# The Electromagnetic Coupling $\alpha_{em}(M_Z)$ and $(g\text{-}2)_\mu$

- Precision Physics (EW fit) at ILC or TLEP needs precise knowledge of $\alpha_{em}(M_Z)$

$$\alpha(M_Z) = \frac{\alpha(0)}{1-\Delta\alpha(M_Z)} \qquad \Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

- Its uncertainty affects the prediction for $M_W$ and $\sin^2\theta^l_{eff}$
- It is dominated by non perturbative hadronic effects ($\Delta\alpha^{(5)}_{had}$) which can be related to measured hadr. cross sections (R(s)) at low energy (below 10 GeV)

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027627 \pm 0.000138$$

$$\alpha^{-1}(M_Z^2) = 128.944 \pm 0.019$$

[HLMNT J. Phys. G 38 (2011) 085003]

- $\delta\Delta\alpha^{(5)}_{had} \sim 1.5\times10^{-4} \rightarrow$ **$5\times10^{-5}$** needed to match ILC/TLEP precision (a **x3** improvement)

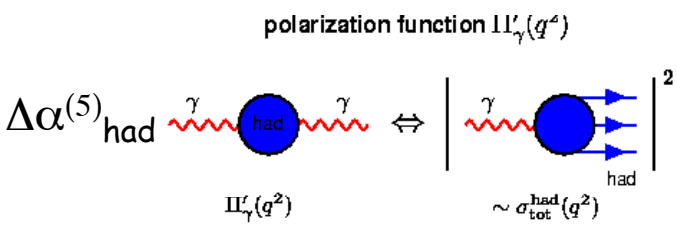
- **Necessity** of an experimental program of precise measurement of R(s) at low energies

- Similar analysis for the hadronic contribution to the muon g-2 ($a_\mu^{had}$). Very important the region below 2.5 GeV!

# Hadronic contribution to $\alpha_{em}(M_Z)$ and g-2

$$\alpha(M_Z) = \frac{\alpha(0)}{1-\Delta\alpha(M_Z)} \qquad \Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

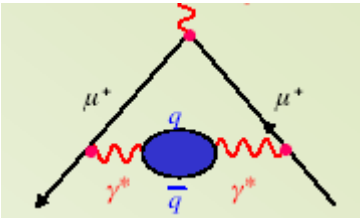
- The contribution of 5 light quarks not computable by perturbative QCD
- "Way out": Optical Theorem -> dispersion integral on $\sigma(e^+e^- \to \text{hadr.})$




$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds\,\frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

$$= -\frac{\alpha M_Z^2}{3\pi}\left(\mathrm{Re}\int_{4m_\pi^2}^{E_{cut}^2} ds\,\frac{R^{data}(s)}{s(s-M_Z^2-i\varepsilon)} + \mathrm{Re}\int_{E_{cut}^2}^{\infty} ds\,\frac{R^{pQCD}(s)}{s(s-M_Z^2-i\varepsilon)}\right)$$

$a_\mu^{had} =$




$$a_\mu^{had,lo} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \sigma_{e^+e^-\to hadr}(s)K(s)ds \qquad K(s) \sim 1/s$$

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

The region below 10 (2.5) GeV very important!!!

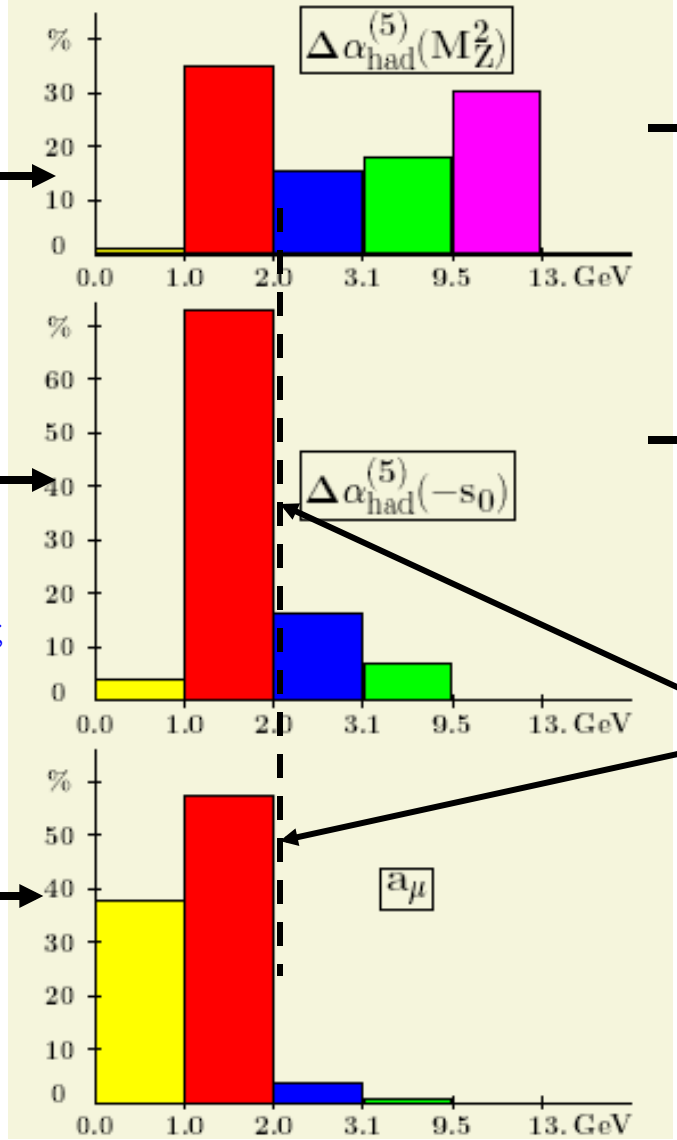
# Error profiles for $\Delta\alpha^{(5)}_{had}(M_Z)$ and $a_\mu^{had}$

Direct integration of energy points for $\Delta\alpha^{(5)}_{had}(M_Z)$

Use of Adler function for $\Delta\alpha^{(5)}_{had}(-s_0)$

[F.Jegerlehner, NPPS. 181-182 (2008) 135-140; NPPS 162 (2006) 22-32]

Direct integration of energy points for $a_\mu^{had}$




$\delta R$ at 1% in $\sqrt{s}$ < 10 GeV ⇒ improvement of ~3 in $\delta\alpha(M_Z)$

$\delta R$ at 1% in the region $1<\sqrt{s}<2.5$ GeV (which is known with 6% accuracy) ⇒ similar improvement on $\delta\alpha(M_Z)$

