Pseudoscalar decays into lepton pairs from rational approximants

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In collaboration with P. Masjuan

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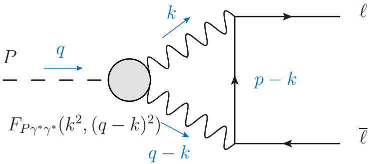
Outline

Section 1

The process and the puzzle

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state



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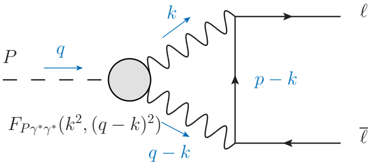

$$\frac{BR(P\to \ell\bar{\ell})}{BR(P\to\gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2,$$

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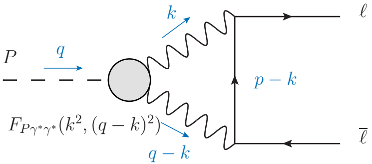

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$$\mathcal{A}(q^2) = \frac{2i}{\pi^2 q^2}\int d^4k \, \frac{q^2k^2-(k\cdot q)^2}{k^2(k-q)^2((p-k)^2-m_\ell^2)} \, \tilde{F}_{P\gamma^*\gamma^*}(k^2,(q-k)^2)$$

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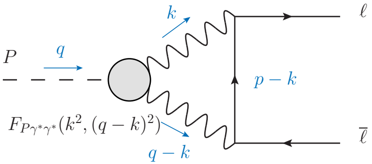

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Normalized $\tilde{F}_{P\gamma\gamma}(0,0) = 1$. It diverges without $\tilde{F}_{P\gamma\gamma}(k_1^2, k_2^2)$

The Puzzle (I): $\pi^0 \to e^+e^-$

$$BR^{KTeV}(\pi^0 \to e^+e^-) = 7.48(38) \times 10^{-8}$$

$$BR^{Th}(\pi^0 \to e^+e^-) = 6.23(09) \times 10^{-8}$$ Dorokhov et al '07

Which represents a $\mathbf{3\sigma}$ deviation

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$$BR^{KTeV+RC}(\pi^0 \to e^+e^-) = 6.87(36) \times 10^{-8}$$ Husek et al '14

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Still, **no model can reproduce such value**

$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ enters in $(g-2)_\mu^{HLbL} \Rightarrow$ impact?

—New Physics?—

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The Puzzle (II): $\eta \to \mu^+\mu^-$

From approximated calculations

$BR^{Exp}(\eta \to \mu^+\mu^-) = 5.8(8) \times 10^{-6}$

$BR^{Th}(\eta \to \mu^+\mu^-) = 5.35(27) \times 10^{-6}$ Dorokhov et al '07

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The Puzzle (II): $\eta \to \mu^+\mu^-$

However, exact calculations are required

$$BR^{Exp}(\eta \to \mu^+\mu^-) = 5.8(8) \times 10^{-6}$$

$$BR^{Th}(\eta \to \mu^+\mu^-) = 4.62(13) \times 10^{-6}$$

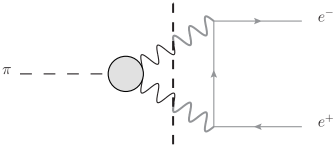
Just a **1.5σ** deviation; but **potentially large**

Section 2

Main properties

A baby problem: Light pseudoscalars

Very light → No hadronic states: Imaginary part from Cutcosky



$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_e(q^2)} ln\left(\frac{1-\beta_e(q^2)}{1+\beta_e(q^2)}\right)$$

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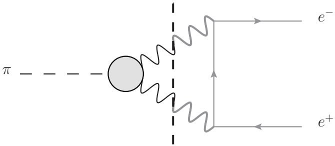

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This result is model-independent → Unitary Bound Drell '59

$$|\mathcal{A}|^2 \geq Im(\mathcal{A})^2 = (-17.52)^2; \quad BR(\pi \to e^+e^-) \geq 4.7 \times 10^{-8}$$

This is the further we can go without any information on $F_{\pi\gamma\gamma}(Q_1^2, Q_2^2)$

