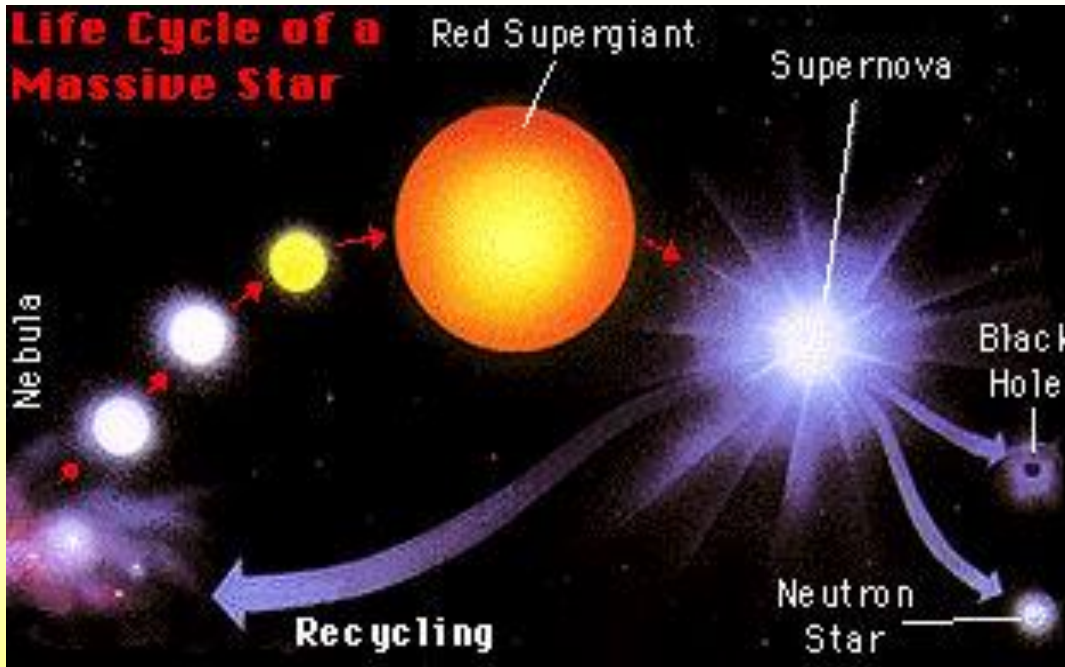


**Investigating strangeness:
from accelerators to compact stellar objects
LNF – INFN Frascati (Roma), 14 Maggio 2014**

Strangeness in compact stars

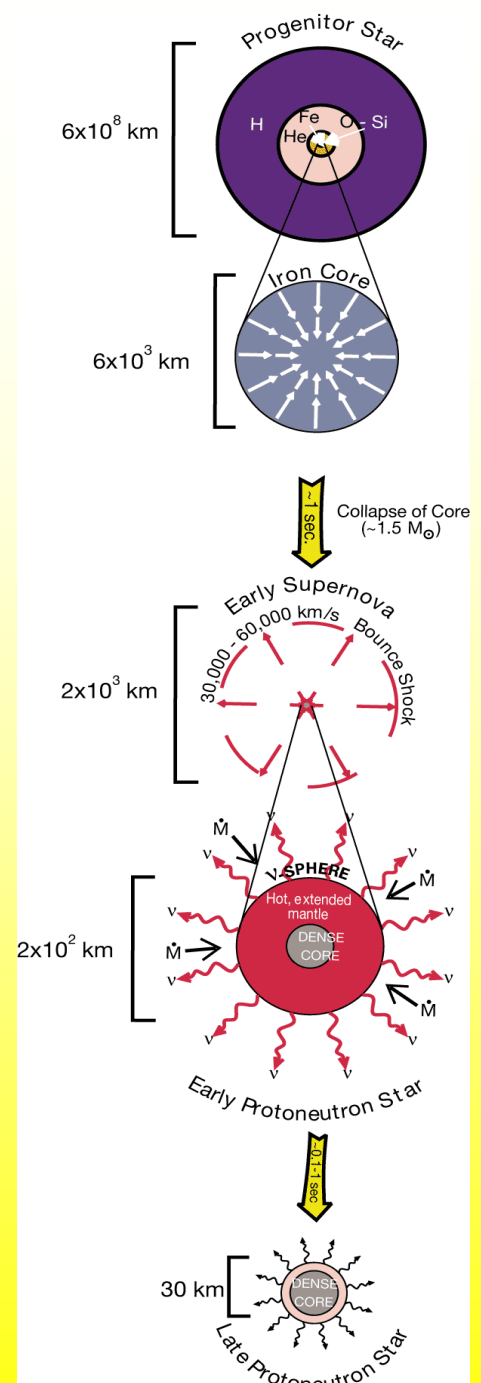
**Ignazio Bombaci
Dipartimento di Fisica “E. Fermi”, Università di Pisa
INFN Sezione di Pisa**

The birth of a Neutron Star



Neutron stars are the **compact remnants** of type II **Supernova explosions**, which occur at the end of the evolution of massive stars ($8 < M/M_{\odot} < 25$).

Composition and Structure of Protoneutron Stars,
 M. Prakash, I. Bombaci, M. Prakash, P.J. Ellis, J.M. Lattimer, R. Knorren
 Physics Reports 280 (1997) 1.



Neutron Stars: bulk properties

Mass	$M \sim 1.5 M_{\odot}$
Radius	$R \sim 10 \text{ km}$
Centr. Density	$\rho_c = (4 \text{ -- } 10) \rho_0$
Compactness	$R/R_g \sim 2 - 4$
Baryon number	$A \sim 10^{57}$
Binding energy	$B \sim 10^{53} \text{ erg}$
$B/A \sim 100 \text{ MeV}$	$B/(Mc^2) \sim 10\%$

Stellar structure:
General Relativity

Giant “atomic nucleus”
bound by **gravity**

$$M_{\odot} = 1.989 \times 10^{33} \text{ g} \quad R_{\odot} = 6.96 \times 10^5 \text{ km}$$
$$\rho_0 = 2.8 \times 10^{14} \text{ g/cm}^3 \quad (\text{nuclear saturation density})$$

$$R_g_{\odot} = 2.95 \text{ km}$$

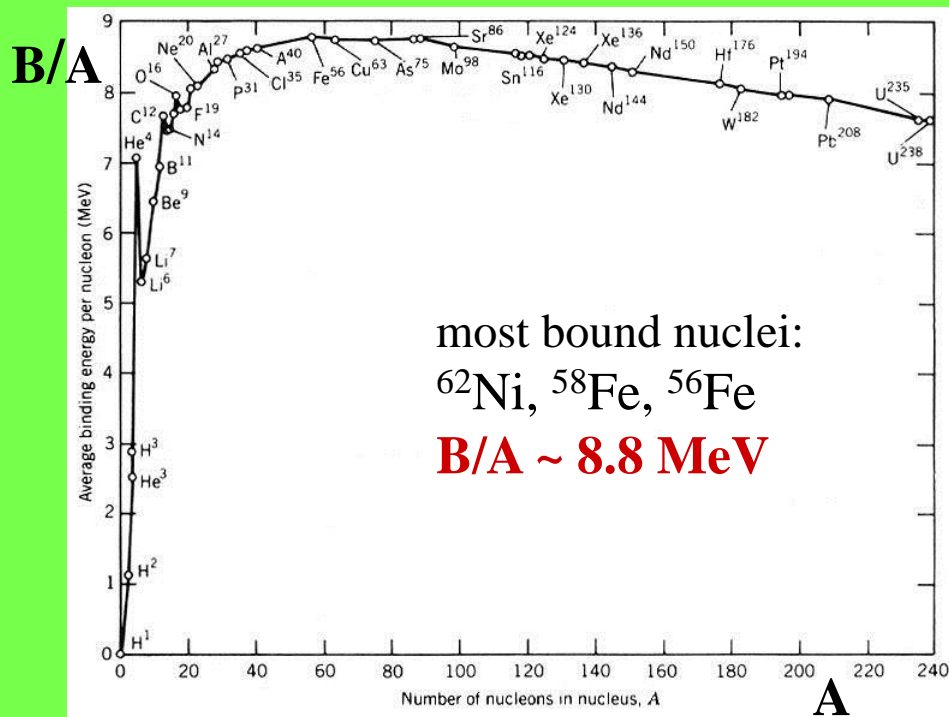
$$R_g \equiv 2GM/c^2 \quad (\text{Schwarzschild radius})$$

Atomic Nuclei: bulk properties

Mass number $A = 1 - 238$ (natural stable isotopes)

Radius $R = r_0 A^{1/3} \sim (2 - 10) \text{ fm}$

Density $\rho \sim \rho_0 = 2.8 \times 10^{14} \text{ g/cm}^3$



$B/(Mc^2)$
 $\sim (0.1-1)\%$

bound by
nuclear
interactions

The Neutron Star idea (Baade and Zwicky, 1934)

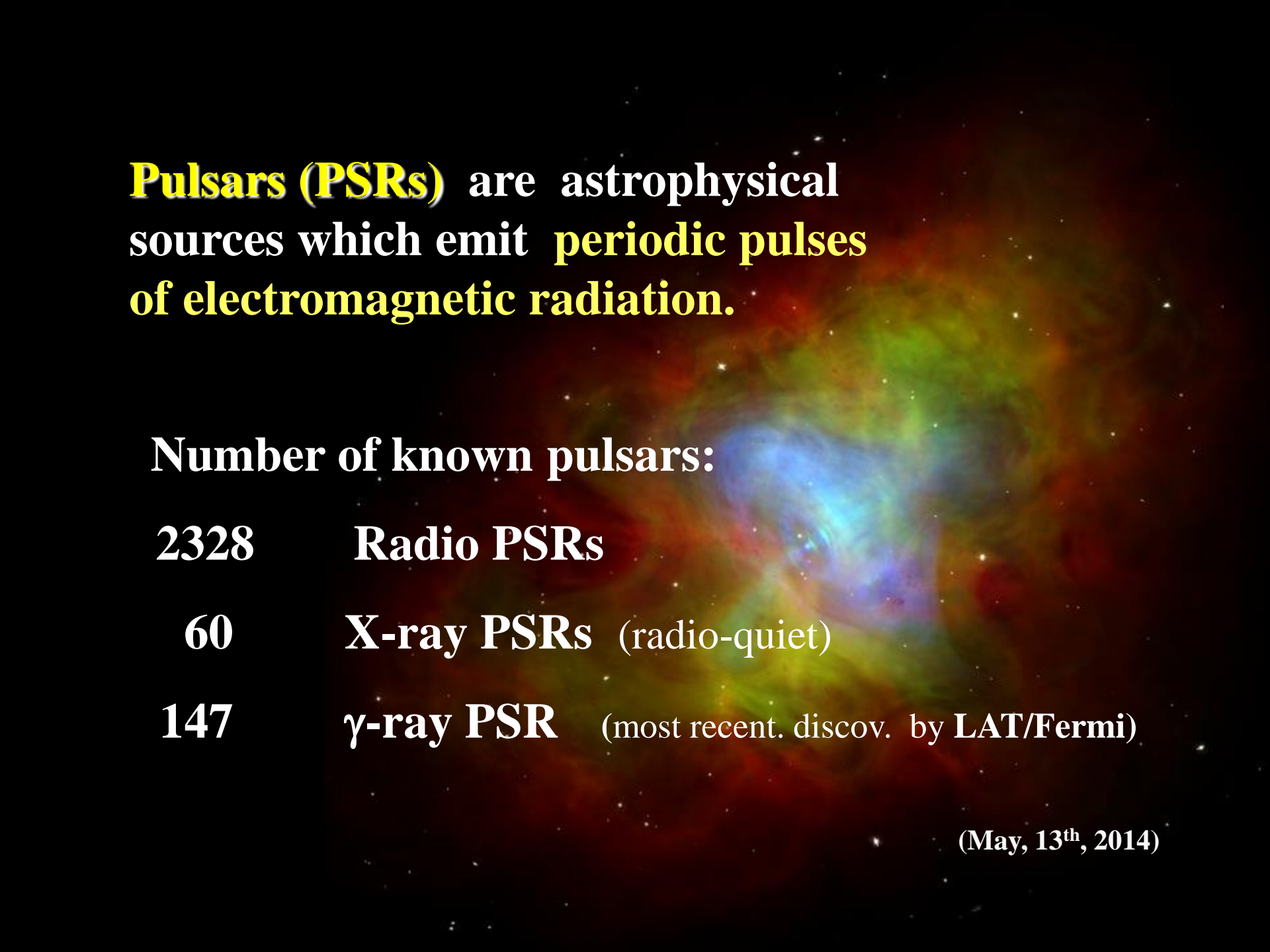
“With all reserve we advance the view that **supernovae** represent the transition from ordinary stars into **neutron stars**, which in their final stages consist of extremely closely packed neutrons.”

