Gravity and Thermodynamics
I. Fundamental principles

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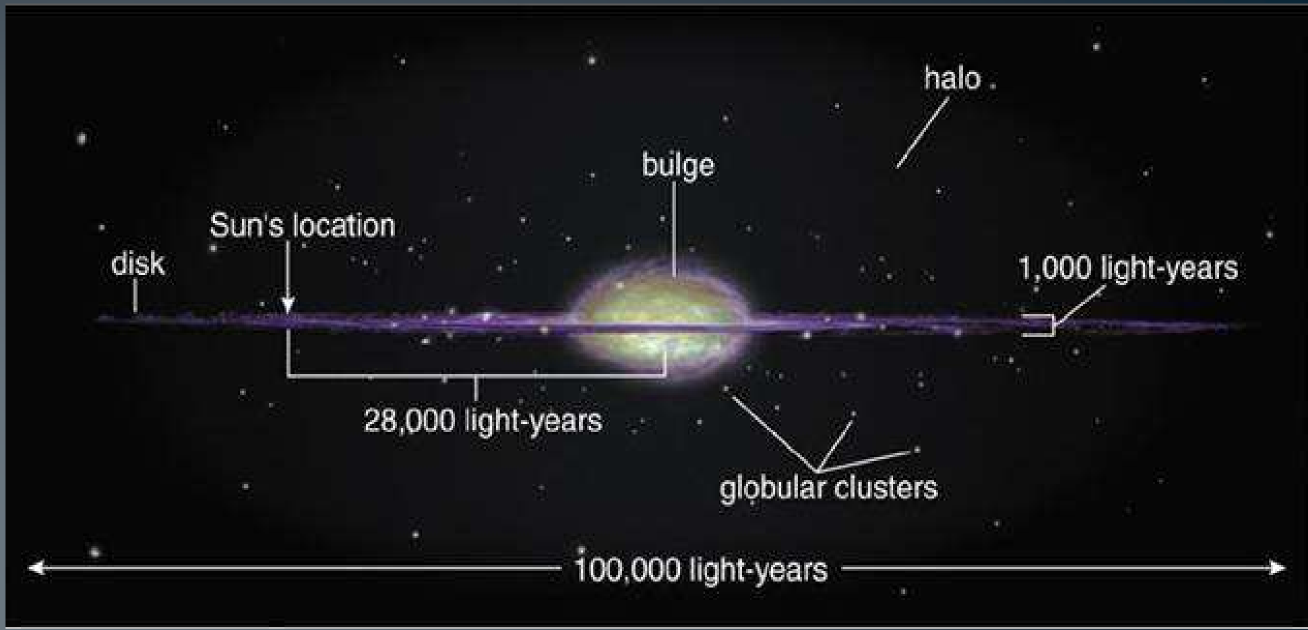
Globular Clusters



Absolute age T: $10 \div 13\ Gyr$

Stars number N: $10^5 \div 10^6$

Mass M: $10^4 \div 10^6\ \mathrm{M}_\odot$

Eccentricity e: < 0.2 (from latin '*globulus*'=*little sphere*)

Central density ρ_c: $10^4\ \mathrm{M}_\odot/pc^3$

Core radius r_c: $0.3 \div 10\ pc$ (radial coordinate where the brightness becomes one half of the central value)

Tidal radius r_t: $\sim 50\ pc$ (extension of the globular cluster)

Concentration r_t/r_c: $10 \div 100$

Radial velocity v_r: $3\ km/s$ (outer) $\div$ $10\ km/s$ (inner)

From empirical formula to…

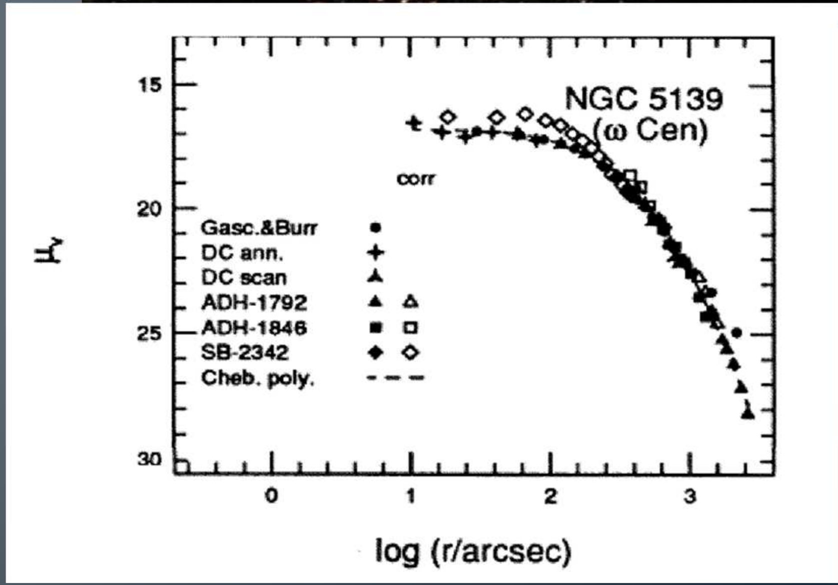
King empirical brightness formula

$$f = \frac{f_0}{1 + (r/r_c)^2}$$

f = surface brightness
f_0 = central surface brightness
r_c = core radius

The empirical law is the same for all the globular cluster, differing only for stars concentration for different GCs