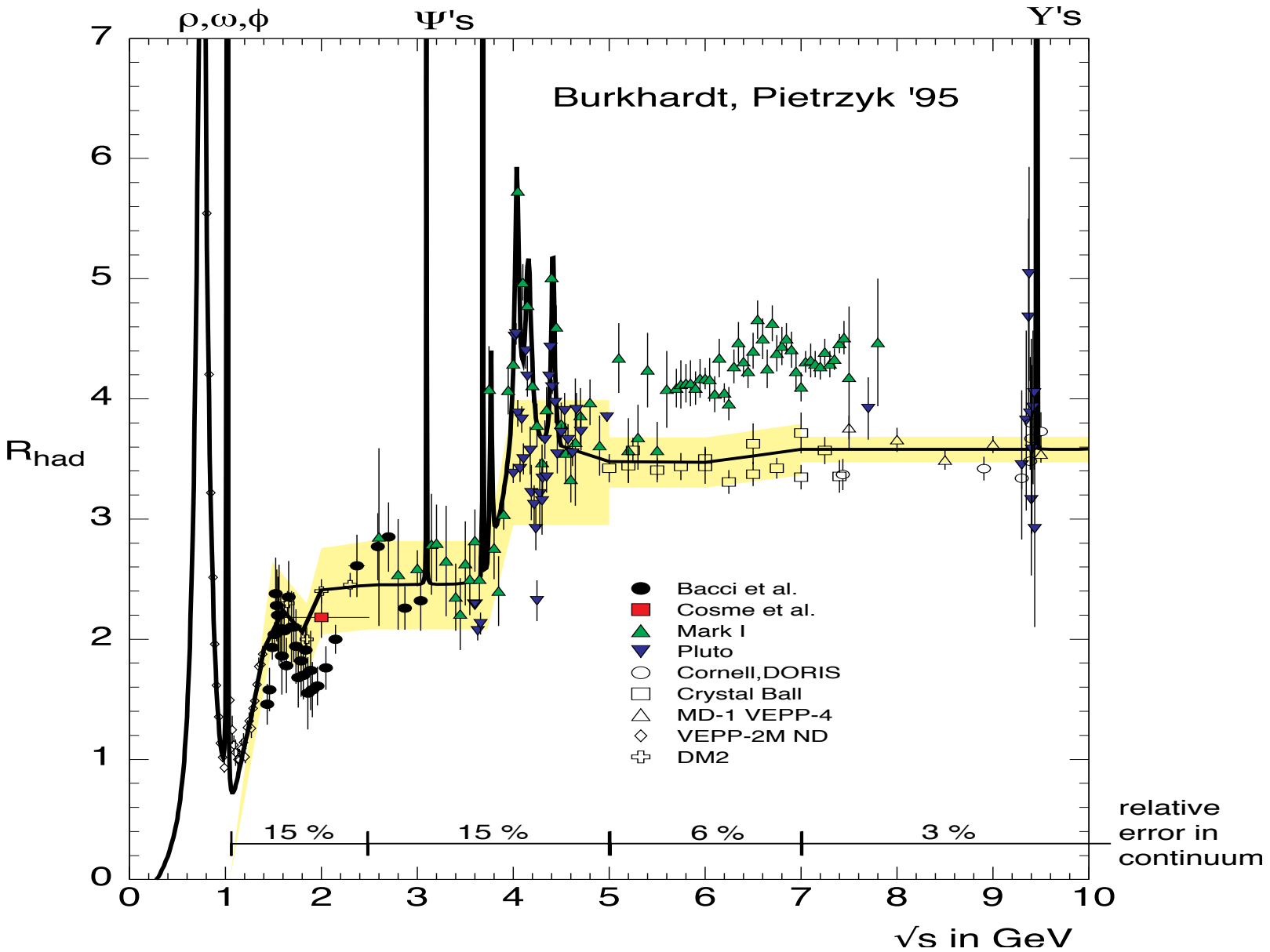
**Region below 2.5 GeV Extremely important!!!**

*-80% of the total error on $\Delta\alpha^{(5)}_{had}$ (using Adler function)*

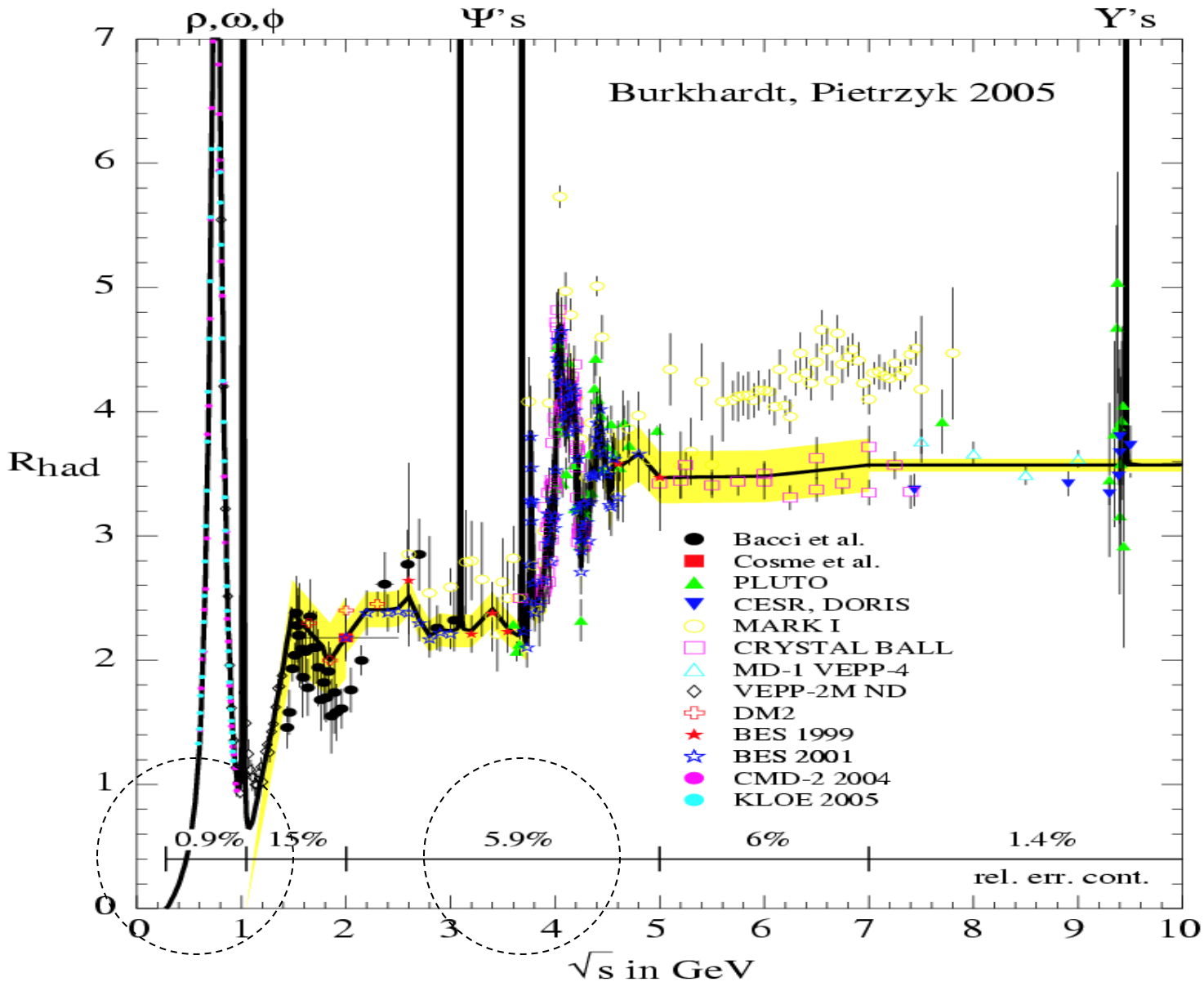
*-95% of the tot error on $a_\mu^{had}$*

Needs a dedicated experimental effort with e+e- machines at low energy!!!