1st calculation of Neutron Star properties (Oppenheimer and Volkov, 1939)

Discovery of Pulsars (J. Bell, A. Hewish et al. 1967)

Interpretation of Pulsars as rotating magnetized Neutron Star

(Pacini, 1967, Nature 216), (Gold, 1968, Nature 218)



Pulsars (PSRs) are astrophysical sources which emit **periodic pulses of electromagnetic radiation.**

Number of known pulsars:

2328 **Radio PSRs**

60 **X-ray PSRs** (radio-quiet)

147 **γ -ray PSR** (most recent. discov. by **LAT/Fermi**)

(May, 13th, 2014)

Relativistic equations for stellar structure

Static and spherically symmetric self-gravitating mass distribution

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\Phi(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

$\Phi = \Phi(r)$, $\lambda = \lambda(r)$ metric functions

$$e^{\lambda(r)} = \left[1 - \frac{2G m(r)}{c^2 r} \right]^{-1/2}$$

for the present case the Einstein's field equations take the form called the

Tolman – Oppenheimer – Volkov equations (TOV)

$$\frac{dP}{dr} = -G \frac{m(r) \rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)} \right) \left(1 + 4\pi \frac{r^3 P(r)}{m(r) c^2} \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

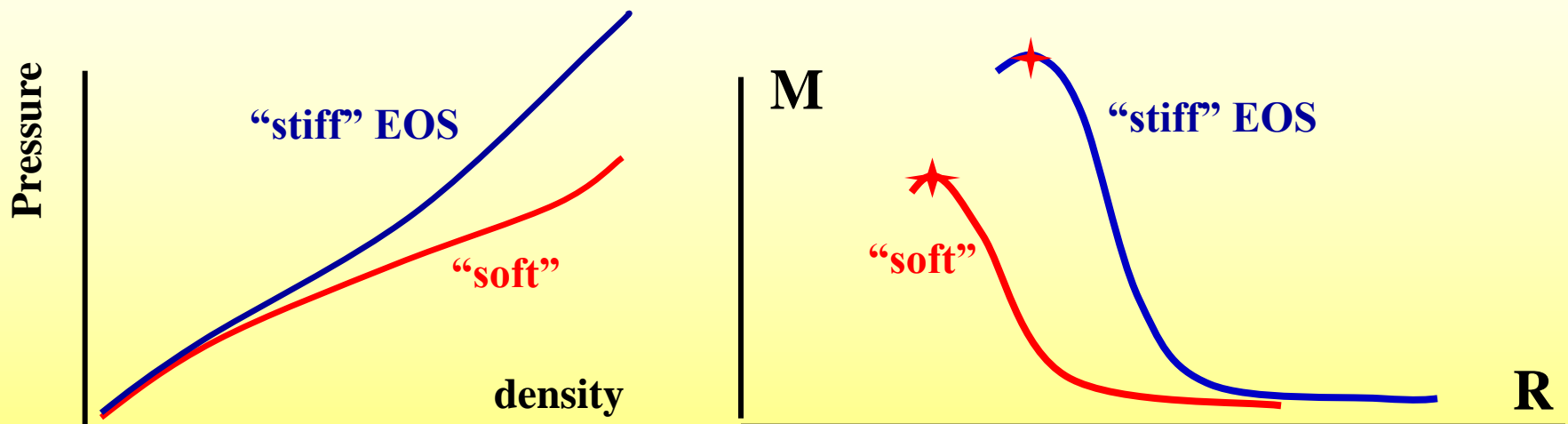
$$\frac{d\Phi}{dr} = -\frac{1}{\rho(r) c^2} \frac{dP}{dr} \left(1 + \frac{P(r)}{\rho(r) c^2} \right)^{-1}$$

One needs the **equation of state (EOS) of dense matter, $P = P(\rho)$, up to very high densities**

The Oppenheimer-Volkoff maximum mass

There is a maximum value for the gravitational mass of a Neutron Star that a given EOS can support. This mass is called the **Oppenheimer-Volkoff mass**

$$M_{\max} = (1.4 - 2.5) M_{\odot}$$



The OV maximum mass represent the key physical quantity to separate (and distinguish) Neutrons Stars from Black Holes.

$$M_{\max}(\text{EOS}) \geq \text{all measured neutron star masses}$$

Measured Neutron Star masses in Relativistic binary systems

Measuring **post-Keplerian parameters**:

- * **very accurate NS mass measurements**
- * **model independent measurements within GR**

● **PSR B1913+16** NS (radio PSR) + NS (“silent”) (Hulse and Taylor 1974)

$$P_{\text{PSR}} = 59 \text{ ms}, P_{\text{b}} = 7 \text{ h } 45 \text{ min} \quad \dot{\omega} = 4.22^{\circ} / \text{yr}$$

$$M_{\text{p}} = 1.4408 \pm 0.0003 M_{\odot} \quad M_{\text{c}} = 1.3873 \pm 0.0003 M_{\odot}$$

Orbital period decay in agreement with GR predictions over about 40 yr
→ indirect evidence for gravitational waves emission

● **PSR J0737-3039** NS(PSR) + NS(PSR) (Burgay, et al 2003)

$$P_1 = 22.7 \text{ ms}, P_2 = 2.77 \text{ s} \quad P_{\text{b}} = 2 \text{ h } 24 \text{ min} \quad \dot{\omega} = 16.88^{\circ} / \text{yr}$$

$$M_1 = 1.34 M_{\odot} \quad M_2 = 1.25 M_{\odot}$$

Two “heavy” Neutron Stars

PSR J1614–2230

$$M_{\text{NS}} = 1.97 \pm 0.04 M_{\odot}$$

NS – WD binary system (He WD)

$M_{\text{WD}} = 0.5 M_{\odot}$ (companion mass)

$P_b = 8.69$ hr (orbital period) $P = 3.15$ ms (PSR spin period)

$i = 89.17^{\circ} \pm 0.02^{\circ}$ (inclination angle)

P. Demorest et al., Nature 467 (2010) 1081

PSR J0348+0432

$$M_{\text{NS}} = 2.01 \pm 0.04 M_{\odot}$$

NS – WD binary system

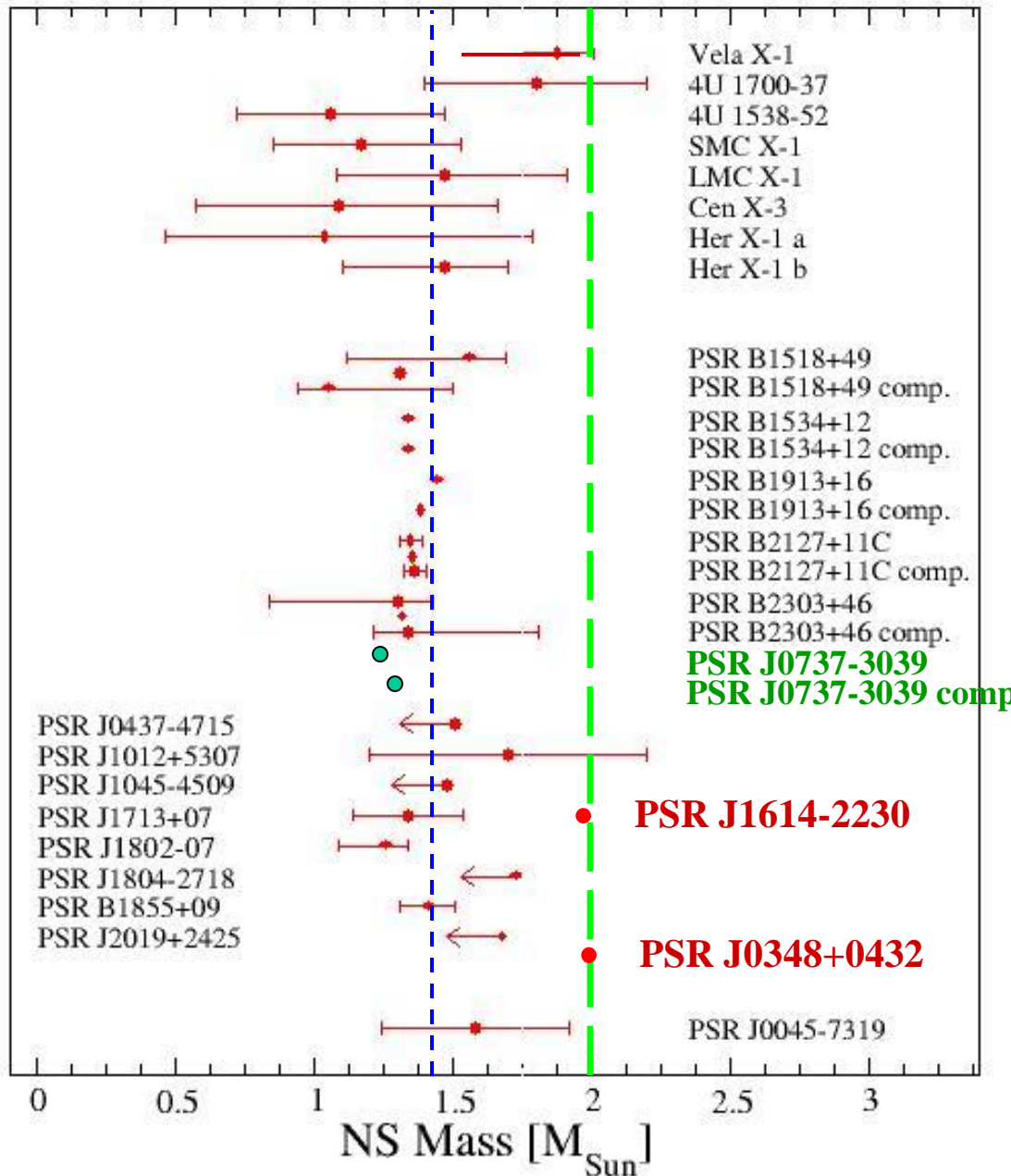
$M_{\text{WD}} = 0.172 \pm 0.003 M_{\odot}$ (companion mass)

$P_b = 2.46$ hr (orbital period) $P = 39.12$ ms (PSR spin period)

$i = 40.2^{\circ} \pm 0.6^{\circ}$ (inclination angle)

Antoniadis et al., Science 340 (2013) 448

Measured Neutron Star Masses



$$M_{\text{max}} \geq M_{\text{measured}}$$

$$M_{\text{max}} \geq 2 M_{\odot}$$



**Very stringent
constraint on the
EOS**

Neutron Stars in the QCD phase diagram

Lattice QCD at $\mu_b=0$ and finite T

► The transition to Quark Gluon Plasma is a crossover
Aoki et al., *Nature*, 443 (2006) 675

► Deconfinement transition temperature T_c

HotQCD Collaboration

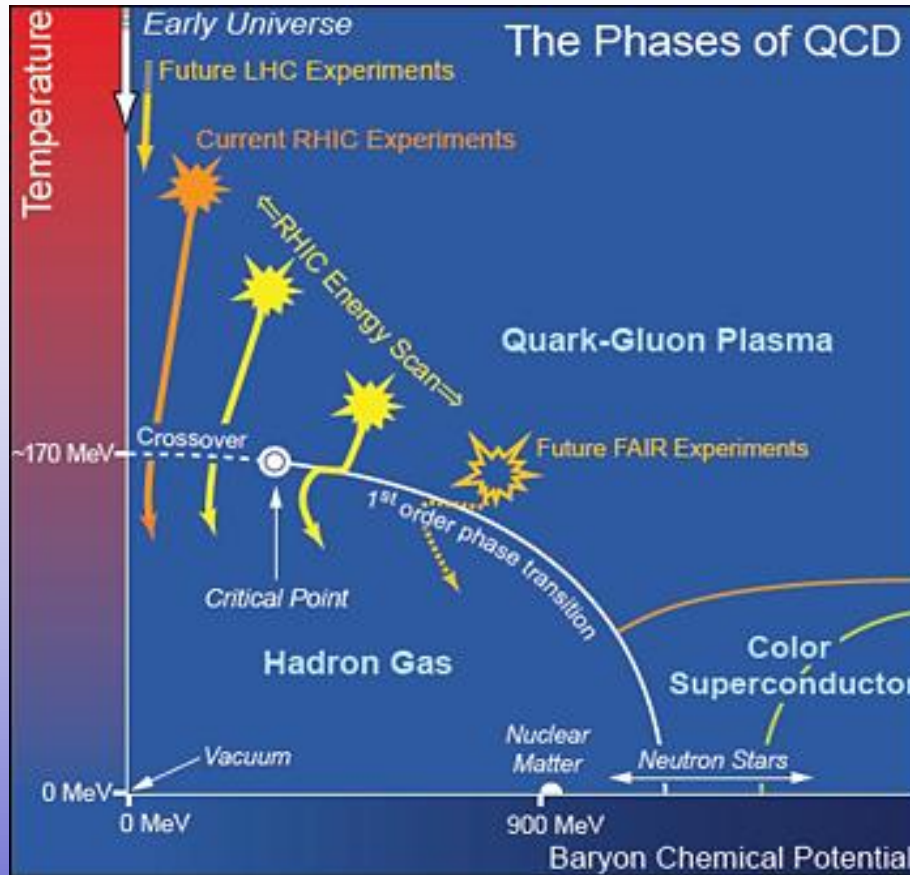
$$T_c = 154 \pm 9 \text{ MeV}$$

Bazarov et al., *Phys.Rev. D85* (2012) 054503

Wuppertal-Budapest Collab.

