Chiral two-nucleon dynamics, analyticity and dispersion relations

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Introduction

QCD → Chiral Effective Theory → hadron dynamics

Effective Lagrangian

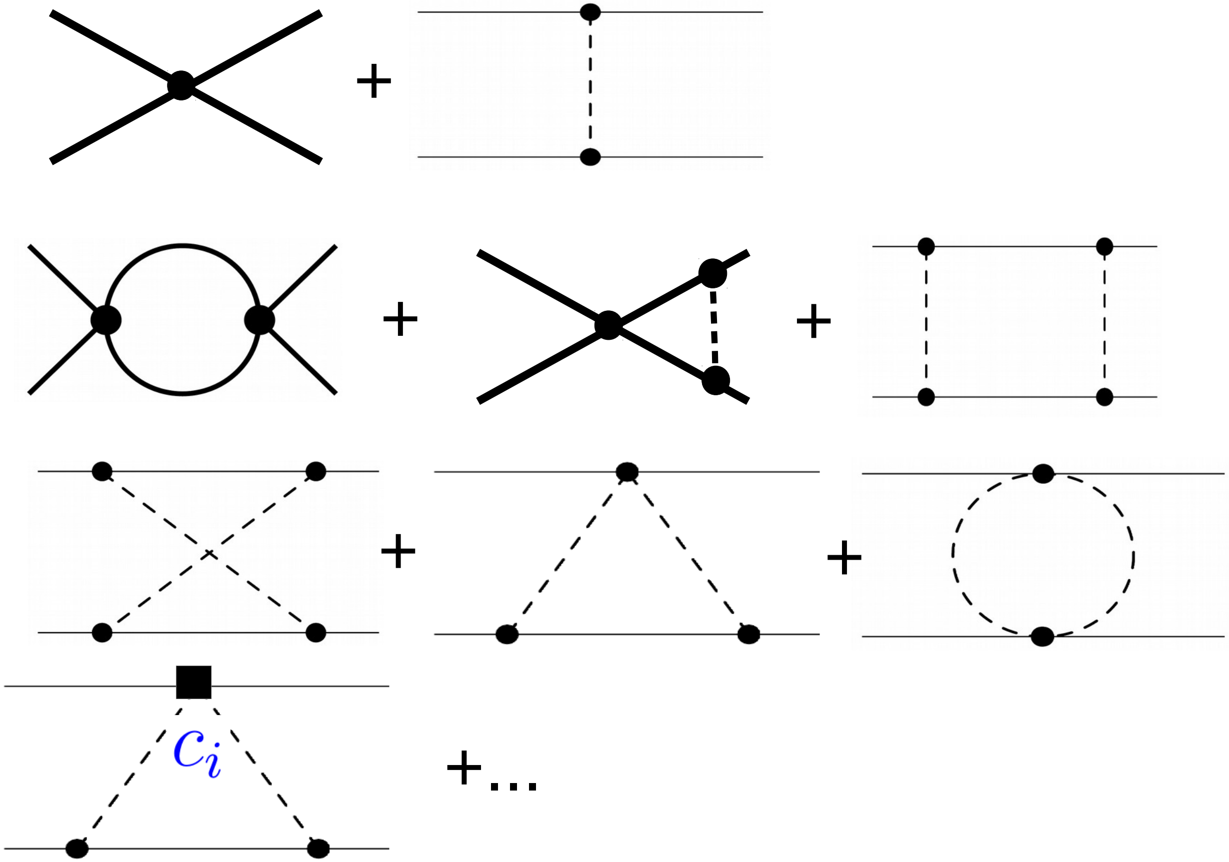
$$\mathcal{L}(\Psi_N, U = e^{(i\vec{\tau}\cdot\vec{\pi})/f}, D_\mu) = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

→ The most general S-matrix, consistent with perturbative unitarity, analyticity, symmetries

(Weinberg '79)

Power Counting

expansion in small parameter Q $\left(|\vec{p}_i|,\, M_\pi\right)/\left(\Lambda_\chi,\, m_N\right)$



