RUHR UNIVERSITÄT BOCHUM RUB

Application of low-energy theorems to NN scattering at unphysical pion masses

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in collaboration with

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Related works: arXiv:1504.07852 [nucl-th], to appear in PRC

Introduction

How the nuclear observables depend on the fundamental parameters of the SM?

- How much fine-tuning in the quark masses is needed for life to emerge on Earth?

ab initio nuclear lattice simulations: Epelbaum, Krebs, Lähde, Lee, Meißner PRL110 (2013)

  - The excited Hoyle state of ^{12}C: $\varepsilon = m_{\rm Hoyle} - 3\, m_{^4\rm He} \simeq 379\,{\rm KeV}$

☛ enhanced resonance production of ^{12}C and ^{16}O

☛ Changing ε by 25% ⇒ strong decrease in production of ^{12}C and ^{16}O

Oberhammer et al. NPA689, (2001)

☛ Converting this to the quark-mass variation δm_q requires knowledge of the NN scatt. lengths: $a_{^3S_1}(m_\pi)$ and $a_{^1S_0}(m_\pi)$

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- Impact of quark-mass variation on Big Bang nucleosynthesis (BBN)

  ☛ Input: $a_{^3S_1}(m_\pi), a_{^1S_0}(m_\pi)$ + BE of ^{3}He, ^{4}He, ..., ^{7}Be

  ☛ Output: quark-mass variation at the time of BBN

  Berengut et al., PRD87 (2013), Bedaque et al. PRC83 (2011),...

⇒ m_π-dependence of the NN parameters is a crucial pre-requisite

m_π-dependence of the NN parameters. Methods

- Lattice QCD simulations at unphysical pion masses

  ☛ much progress in recent years

  Beane et al. (NPLQCD) PRD 85 (2012), PRD 87, PRC 88 (2013)
  Yamazaki et al. PRD 81 (2010), PRD 86 (2012), (2015)
  ...

- Chiral Extrapolations based on Chiral EFT

  ☛ assuming scat. length a is known at some $m_{\pi 1}$ and $m_{\pi 2}$
  ⇒ fix short-range parameters ⇒ extrapolate to physical world

- Low-energy theorems: complementary model independent info about NN

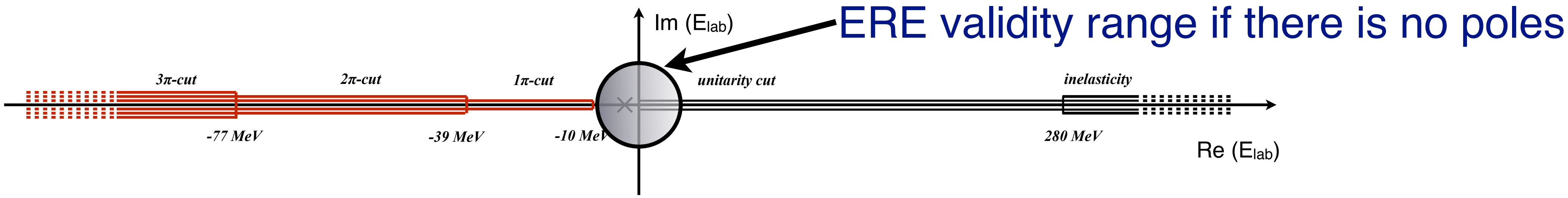
  ☛ assuming a (or the effective range r) is known at some m_π
  ⇒ employ correlations caused by long-range pion physics
  ⇒ predict all other parameters in the effective range expansion at this m_π

Modified effective range expansion (MERE)

ERE and its validity

$$T_l(k) = -\frac{16\pi^2}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}} \qquad\qquad F_l(k) \equiv k^{2l+1}\cot\delta_l(k)$$

$$k^{2l+1}\cot\delta_l(k) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \ldots$$ — ERE



$$V_{\rm OPE} \sim \log\left(4k^2 + m_\pi^2\right) \quad \Rightarrow \quad k = \pm i\, m_\pi/2$$ — branch point

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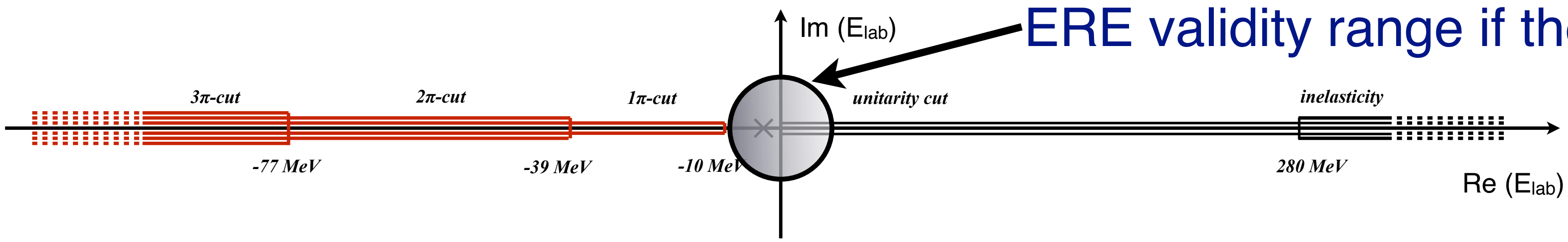
ERE validity range if there is no poles



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Idea of MERE

v.Haeringen and Kok PRA 26 (1982), Cohen and Hansen PRC 59 (1999), Epelbaum and Gegelia EPJA 41(2009)