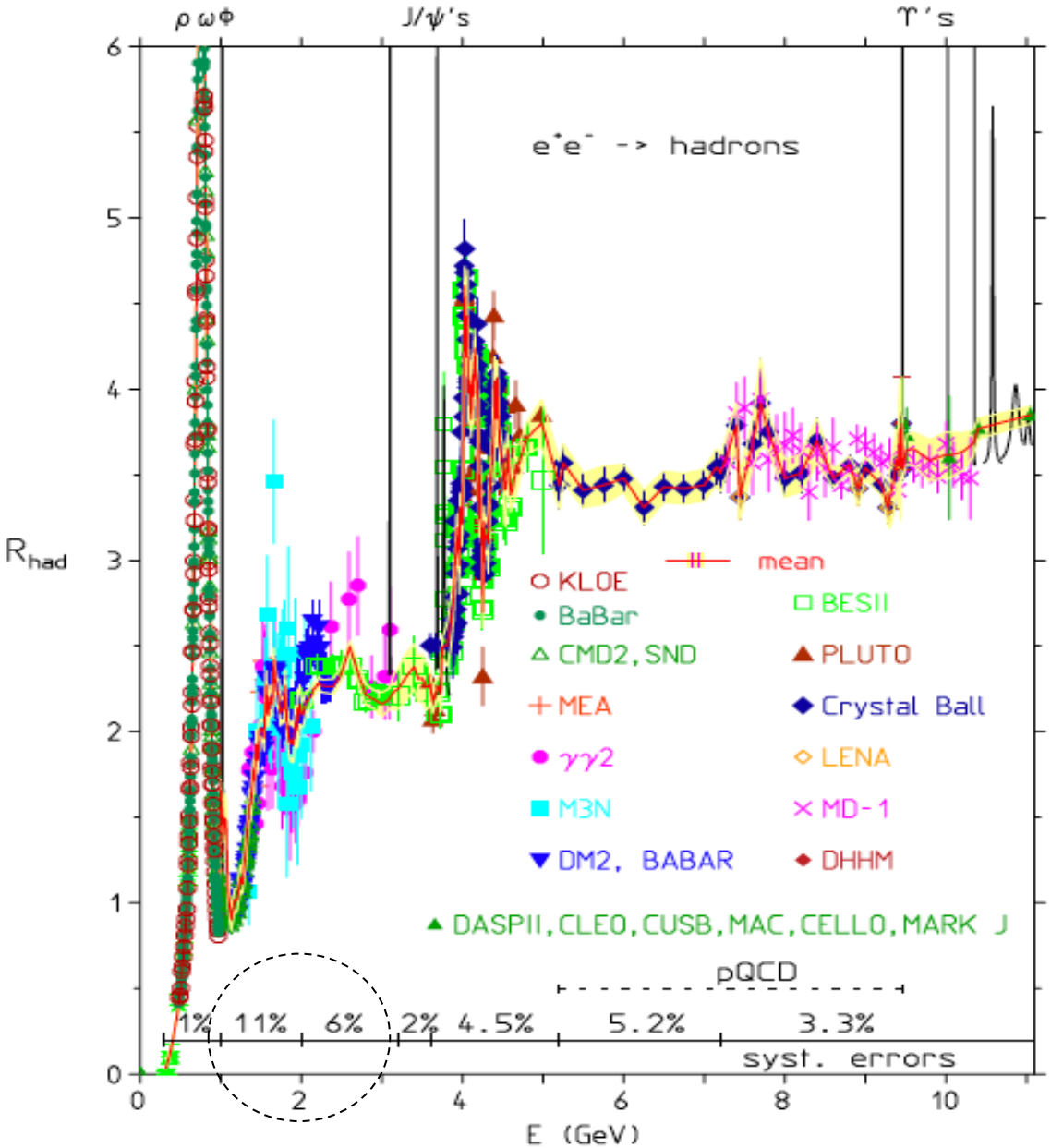
# Data at '95


ρ,ω,φ
Ψ's
Υ's
Burkhardt, Pietrzyk '95
R_had
√s in GeV
Bacci et al.
Cosme et al.
Mark I
Pluto
Cornell,DORIS
Crystal Ball
MD-1 VEPP-4
VEPP-2M ND
DM2
15 %
15 %
6 %
3 %
relative error in continuum

# Data at '05

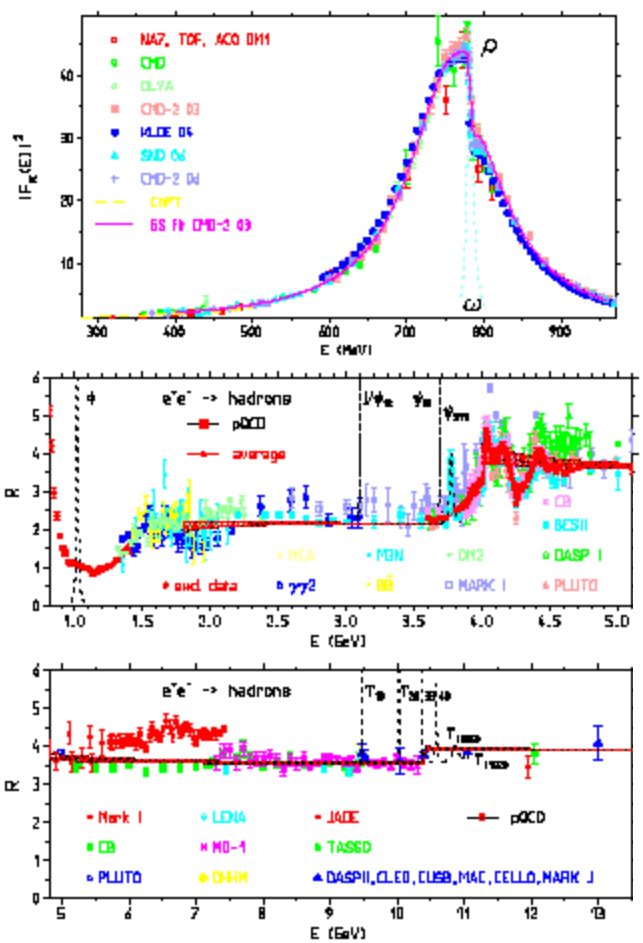

ρ,ω,φ
Ψ's
Υ's
Burkhardt, Pietrzyk 2005
Rhad
Bacci et al.
Cosme et al.
PLUTO
CESR, DORIS
MARK I
CRYSTAL BALL
MD-1 VEPP-4
VEPP-2M ND
DM2
BES 1999
BES 2001
CMD-2 2004
KLOE 2005
0.9%
15%
5.9%
6%
1.4%
rel. err. cont.
√s in GeV

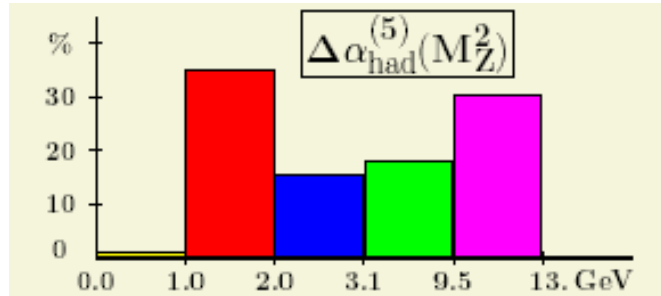
# Data at '13




*Many improvements in the last 15 years However the region between 1 and 2.5 GeV is still poorly known ($\delta R$~5-15%)*