A baby problem: Light pseudoscalars

Given the relevant QCD scale $M_V \gg m_P, m_\ell$, approximations are possible

$$\mathcal{A}(m_{\pi^0}^2) \simeq \frac{i\pi}{2\beta_\ell} L + \frac{1}{\beta_\ell}\left(\frac{1}{4}L^2 + \frac{\pi^2}{12} + Li_2\left(\frac{\beta_\ell - 1}{1+\beta_\ell}\right)\right) - \frac{5}{4} + \int_0^\infty dQ \frac{3}{Q}\left(\frac{m_\ell^2}{m_\ell^2 + Q^2} - \tilde{F}_{\pi^0\gamma^*\gamma^*}(Q^2, Q^2)\right)$$

P. Masjuan, P. Sanchez, arXiv:15040.07001 [hep-ph]

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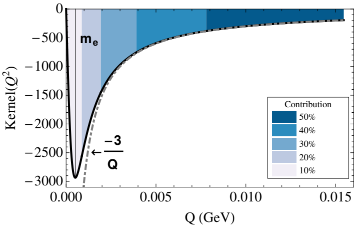

- Singularity from $\gamma\gamma$ $\frac{1}{Q}$ suppression
  Low space-like energies peak
- Peak at lepton mass
  IR regulator $\sim ln(m_e^2)$
- High energies, $F_{\pi\gamma\gamma}$ dominates
  UV regulator $\sim -ln(\Lambda^2)$

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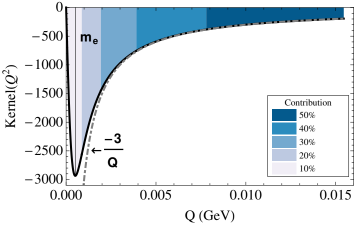

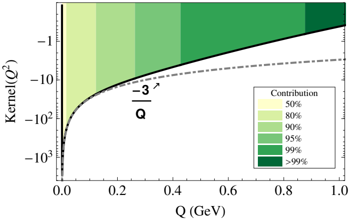

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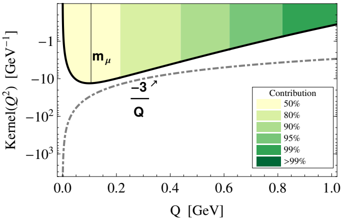

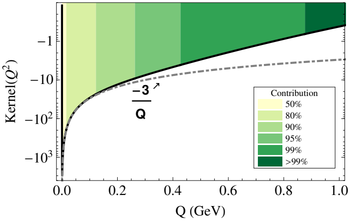

—Calculation Requires

$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ description precise at low space-like energies

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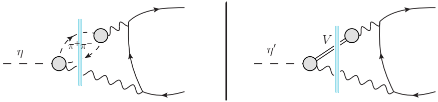
A grown-up problem: Heavier pseudoscalars

Unitary Bound



New hadronic contributions

P. Masjuan, P. Sanchez, In preparation

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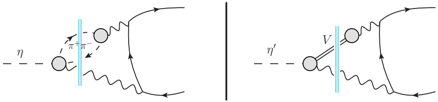

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First $Im(\mathcal{A})$ estimation ever: realistic (toy) models

$$\eta : Im(\pi\pi)/Im(\gamma\gamma) = -0.5\%$$
$$\eta' : Im(\rho,\omega)/Im(\gamma\gamma) = -20\%$$

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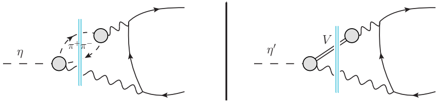

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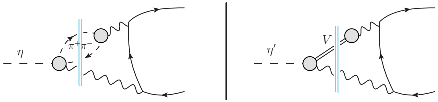

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Integral sneaks into time-like

- Only up to $m_P^2 \to$ Dalitz decay
- Ensure a proper description there

P. Masjuan, P. Sanchez, In preparation

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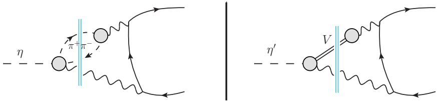