King, AJ, vol 67, p.471, 1962



Trager-King-Djorgoski, AJ, vol 109, p.218, 1995

...King distribution function

THE velocity distribution in a star cluster is determined by two competing processes: relaxation through stellar encounters drives the distribution toward a Maxwellian form, but stars beyond the finite escape velocity continually disappear from the cluster. Since escaping stars traverse the cluster in a small fraction of a relaxation time, the resultant velocity distribution should be zero beyond the escape velocity but otherwise nearly Maxwellian. Such a distribution can be found by seeking a steady-state solution of the Fokker–Planck equation, subject to the cutoff condition.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \vec{\nabla}\Phi \cdot \frac{\partial f}{\partial \vec{v}} = \Gamma(f)$$

$$\Gamma(f) = -\frac{\partial}{\partial v_i}\left[f \cdot \langle \Delta v_i \rangle\right] + \frac{1}{2}\frac{\partial^2}{\partial v_i \partial v_j}\left[f \cdot \langle \Delta v_i \Delta v_j \rangle\right]$$

King, AJ, vol 70, p.376, 1965

$$f(v,t) = e^{-\lambda t} g(v)$$

λ= evaporation rate

$$g(\varepsilon) = \begin{cases} A\left[e^{-\frac{\varepsilon}{kT}} - e^{-\frac{\psi}{kT}}\right] & \varepsilon \le \psi \\ 0 & \varepsilon > \psi \end{cases}$$

ψ= cutoff energy (depends on r)

If we assume the existence of a unique sequence of cluster models, then the evolution of an individual cluster can be followed as it progresses down the sequence. The loss of stars causes (a) a decrease in the mass M, (b) a decrease in r_t, which is proportional to $M^{\frac{1}{3}}$, and (c) a calculable change in the total energy of the cluster. The problem is then simply to find the model that fits the changed values of M, r_t, and the total energy H.

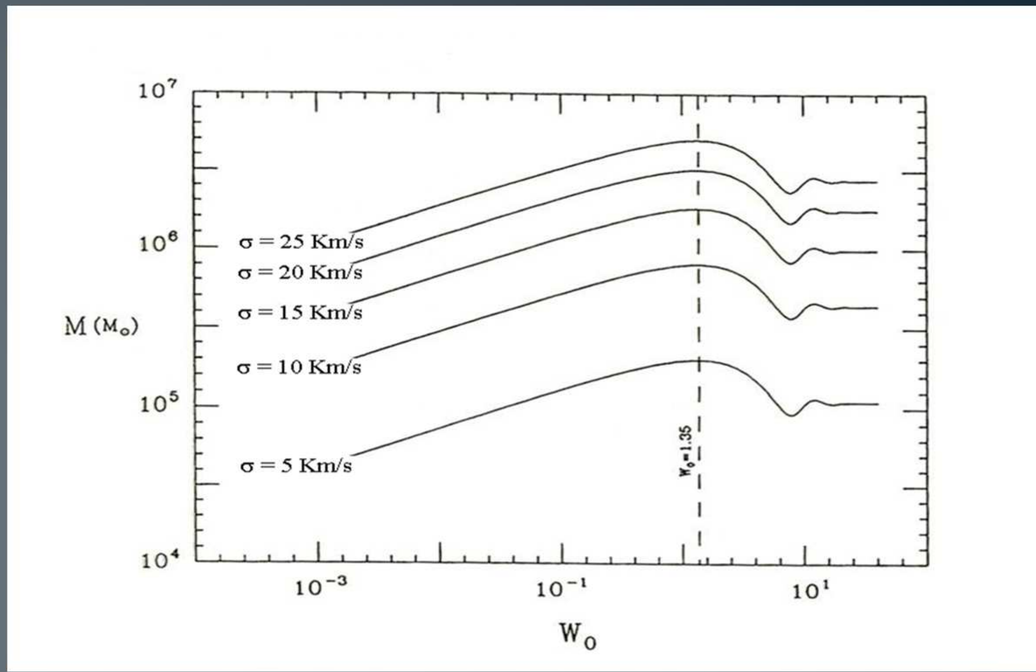
Equilibrium and dynamics of GCs

Equilibrium from Poisson equation

$$\frac{d^2W}{dR^2} + \frac{2}{R}\frac{dW}{dR} = -9\frac{\rho}{\rho_0}$$

$$\frac{dM}{dR} = 4\pi R^2 \rho$$

$W = m[\phi_R - \phi(r)]/kT = \psi/kT =$ adimensional potential
$R = r/r_c =$ adimensional radius
$r_c = (9\sigma^2/4\pi G\rho_0) =$ core radius
$\rho_0 =$ central density
$\sigma^2 = kT/m =$ dispersion of velocity



Merafina-Ruffini, A&A, vol 221, p.4, 1989

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Dynamics leads the system to change its parameters during the evolution