# Hadronic contribution to $\alpha_{em}(M_Z)$ (direct integration)



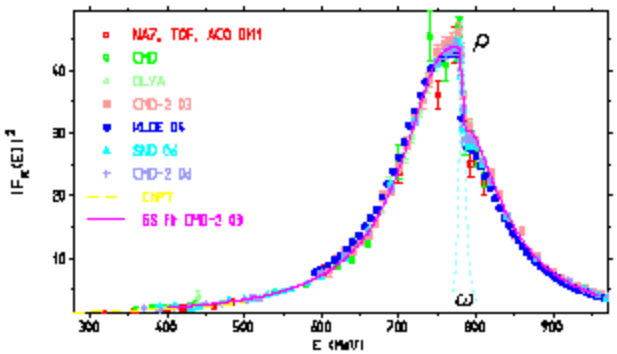

$\delta^2\Delta\alpha^{(5)}_{had}(M_Z)$

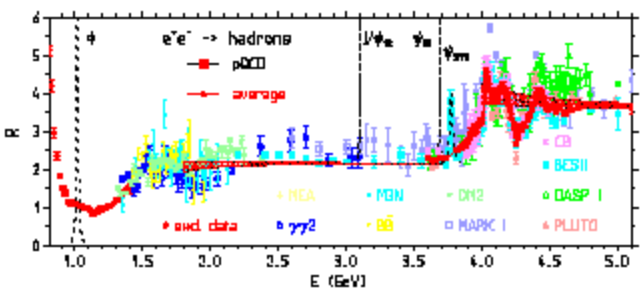
| Energy range | $\Delta\alpha^{(5)}_{had}$[%](error) $\times 10^4$ | rel. err. | abs. err. |
|---|---|---|---|
| $\rho,\omega$ $(E < 2M_K)$ | 36.99 [ 13.4](0.21) | 0.6 % | 1.7 % |
| $2M_K < E < 2$ GeV | 20.89 [ 7.5](0.94) | 4.5 % | 33.6 % |
| 2 GeV $< E < M_{J/\psi}$ | 15.34 [ 5.5](0.62) | 4.0 % | 14.4 % |
| $M_{J/\psi} < E < M_\Upsilon$ | 67.77 [ 24.5](0.92) | 1.4 % | 31.8 % |
| $M_\Upsilon < E < E_{cut}$ | 12.43 [ 4.5](0.70) | 5.6 % | 18.4 % |
| $E_{cut} < E$ pQCD | 123.29 [ 44.6](0.05) | 0.0 % | 0.1 % |
| $E < E_{cut}$ data | 153.42 [ 55.4](1.63) | 1.1 % | 99.9 % |
| total | 276.71 [100.0](1.63) | 0.6 % | 100.0 % |

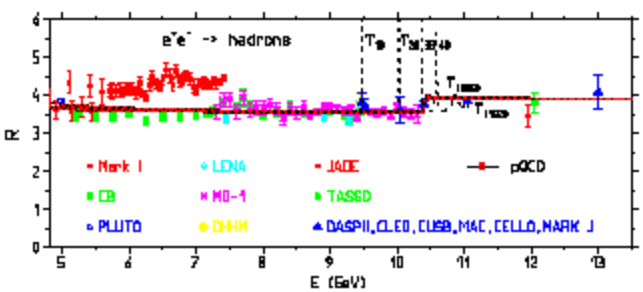
$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027671 \pm 0.000163$$

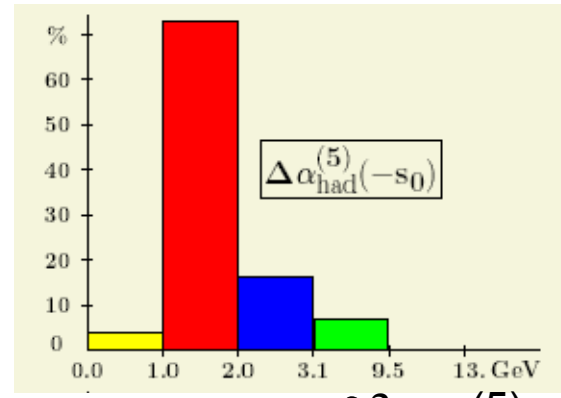
*Courtesy of F. Jegerlehner*

# Hadronic contribution to $\alpha_{em}(M_Z)$ (Adler function)



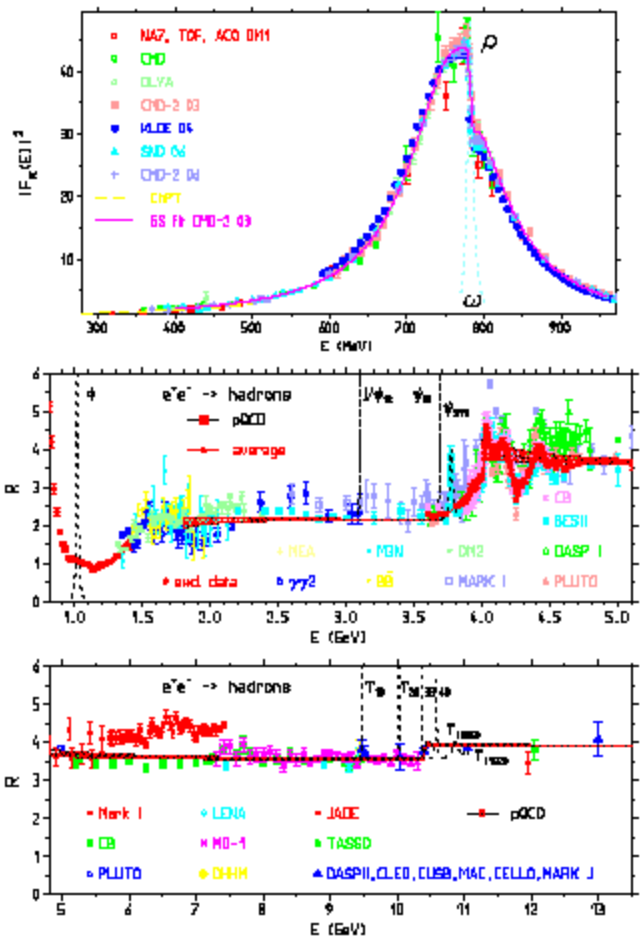

$\delta^2\Delta\alpha^{(5)}_{had}(-s_0)$

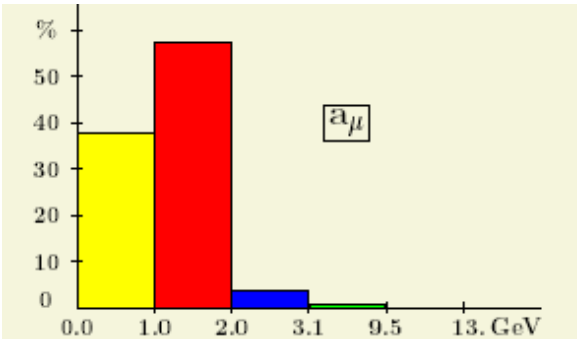
| Energy range | $\Delta\alpha^{(5)}_{had}$[%](error) $\times 10^4$ | rel. err. | abs. err. |
|---|---|---|---|
| $\rho,\omega$ $(E < 2M_K)$ | 32.45 [ 50.9](0.18) | 0.6 % | 7.2 % |
| $2M_K < E < 2$ GeV | 13.90 [ 21.8](0.56) | 4.1 % | 68.7 % |
| 2 GeV $< E < M_{J/\psi}$ | 6.06 [ 9.5](0.25) | 4.2 % | 13.8 % |
| $M_{J/\psi} < E < M_\Upsilon$ | 10.04 [ 15.7](0.22) | 2.2 % | 10.2 % |
| $M_\Upsilon < E < E_{\rm cut}$ | 0.44 [ 0.7](0.02) | 5.1 % | 0.1 % |
| $E_{\rm cut} < E$ pQCD | 0.90 [ 1.4](0.00) | 0.0 % | 0.0 % |
| $E < E_{\rm cut}$ data | 62.89 [ 98.6](0.68) | 1.1 % | 100.0 % |
| total | 63.79 [100.0](0.68) | 1.1 % | 100.0 % |