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Approximations are poor

- $m_{P,\ell}/M_V$ or m_ℓ/m_P corrections relevant for η, η', μ
- Exact calculation required

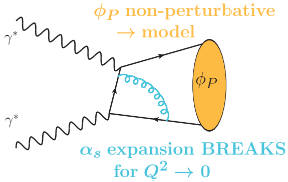
P. Masjuan, P. Sanchez, In preparation

Section 3

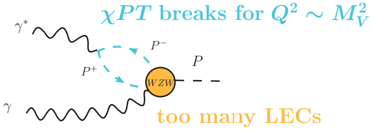
A rational description for the transition form factor

The problem: a first principle QCD description for the TFF

—High Energies: pQCD



—Low Energies: χPT



$Q^2 \to \infty$

$F_{\pi\gamma\gamma^*}(0,\infty)=2F_\pi Q^{-2}$
$F_{\pi\gamma^*\gamma^*}(\infty,\infty)=2/3F_\pi Q^{-2}$

?

$Q^2 \to 0$

$F_{\pi\gamma\gamma}(0,0)=(4\pi^2 F_\pi)^{-1}$

Objectives and strategies

—What do we need?

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Generalize our approach to bivariate functions: Chisholm Approximants

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Padé Approximants: Introduction to the method

Given a function with known series expansion

$$F_{P\gamma\gamma^*}(Q^2) = F_{P\gamma\gamma^*}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots) \quad \text{i.e. } \chi PT$$

Its Padé approximant is defined as

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = F_{P\gamma\gamma^*}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots + \mathcal{O}(Q^2)^{N+M+1})$$

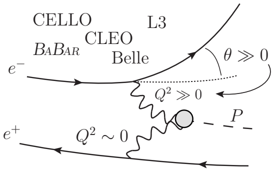
Convergence th. $\Rightarrow$ Model-independency

Increase$\{N, M\}$ $\Rightarrow$ Systematic error estimation

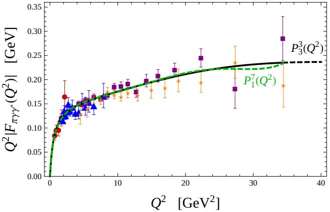
$$P^0_1 = \frac{F_{P\gamma\gamma^*}(0)}{1 - b_P Q^2} = F_{P\gamma\gamma^*}(0)(1 + b_P Q^2 + \mathcal{O}(Q^4)) \; \text{~~VMD~~} \rightarrow \chi PT + pQCD$$

Correct low energy implementation!

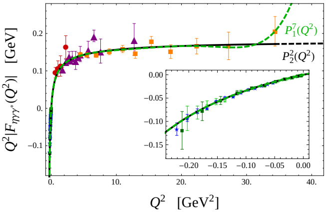
Padé Approximants: Results



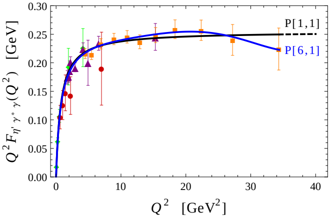
π



η



η'



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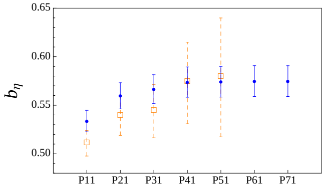

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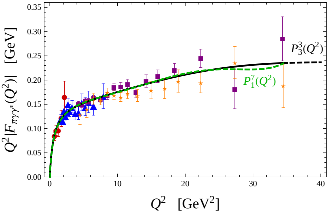
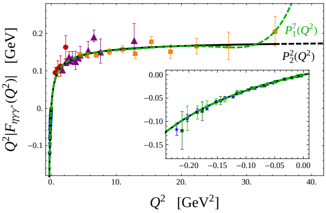


η



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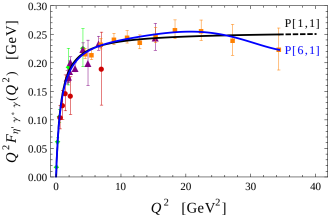

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What about the double virtual $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Extend Padé approximants to bivariate case (Chisholm '73)