Nearly thermodynamic equilibrium

The verification of the fundamental question as to whether clusters of particles interacting only gravitationally evolve toward thermodynamic equilibrium and at what rate, is quite outside the scope of the present work. Computer experiments (Gott 1974; see also recent reviews in Aarseth and Lecar 1975; Spitzer 1975; Wielen 1974), theoretical calculations (Lynden-Bell 1967*b*), and direct observation of globular clusters and nuclei of galaxies (Ogorodnikov 1965) indicate that there is a substantial achievement of nearly thermodynamic equilibrium states for all or parts of such systems. In the present work, we present results for a model and we assume the connection

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to a maximum of the entropy of the model, regarded as a closed system. Its evolution is simulated by a progression of thermodynamic equilibrium configurations with different equilibrium parameters. There is thus no *a priori* reason that the evolution should proceed through configurations corresponding to increasing entropy of the model

Katz, ApJ, vol 211, p.226, 1977

Thermodynamics is important since $\tau_{rel} < \tau_{age}$

Thermodynamical transformations are due to the evaporations of stars

During GC evolution, the King distribution function mantains unchanged its functional form: the evolution can be described by a sequence of King models with increasing values of W_0, until $W_0 \approx 9$ (King 1966, Cohn 1980)

Over this value the system undergoes gravothermal catastrophe (Lynden-Bell & Wood 1968). So it are not able to mantain quasi-thermodynamical equilibrium and the distribution function changes

A new point of view

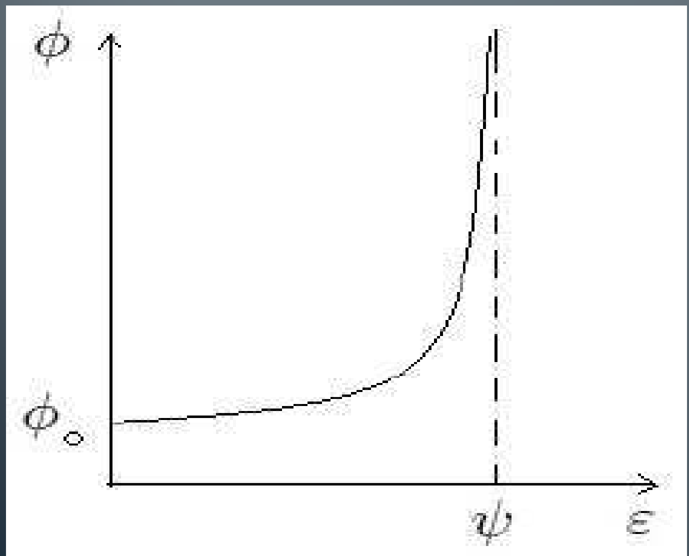
It is possible to include the competing effects of stellar encounters and evaporation of stars in a Boltzmann-like distribution function thanks to an effective potential ϕ, which reduces the phase space accessible to particles

$$f(\varepsilon) = Be^{-H/k\theta}$$

The Hamiltonian is made up of three terms

$$H = \varepsilon + m\varphi + \phi$$

- kinetic energy
- gravitational field
- effective potential



Characteristics of ϕ

- reduces the phase space
- depends on the energy of the particles
- depends on the radial coordinate
- goes to infinity in ψ

Splitting of thermodynamical variables

King effective potential

$$\phi = -k\theta \ln\left(1 - e^{(\varepsilon-\psi)/k\theta}\right)$$

1

The King distribution can be rewritten as a Boltzmann-like distribution function

Thermodinamical variables

2

The reduction of the phase space leads to change the meaning of some parameters and variables

Kinetic variables

The introduction of an effective potential naturally leads to the splitting of the variables into two different classes

Thermodynamical and kinetic variables

Intensive kinetic and thermodynamical variables

-Temperature

$$kT = \left\langle q_i \frac{\partial \varepsilon}{\partial q_i} \right\rangle$$

$$k\theta = \left\langle q_i \frac{\partial H}{\partial q_i} \right\rangle$$

-Pressure

$$P = \frac{1}{3} A \int_0^{\psi} f q \frac{d\varepsilon}{dq} d^3 q$$

$$\Pi = \frac{1}{3} A \int_0^{\psi} f q \frac{dH}{dq} d^3 q$$

-Chemical potential

$$\mu = \left. \frac{\partial U_k}{\partial N} \right|_{S,V}$$

$$\alpha = \left. \frac{\partial U}{\partial N} \right|_{S,V}$$

$\mu = \mu_0 + m\varphi$;
$\alpha = \alpha_0 + m\varphi$
(Landau-Lifsits-Pitaevskij 1976)

Extensive variables

-Number of particles

$$N = AV \int_0^{\psi} f \varepsilon^{1/2} d\varepsilon$$

-Entropy

$$S = AVk \int_0^{\psi} f[1 - \ln(f)] \varepsilon^{1/2} d\varepsilon$$

Energy

-Kinetic energy

$$U_k = AV \int_0^{\psi} f\ \varepsilon\ \varepsilon^{1/2} d\varepsilon$$

-Thermodynamical energy

$$U = AV \int_0^{\psi} f\ H\ \varepsilon^{1/2} d\varepsilon$$

Theory Symmetries

Kinetic and hermodynamical variables are linked through

$$R = 1 - \frac{1}{3k\theta}\left\langle q\frac{\partial\phi}{\partial q}\right\rangle$$