$$T_c = 147 \pm 5 \text{ MeV}$$

Borsanyi et al., *J.H.E.P.* 09 (2010) 073



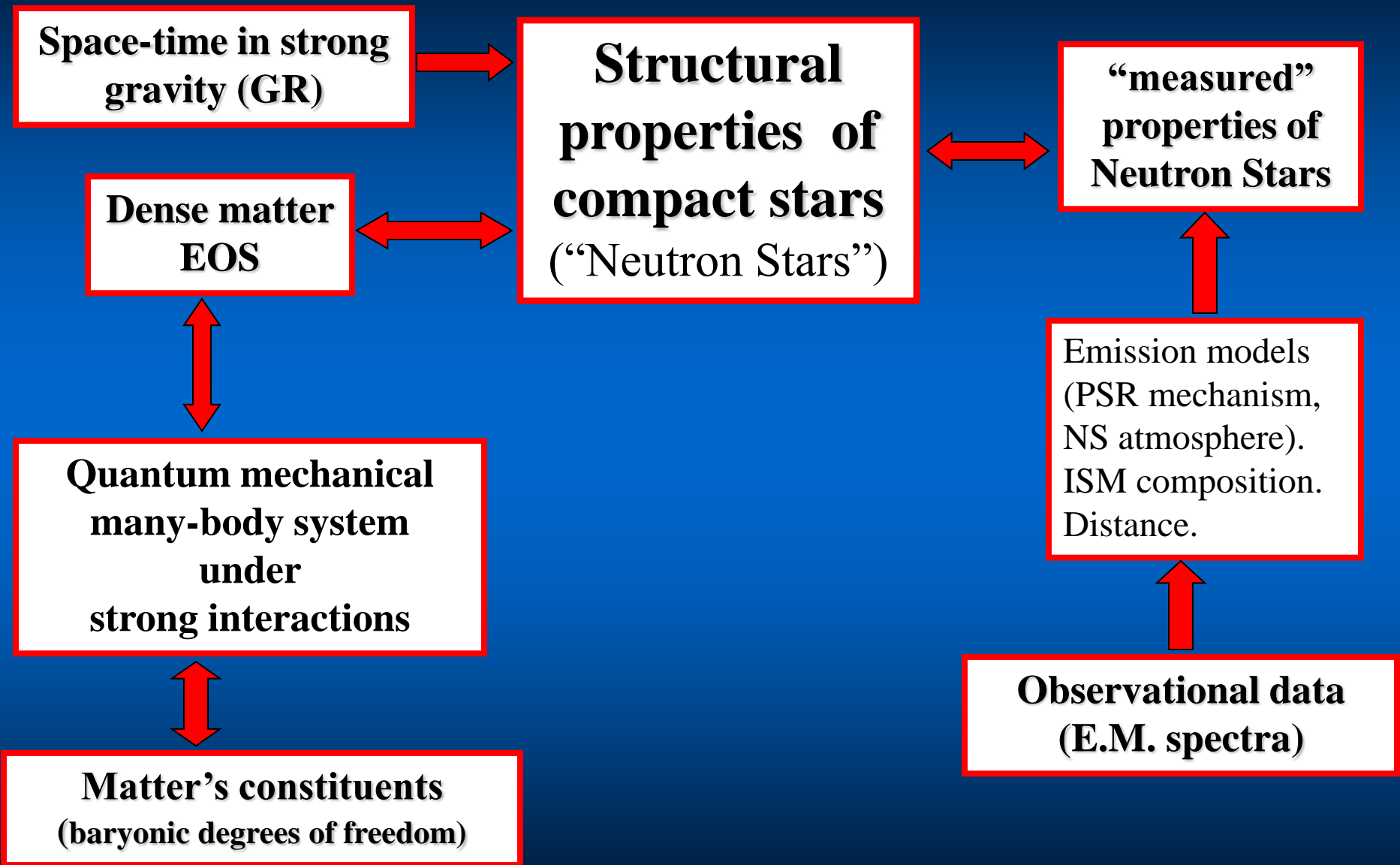
Cristalline Color
superconductor

Neutron Stars: high μ_b and low T

Quark deconfinement transition
expected of the first order

Z. Fodor, S.D. Katz, *Prog. Theor. Suppl.* 153 (2004) 86

Lattice QCD calculations are
presently not possible



Neutron star physics in a nutshell

1) **Gravity** compresses matter at very high density

2) **Pauli principle**

Stellar constituents are different species of **identical fermions** (n, p, ..., e⁻, μ⁻)
→ antisymmetric wave function for particle exchange → Pauli principle

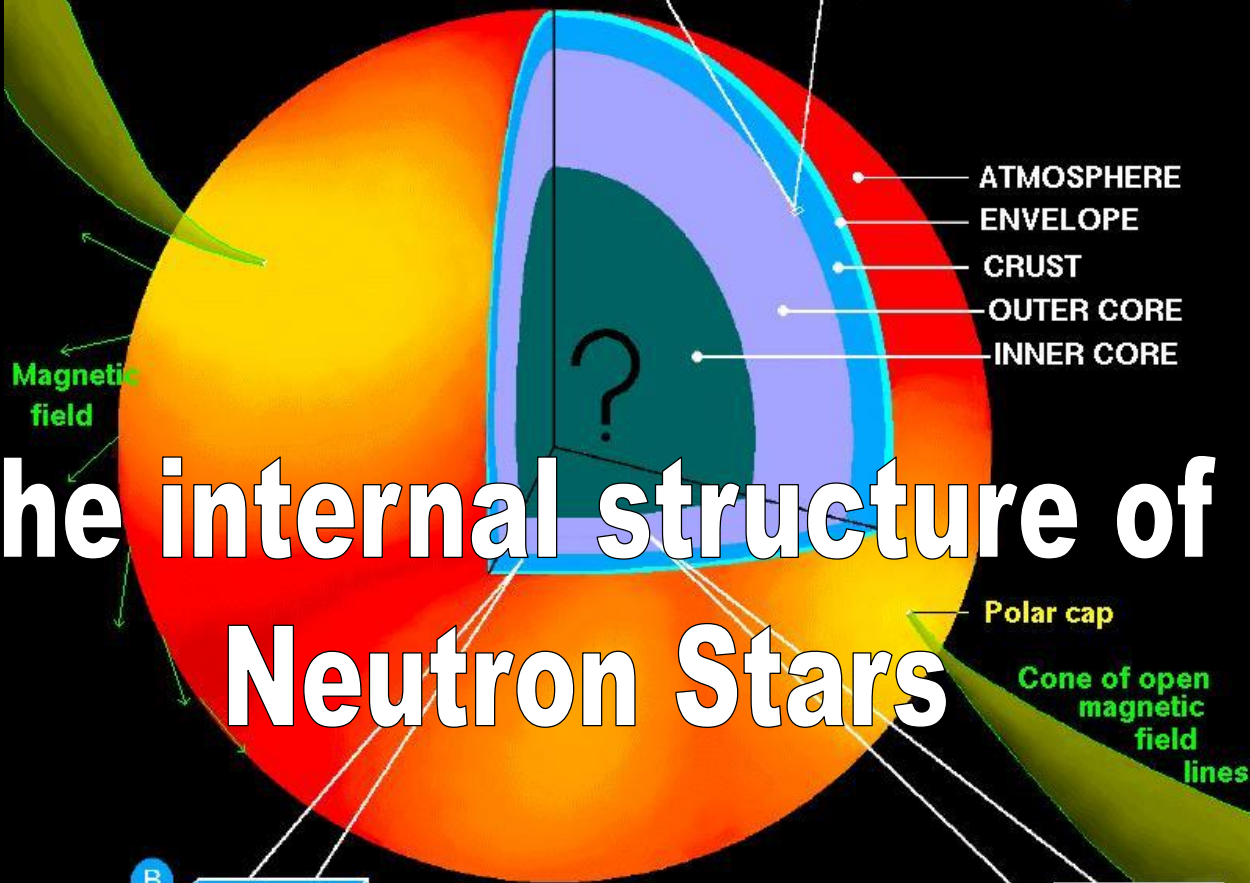
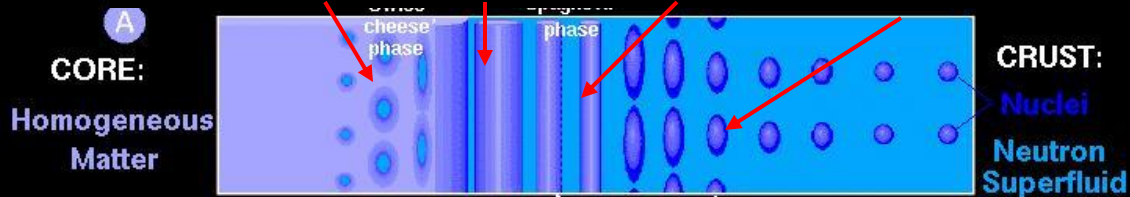
Chemical potentials $\mu_n, \mu_p, \dots, \mu_e$ rapidly increasing functions of density

3) **Weak interactions** change the isospin and strangeness content of dense matter to minimize energy

Cold catalyzed matter (Harrison, Wakano, Wheeler, 1958)

The ground state (minimum energy per baryon) of a system of **hadrons** and **leptons** with respect to their mutual **strong** and **weak interactions** at a given total baryon density n and temperature T = 0.

swiss cheese lasagne spaghetti meet-balls



The internal structure of Neutron Stars



Nucleon Stars

β -stable nuclear matter

$$p + e^- \leftrightarrow n + \nu_e$$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

$$\mu_e \geq m_\mu$$

$$e^- \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

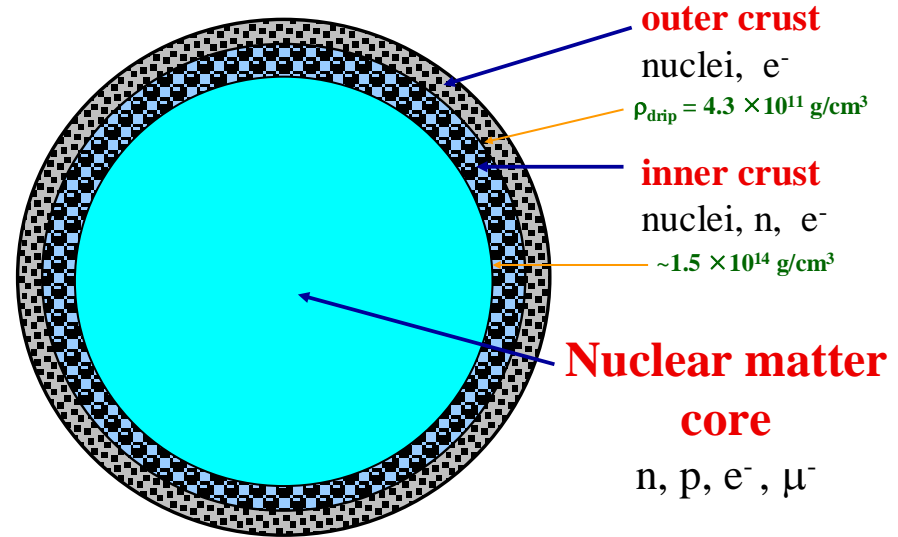
Equilibrium with respect to the weak interaction processes

$$\mu_n - \mu_p = \mu_e$$

$$\mu_\mu = \mu_e$$

Charge neutrality

$$n_p = n_e + n_\mu$$



neutrino-free matter

$$\mu_\nu = \mu_{\bar{\nu}} = 0$$

To be solved for any given value of the total baryon number density n_B

Proton fraction in β -stable nuclear matter and role of the nuclear symmetry energy

$$\hat{\mu} \equiv \mu_n - \mu_p = - \left. \frac{\partial(E/A)}{\partial x} \right|_n = 2 \left. \frac{\partial(E/A)}{\partial \beta} \right|_n$$

$$\beta = (n_n - n_p)/n = 1 - 2x$$

$$n = n_n + n_p$$

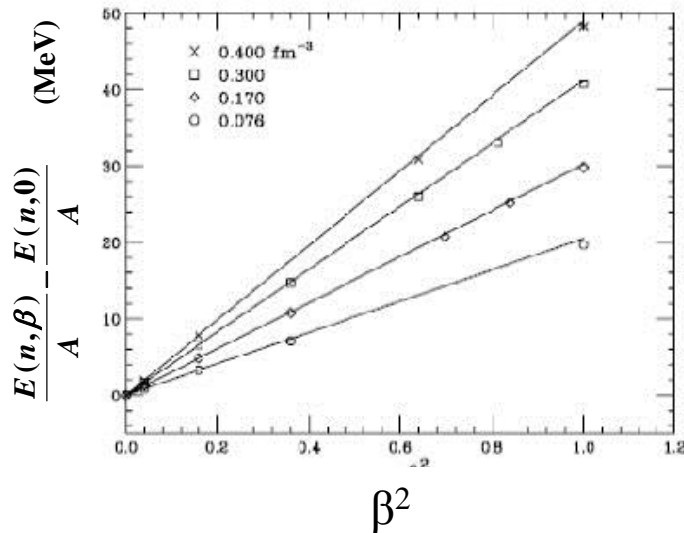
$x = n_p/n$ proton fraction

Energy per nucleon for asymmetric nuclear matter

$$\frac{E(n, \beta)}{A} = \frac{E(n, 0)}{A} + E_{sym}(n) \beta^2 + S_4(n) \beta^4 + \dots$$

Symmetry energy

$$E_{sym}(n) \equiv \left. \frac{1}{2} \frac{\partial^2(E/A)}{\partial \beta^2} \right|_{\beta=0}$$



The “parabolic approximation” (*)

$$\frac{E(n, \beta)}{A} = \frac{E(n, 0)}{A} + E_{sym}(n) \beta^2$$

(*) Bombaci, Lombardo, Phys. Rev: C44 (1991)

Proton fraction in β -stable nuclear matter and role of the nuclear symmetry energy