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 - b_P(Q_1^2 + Q_2^2) + (2b_P^2 - a_{1,1})(Q_1^2 Q_2^2)}$$

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—Properties

1.Reproduce original series expansion $\Rightarrow$ low energies

$$C_1^0(Q_1^2, Q_2^2) = F_{P\gamma\gamma}(0,0)(1 + b_P(Q_1^2 + Q_2^2) + a_{1,1} Q_1^2 Q_2^2 + ...)$$

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—Properties

1.Reproduce original series expansion $\Rightarrow$ low energies

2.Reduce to Padé Approximants (already determined)

$$C_1^0(Q^2, 0) = \frac{F_{P\gamma\gamma}(0,0)}{1 - b_P Q^2} = P_1^0(Q^2)$$

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1.Reproduce original series expansion $\Rightarrow$ low energies

2.Reduce to Padé Approximants (already determined)

3.Can incorpore QCD constrains from OPE

$$C_1^0(Q_1^2, Q_2^2)|_{OPE} = \frac{F_{P\gamma\gamma}(0,0)}{1 + b_P(Q_1^2 + Q_2^2)}; \quad (a_{1,1} \equiv 2b_P^2) \ OPE\checkmark$$

What about the double virtual $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Extend Padé approximants to bivariate case (Chisholm '73)

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 - b_P(Q_1^2 + Q_2^2) + (2b_P^2 - a_{1,1})(Q_1^2 Q_2^2)}$$

—Properties

1.Reproduce original series expansion $\Rightarrow$ low energies

2.Reduce to Padé Approximants (already determined)

3.Can incorpore QCD constrains from OPE

4.Can be factorized as hinted by χPT (low-energies) leading logs

$$C_1^0(Q_1^2, Q_2^2)|_{OPE} = \frac{F_{P\gamma\gamma}(0,0)}{(1 + b_P Q_1^2)(1 + b_P Q_2^2)}; \quad (a_{1,1} \equiv b_P^2) \ \textit{Factorization}$$

What about the double virtual $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

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—Properties

1.Reproduce original series expansion $\Rightarrow$ low energies

2.Reduce to Padé Approximants (already determined)

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Parameter $a_{1,1}$ from data $\Rightarrow$ Low-energies ... But not available!
Take generous range, i.e. $a_{1,1} \in \{0 \div 2b_P^2\}$ (includes OPE, fact)

Section 4

Results

$\pi^0 \to e^+e^-$

Our Result

$$BR(\pi^0 \to e^+e^-) = (6.20 \div 6.41)(5) \times 10^{-8}; a_{1,1} \in \{2b_P^2 \div 0\}$$

Accepted value: $6.23(9) \times 10^{-8}$(Dorokhov et.al. '07) $\Rightarrow$ UNDERESTIMATED

P. Masjuan, P. Sanchez, arXiv:1504:07001[hep-ph]

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Hypothetic Double-Virtual Data below 1GeV 30% Error

$$BR(\pi^0 \to e^+e^-) = 6.36(5)_{b_\pi}(4)_{a_{11}}(6)_{sys} \times 10^{-8} \to 6.36(8) \times 10^{-8}$$

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Fix $a_{1,1}$ To Experiment $\Rightarrow (g-2)_\mu^{HLbL;\pi^0}$ Impact?

$$a_\mu^{HLbL;\pi^0} = (5.10 \div 6.64)10^{-10} \Rightarrow 2.85 \times 10^{-10} \quad (1.8\sigma^{FNAL})$$

P. Masjuan, P. Sanchez, arXiv:1504:07001[hep-ph]

$\eta \to \bar{\ell}\ell$

Our C_1^0 Result [exact] - (Preliminary Results)

$$\eta \to e^+e^- = (5.31 \div 5.44)(^{+4}_{-5})10^{-9}$$
$$\eta \to \mu^+\mu^- = (4.52 \div 4.72)(^{+4}_{-8})10^{-6}$$

P. Masjuan, P. Sanchez, In preparation

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P. Masjuan, P. Sanchez, In preparation