$E = \varepsilon + m\varphi$

$$T = R\theta$$
$$P = R\Pi$$
$$\mu - E = R(\alpha - H)$$

Intensive Variable	Extensive Conjugate
$T = R\theta$	S
$P = R\Pi$	V
$\langle\mu_0 - \varepsilon\rangle = R\langle\alpha_0 - H_0\rangle$	N

$H = H_0 + m\varphi$
$H_0 = \varepsilon + \phi$

Equation of state

$\Pi V = Nk\theta$
$PV = NkT$

First principle

$dU = \theta dS - \Pi dV + \alpha dN + N\langle d\phi\rangle$
$dU_k = TdS - PdV + \langle\mu_0\rangle dN + N(d\langle\mu_0\rangle - \langle d\mu_0\rangle)$

Gibbs-Duhen relation

$N\langle d\phi\rangle = Sd\theta - Vd\Pi + Nd\alpha$
$N(d\langle\mu_0\rangle - \langle d\mu_0\rangle) = SdT - VdP + Nd\langle\mu_0\rangle$

Euler equation

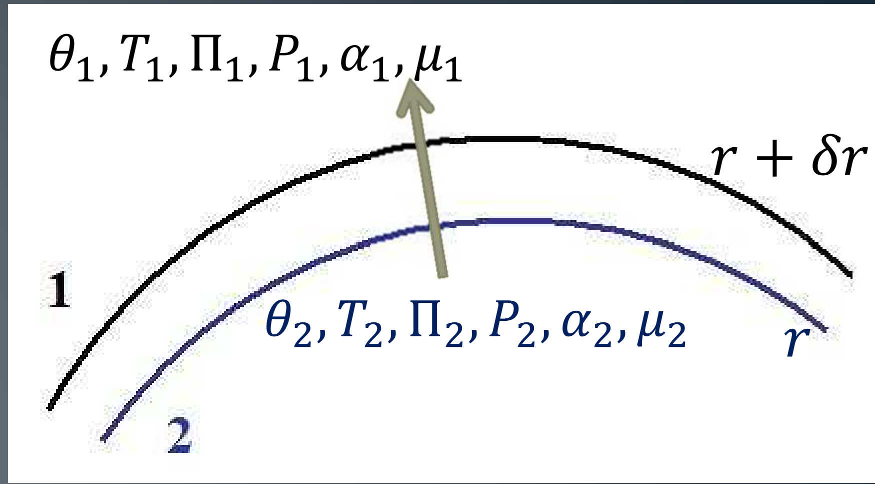
$U = \theta S - \Pi V + \alpha N$
$U_k = TS - PV + \langle\mu_0\rangle N$

Thermodynamical equilibria

Two types of variations

-d along a thermodynamical transformation

-δ along radial coordinate



Thermal equilibrium

$$dS_{tot} = 0 \; ; N, V = const \;\Rightarrow\; \delta\theta = 0 \;\Rightarrow\; \mathrm{k}\delta T = R\langle\delta\phi\rangle - (1-R)m\delta\varphi$$

Mechanical equilibrium

$$dS_{tot} = 0 \; ; \mathrm{S}, N = const \;\Rightarrow\; \delta\Pi + \frac{N}{V}[\langle\delta\phi\rangle + m\delta\varphi] = 0 \;\Rightarrow\; \delta P = -\rho\delta\varphi$$

Chemical equilibrium

$$dS_{tot} = 0 \; ; \mathrm{V}, S = const \;\Rightarrow\; \delta\alpha = 0 \;\Rightarrow\; \mathrm{N}\delta\mu = -S\delta T$$

Hydrostatic equilibrium

Specific heat ($m\varphi = 0$)

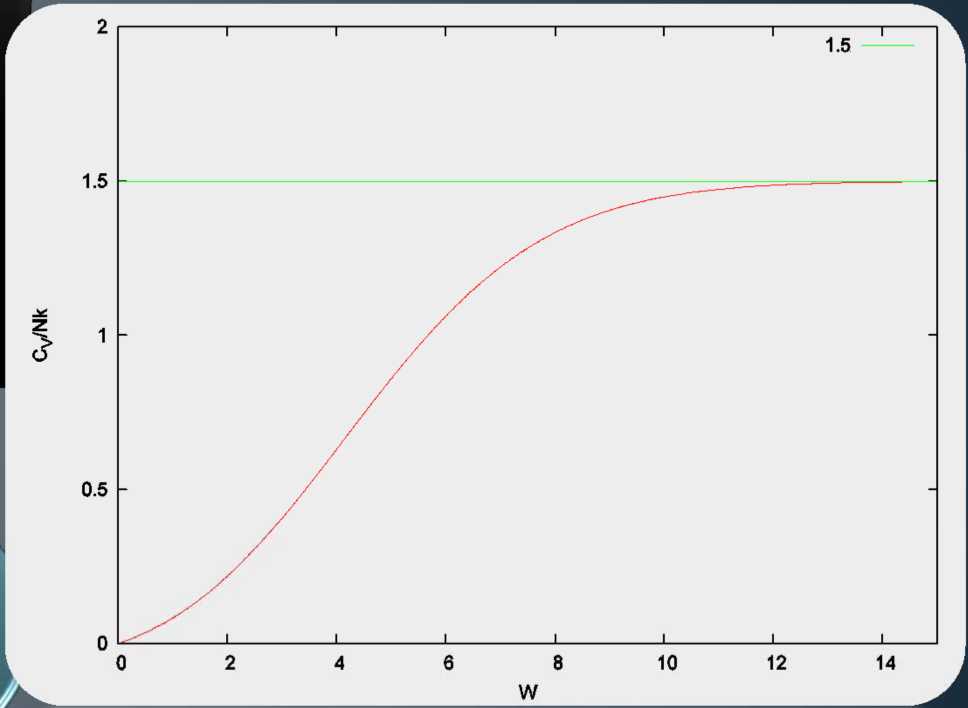
From first principle ($N = const$)