In the “parabolic approximation”:

$$E_{sym}(n) = \frac{E(n, \beta=1)}{A} - \frac{E(n, \beta=0)}{A}$$

$\beta=0$ symm nucl matter
 $\beta=1$ pure neutron matter



$$\hat{\mu} = 4 E_{sym}(n) [1 - 2x]$$

Chemical equil.+charge neutrality
 (no muons)

$$3\pi^2 (\hbar c)^3 n x(n) - [4E_{sym}(n)(1 - 2x(n))]^3 = 0$$

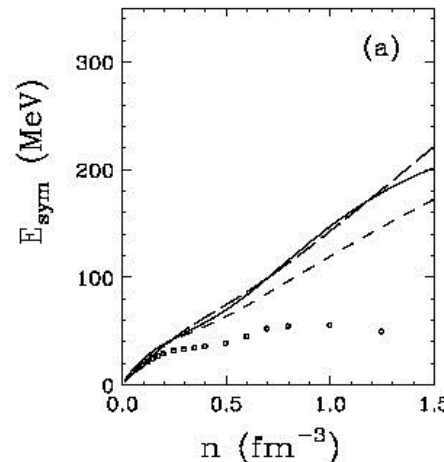
if $x \ll 1/2$



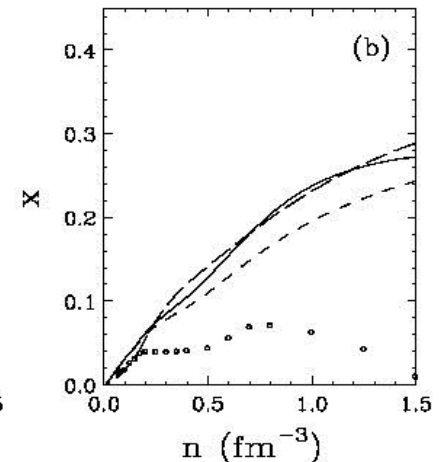
$$x_{eq}(n) \approx \frac{1}{3\pi^2} \frac{1}{n} \left(\frac{4E_{sym}(n)}{\hbar c} \right)^3$$

The composition of β -stable nuclear matter is strongly dependent on the nuclear symmetry energy.

Symmetry en.



proton fraction



Microscopic approach to nuclear matter EOS

input

Two-body nuclear interactions: V_{NN}

“realistic” interactions: e.g. Argonne, Bonn, Nijmegen interactions.

Parameters fitted to NN scattering data with $\chi^2/\text{datum} \sim 1$

Three-body nuclear interactions: V_{NNN}

semi-phenomenological. Parameters fitted to

- binding energy of $A = 3, 4$ nuclei or
- empirical saturation point of symmetric nuclear matter: $n_0 = 0.16 \text{ fm}^{-3}$, $E/A = -16 \text{ MeV}$

	AV18	AV18/UIX	Exp.
B(3H)	7.624	8.479	8.482
B(3He)	6.925	7.750	7.718
B(4He)	24.21	28.46	28.30

Values in MeV

Nuclear Matter at $n = 0.16 \text{ fm}^{-3}$

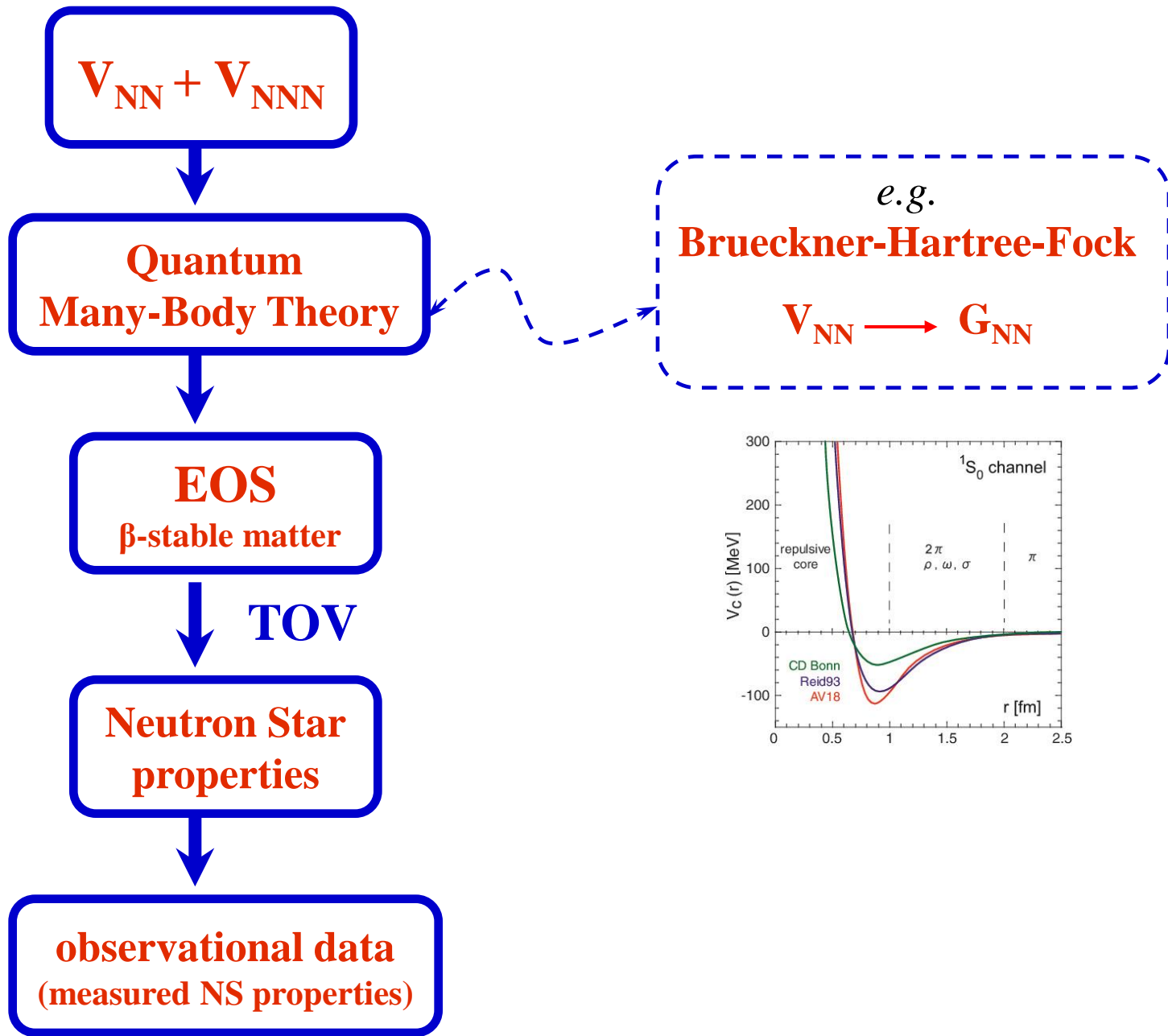
$E^{\text{pot}}(2\text{BF})/A \sim -40 \text{ MeV}$

$E^{\text{pot}}(3\text{BF})/A \sim -1 \text{ MeV}$

A. Kievsky, S. Rosati, M. Viviani, L.E. Marcucci, L. Girlanda, *Jour. Phys. G* 35 (2008) 063101

A. Kievsky, M. Viviani, L. Girlanda, L.E. Marcucci, *Phys. Rev. C* 81 (2010) 044003

Z.H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, *Phys. Rev. C* 77 (2008) 034316



Microscopic EOS for nuclear matter: Brueckner-Bethe-Goldstone theory

$$G_{\tau\tau'}(\omega) = V + V \sum_{k_a k_b} \frac{|k_a k_b\rangle Q_{\tau\tau'} \langle k_a k_b|}{\omega - e_{\tau}(k_a) - e_{\tau'}(k_b)} G_{\tau\tau'}(\omega)$$

$$e_{\tau}(k) = \frac{\hbar^2 k^2}{2M} + U_{\tau}(k)$$

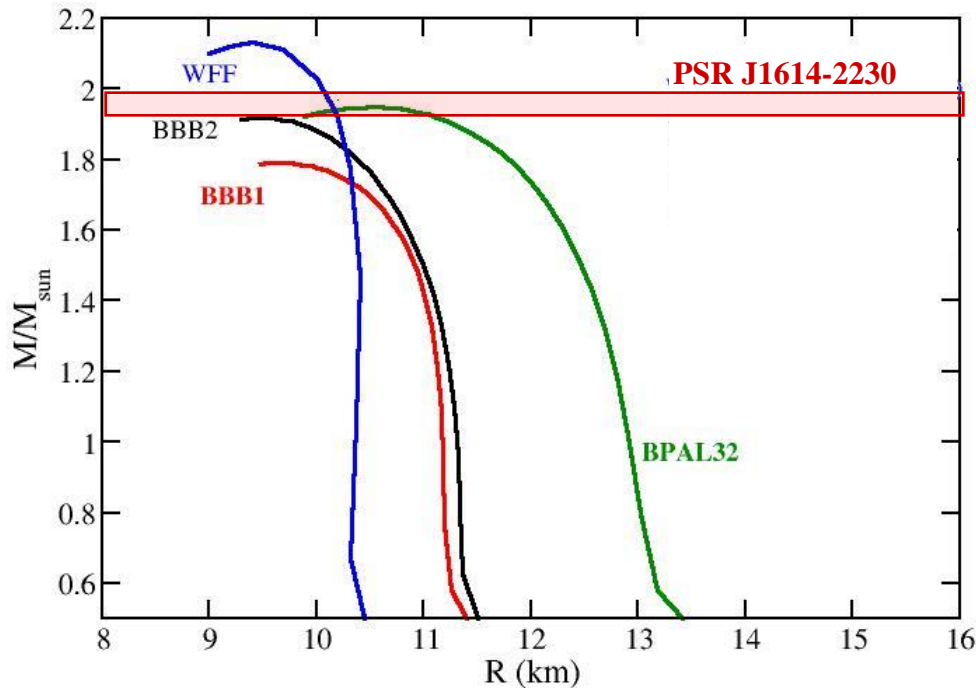
$$U_{\tau}(k) = \sum_{\tau'} \sum_{k'} \langle \vec{k} \vec{k}' | G_{\tau\tau'}(e_{\tau} + e_{\tau'}) | \vec{k} \vec{k}' \rangle$$

V is the **nucleon-nucleon interaction** (e.g. the **Argonne v14, Bonn, ... potential**) plus a density dependent term which represents the average of **Three-Body Force (TBF)** over one of the interacting nucleons.

- Energy per baryon in the Brueckner-Hartree-Fock (BHF) approximation

$$\frac{E}{A} = \frac{1}{A} \sum_{\tau} \sum_k \frac{\hbar^2 k^2}{2M} + \frac{1}{2A} \sum_{\tau} \sum_k U_{\tau}(k)$$

Mass-Radius relation for Nucleon Stars



$$M_{\text{max}} = (1.8 \text{ } \square \text{ } 2.3) M_{\odot}$$

Maximum mass configuration for Nucleon Stars

EOS	M_G/M_{\odot}	R(km)	n_c / n_0
BBB1	1.79	9.66	8.53
BBB2	1.92	9.49	8.45
WFF	2.13	9.40	7.81
APR	2.20	10.0	7.25
BPAL32	1.95	10.54	7.58
KS	2.24	10.79	6.30

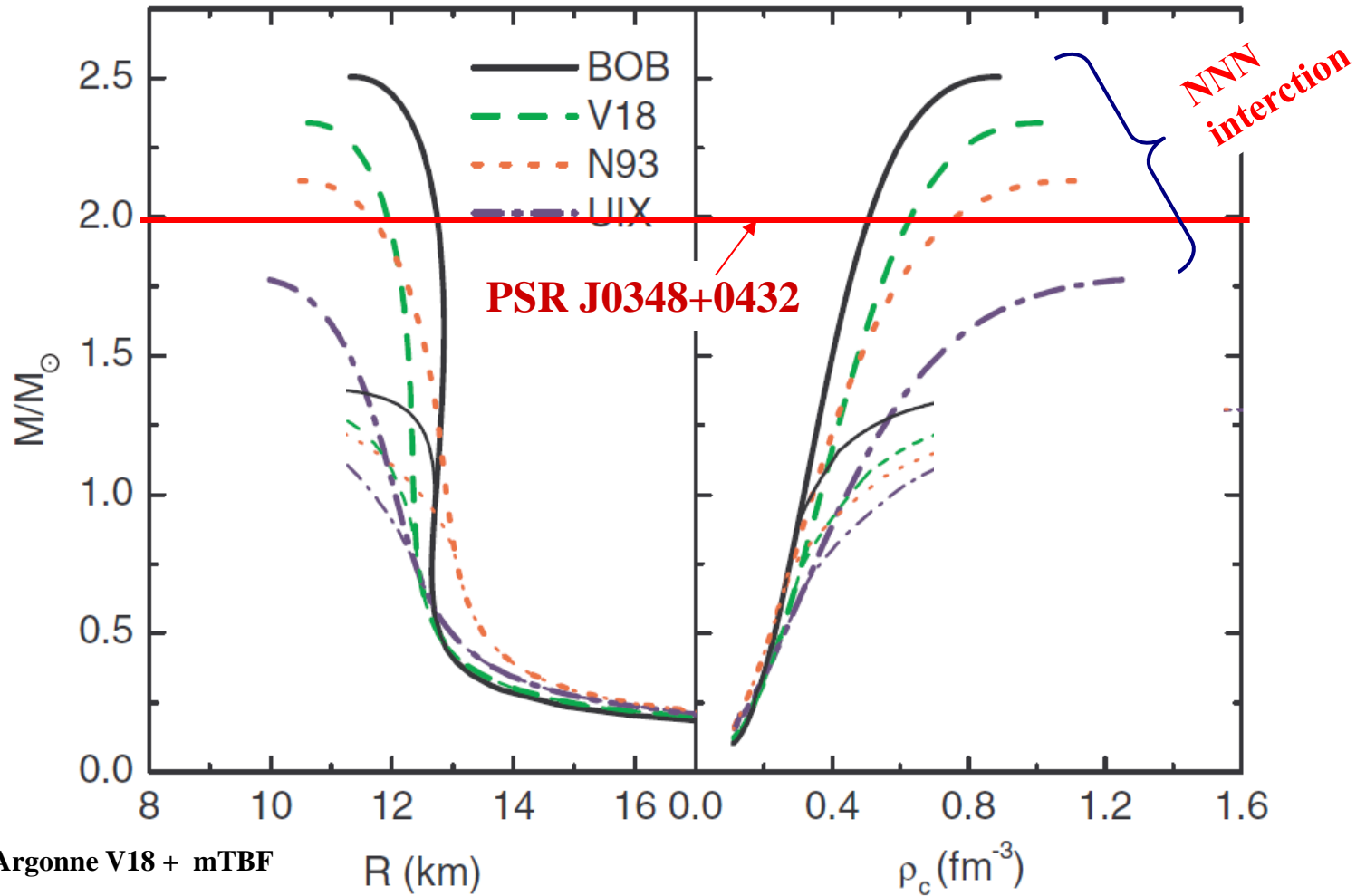
WFF: Wiringa-Ficks-Fabrocini, 1988.