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Compare to Experiment

$\eta \to e^+e^- \leq 2.3 \times 10^{-6}$ HADES '14

$\eta \to \mu^+\mu^- = 5.8(8) \times 10^{-6}$ SATURNE-II '94

P. Masjuan, P. Sanchez, In preparation

$\eta' \to \bar{\ell}\ell$

Our C_1^0 & Combined Results [exact] - Preliminary Results

$$\eta' \to e^+e^- = (1.82 \div 1.86)(11)10^{-10}$$
$$\eta' \to \mu^+\mu^- = (1.36 \div 1.49)(10)10^{-7}$$

P. Masjuan, P. Sanchez, In preparation

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P. Masjuan, P. Sanchez, In preparation

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Compare to Experiment

$$\eta' \to e^+e^- \leq 5.6 \times 10^{-9} \text{ (SND+CMD-III) '15}$$
$$\eta' \to \mu^+\mu^- = \text{—}$$

P. Masjuan, P. Sanchez, In preparation

Section 5

Summary & Outlook

Summary & Outlook

- Rational Approximants have been used to describe the TFF
- Is data driven: better data, better description and easy to apply
- Precise low-energies but QCD constraints as well
- We updated $P \to \bar{\ell}\ell$ decays and included systematics
- $\pi \to e^+e^-$, $\eta \to \mu^+\mu^-$ discrepancy $\Rightarrow (g-2)_\mu^{HLbL;P}$, New Phys.?
- $\gamma^*\gamma^* \to P$ (allows $C_1^0 \to C_2^1$) and $P \to \bar{\ell}\ell$ required

- Future: pQCD matching, including cuts and resonance appropriately

Section 6

Bacukp

Low Energies *vs* High Energies

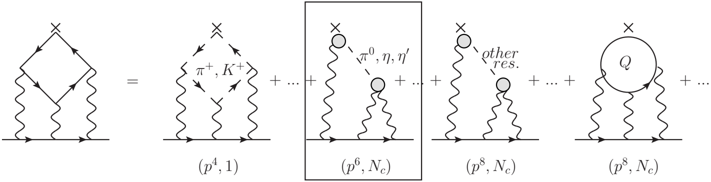
As an illustration: LMD+V model formerly used in $(g-2)_\mu^{HLbL;\pi^0}$

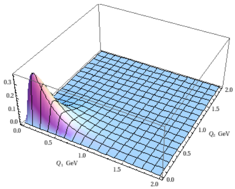
$$F_{\pi^0\gamma^*\gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{h_7 + h_5(q_1^2+q_2^2) + h_2 q_1^2 q_2^2 + (1) q_1^2 q_2^2 (q_1^2+q_2^2)}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

$$h_7 = -\frac{N_c}{4\pi^2}\frac{M_{V_1}^4 M_{V_2}^4}{F_\pi^2} \quad , \quad h_2 \in (-10,10) \quad , \quad (1) \to OPE$$

Process	h_2	Exact	C_1^0	$a_{1,1} \in (0, 2b_P^2)$
$BR(\pi^0 \to e^+e^-) \times 10^8$	10	6.38	6.36	6.19-6.40
	0	6.32	6.34	
	-10	6.26	6.32	
$a_\mu^{HLbL;\pi^0} \times 10^{10}$	10	5.3	5.5	5.3-6.8
	0	5.8	5.6	
	-10	6.3	5.8	

$(g-2)_\mu$: hadronic light-by-light





Knecht & Nyffeler: π^0, η, η'-exchange

- Loop integral involving $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$
- SL low energy regime $\rightarrow$ our PAs are good
- Multiscale: low-high energies

$(g-2)_\mu$: hadronic light-by-light

Our Results from Bivariate Padé Approximants

Units of 10^{-10}	π^0	η	η'	Total
$a_{1,1} = 2b_P^2$ [OPE]	6.64(33)	1.69(6)	1.61(21)	$9.94(40)_{stat}(50)_{sys}$
$a_{1,1} = b_P^2$ [Fact]	5.53(27)	1.30(5)	1.21(12)	$8.04(30)_{stat}(40)_{sys}$
$a_{1,1} = 0$	5.10(23)	1.16(7)	1.07(15)	$7.33(28)_{stat}(37)_{sys}$

$$a_\mu^{HLbL;P} = (9.94(40)(50) \div 7.33(28)(37)) \times 10^{-10} \quad (1.6\sigma^{FLab})$$