$$C_V = \theta \frac{dS}{d\theta}\bigg|_V = \frac{dU}{d\theta} - N\left\langle\frac{d\phi}{d\theta}\right\rangle$$

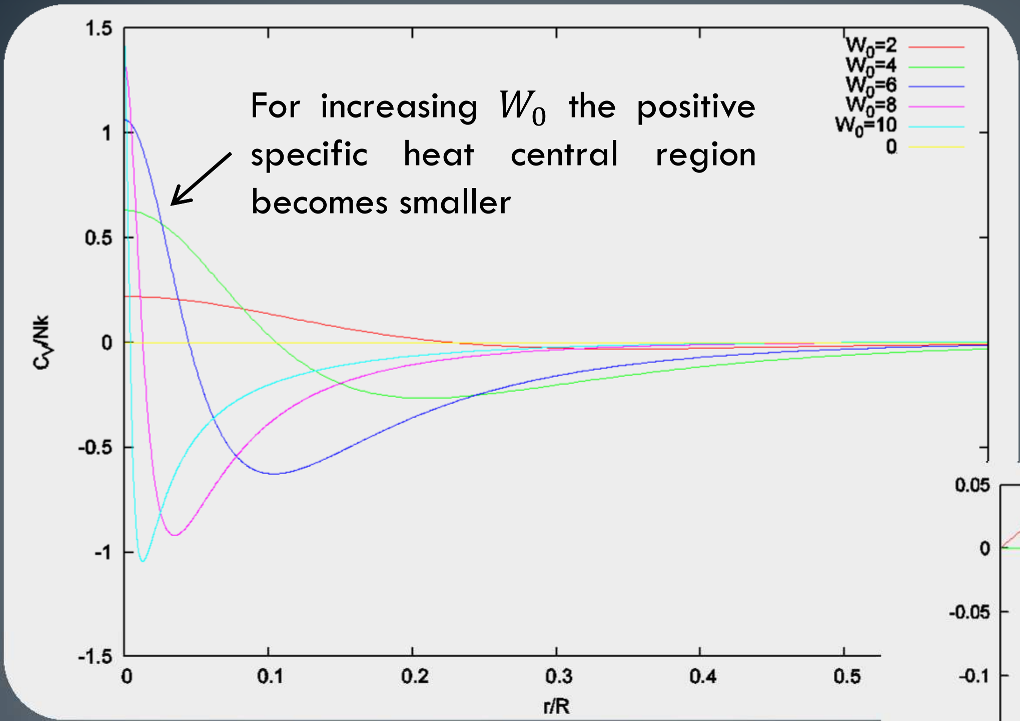
Mayer relation

$$C_P = C_V + Nk$$

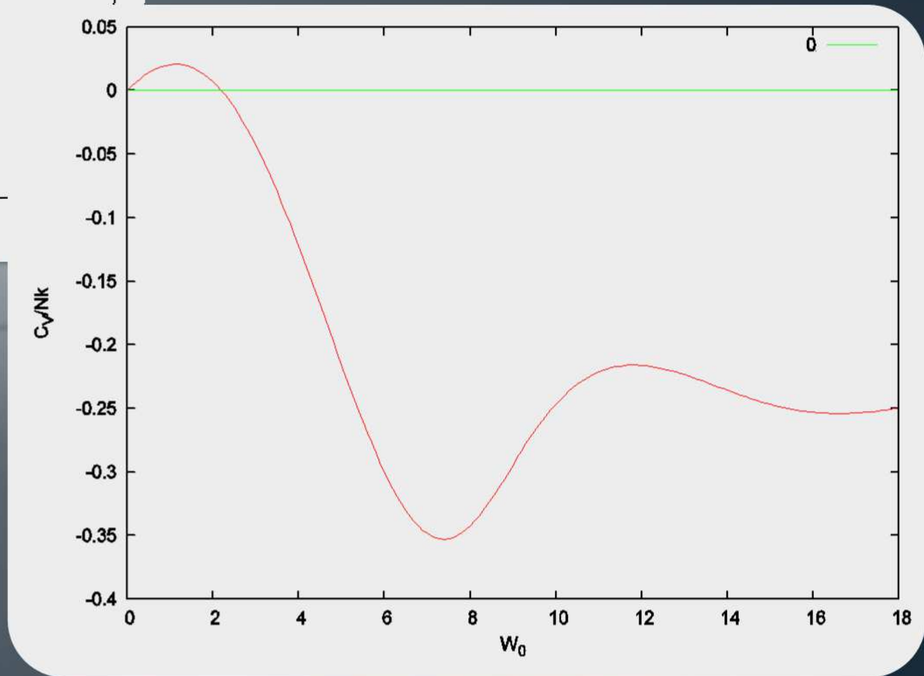
When $W \to \infty$ the King DF tends to the Boltzmann DF and $C_V \to 3/2\, Nk$