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$\sim Q^2$

$\sim Q^3$

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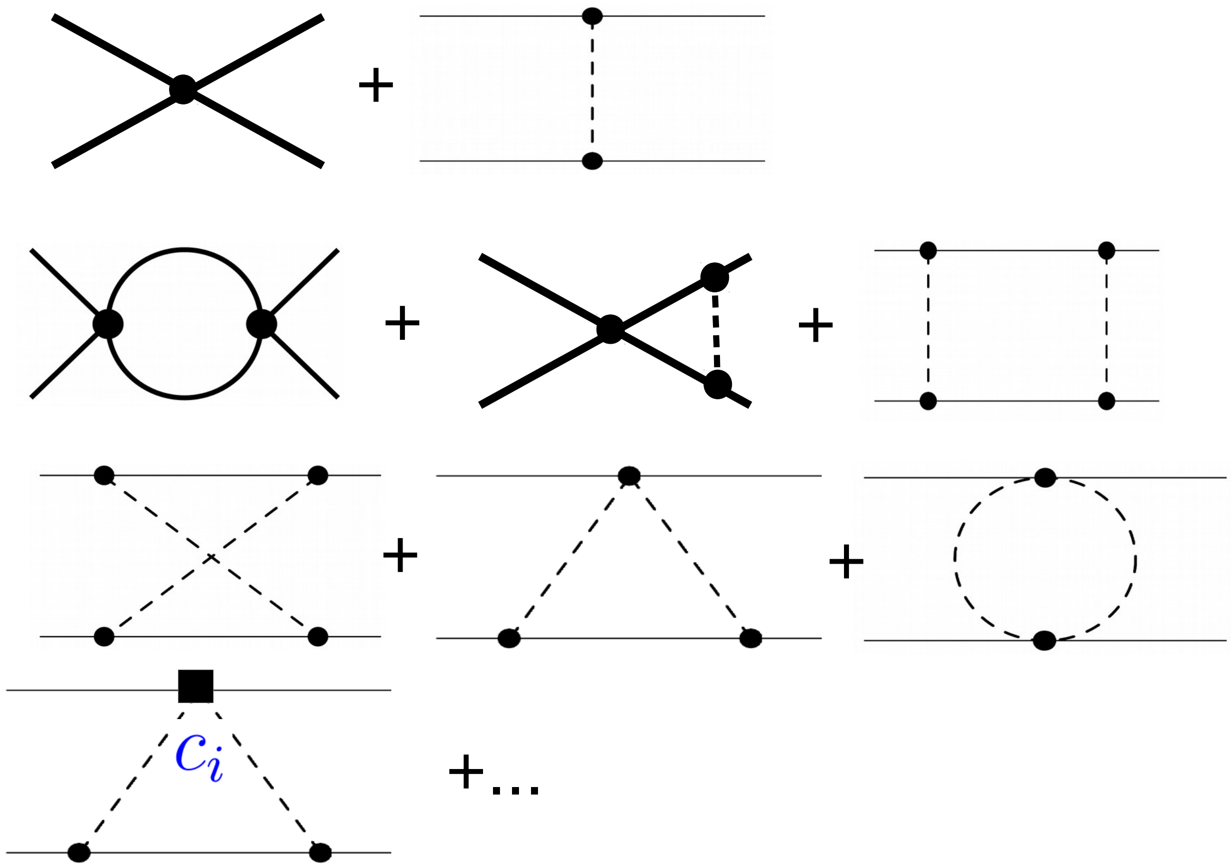

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Perturbative expansion does not converge close to threshold: bound state (deutron)

Non-perturbative approaches

KSW: resummation of leading contact terms   Kaplan, Savage, Wise '97

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Potential (Weinberg) approach:
Lippmann-Schwinger Equation with a cut-off:
resummation of all 2N reducible diagrams — Weinberg '90,'91; Entem, Machleidt '03; Epelbaum, Glöckle, Meißner '05

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The most general S-matrix, consistent with ~~perturbative~~ unitarity, analyticity, symmetries?

Combining ChPT with dispersive approach

Gasparyan, et al. , '12, Oller et al., '13, Goldberger et al.,'60

Take into account the analyticity along the right-hand cut nonperturbatively:

$$\frac{1}{2\,i}\left(T^{(JP)}(s+i\epsilon)-T^{(JP)}(s-i\epsilon)\right)=T^{(JP)}(s+i\epsilon)\,\rho^{(JP)}(s\,)\,T^{(JP)}(s-i\epsilon)$$