Big uncertainty from double-virtual term often non-considererd
High-energies *vs.* Low-energies

To be compared with pseudoscalar-pole contributions in the literature
BPP: 8.5(1.3); **HKS**: 8.6(0.6); **KN**: 8.3(1.2)

Toy Model for $P \to \bar{\ell}\ell$: Unitarity Analtiticity & Cuts

$$F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma\gamma^*}(q_1^2) \times F_{P\gamma\gamma^*}(q_2^2)$$

$$F_{P\gamma\gamma^*}(q^2) = c_{P\rho} G_\rho(q^2) + c_{P\omega} G_\omega(q^2) c_{P\phi} G_\phi(q^2)$$

Based on Dumm, Pich, Portoles PRD62 and Dumm, Roig, EPJC73

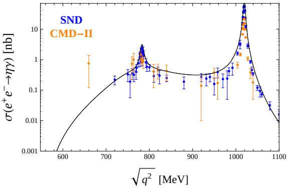
$$G_\rho(s) = \frac{M_V^2}{M_\rho^2 - s + \frac{sM_\rho^2}{96\pi^2 F_\pi^2}\left(ln\left(\frac{m_P^2}{\mu^2}\right) + \frac{8m_P^2}{s} - \frac{5}{3} - \sigma(s)^3 ln\left(\frac{\sigma(s)-1}{\sigma(s)+1}\right)\right)}$$

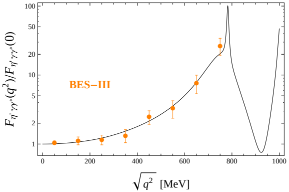
For narrow resonances

$$G_{\omega,\phi} = \frac{M_{\omega,\phi} + M_{\omega,\phi}\Gamma_{\omega,\phi}\sqrt{s_{th}/M_{\omega,\phi}}}{M_{\omega,\phi} - s + M_{\omega,\phi}\Gamma_{\omega,\phi}\sqrt{(s_{th} - s)/M_{\omega,\phi}}}$$

Toy Model for $P \to \bar{\ell}\ell$: Unitarity Analtiticity & Cuts

$$F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma\gamma^*}(q_1^2) \times F_{P\gamma\gamma^*}(q_2^2)$$
$$F_{P\gamma\gamma^*}(q^2) = c_{P\rho} G_\rho(q^2) + c_{P\omega} G_\omega(q^2) c_{P\phi} G_\phi(q^2)$$





Toy Model for $P \to \bar{\ell}\ell$: Unitarity Analtiticity & Cuts

Integration is easy through Cauchy's integral Formula

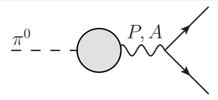
$$G(q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} dM^2 \frac{Im\left[G(M^2)\right]}{M^2 - q^2 - i\epsilon}$$

Then our loop integral can be solved through standard procedures

$$\mathcal{A} = \frac{1}{\pi^2}\int_{s_{th}}^{\infty}\int_{s_{th}}^{\infty} dM_1^2 dM_2^2 Im\left[G(M_1^2)\right] Im\left[G(M_2^2)\right] \times$$
$$\times \int_{Loop} d^4k[...] \frac{1}{k^2 - M_1^2 + i\epsilon}\frac{1}{(q-k)^2 - M_2^2 + i\epsilon}$$

New Physics contributions

From quantum numbers either axial or pseudoscalar interaction



$$\mathcal{L}^A = \frac{g}{4m_W} m_A c_f^A (\bar{f}\gamma^\mu\gamma_5 f) A_\mu$$

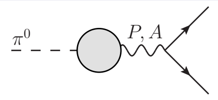
$$\mathcal{L}^P = \frac{g}{2m_W} m_f c_f^P (\bar{f} i\gamma_5 f) P$$