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027498 \pm 0.000135$$

*Courtesy of F. Jegerlehner*

# Hadronic contribution to $\alpha_\mu^{HAD}$



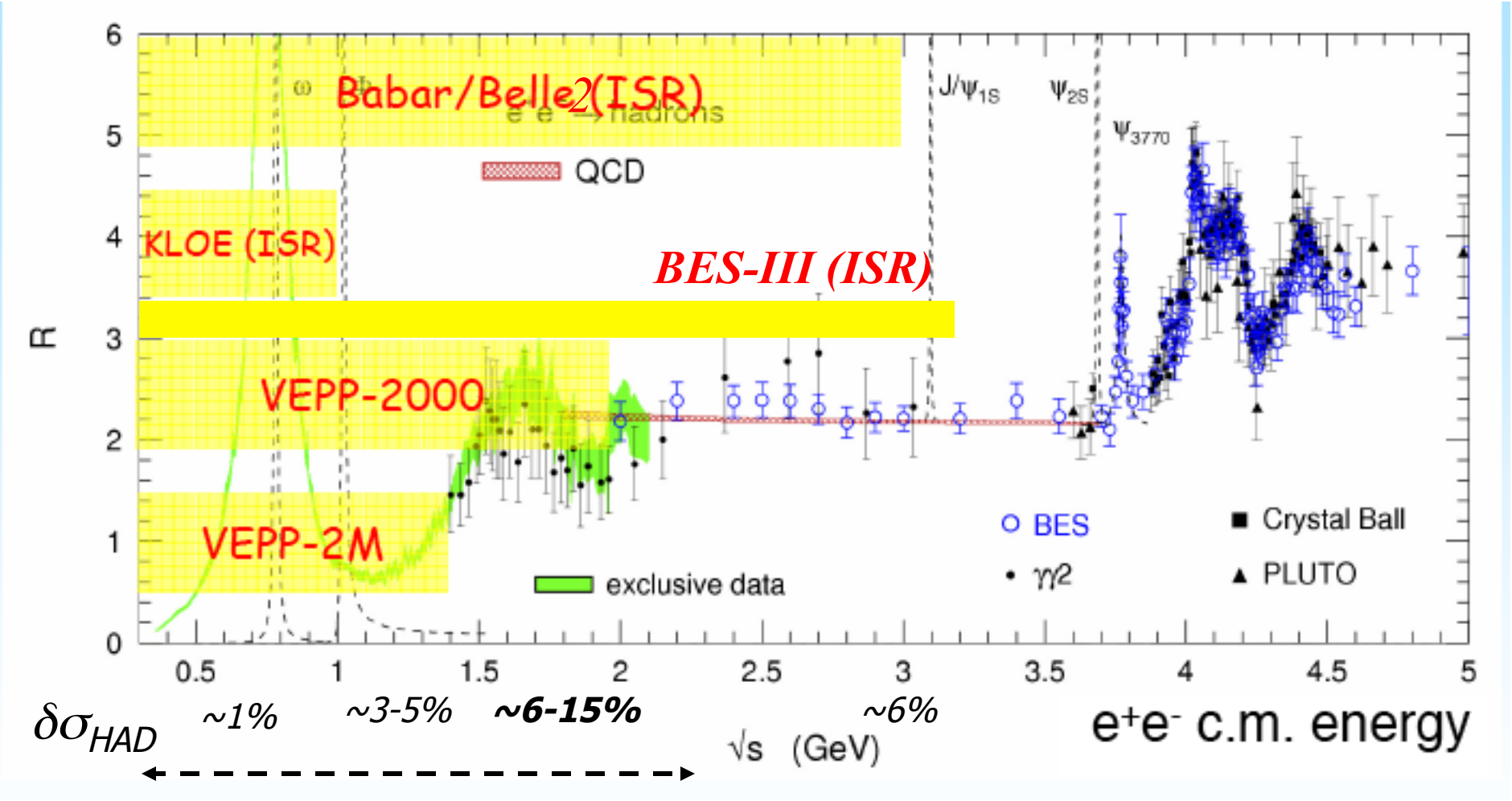

$\delta^2\alpha_\mu^{HAD}$

| Energy range | $a_\mu^{had}[\%](\text{error}) \times 10^{10}$ | rel. err. | abs. err. |
|---|---|---|---|
| $\rho, \omega$ $(E < 2M_K)$ | 543.60 [ 78.8](2.95) | 0.5 % | 44.1 % |
| $2M_K < E < 2$ GeV | 95.05 [ 13.8](3.13) | 3.3 % | 49.8 % |
| 2 GeV $< E < M_{J/\psi}$ | 21.63 [ 3.1](0.93) | 4.3 % | 4.4 % |
| $M_{J/\psi} < E < M_\Upsilon$ | 26.49 [ 3.8](0.60) | 2.3 % | 1.8 % |
| $M_\Upsilon < E < E_{\text{cut}}$ | 0.98 [ 0.1](0.05) | 5.2 % | 0.0 % |
| $E_{\text{cut}} < E$ pQCD | 1.96 [ 0.3](0.00) | 0.0 % | 0.0 % |
| $E < E_{\text{cut}}$ data | 687.74 [ 99.7](4.44) | 0.6 % | 100.0 % |
| total | 689.70 [100.0](4.44) | 0.6 % | 100.0 % |

$$a_\mu^{had} = (689.7 \pm 4.4) \cdot 10^{-10}$$

*Courtesy of F. Jegerlehner*

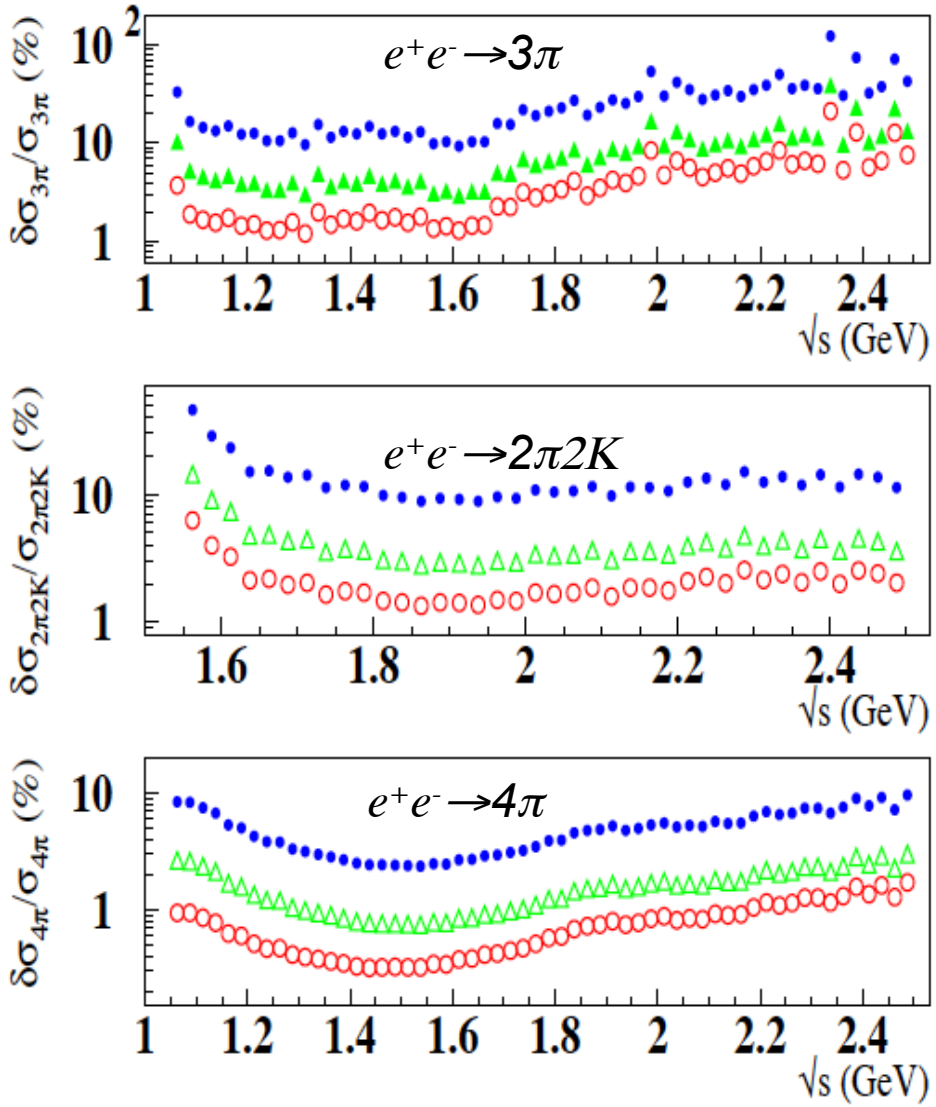
# $e^+e^-$ data into hadrons: current and future activities