ρ(s)-phase space

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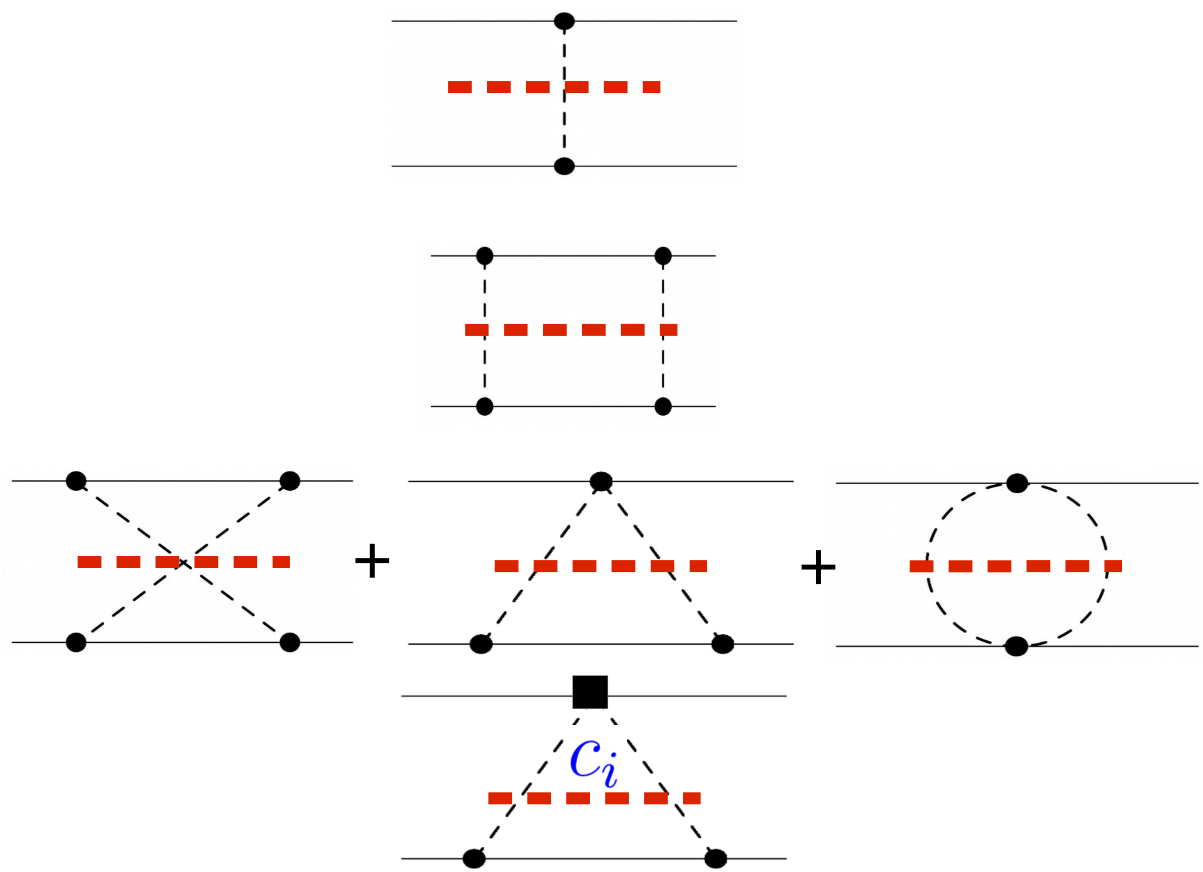
ρ(s)-phase space

$$T(s\,)=U(s\,)+\int_{4m_N^2}^{\infty}\frac{ds'}{\pi}\,\frac{T(s)\,\rho(s')\,T^*(s')}{s'-s-i\epsilon}\,\frac{s-\mu_M^2}{s'-\mu_M^2}$$

Left hand (t-channel) cuts: perturbative



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Left hand (t-channel) cuts: perturbative

$$U(s) = T_L(s) = \int_{\Lambda_t=4m_N^2-(3M_\pi)^2}^{4m_N^2-M_\pi^2} \frac{\Delta T(s')}{s'-s}\frac{ds'}{\pi} + \sum_i C_i\,\xi(s)^i$$