Specific heat and gravity

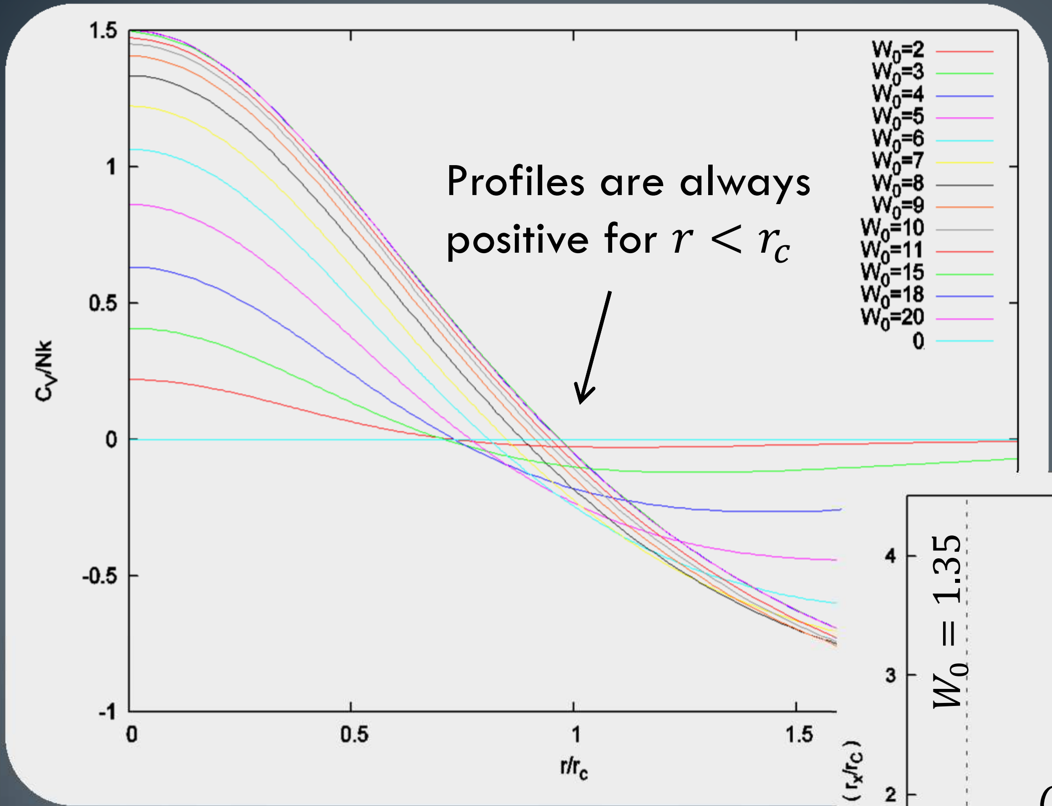


The total specific heat is negative only for systems with $W_0 > 2.3$ unlikely the case of Boltzmann DF

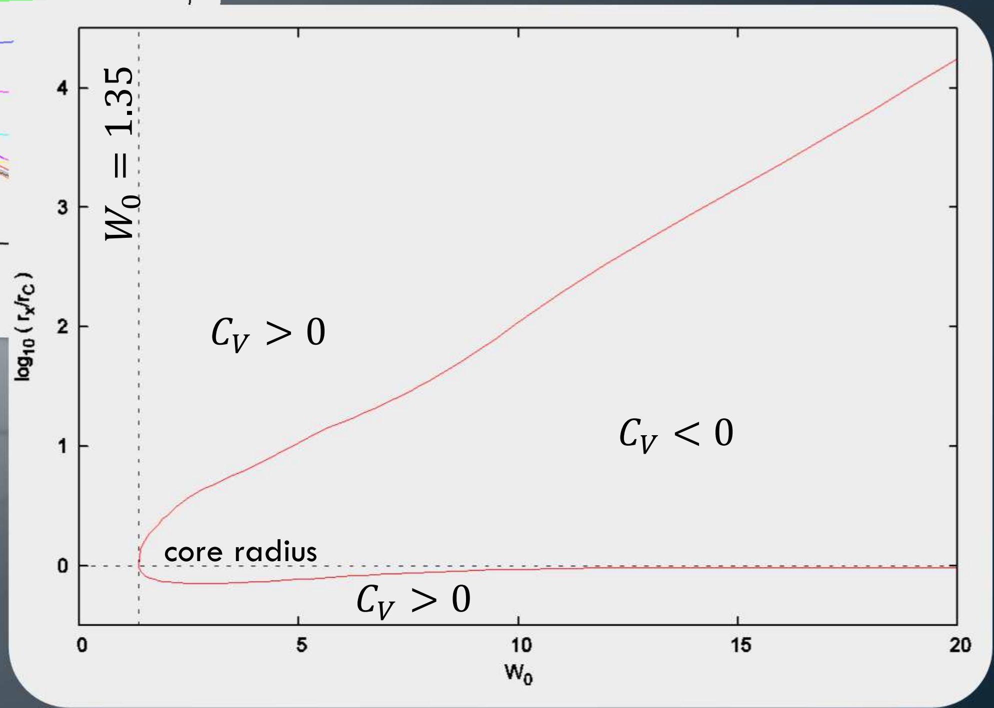


The core is a positive specific heat region with a subsequent negative specific heat region