$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2) \left| \mathcal{A}(m_\pi^2) + \frac{\sqrt{2} F_\pi G_F}{4\alpha^2 F_{\pi\gamma\gamma}} (f^A + f^P) \right|^2$$

$$\frac{BR(\eta \to \mu^+\mu^-)}{BR(\eta \to \gamma\gamma)} = 2\left(\frac{\alpha m_\mu}{\pi m_\eta}\right)^2 \beta_\mu(m_\eta^2) \left| \mathcal{A}(m_\eta^2) + \frac{\sqrt{2} G_F}{4\alpha^2} (F_\eta^q (f_q^A + f_q^P) + \sqrt{2} F_\eta^s (f_s^A + f_s^P)) \right|^2$$

New Physics contributions

From quantum numbers either axial or pseudoscalar interaction



$$\mathcal{L}^A = \frac{g}{4m_W} m_A c_f^A (\bar{f}\gamma^\mu\gamma_5 f) A_\mu$$

$$\mathcal{L}^P = \frac{g}{2m_W} m_f c_f^P (\bar{f} i\gamma_5 f) P$$

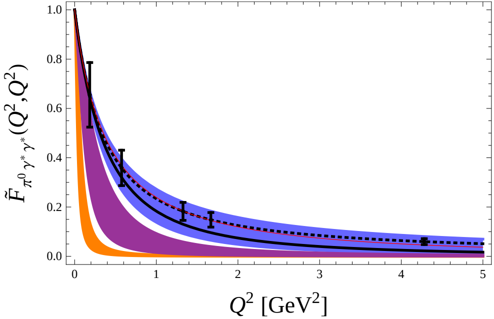
$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2) \left| -17.5i + 10.5 + 0.025(f^A + f^P) \right|^2$$

$$\frac{BR(\eta \to \mu^+\mu^-)}{BR(\eta \to \gamma\gamma)} = 2\left(\frac{\alpha m_\mu}{\pi m_\eta}\right)^2 \beta_\mu(m_\eta^2) \Big| -1.6 - 5.5i + 0.025\left(\frac{F_\eta^q}{F_\pi}(f_q^A + f_q^P) + \sqrt{2}\frac{F_\eta^s}{F_\pi}(f_s^A + f_s^P)\right) \Big|^2$$

$$f^A_{\pi,q,s} = c_e^A[(c_u^A - c_d^A), (c_u^A + c_d^A), (c_s)] \quad f^P_{\pi,q,s} = c_e^P[(c_u^P - c_d^P), (c_u^P + c_d^P), (c_s)]\frac{m^2_{\pi,\eta}}{m^2_{\pi,\eta} - m_P^2}$$

Double virtual transition form factor

Check $BR(\pi^0 \to e^+e^-)$ implications on $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$
More relevant at the $Q_1^2 = Q_2^2 = Q^2$ region



KTeV, Radiative corrections, Our prediction

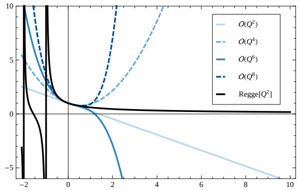
— Factorization — Regge large-N_c model - - - Bivariate P_1^0 fit;

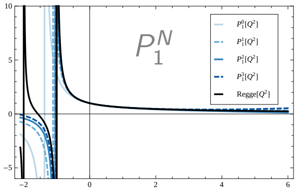
Padé Approximants: Convergence properties

Convergence known for meromorphic (large-N_c) and Stieltjes (DR)

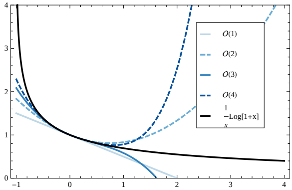
for the last $\lim_{N\to\infty} P^N_{N+1}(x) \leq f(x) \leq P^N_N(x)$

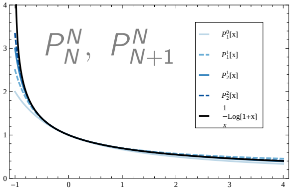
$$\Big| \quad F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{Q^2} \frac{a}{\psi^{(1)}\left(\frac{M^2}{a}\right)} \left(\psi^{(0)}\left(\frac{M^2+Q^2}{a}\right) - \psi^{(0)}\left(\frac{M^2}{a}\right)\right) \quad \Big|$$





