Pion production in NN collisions near threshold: complete NNLO calculation in chiral EFT

in collaboration with
V. Baru, E. Epelbaum, C. Hanhart, H. Krebs, and F. Myhrer

Outline:

• Why is pion production interesting?
• High accuracy pion production operator
• Convolution with NN wave functions
• Summary and outlook
Why is pion production interesting?

- First inelastic process in nucleon-nucleon interactions
- Several channels:
  \( pp \rightarrow pp\pi^0 \) and \( pp \rightarrow d\pi^+ \) cross sections differ by an order of magnitude
  \[ \sigma_{\text{tot}}(pp \rightarrow pp\pi^0) \simeq 3 \mu b \quad \sigma_{\text{tot}}(pp \rightarrow d\pi^+) \simeq 43 \mu b \]
  \( T_{\text{lab}} = 293.5 \text{ MeV} \)
- Charge symmetry breaking (CSB) in \( pn \rightarrow d\pi^0 \)
- Important prerequisite for more complicated processes:

\[ \text{CSB in } dd \rightarrow \alpha\pi^0 \quad \text{3N forces} \quad \text{pionic deuterium} \]

Goal: Study \( NN \rightarrow NN\pi \) in chiral effective field theory
Strategy of NN→NNπ study

We use ‘Hybrid’ chiral EFT method:

1. Calculate irreducible production operator perturbatively in chiral EFT

   \[ \text{production operator} = \ldots + \text{non-perturbative NN wave functions} + \ldots \]

2. Convolute it with non-perturbative NN wave functions

   Phenomenological NN wave functions: CDBonn, AV18, Nijmegen, …

Chiral NN wave functions [Epelbaum, Krebs, Meißner 2014]
   - calculated up to N4LO
   - describe phase shifts up to pion production
   - several versions available with different R-space cutoff: 0.8-1.2fm (approximately correspond to momentum cutoff 330-500 MeV)
Main specifics:

- NN momenta in CMS are large enough to produce a pion

$$|\vec{p}| \sim \sqrt{m_\pi m_N} \sim 360 \text{ MeV} \quad \text{– new soft scale}$$

Special counting: Momentum Counting Scheme (MCS)

expansion parameter: $$\chi_{\text{MCS}} \sim \frac{|\vec{p}|}{\Lambda_\chi} \sim \sqrt{\frac{m_\pi}{m_N}}$$

- Delta(1232)-resonance is close $$m_\Delta - m_N \sim 280 \text{ MeV} \sim |\vec{p}|$$

=> Include additional operators with explicit delta

$$+ + \ldots$$
Part 1: s-wave pion production operators

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<td>pp→dπ⁺</td>
<td>big contribution</td>
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⇒ NNLO effects are crucial to understand cross section puzzle
**Part 1: s-wave pion production operators**

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⇒ **NNLO** effects are crucial to understand cross section puzzle
Pion production operator at NNLO — Results

- We calculated full pion production operator at NNLO
  - Lots of cancellations found
  - Finite contribution remains

\[ \Delta_{\text{IIIb}} \Delta_{\text{IV}} \Delta_{\text{Box a}} \Delta_{\text{Box b}} \]

\[ \Delta_{\text{II}} \Delta_{\text{IIIa}} \Delta_{\text{IIIb}} \Delta_{\text{IV}} \Delta_{\text{Box a}} \Delta_{\text{Box b}} \]

\[ \Delta_{\text{V}} \Delta_{\text{VIa}} \Delta_{\text{VIIa}} \Delta_{\text{VIIIa}} \Delta_{\text{IXa}} \]

\[ \Delta_{\text{VIb}} \Delta_{\text{VIIb}} \Delta_{\text{VIIIb}} \Delta_{\text{IXb}} \]

\[ + \ldots \]

\[ A_{\text{CT}}, B_{\text{CT}} \]
Convolution: $pp \rightarrow d\pi^+ \text{ channel}$

Threshold amplitude in $pp \rightarrow d\pi^+ \text{ channel}$

Contribution of the longest range LO rescattering operator

- Cutoff-independence for $\Lambda > 600$ MeV (Soft scale is 360 MeV)
- Agreement for all phenomenological as well as chiral potentials
Convolution: $pp \rightarrow d\pi^+ + \pi^+$ channel

Threshold amplitude in $pp \rightarrow d\pi^+$ channel

Contribution of full LO operator

- Consistent result for all phenomenological potentials
- Chiral WF cut intermediate momenta, which are kept in phenomenological WF

Amplitude extracted from experiment [Strauch et al. 2011]

Required contribution of NNLO

Preliminary
NNLO corrections give small (natural) contribution — consistent with our power counting
Summary and outlook

Study of NN→NNπ in Chiral EFT

• Test of chiral EFT at intermediate energies
• Tool to study charge symmetry breaking (pn→dπ^0)
• Building block for more complicated reactions (dd→απ^0, 3NF,...)

Current results:

• s-wave pion production operator at threshold up to N^2LO MCS (6%) including explicit Delta(1232)
• convolution with nucleon-nucleon wave functions in pp→dπ^+
• hybrid calculations with all phenomenological potentials give consistent results
• since pion production probes intermediate distances the use of phenomenological WF for pion production is preferred

Next step

• Convolution with nucleon-nucleon wave functions and calculation of the observables in pp→ppπ^0
spares
Convolution: $pp \rightarrow d\pi^+\pi^+$ channel

Threshold amplitude in $pp \rightarrow d\pi^+\pi^+$ channel
Contribution of full NNLO operator (w/o CT)

Preliminary