Specific heat and GC core

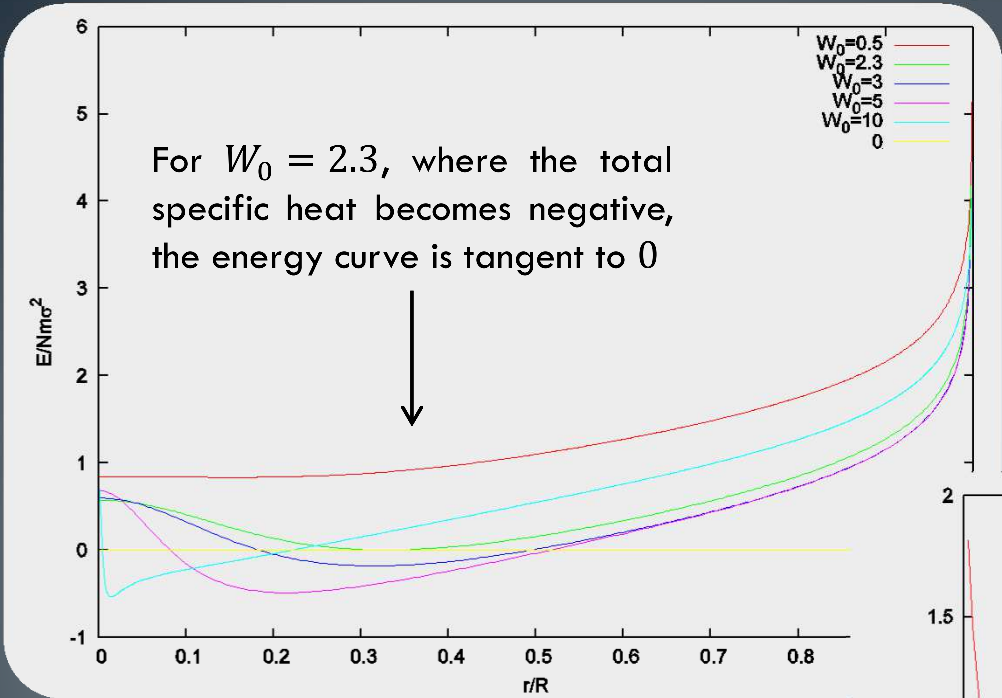


The $(r/r_c, W_0)$ plane can be subdivided in regions of positive and negative specific heat



The core is a positive specific heat region with a subsequent negative specific heat region

Energy

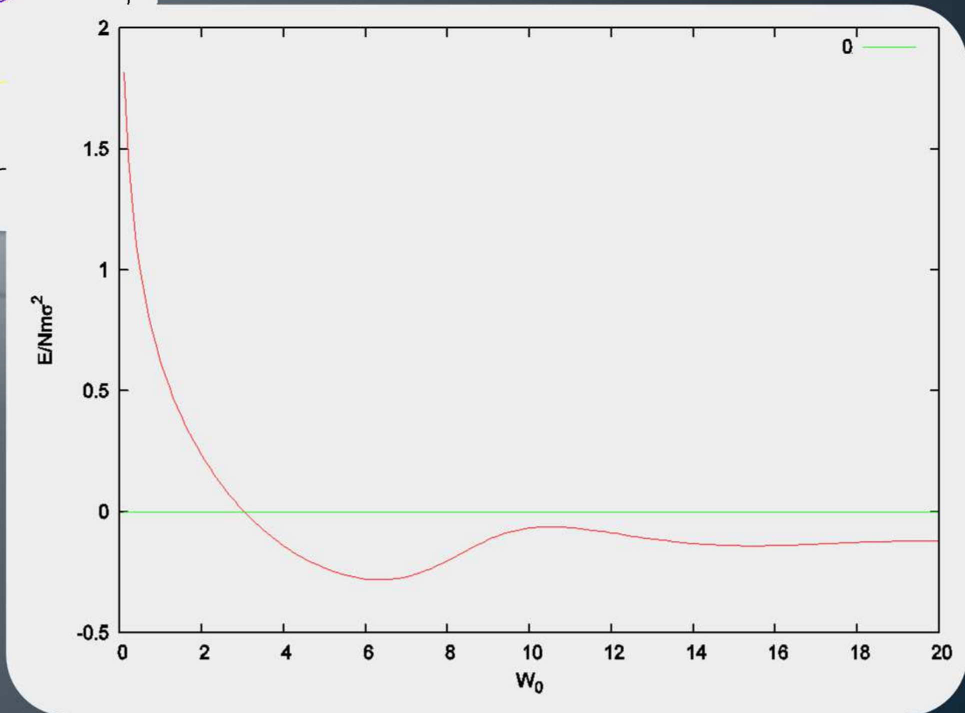


The total energy is negative only for systems with $W_0 > 3$ unlikely the case of Boltzmann DF