BPAL: Bombaci, 1995.

BBB: Baldo-Bombaci-Burgio, 1997.

APR: Akmal-Pandharipande-Ravenhall, 1988.

KS: Krastev-Sammarruca, 2006



V18: Argonne V18 + mTBF

BOB: Bonn B + mTBF

N93: Nijmegen 93 + mTBF

UIX: Argonne V18 + Urbana IX

models of **Nucleon Stars**

(i.e. Neutron Stars with a pure **nuclear matter core**)

are able to explain

measured Neutron Star masses

as those of

PSR J1614-2230 and **PSR J0348+0432**

$$M_{\text{NS}} \approx 2 M_{\odot}$$

Happy?

Not the end of the story!

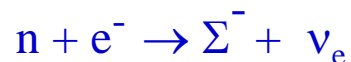
Hyperon Stars

Why is it very likely to have hyperons in the core of a Neutron Star?

- **Pauli principle.** Neutrons (protons) are identical Fermions, thus their chemical potentials (Fermi energies) increase very rapidly as a function of density.

The central density of a Neutron Star is “high”: $n_c \approx (4 - 10) n_0$
($n_0 = 0.17 \text{ fm}^{-3}$)

- above a **threshold density**, $n_{\text{cr}} \approx (2 - 3) n_0$, **weak interactions** in dense matter can produce **strange baryons (hyperons)**



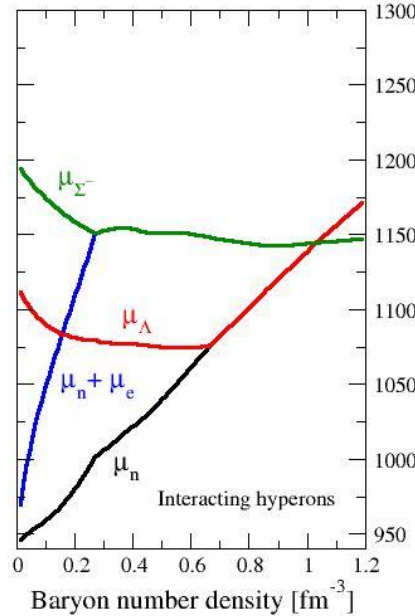
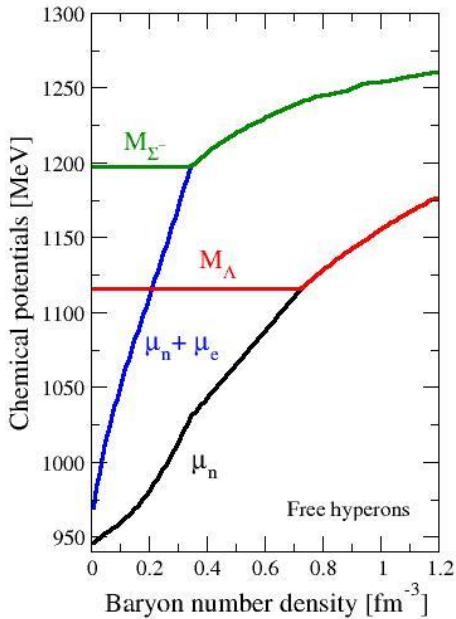
etc.

A. Ambarsumyan, G.S. Saakyan, (1960)
V.R. Pandharipande (1971)

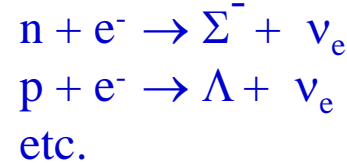
In Greek mythology Hyperion (Ἵπερίων) was one of the twelve Titan son of Gaia and Uranus

Threshold density for hyperons in neutron matter: a simple estimate (ideal non-rel. Fermi gas of neutrons)

$$n_{cr} = \frac{1}{3\pi^2 (\hbar c)^3} \left[2m_n c^2 (m_\Lambda - m_n) c^2 \right]^{3/2}$$



Hyperons appear in the stellar core above a threshold density $\rho_{cr} \approx (2 - 3) \rho_0$



I. Vidaña, Ph.D. thesis (2001)

$$m_\Lambda = 1115.7 \text{ MeV}$$

$$m_{\Sigma^-} = 1197.5 \text{ MeV}$$

$$\mu_n = \mu_\Lambda$$

$$\mu_n + \mu_e = \mu_{\Sigma^-}$$

$$\mu_p = \mu_n - \mu_e = \mu_{\Sigma^+}$$

$$\mu_n = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_\Lambda$$

$$\mu_n + \mu_e = \mu_{\Sigma^-} = \mu_{\Xi^-}$$

$$\mu_\mu = \mu_e$$

$$n_p + n_{\Sigma^+} = n_e + n_\mu + n_{\Sigma^-} + n_{\Xi^-}$$

Microscopic approach to hyperonic matter EOS

input

2BF: nucleon-nucleon (NN), nucleon-hyperon (NY), hyperon-hyperon (YY)

e.g. Nijmegen, Julich models

3BF: NNN, NNY, NYY, YYY

Hyperonic sector: experimental data

- 1. YN scattering** (very few data)
- 2. Hypernuclei**

Hypernuclear experiments

**FINUDA (LNF-INFN),
PANDA and HypHI (FAIR/GSI),
Jeff. Lab, J-PARC**

Phenomenological approaches to hyperonic matter EOS

Relativistic Mean Fields Models (Glendenning 1995, Knorren et al 1995, Schaffner-Bielich Mishustin 1996)

Skyrme-like potential models (Balberg and Gal 1997)

Chiral Effective Lagrangians (Hanuske et al 2000)

Quark-meson coupling model (Pal, Hanuske, Zakout, Stoker, Greiner, 1999)

Microscopic EOS for hyperonic matter: **extended Brueckner theory**

$$G(\omega)_{B_1 B_2 B_3 B_4} = V_{B_1 B_2 B_3 B_4} + \sum_{B_5 B_6} V_{B_1 B_2 B_5 B_6} \frac{Q_{B_5 B_6}}{\omega - e_{B_5} - e_{B_6}} G(\omega)_{B_5 B_6 B_3 B_4}$$

$$e_{B_i}(k) = M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + U_{B_i}(k)$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k' \leq k_{F B_j}} \langle \vec{k} \vec{k}' | \mathbf{G}_{B_i B_j B_i B_j}(\omega = e_{B_i} + e_{B_j}) | \vec{k} \vec{k}' \rangle$$

\mathbf{V} is the **baryon--baryon interaction for the baryon octet** (\mathbf{n} , \mathbf{p} , $\mathbf{\Lambda}$, $\mathbf{\Sigma}^-$, $\mathbf{\Sigma}^0$, $\mathbf{\Sigma}^+$, $\mathbf{\Xi}^-$, $\mathbf{\Xi}^0$) (e.g. the **Nijmegen potential**).

- **Energy per baryon in the BHF approximation**

$$E/N_B = 2 \sum_{B_i} \int_0^{k_{F[B_i]}} \frac{d^3 k}{(2\pi)^3} \left\{ M_{B_i} c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \frac{1}{2} U_{B_i}^N(k) + \frac{1}{2} U_{B_i}^Y(k) \right\}$$

Baldo, Burgio, Schulze, Phys.Rev. C61 (2000) 055801;

Vidaña, Polls, Ramos, Engvik, Hjorth-Jensen, Phys.Rev. C62 (2000) 035801;

Vidaña, Bombaci, Polls, Ramos, Astron. Astrophys. 399, (2003) 687.

Isospin and Strangeness channels

S = 0

S = -1

S = -2

S = -3

S = -4

I = 0

$(NN \rightarrow NN)$

$$\begin{pmatrix} \Lambda\Lambda \rightarrow \Lambda\Lambda & \Lambda\Lambda \rightarrow \Xi N & \Lambda\Lambda \rightarrow \Sigma\Sigma \\ \Xi N \rightarrow \Lambda\Lambda & \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Lambda\Lambda & \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$$

$(\Xi\Xi \rightarrow \Xi\Xi)$

I = 1/2

$$\begin{pmatrix} \Lambda N \rightarrow \Lambda N & \Lambda N \rightarrow \Sigma N \\ \Sigma N \rightarrow \Lambda N & \Sigma N \rightarrow \Sigma N \end{pmatrix}$$

$$\begin{pmatrix} \Lambda\Xi \rightarrow \Lambda\Xi & \Lambda\Xi \rightarrow \Sigma\Xi \\ \Sigma\Xi \rightarrow \Lambda\Xi & \Sigma\Xi \rightarrow \Sigma\Xi \end{pmatrix}$$

I = 1

$(NN \rightarrow NN)$

$$\begin{pmatrix} \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Lambda\Sigma & \Xi N \rightarrow \Sigma\Sigma \\ \Lambda\Sigma \rightarrow \Xi N & \Lambda\Sigma \rightarrow \Lambda\Sigma & \Lambda\Sigma \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Lambda\Sigma & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$$