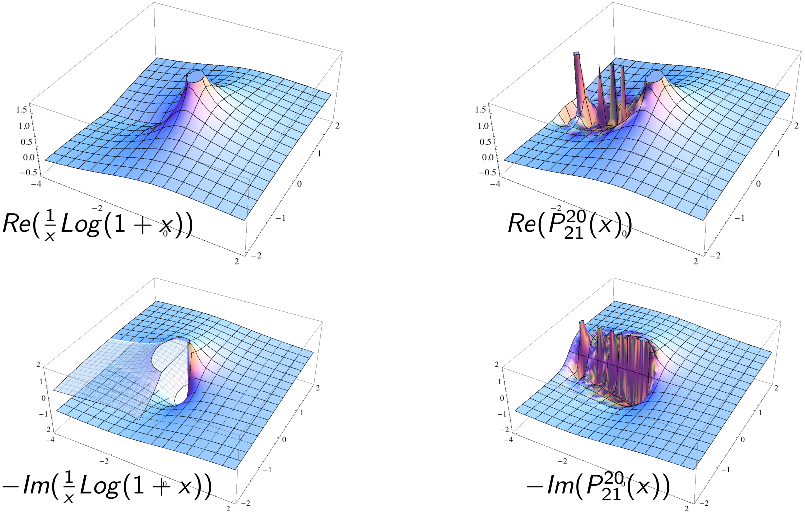
Taylor || $f(x) = \frac{1}{x} Log(1+x)$ || Padé





Padé Approximants: Convergence properties

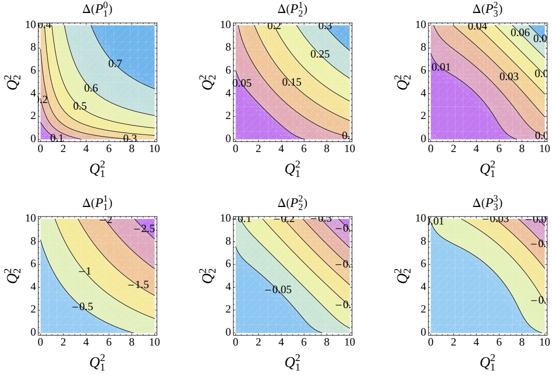
II. Stieljes functions: $(1/x)ln(1+x)$



Our proposal: Bivariate Padé Approximants

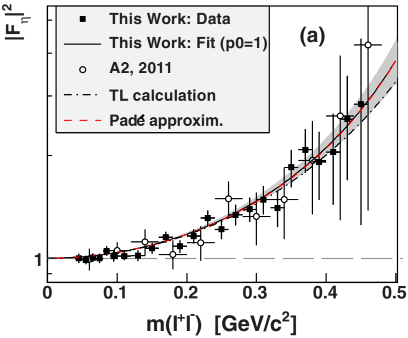
Lets revisit the Regge Model

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{Q_1^2 - Q_2^2} \frac{a}{\psi^{(1)}\left(\frac{M^2}{a}\right)} \left(\psi^{(0)}\left(\frac{M^2+Q_1^2}{a}\right) - \psi^{(0)}\left(\frac{M^2+Q_2^2}{a}\right)\right)$$



Obeys $P^N_{N+1}(x,y) \leq f(x,y) \leq P^N_N(x,y)$ (Stieltjes)

Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$



Compare to A2 Coll. results in Mainz [Phys.Rev. C89 (2014) 044608]
The results are excellent → reasonable to use them in our fit

$\eta' \to \bar{\ell}\ell$

Our C_1^0 & Combined Results [exact] - Preliminary Results

$$\eta' \to e^+e^- = (1.82 \div 1.86)(7)10^{-10} \xrightarrow{C_1^0+Res} (1.73 \div 1.77)(7)10^{-10}$$
$$\eta' \to \mu^+\mu^- = (1.36 \div 1.49)(5)10^{-7} \xrightarrow{C_1^0+Res} (1.22 \div 1.35)(5)10^{-7}$$

P. Masjuan, P. Sanchez, In preparation

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P. Masjuan, P. Sanchez, In preparation