Short range contribution

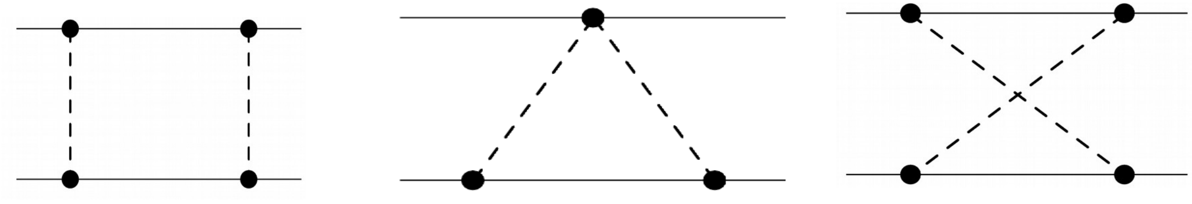
ξ(s)=s, or conformal mapping of s

Matching with ChPT expansion below threshold at $s=\mu_M^2 = 4m_N^2 - 2M_\pi^2$

C_i ⟷ LEC's of $\mathcal{L}_{NN}$

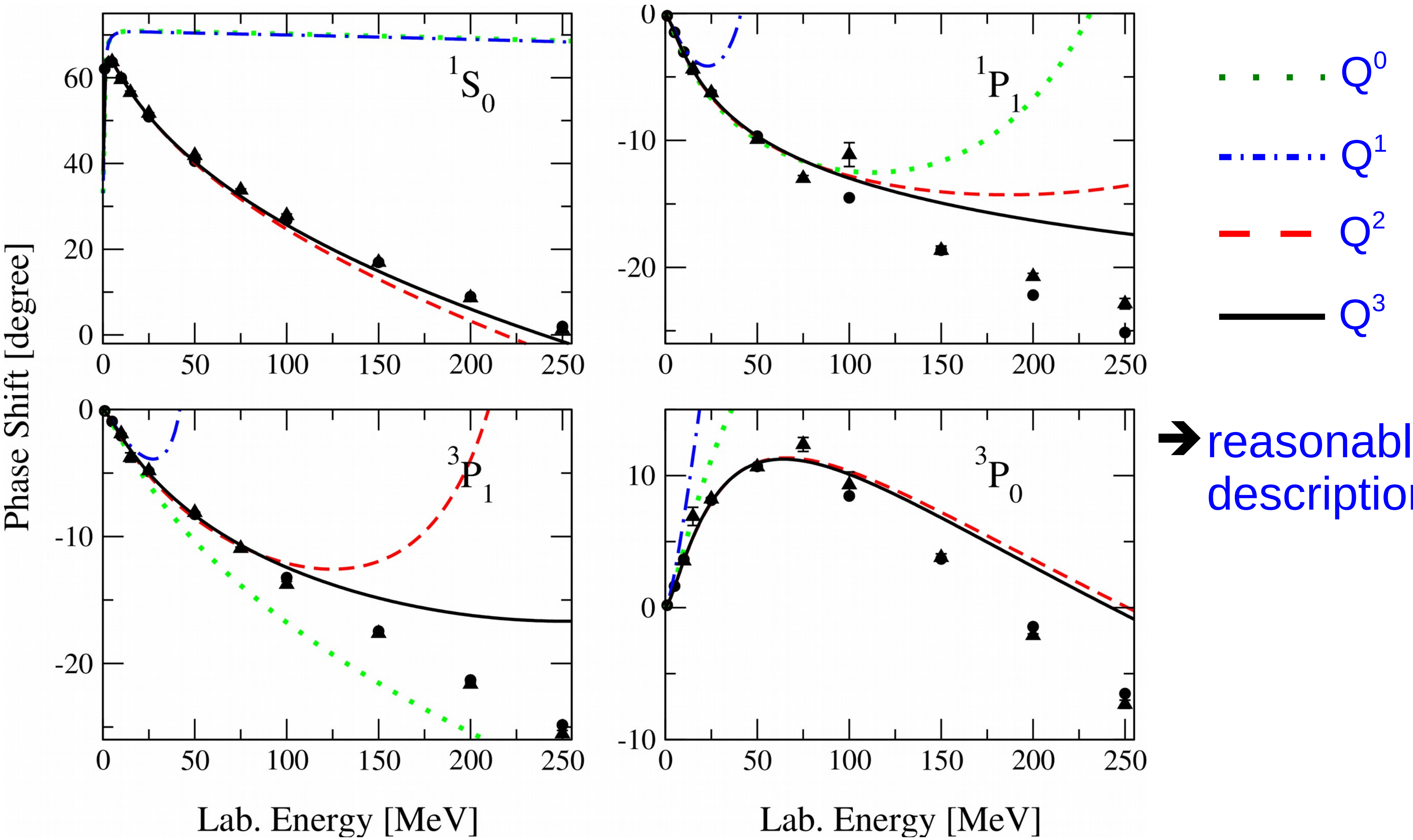
Left hand (t-channel) cuts: perturbative