$(\Xi\Xi \rightarrow \Xi\Xi)$

I = 3/2

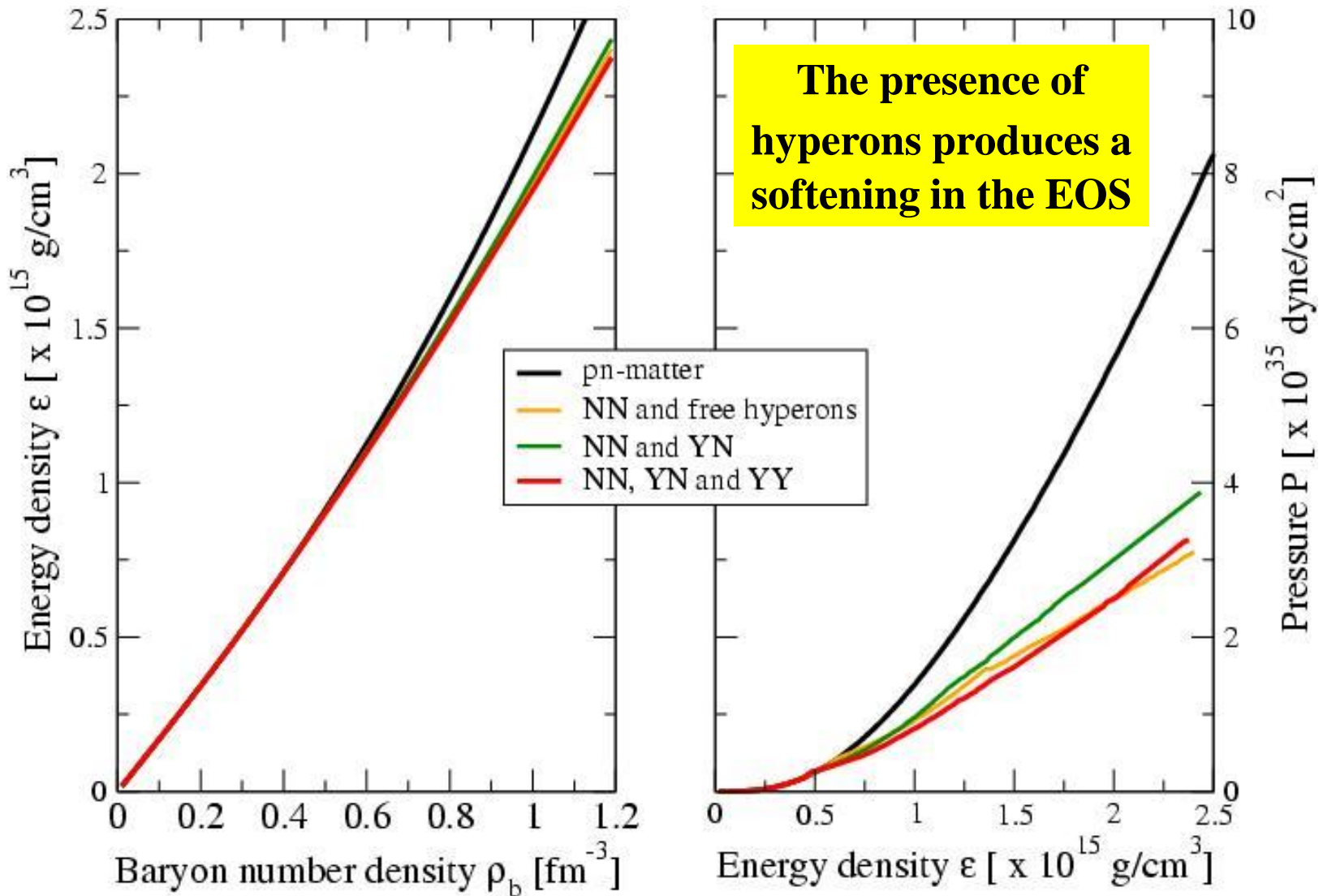
$(\Sigma N \rightarrow \Sigma N)$

$(\Sigma\Xi \rightarrow \Sigma\Xi)$

I = 2

$(\Sigma\Sigma \rightarrow \Sigma\Sigma)$

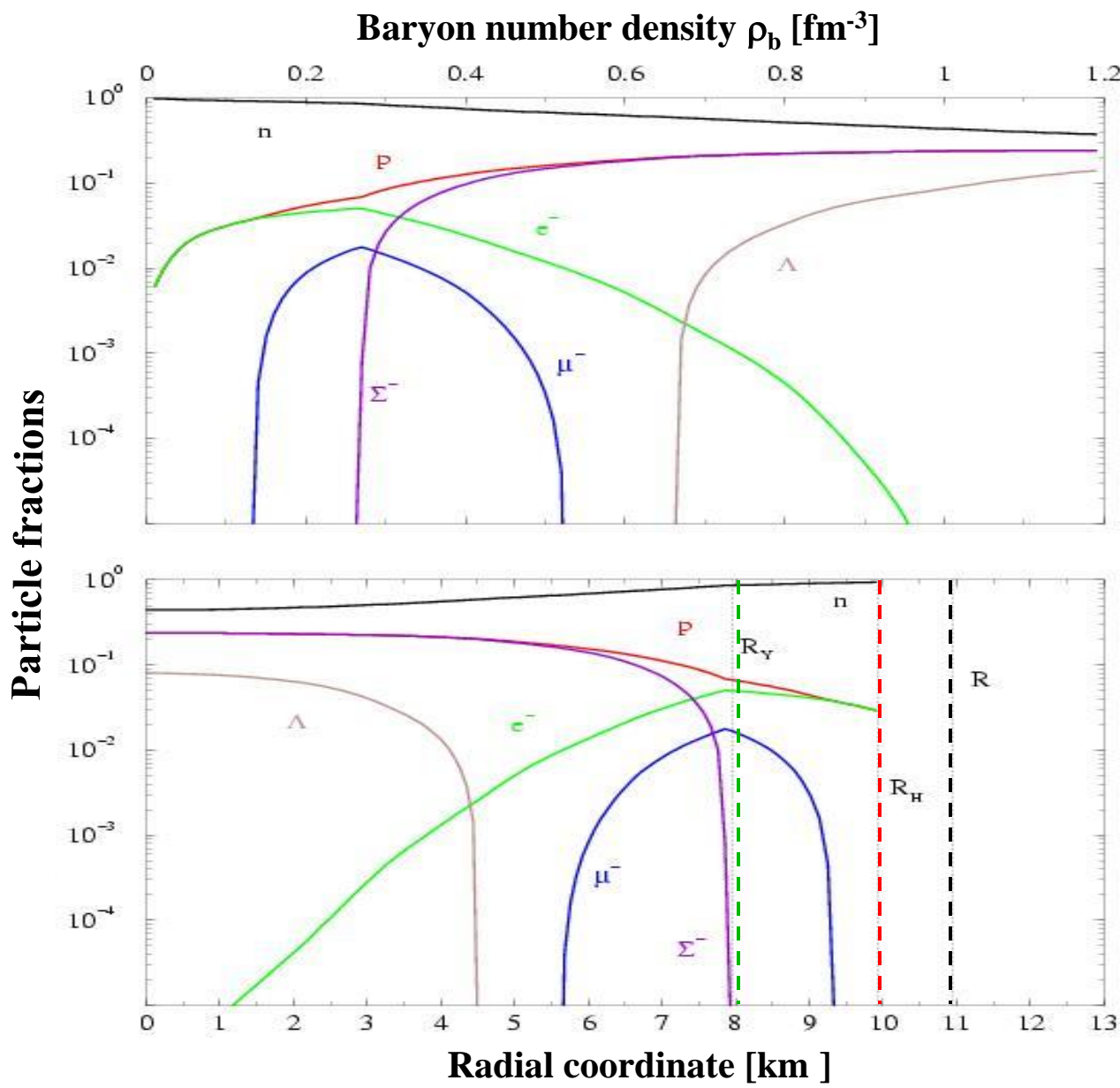
The Equation of State of Hyperonic Matter



NSC97e

I. Vidaña et al., Phys. Rev: C62 (2000) 035801

Composition of hyperonic beta-stable matter

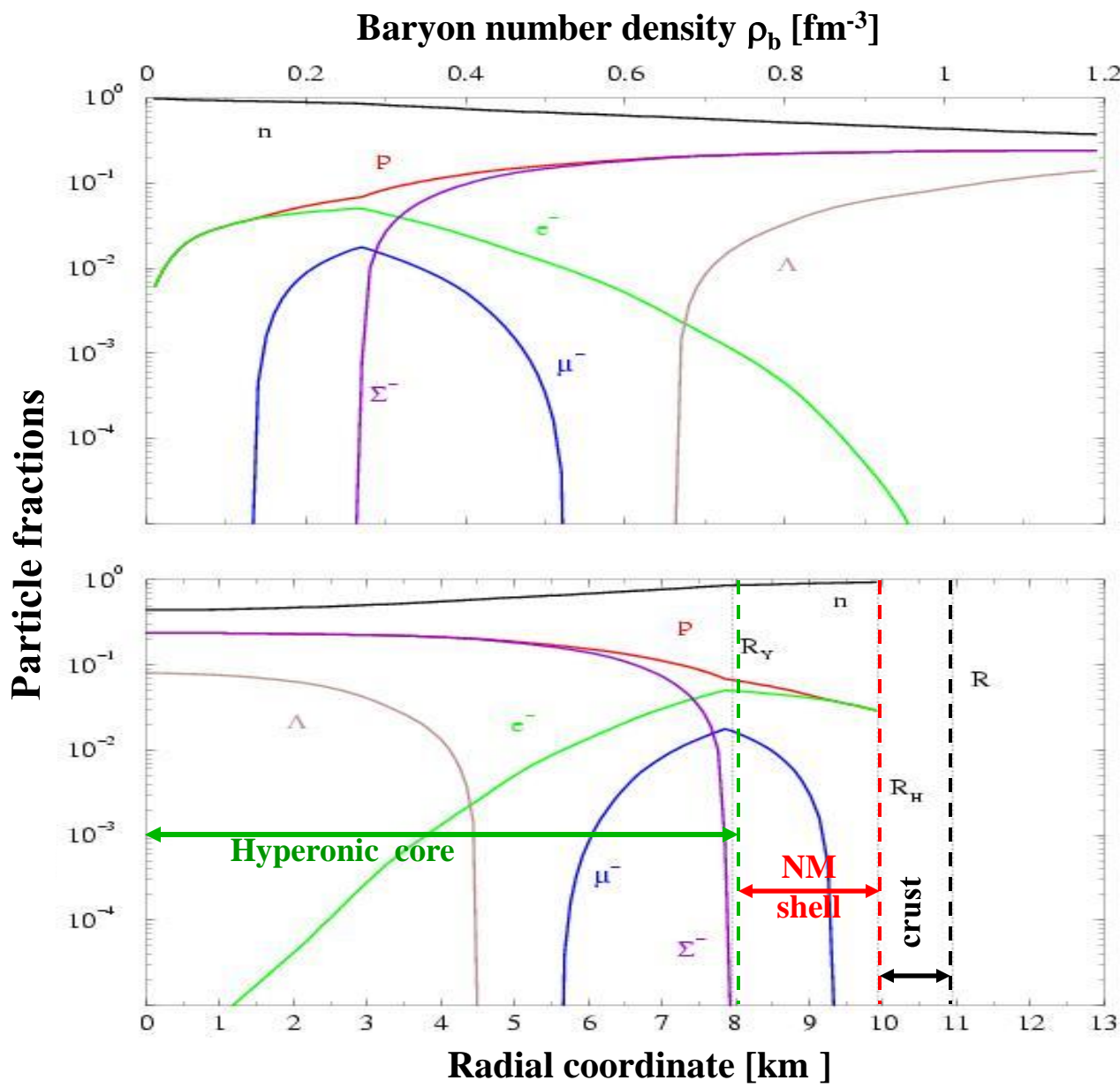


Hyperonic Star

$M_B = 1.34 M_\odot$

I. Vidaña, I. Bombaci,
A. Polls, A. Ramos,
Astron. and Astrophys.
399 (2003) 687

Composition of hyperonic beta-stable matter



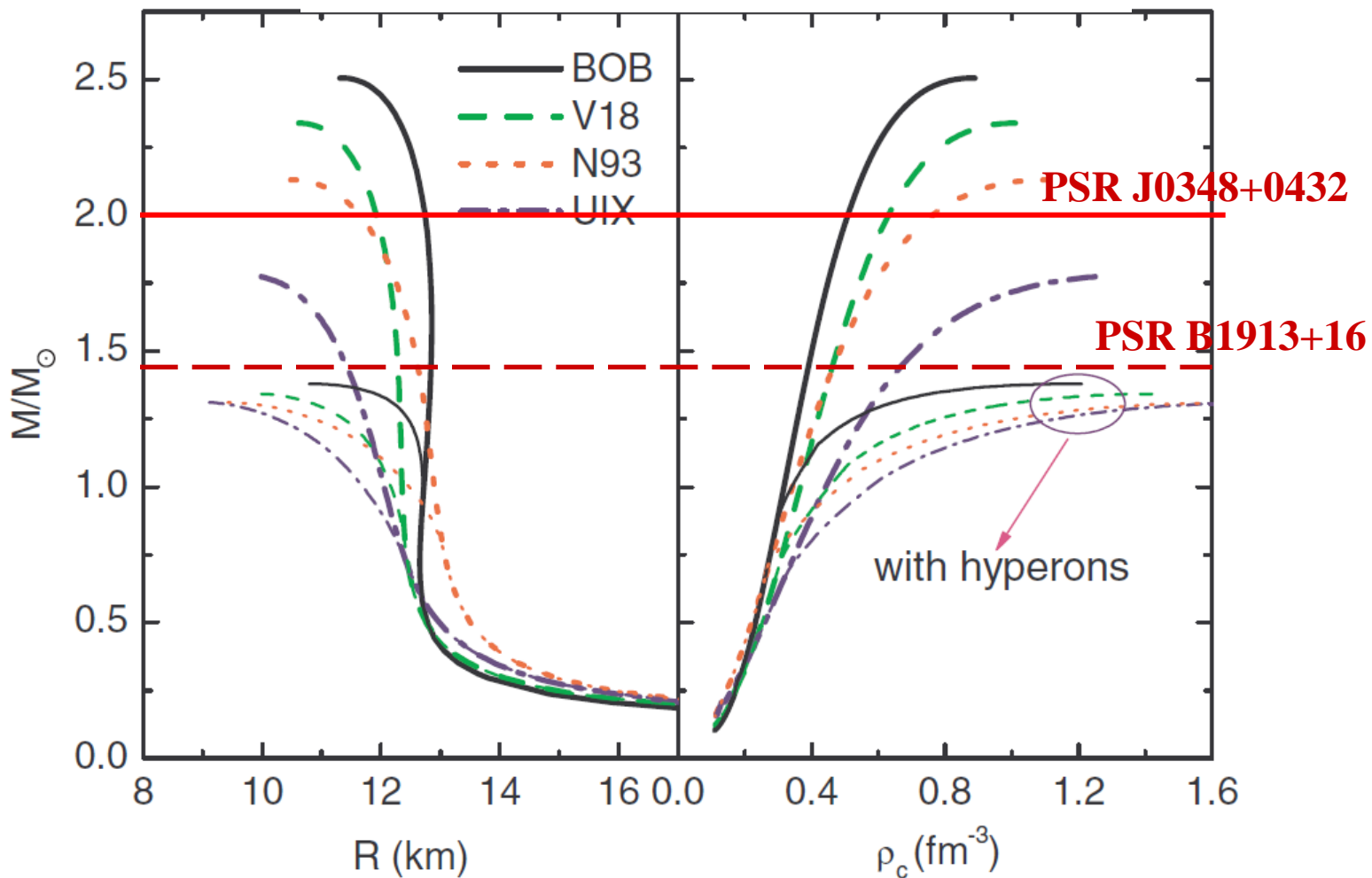
Hyperonic Star

M_B = 1.34 M_⊙

I. Vidaña, I. Bombaci,
 A. Polls, A. Ramos,
 Astron. and Astrophys.
 399 (2003) 687

Z.H. Li, H.-J. Schulze, *PHYSICAL REVIEW C* **78**, 028801 (2008)