*Possible Upgrade in energy of DAFNE*

# *Impact of DAFNE-2 on exclusive channels in the range [1-2.5] GeV with a scan (Statistics only)*

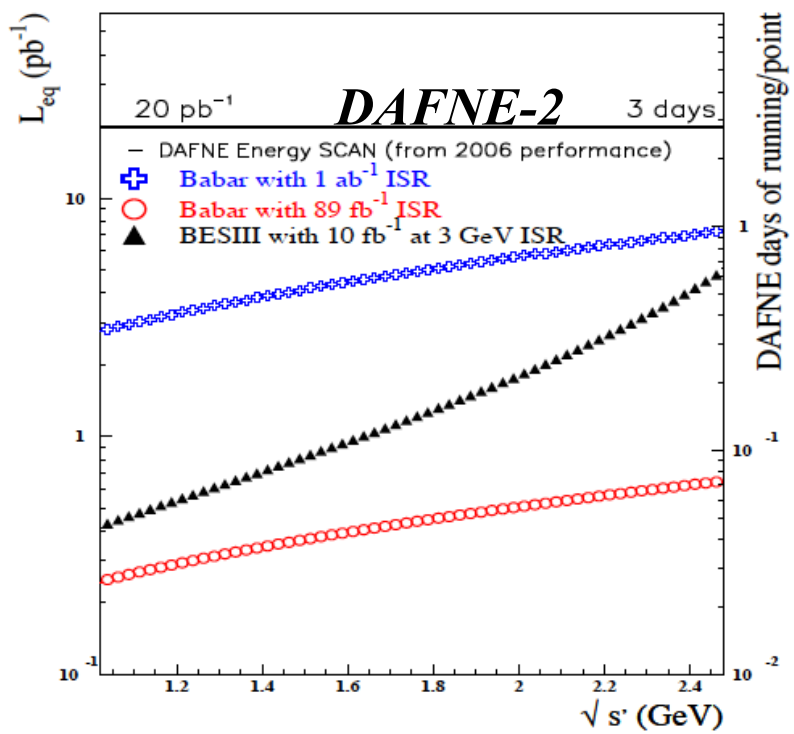
***D. Babsuci et al, arXiv:1007.521***




- ***Published BaBar results:89 fb$^{-1}$(ISR)***

▲ ***"BaBar" × 10 (890 fb$^{-1}$)***

o ***DAFNE-2 energy scan: 20 pb$^{-1}$/point @ L= 10$^{32}$ cm$^{-2}$ s$^{-1}$, 25 MeV bin ⇒ 1 year data-taking***

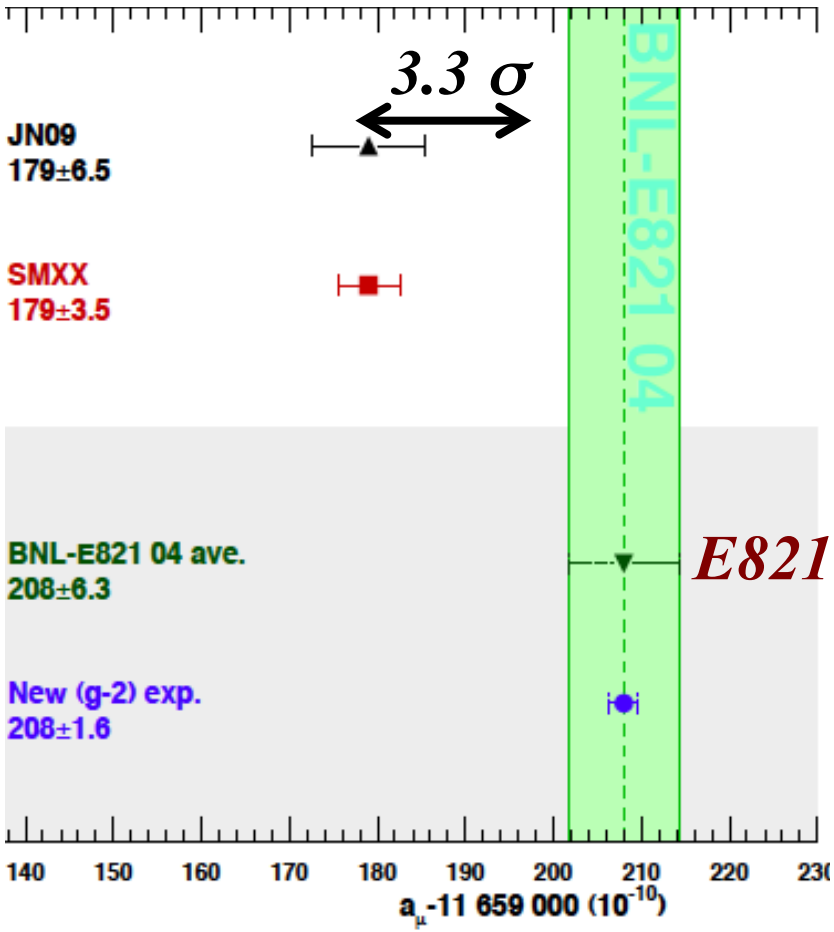



*DAFNE-2 is **statistically** equivalent to 5÷10 ab$^{-1}$ (Super)B-factory*

# A rough estimate for g-2: now

$a_\mu^{exp} - a_\mu^{theo,SM} = (27.7 \pm 8.4)10^{-10}$ $(3.3\sigma)$

$8.4 = \sim 5_{HLO} \oplus \sim 3_{HLbL} \oplus 6_{BNL}$




$\delta a_\mu^{HLO} = 5.3 = 3.3\ (\sqrt{s}<1\text{GeV}) \oplus 3.9\ (1<\sqrt{s}<2\text{GeV}) \oplus 1.2(\sqrt{s}>2\text{GeV})$

# A rough estimate for g-2: …and (possible) future

$a_\mu^{exp} - a_\mu^{theo,SM} = (27.7 \pm 8.4)10^{-10}$ $(3.3\sigma)$

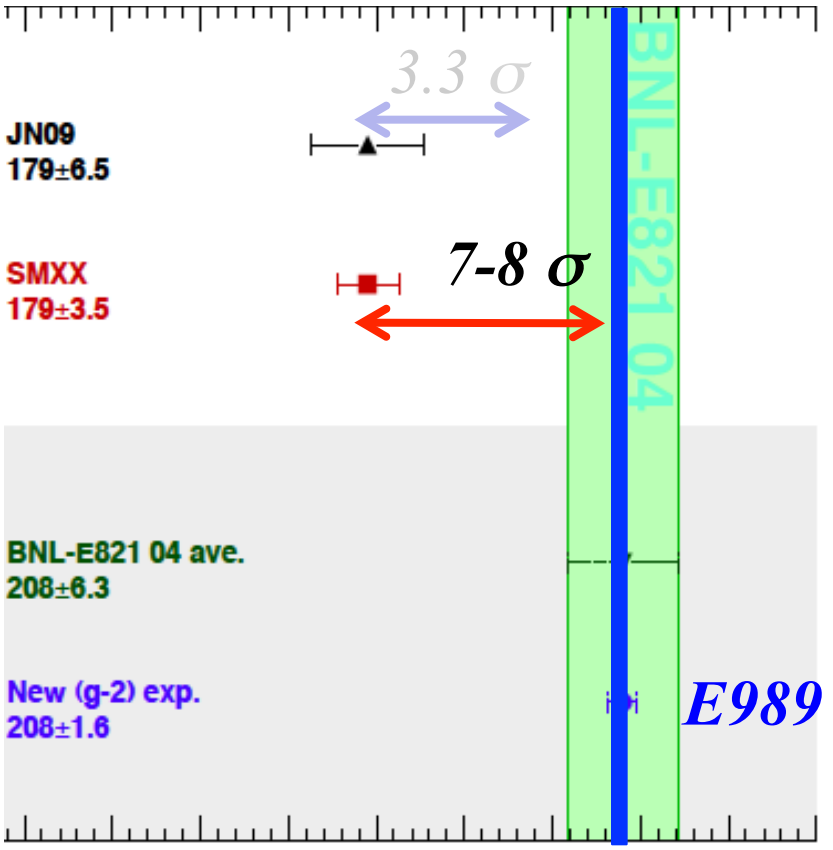
$8.4 = \sim 5_{HLO} \oplus \sim 3_{HLbL} \oplus 6_{BNL}$