Covariant amplitudes (correct singularity structure)



Not so much relevant in the potential approach Epelbaum, '06

Uncoupled S- and P-wave pn phase shifts



➔ reasonable description

● Nijmegen PWA ▲ SAID PWA

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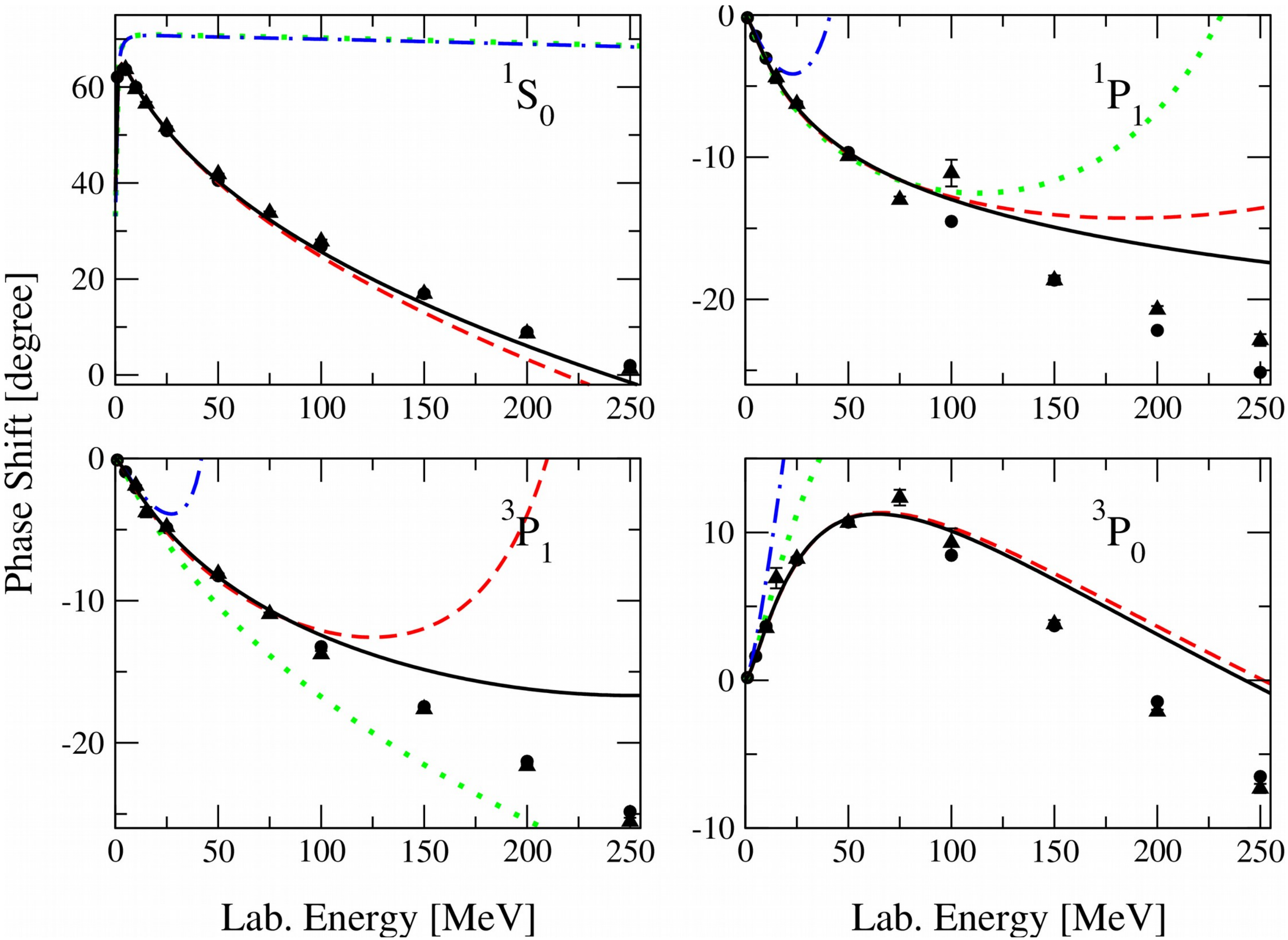