interaction: NN + NY + YY + NNN



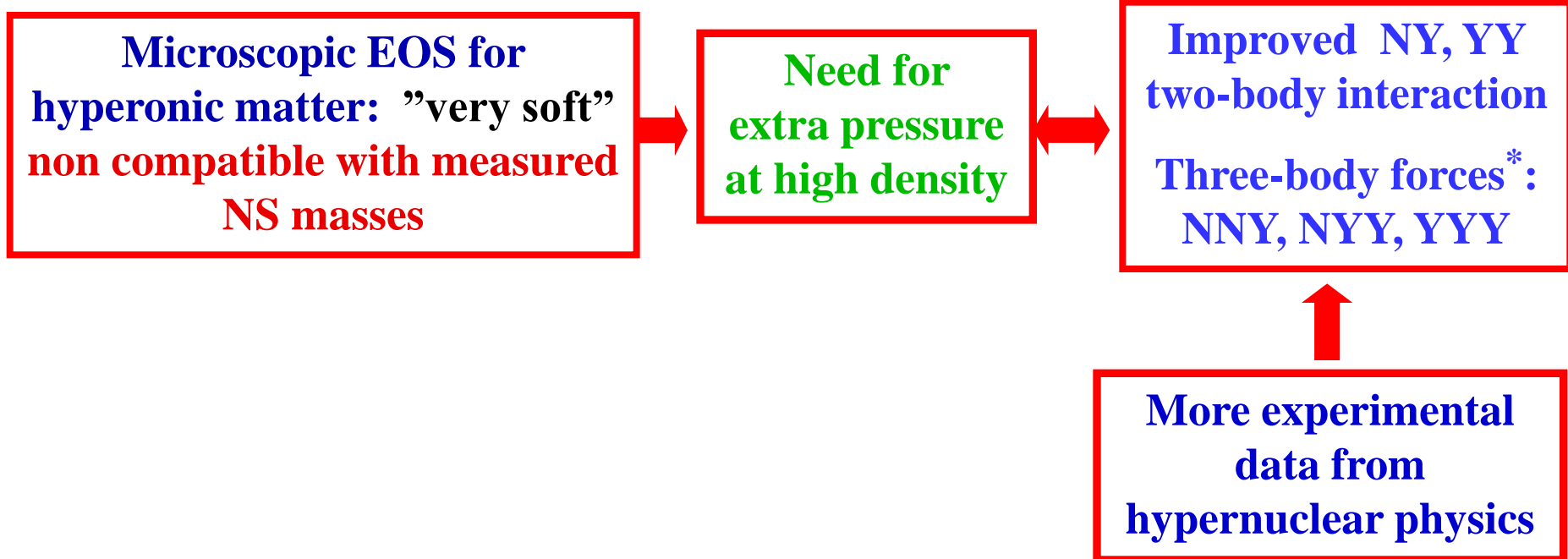
NY,YY: Nijmegen NSC89 potential (Maessen et al, *Phys. Rev. C* **40 (1989))**

Hyperons in Neutron Stars: implications for the stellar structure

The presence of hyperons **reduces the maximum mass of neutron stars:**

$$\Delta M_{\max} \approx (0.5 - 1.2) M_{\odot}$$

Therefore, **to neglect hyperons always leads to an overestimate of the maximum mass of neutron stars**



(*) A preliminary study: I. Vidana, D. Logoteta, C. Providencia, A. Polls, I. Bombaci, EPL 94 (2011) 11002

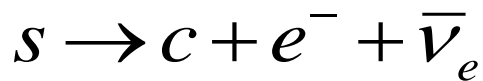
Quark Matter in Neutron Stars

only u, d, s quark flavors are expected in Neutron Stars (SQM)

$$m_c = 1275 \pm 25 \text{ MeV}$$

$$q_c = \frac{2}{3}|e|$$

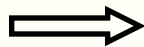
● Threshold density for the c quark



perfect Fermi gas
massless u, d, s quarks
in beta-equil.
 $Q_{\text{tot}} = 0$

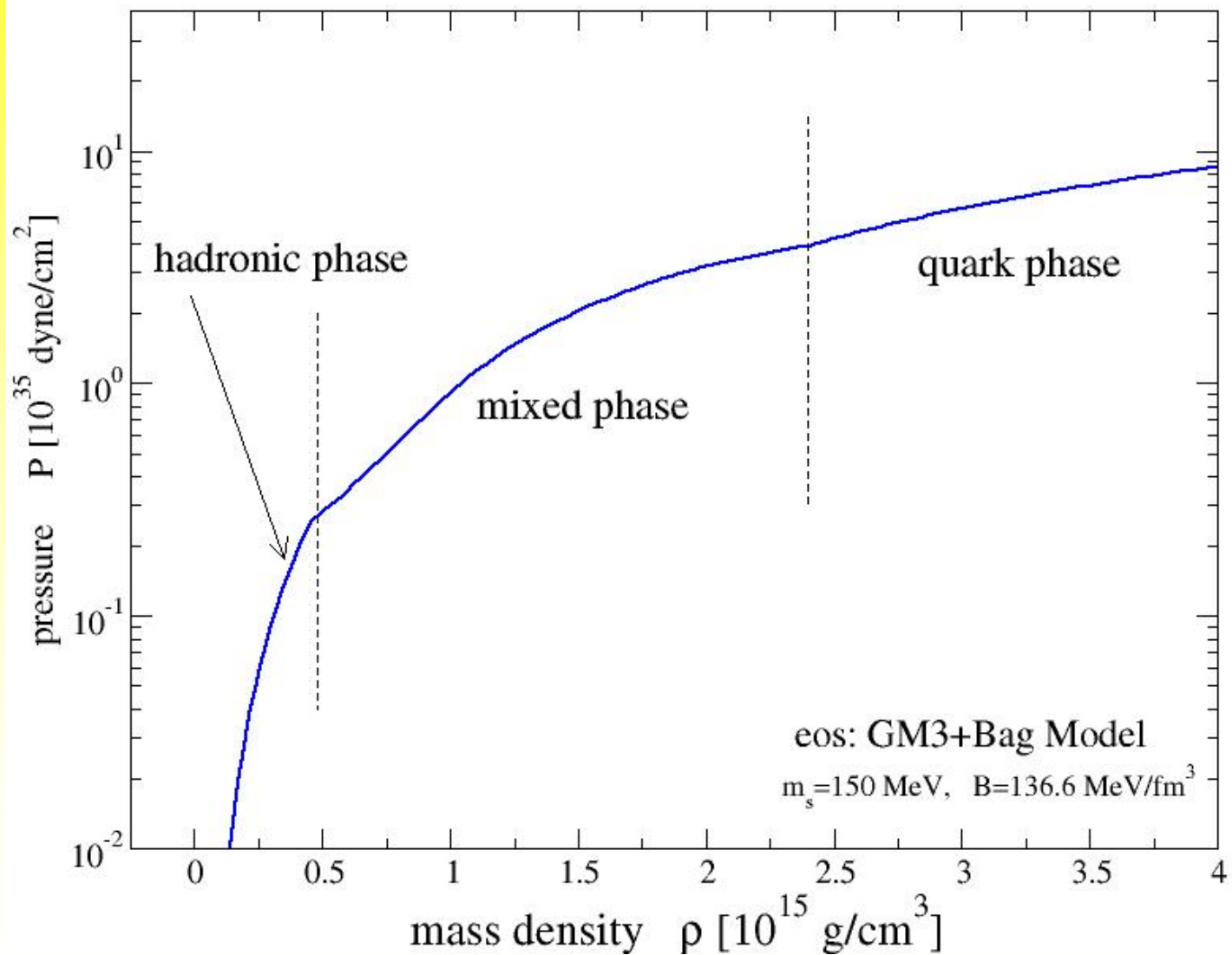
$$\mathbf{n}_B = \mathbf{n}_u = \mathbf{n}_d = \mathbf{n}_s$$

$$\begin{aligned} E_{Fq} &= \hbar c k_{Fq} = \hbar c (\pi^2 n_q)^{1/3} = \\ &= \hbar c (\pi^2 n_B)^{1/3} \geq m_c \end{aligned}$$