4 3 3 1.6 NEW G-2

**7-8σ**

(if 27.7 will remain the same)

$\delta a_\mu^{HLO} \rightarrow 2.6 = 1.9\ (\sqrt{s}<1GeV) \oplus 1.3\ (1<\sqrt{s}<2GeV) \oplus 1.2(\sqrt{s}>2GeV)$




This requires:

$\delta\sigma_{HAD} \sim 0.4\%\ \sqrt{s}<1GeV$ (instead of 0.7% as now) *With ISR at 1 GeV*

$\delta\sigma_{HAD} \sim 2\%\ 1<\sqrt{s}<2GeV$ (instead of 6% as now) *With Energy Scan 1-2 GeV*

# Prospects on $\alpha_{em}(M_Z)$

- Actual status

| **Energy (GeV)** | **< 1** | **1-2** | **2-3** | **3-10** |
|---|---|---|---|---|
| $\delta_{tot}R/R$ | ~0.7% | 6% | 4% | 2% |
| $\delta^2\Delta\alpha^{(5)}_{had}(M_Z^2)$ | ~1% | 34% | 14% | 50% |
| $\delta^2\Delta\alpha^{(5)}_{had}(-2.5GeV)$ | ~7% | 69% | 14% | 10% |

Canonical Integration

Adler

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027671 \pm 0.00016$$

$$\alpha^{-1}(M_Z^2) = 128.961 \pm 0.030$$

Standard: 1.6 x$10^{-4}$

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027498 \pm 0.000135$$

$$\alpha^{-1}(M_Z^2) = 128.962 \pm 0.018$$

Adler: 1.3 x$10^{-4}$

# Prospects on $\alpha_{em}(M_Z)$

- 1% between 1 and 2 GeV

The goal is $\delta\Delta\alpha(5)_{had} = 5\times10^{-5}$

| **Energy (GeV)** | **< 1** | **1-2** | **2-3** | **3-10** |
|---|---|---|---|---|
| $\delta_{tot}R/R$ | ~0.7% | 1% | 4% | 2% |
| $\delta^2\Delta\alpha^{(5)}_{had}(M_Z^2)$ | ~1% | 2% | 21% | 75% |
| $\delta^2\Delta\alpha^{(5)}_{had}(-2.5GeV)$ | ~10% | 12% | 37% | 29% |

← Canonical Integration

← Adler

$\delta\Delta\alpha(5)_{had}$:1.6 $10^{-4}\rightarrow$1.3 $10^{-4}$
(20% reduction)

Standard

$\delta\Delta\alpha(5)_{had}$:1.3 $10^{-4}\rightarrow$0.8 $10^{-4}$
(40% reduction)

Adler

# Prospects on $\alpha_{em}(M_Z)$

- 0.4% below 1 GeV; 1% between 1 and 3 GeV

The goal is $\delta\Delta\alpha(5)_{had} = 5\times10^{-5}$

| **Energy (GeV)** | **< 1** | **1-2** | **2-3** | **3-10** |
|---|---|---|---|---|
| $\delta_{tot}R/R$ | ~0.4% | 1% | 1% | 2% |
| $\delta^2\Delta\alpha^{(5)}_{had}(M_Z^2)$ | ~1% | **3%** | 2% | 90% |
| $\delta^2\Delta\alpha^{(5)}_{had}(-2.5GeV)$ | ~10% | **23%** | 4% | 63% |

← Canonical Integration

← Adler

$\delta\Delta\alpha(5)_{had}$: $1.6\ 10^{-4} \rightarrow 1.1\ 10^{-4}$
(30% reduction)

Standard

$\delta\Delta\alpha(5)_{had}$: $1.3\ 10^{-4} \rightarrow$ **$0.6\ 10^{-4}$** ✔
(55% reduction)

Adler

# Prospects on $\alpha_{em}(M_Z)$

- 0.4% below 1 GeV; 1% between 1 and 10 GeV

The goal is $\delta\Delta\alpha(5)_{had} = 5\times10^{-5}$

| **Energy (GeV)** | **< 1** | **1-2** | **2-3** | **3-13** |
|---|---|---|---|---|
| $\delta_{tot}R/R$ | ~0.4% | 1% | 1% | 1% |
| $\delta^2\Delta\alpha^{(5)}_{had}(M_Z^2)$ | ~2% | **10%** | 5% | 83% |
| $\delta^2\Delta\alpha^{(5)}_{had}(-2.5GeV)$ | ~20% | **50%** | 10% | 25% |

Canonical Integration

Adler

$\delta\Delta\alpha(5)_{had}$:1.6 $10^{-4}\rightarrow$0.6 $10^{-4}$ ✔
(70% reduction)

Standard

$\delta\Delta\alpha(5)_{had}$:1.3 $10^{-4}\rightarrow$0.4 $10^{-4}$ ✔
(70% reduction)

Adler

# Conclusion

- Precision Tests of the Standard Model at future machines (ILC,TLEP, g-2) need precise knowledge of the input parameters ($\alpha_{em}(M_Z)$, $(g\text{-}2)_\mu$)
- To match the required accuracy an experimental effort of measuring the hadronic cross section with 1% accuracy at low energy (<10 GeV) is necessary!
- By reaching 1% accuracy in the region below 2.5 GeV we would match the requirements on $\delta\Delta\alpha^{(5)}_{had} \sim 5 \times 10^{-5}$ (using the Adler function) and in addition bring a possible $(g\text{-}2)_\mu$ discrepancy to 7-8$\sigma$

# SPARES

# Confrontation of measurement and prediction

- Breakdown of individual contributions to errors of $M_W$ and $\sin^2\theta^l_{eff}$
- Parametric uncertainties (not the full fit).

| Parameter | Scenario | $\delta_{meas}$ | $\delta_{pred}$ | $\delta_{exp}$ | $\delta M_Z$ | $\delta m_t$ | $\delta\Delta\alpha_{had}$ | $\delta\alpha_s$ | $\delta_{theo}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | error due to uncertainty ($\pm 1\sigma$) | | | | | |
| $M_W$ [MeV] | Present | 15 | 10.4 | 6.4 | 2.6 | 5.2 | 1.8 | 1.7 | 4.0 |
| | LHC | 8 | 5.8 | 4.8 | 2.6 | 3.6 | 0.9 | 1.7 | 1.0 |
| | ILC | 5 | 3.8 | 2.8 | 2.6 | 0.6 | 0.9 | 0.4 | 1.0 |
| | Future | 1.3 | 2.0 | 1.0 | 0.1 | 0.5 | 0.9 | 0.3 | 1.0 |
| $\sin^2\theta^\ell_{\rm eff}$ [(o)] | Present | 16 | 9.5 | 4.8 | 1.5 | 2.8 | 3.5 | 1.0 | 4.7 |
| | LHC | 16 | 4.1 | 3.1 | 1.5 | 1.9 | 1.6 | 1.0 | 1.0 |
| | ILC | 1.3 | 3.2 | 2.2 | 1.5 | 0.3 | 1.6 | 0.2 | 1.0 |
| | Future | 0.3 | 2.7 | 1.7 | 0.1 | 0.3 | 1.6 | 0.2 | 1.0 |

[(o)] In units of $10^{-5}$.