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➔ 2-pion cut contribution is not important

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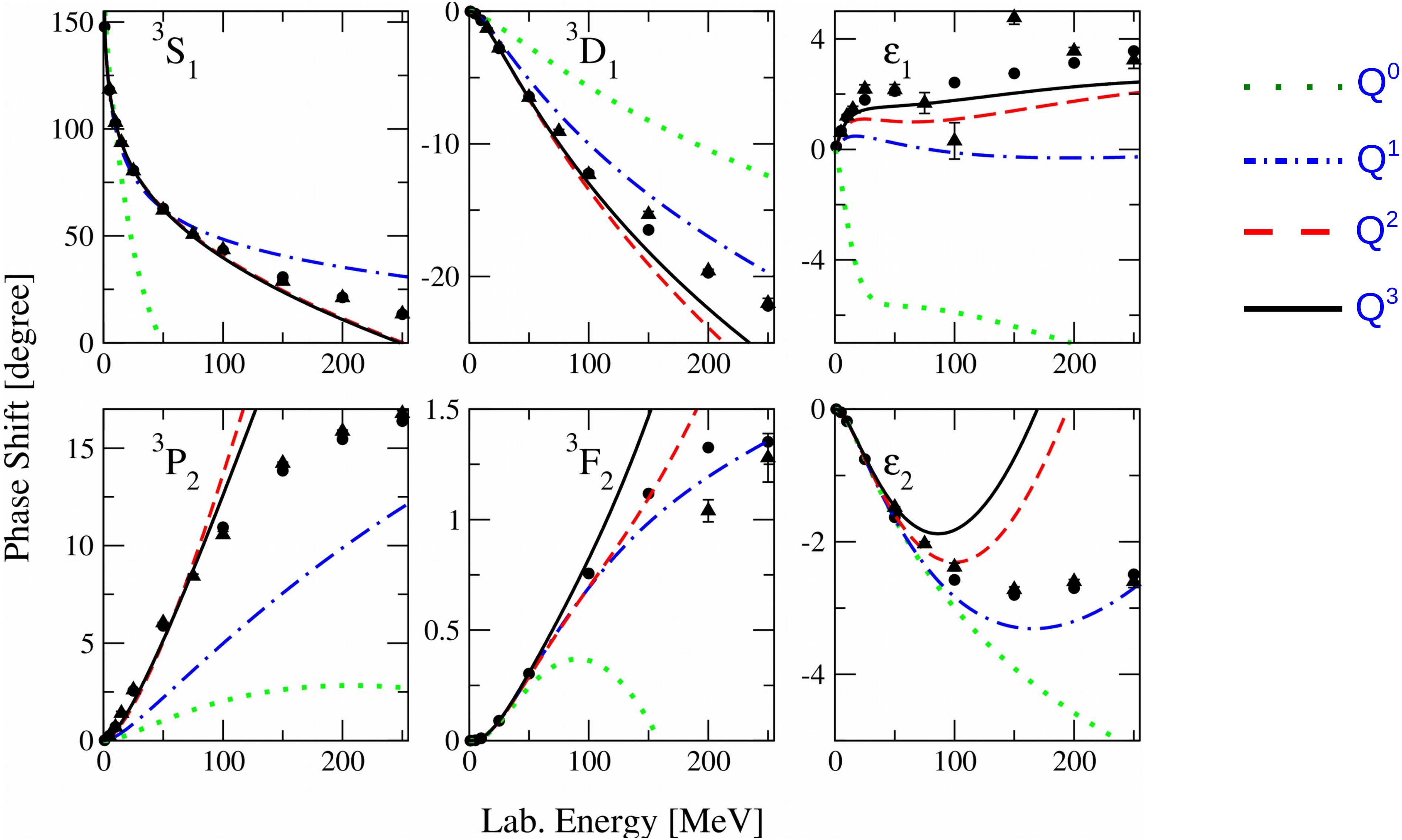
Coupled partial waves