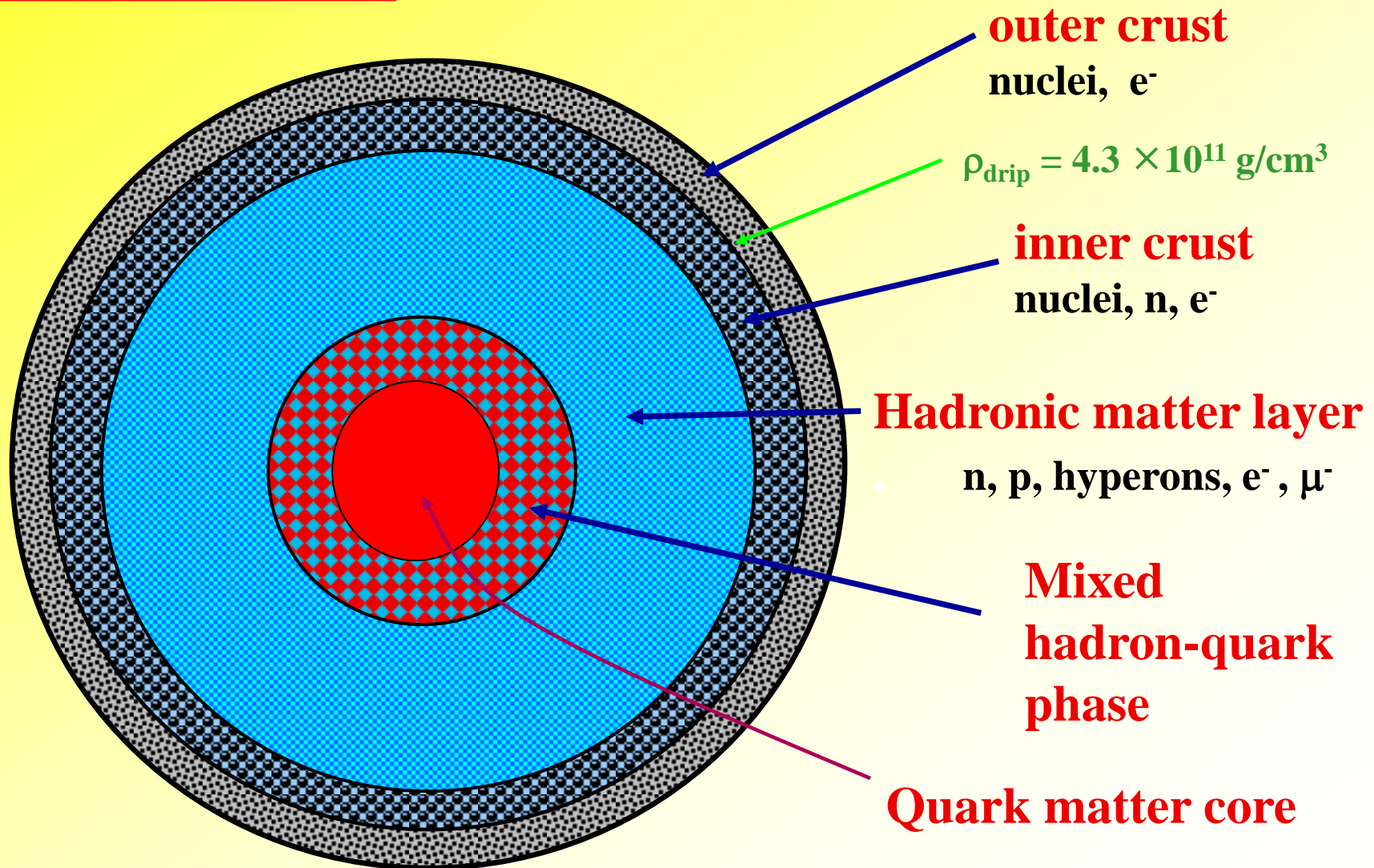


$$n_B \geq 27.3 \text{ fm}^{-3} \approx 171 n_0$$

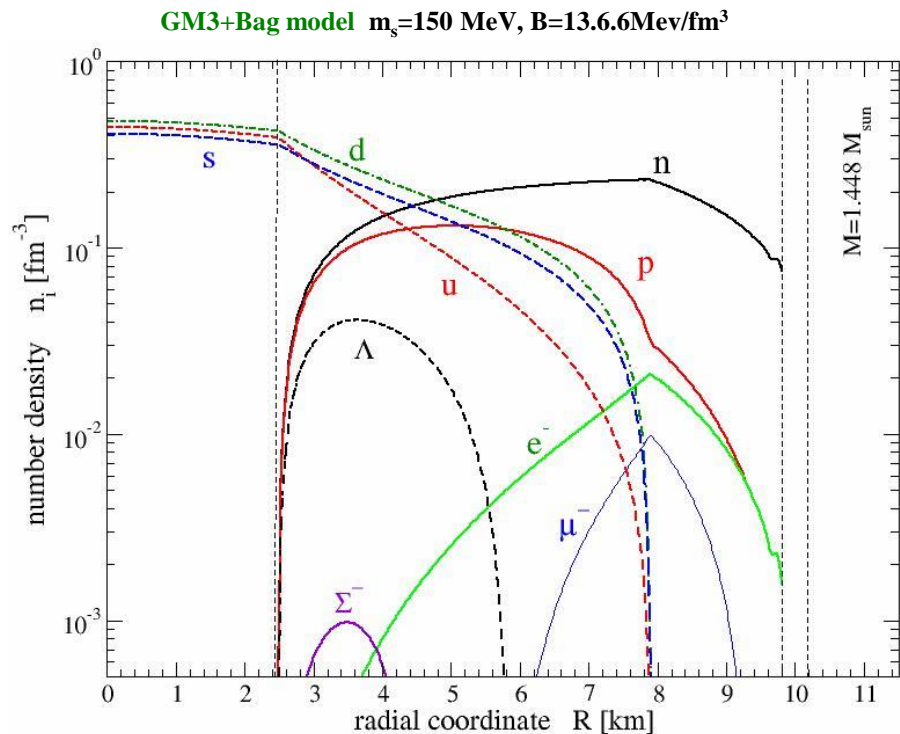
The EOS for Hybrid Stars



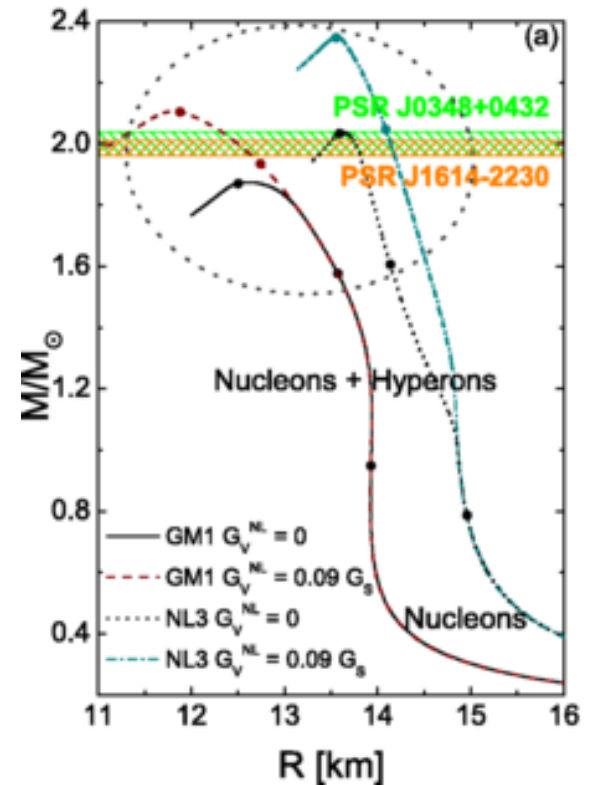
Hybrid Stars (neutron stars with a quark matter core)



Hybrid Stars (neutron stars with a quark matter core)



I. Bombaci, I. Parenti, I. Vidaña (2004)



M. Orsaria, H. Rodrigues, F. Weber, G.A. Contrera, Phys. Rev. C 89 (2014) 015806

perturbative QCD calculations up to α_s^2

A. Kurkela et al., Phys. Rev. D 81, (2010) 105021

M_{max} up to $\sim 2 M_{\odot}$

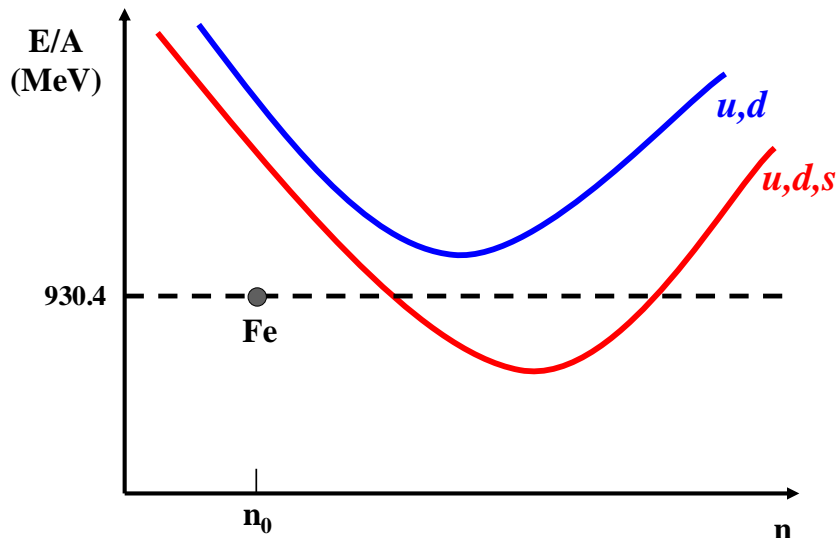
**Present measured NS masses
do not exclude the possibility of
having QM in the stellar core**

Strange Stars (compact stars made of strange quark matter)

The Strange Matter hypothesis (Bodmer (1971), Terazawa (1979), Witten (1984))

Three-flavor *u,d,s* quark matter, in equilibrium with respect to the weak interactions, could be the **true ground state of strongly interacting matter**, rather than ^{56}Fe

$$E/A|_{\text{SQM}} \leq E(^{56}\text{Fe})/56 \sim 930.4 \text{ MeV}$$



Stability of atomic nuclei
with respect to *u,d* QM decay

$$E/A|_{ud} \geq E(^{56}\text{Fe})/56$$

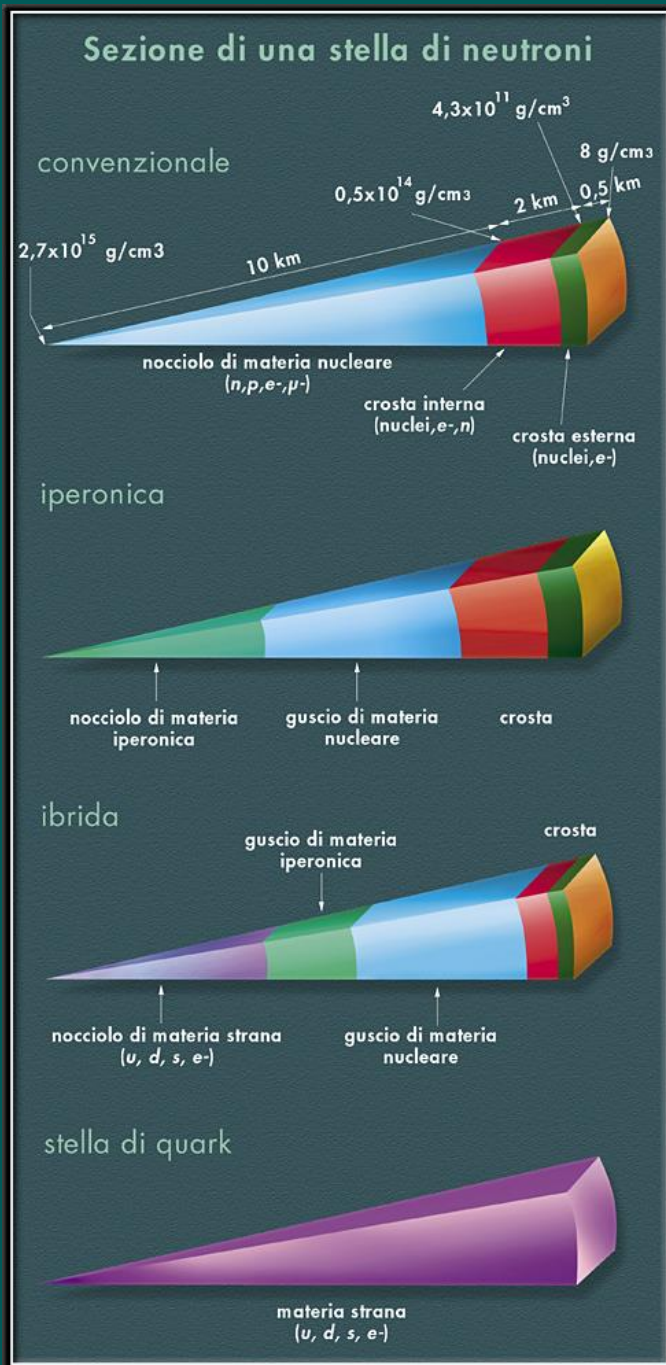
“Neutron Stars”

Nucleon Stars

Hyperon Stars

Hybrid Stars

Strange Stars



Dense matter EOS: open problems

(1) The Hadronic matter phase

(1a) uncertainties in the strength of the **NNN** interactions at high densities

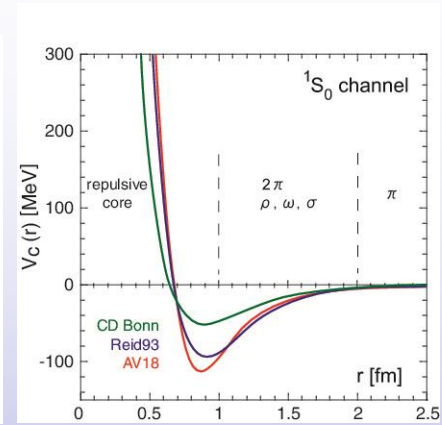
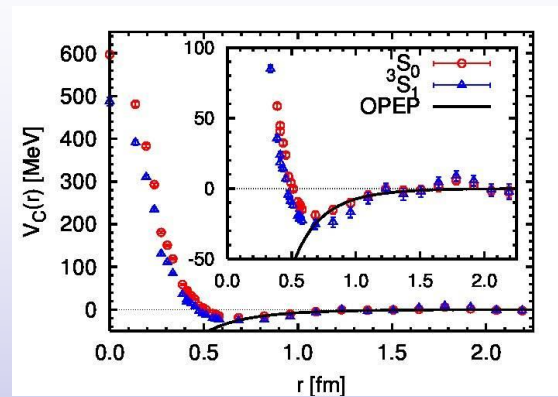
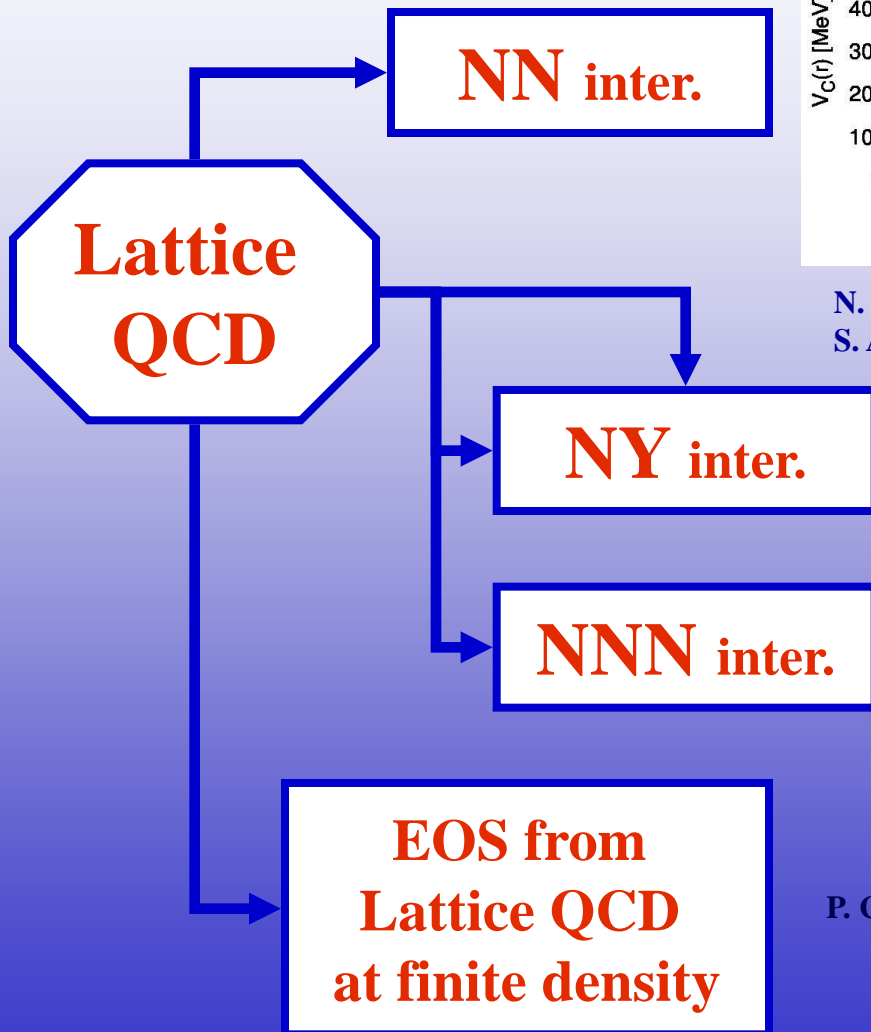
(1b) Poor knowledge of the **NY, YY** and **NNY, NYY, YYY** interactions

(2) The Quark matter phase

(2a) (1a) + (1b) \longrightarrow crucial to determine ρ_{crit}

(2b) inclusion of **non-perturbative QCD effects** which are crucial to determine the nature of the deconfinement transition and the stiffness of the quark matter phase EOS

Dense matter EOS: a true microscopic approach



N. Ishii, S. Aoki, T. Hatsuda, *Phys. Rev. Lett.* **99**, 022001 (2007)
 S. Aoki, T. Hatsuda, N. Ishii, *Prog.Theor. Phys.* **123**, 89 (2010)

H. Nemura, N. Ishii, S. Aoki, T. Hatsuda, *arXiv:0806:1096 nucl-th*

T. Doi et al. (HAL QCD collaboration), *arXiv:1106.2276 hep-lat*

P. Cea, et al., *Phys. Rev. D* **85**, 094512 (2012)