- $M_W$ and $\sin^2\theta^l_{eff}$ are sensitive probes of new physics! In all scenarios.
- At ILC/GigaZ, precision of $M_Z$ will become important again.
- At TLEP ('Future'), limited by external inputs: theory errors and $\Delta\alpha_{had}$

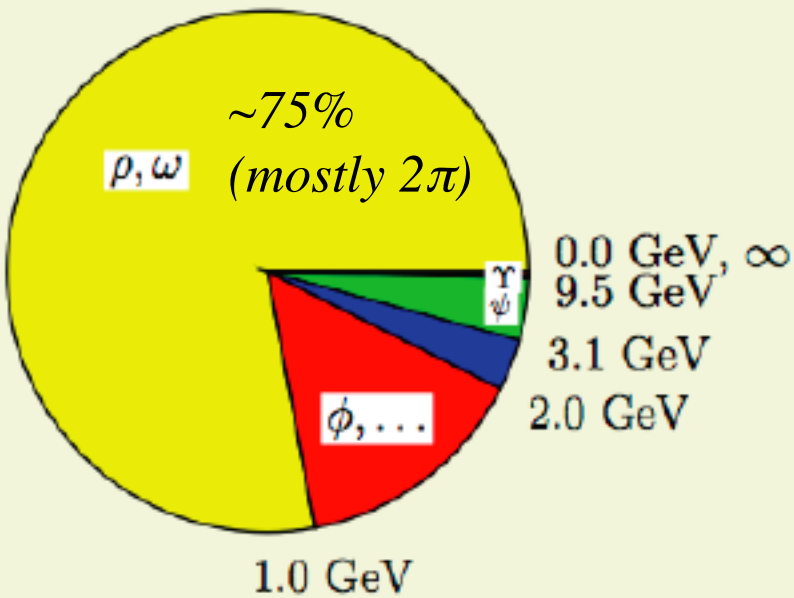
# Dispersion Integral for G-2:

$$a_\mu^{HLO} = \int_{4m_\pi^2}^{\infty} \sigma_{had}(s) K(s) ds$$

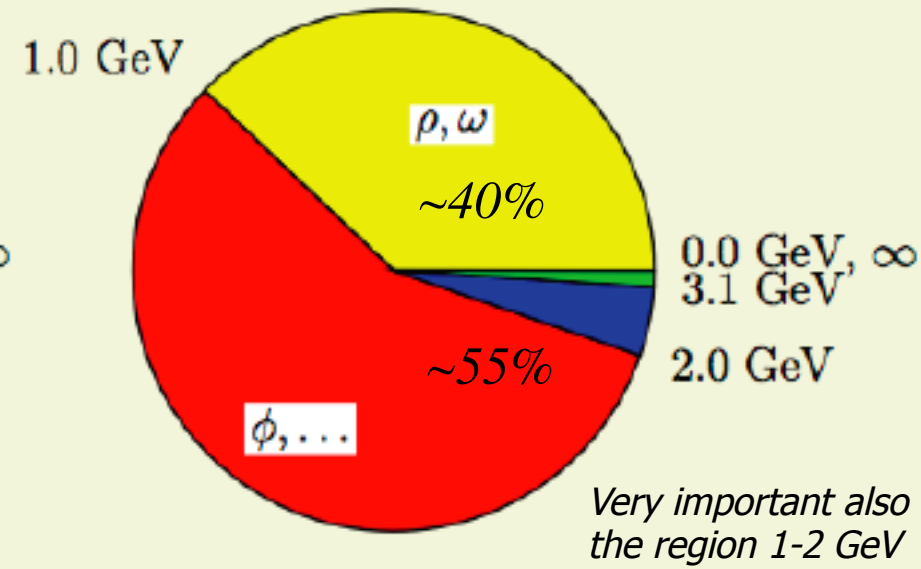
$K(s) \sim 1/s$

*Contribution of different energy regions to the dispersion integral and the error to $a_\mu^{HLO}$*

F. Jegerlehner, Talk at PHIPSI08




*contributions*




*Very important also the region 1-2 GeV*

*error²*

Experimental errors on $\sigma^{had}$ translate into theoretical uncertainty of $a_\mu^{had}$!

→ Needs precision measurements!

$\delta a_\mu^{exp} \rightarrow 1.5\ 10^{-10} = 0.2\%$ *on* $a_\mu^{HLO}$

***NEW G-2 at FNAL***

# Impact of DAFNE-2 on $(g-2)_\mu$

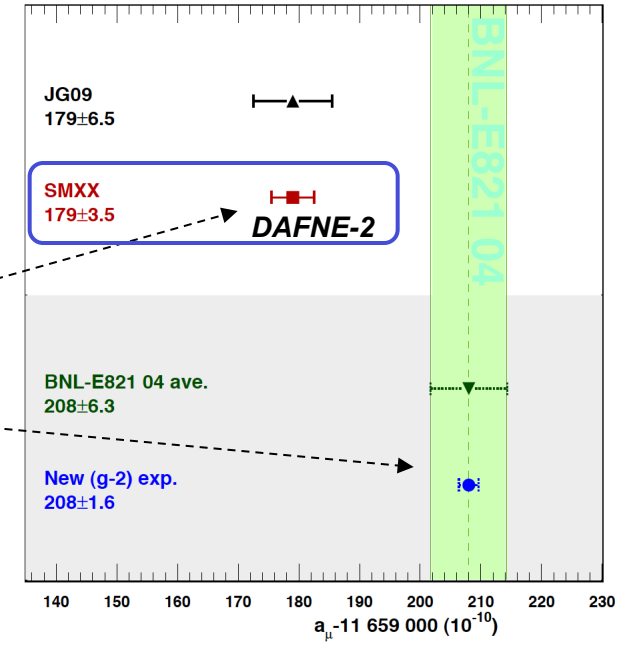
$a_\mu^{exp} - a_\mu^{theo,SM} = (27.7 \pm 8.4)10^{-10}$ $(3.3\sigma)$ [Eidelman, TAU08]

$8.4 = \sim 5_{HLO} \oplus \sim 3_{HLbL} \oplus 6_{BNL}$

**4** **$2.6_{DAFNE-2}$** $2.5$ $1.6_{NEW\ G-2}$

**$7\text{-}8\sigma$**

(if 27.7 will remain the same)




$\delta a_\mu^{HLO} = 5.3 = 3.3(\sqrt{s}<1GeV) \oplus 3.9(1< \sqrt{s}<2GeV) \oplus 1.2(\sqrt{s}>2GeV)$

$\delta a_\mu^{HLO} \rightarrow$ **2.6**$=1.9\ (\sqrt{s}<1GeV) \oplus$ **1.3 ($\sqrt{s}$<1GeV)** $\oplus 1.2(\sqrt{s}>2GeV)$

This means:

**$\delta\sigma_{HAD} \sim 0.4\%$ $\sqrt{s}$<1GeV (instead of 0.7% as now)** *With ISR at 1 GeV*

**$\delta\sigma_{HAD} \sim 2\%$ 1<$\sqrt{s}$<2GeV (instead of 6% as now)** *With Energy Scan 1-2 GeV*

*Possible at DAFNE-2!*

***Precise measurement of $\sigma_{HAD}$ at low energies very important also for $\alpha_{em}(M_Z)$ (necessary for ILC) !!!***

- $\delta\sigma_{HAD} \sim 0.4\%$ $\sqrt{s}<1GeV$ *(instead of 0.7% as now)*

- $\delta\sigma_{HAD} \sim 2\%$ $1<\sqrt{s}<2GeV$ *(instead of 6% as now)*

***(Possible with direct scan or ISR at Flavour factories)***

$\delta a_{\mu}^{HLO} = 2.6\ 10^{-10}$ *(instead of ~5 as now)*

*A similar improvement on* $\delta\alpha_{em}(Mz)$ *using Adler function method*