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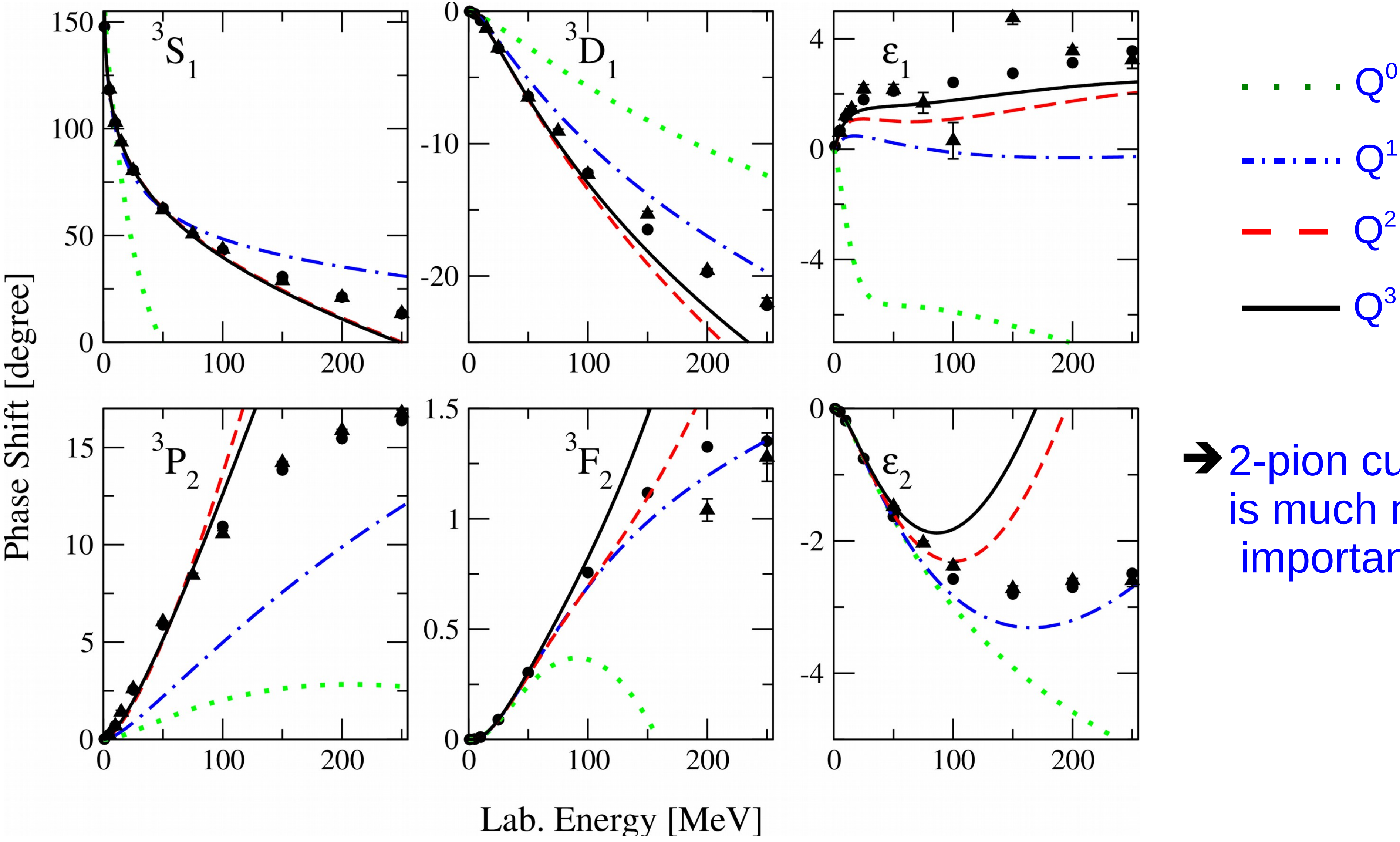
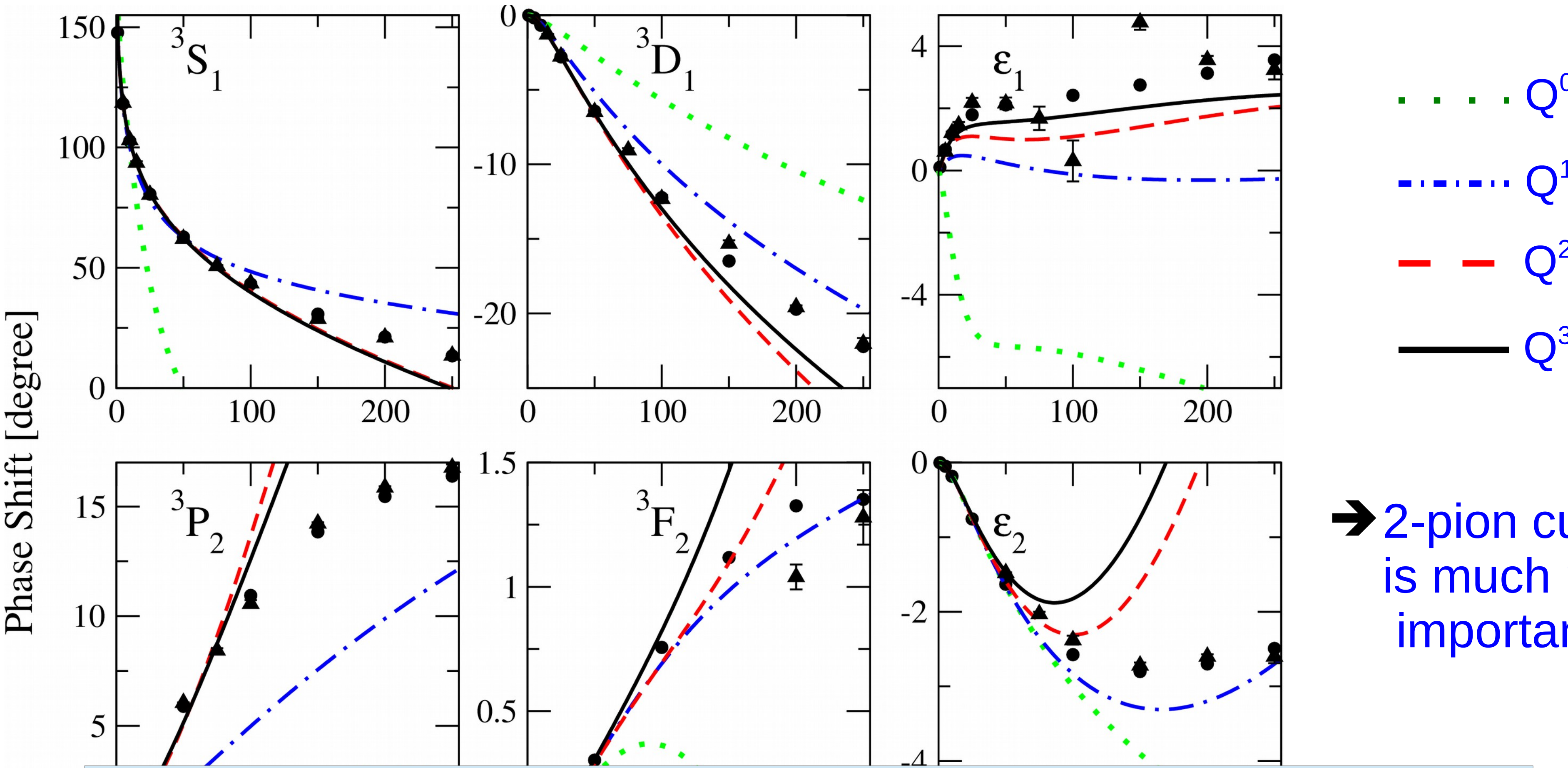


