i\omega t}$.
- A_x, B_x automatically coupled on charged backgrounds: "spintronic effects" are generic.

A Holographic Model

UV asymptotics:

$$
\begin{aligned}
A_x(r) &= A_x^{(0)} + \frac{1}{r} A_x^{(1)} + \dots \\
B_x(r) &= B_x^{(0)} + \frac{1}{r} B_x^{(1)} + \dots
\end{aligned}
$$

IR asymptotics (pick incoming wave boundary conditions):

$$
\begin{aligned}
A_x(r) &= \left(1 - \frac{r_H}{r}\right)^{ia\omega} \left[a_0 + a_1 \left(1 - \frac{r_H}{r}\right) + \dots\right] \\
B_x(r) &= \left(1 - \frac{r_H}{r}\right)^{ia\omega} \left[b_0 + b_1 \left(1 - \frac{r_H}{r}\right) + \dots\right]
\end{aligned}
$$

A Holographic Model

On-shell renormalized action:

$$S_{quad} = \int d^3x \left(\frac{1}{2} A_x^{(0)} A_x^{(1)} + \frac{1}{2} B_x^{(0)} B_x^{(1)} \right)$$

Holographic dictionary:

$$J^A = \frac{\delta S_{quad}}{\delta A_x^{(0)}}$$
$$J^B = \frac{\delta S_{quad}}{\delta B_x^{(0)}}$$

The conductivity matrix:

$$\begin{pmatrix} J^A \\ J^B \end{pmatrix} = \begin{pmatrix} \sigma_A & \gamma \\ \gamma & \sigma_B \end{pmatrix} \cdot \begin{pmatrix} E^A \\ E^B \end{pmatrix}$$

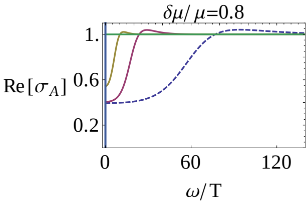
Extract:

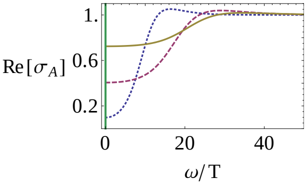
$$\sigma_A = \frac{J^A}{E^A}\Big|_{B_x^{(0)}=0} = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}\Big|_{B_x^{(0)}=0} \quad \text{“Electric cond.''}$$

$$\sigma_B = \frac{J^B}{E^B}\Big|_{A_x^{(0)}=0} = -\frac{i}{\omega}\frac{B_x^{(1)}}{B_x^{(0)}}\Big|_{A_x^{(0)}=0} \quad \text{“Spin cond.''}$$

$$\gamma = \frac{J^B}{E^A}\Big|_{B_x^{(0)}=0} = -\frac{i}{\omega}\frac{B_x^{(1)}}{A_x^{(0)}}\Big|_{B_x^{(0)}=0} \quad \text{“Spin} - \text{electric cond.''}$$

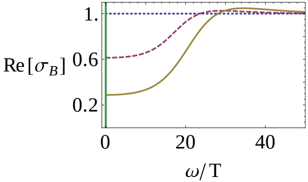
Conductivities in the normal phase.

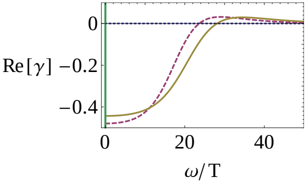




Left: increasing T tend to constant value; infinite DC conductivity: translational invariance.

Right: increasing $\delta\mu/\mu$ (dotted $\to$ solid) tend to constant value.

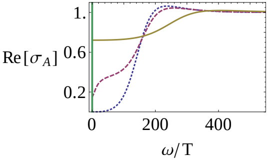


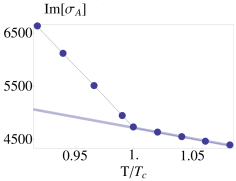


Left: increasing $\delta\mu/\mu$ enhance depletion (opposite to σ_A, e.g. [Bellazzini-Burrello-Mintchev-Sorba 2008]).

Right: increasing $\delta\mu/\mu$ non-monotonic behavior.

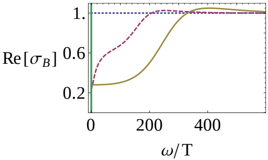
Conductivities in the superconducting phase.

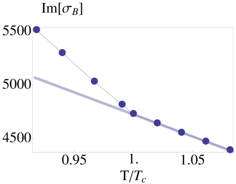




Left: increasing $\delta\mu/\mu$ (dotted $\rightarrow$ solid) tend to normal value.

Right: infinite DC conductivity "jumps", superconductivity.

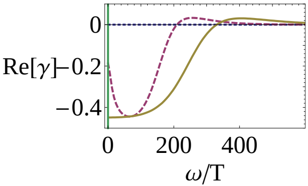


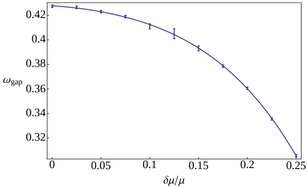


Left: increasing $\delta\mu/\mu$ enhance depletion.

Right: infinite DC conductivity "jumps"! Effect of mixing, opposite to weak coupling [Takahashi-Maekawa 2003].

Conductivities in the superconducting phase.





Left: non-trivial behavior. New energy scale $\delta\mu$ heavily modifies small frequency regime.

Right: pseudo-gap in σ_A non-linear with $\delta\mu/\mu$.

Summary

- Simplest model of unbalanced holographic superconductor shows no evidence of inhomogeneous (LOFF-like) phases.
- Phase diagram not universal: room for developments.
- Holography implements naturally spintronics: charge-spin field/current interplay is generic.
- Prediction: spin DC conductivity enhanced in superconducting phase at strong coupling.

Other main topics in AdS/Condensed Matter

- Construction and study of Non-Fermi Liquids.
- Study of "strange metal" behavior (e.g. $\sigma_A \sim 1/T$).
- Lattice effects.
- Hall effect, topological order, entanglement entropy, quantum quench, ...