The core eneregy is always positive. For subsequent regions:

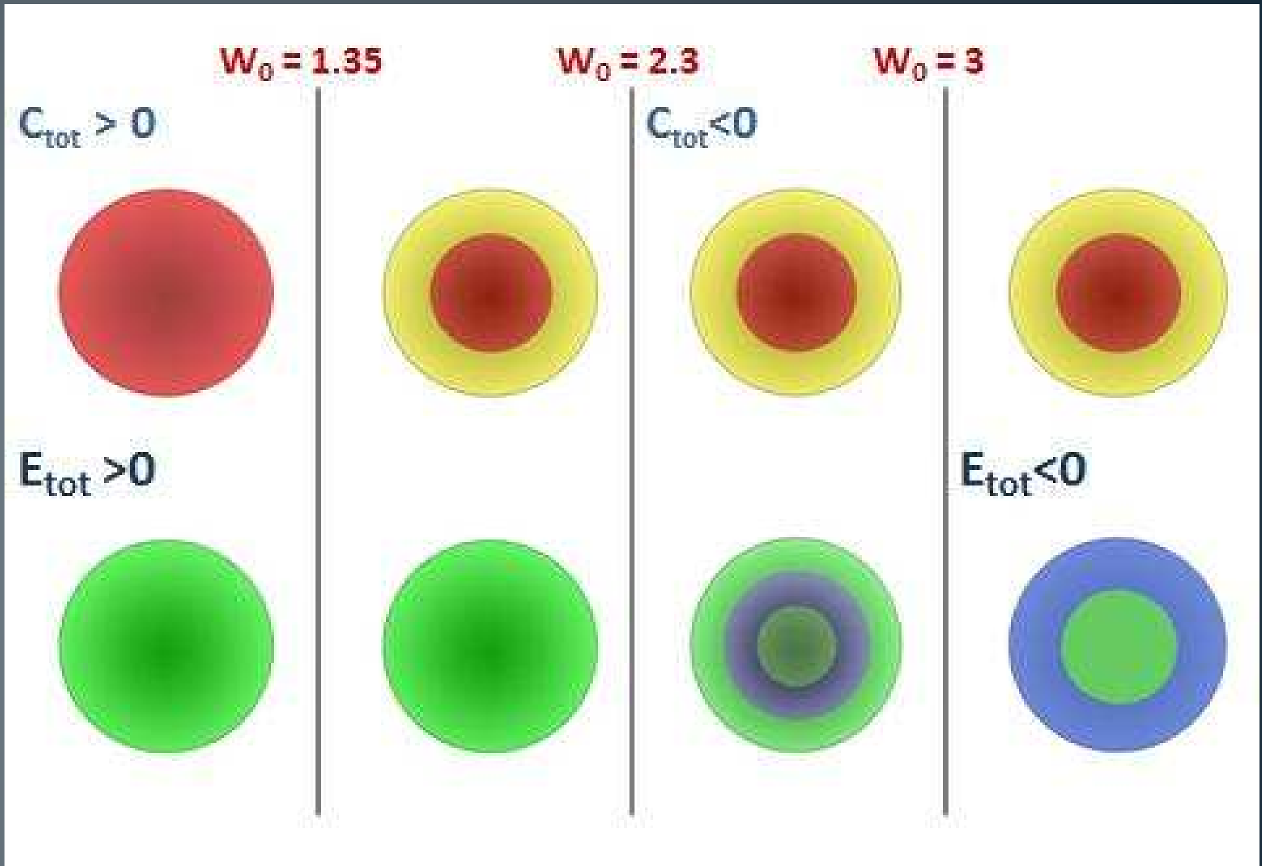
- $2.3 < W_0 < 3$ negative and positive energy regions

-$W_0 > 3$ negative energy region



Configurations and evolution

- $W_0 > 1.35$ regions with negative specific heat
- $W_0 > 2.3$ intermediate regions with negative energy
- $W_0 > 3$ negative total energy



$W_0 < 1.35$ GCs don't evolve towards gravothermal catastrophe

$W_0 < 3$ GCs evolve towards disruption

Conclusions

The model predicts a positive specific heat core with subsequent negative specific heat regions: the model is **self-consistent** since these regions can exchange energy and produce gravothermal instability without the presence of an external bath (Lynden-Bell & Wood model, 1968).

The positive specific heat core is able to justify the possibility of a survival of the system from gravothermal cathastrophe (**post core collapsed objects**)

Problems and perspectives

-The model is not multimass and does not take into account the effects of binary stars formation.

-The new possibility of measuring transverse velocities of GCs stars could lead to the knowledge of the distribution of star orbits and eccentricity, and so to better develop N-body simulations for supporting the validity of the model.

-Construction of thermodynamic ensembles (microcanonical, canonical and grand canonical) in order to develop an evolutive theory by considering the evaporation of stars.

Thanks for the attention

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