➔ 2-pion cut is much more important

Coupled partial waves



➔ 2-pion cut is much more important

It is preferable not to destroy the left-hand cut in a potential approach (r-space regularization) (Epelbaum et al. '15)

Convergence of chiral expansion below threshold

	$^1S_0$	$^1P_1$	$^3P_1$	$^3P_0$	$^3S_1$	$^3P_2$
Q^0	5.79×10^2	0	0	0	-2.95×10^2	0
Q^1	5.82×10^2	4.91×10^3	4.33×10^3	12.41×10^3	6.14×10^2	2.44×10^2
Q^2	8.53×10^2	1.42×10^3	3.46×10^3	-6.43×10^3	2.08×10^2	5.59×10^2
Q^3	8.63×10^2	2.19×10^3	4.81×10^3	-5.70×10^3	2.07×10^2	7.50×10^2

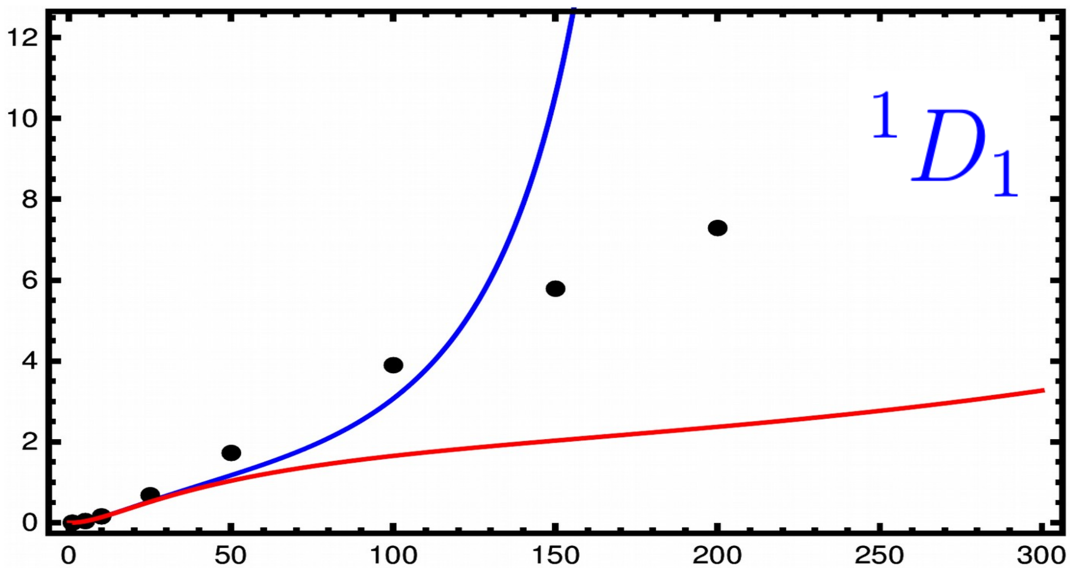
Amplitude at the subthreshold matching point $T(\mu_M^2 = 4m_N^2 - 2M_\pi^2)$ at different chiral orders after subtracting one-pion exchange contribution.

Scheme dependence

Non-uniqueness of the solution to the dispersion relation

(CDD poles ⟷ poles on an unphysical sheet)

Castillejo, Dalitz, Dyson '56



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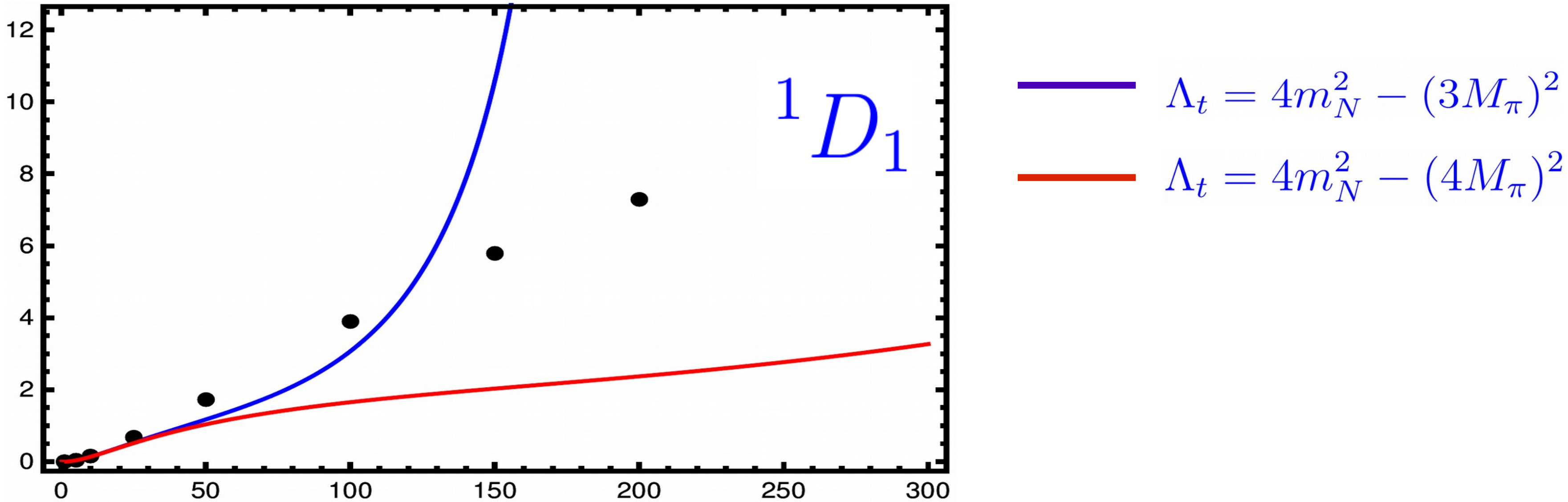

Cut off (renormalization scale) dependence in approaches with resummation

Summary

- The unitarity and analyticity constraints are used to extrapolate the NN amplitude from the subthreshold to the physical region
- Matching to the perturbative (ChPT) amplitude is applied in the subthreshold region
- Indication of convergence below threshold
- Solution to the dispersion relation is not unique – scheme dependence

Outlook

- Implement symmetry constraints and look at other reactions