— keep long-range physics explicitly ⇒ extend the range of validity

$$V = V_L + V_S \qquad r_L \sim M_L^{-1} \qquad r_S \sim M_S^{-1} \qquad M_L \ll M_S$$

$$F_l^M(k^2) \equiv R_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|}\cot[\delta_l(k) - \delta_l^L(k)]$$

$$-\frac{1}{a_M} + \frac{1}{2}r_M k^2 + \ldots$$

known from solutions of Schrödinger Eq. for V_L

- meromorphic for $|k| < M_S/2$
- systematically parameterizes short-range physics: $1/M_S$ expansion

Low-energy theorems (LETs). Formulation

$$k \cot \delta = \text{fun}(\underbrace{f^L(k), \delta^L(k)}_{\text{long-range part}};\ \underbrace{F^M(k)}_{\text{short-range, MERE expansion}})$$

$$\Downarrow$$

$$r, v_2, v_3, \ldots$$

LET $\equiv$ correlations between a, r and v_i caused by long-range interactions

LO LETs

$$k \cot \delta = \text{fun}_{\text{LO}}(f^L(k), \delta^L(k);\ a^M)$$

$$\Rightarrow \quad r = \alpha_0 / M_L, \quad v_2 = \beta_0 / M_L^3, \quad v_3 = \gamma_0 / M_L^5, \quad v_4 = \delta_0 / M_L^7,$$

NLO LETs

$$k \cot \delta = \text{fun}_{\text{NLO}}(f^L(k), \delta^L(k);\ a^M, r^M)$$

$\Rightarrow$ corrections to $\quad r, v_i \sim M_L/M_S \quad \text{from} \quad r^M$

M_L/M_S expansion of ERE $\equiv$ chiral expansion $\quad \chi = m_\pi / \Lambda_\chi$

LETs for NN scattering

LETs are caused by long-range interactions ⇒ model and scheme independent

Framework: modified Weinberg chiral EFT

Epelbaum and Gegelia PLB 716 (2012)

$$T(p,p',E) = V(p,p') + m_N^2 \int_0^\Lambda \frac{dq}{2\pi^2} V(p,q) \frac{q^2}{(q^2+m_N^2)\left(E - 2\sqrt{q^2+m_N^2} + i0\right)} T(q,p',E)$$

Kadyshevsky NPB6 (1968)

LO LETs

$$V_{\rm LO} = V_{1\pi}(\vec q\,) + C_0, \qquad V_{1\pi}(\vec q\,) = -\frac{g_A^2}{4F_\pi^2}\frac{\vec\sigma_1\cdot\vec q\,\vec\sigma_2\cdot\vec q}{\vec q^{\,2}+M_\pi^2}\,\tau_1\cdot\tau_2$$

- explicitly renormalizable ($\Lambda \to \infty$)
- all divergencies can be absorbed to C_0 ; C_0^R is adjusted to a (or a_M)

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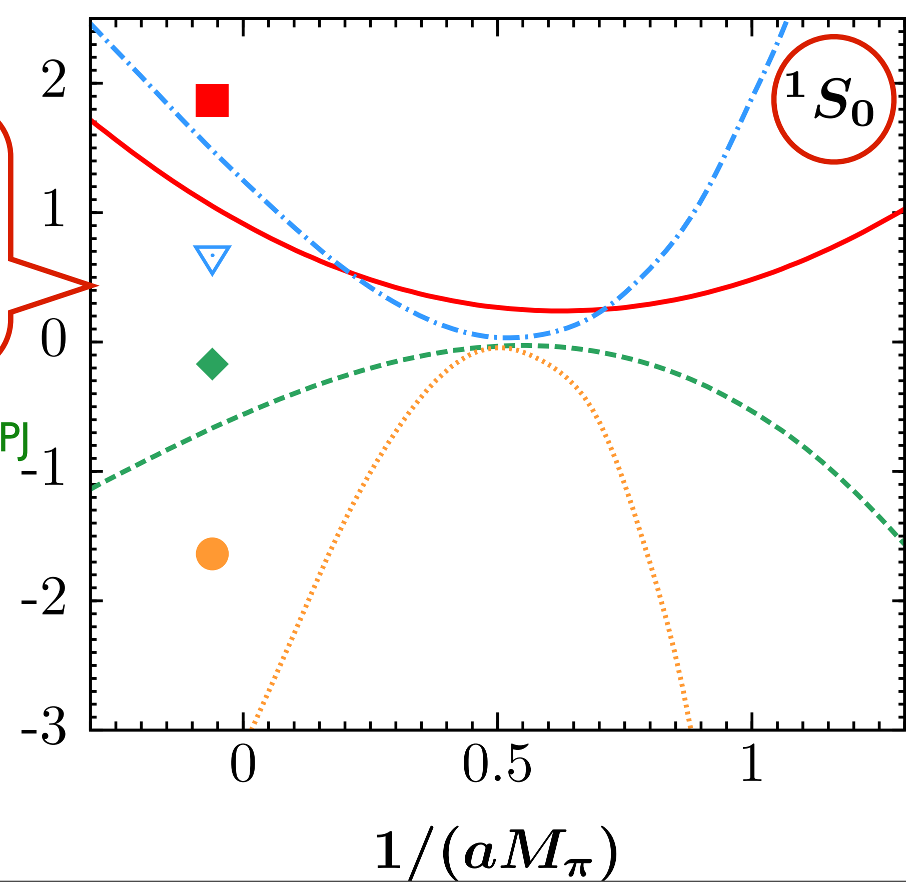
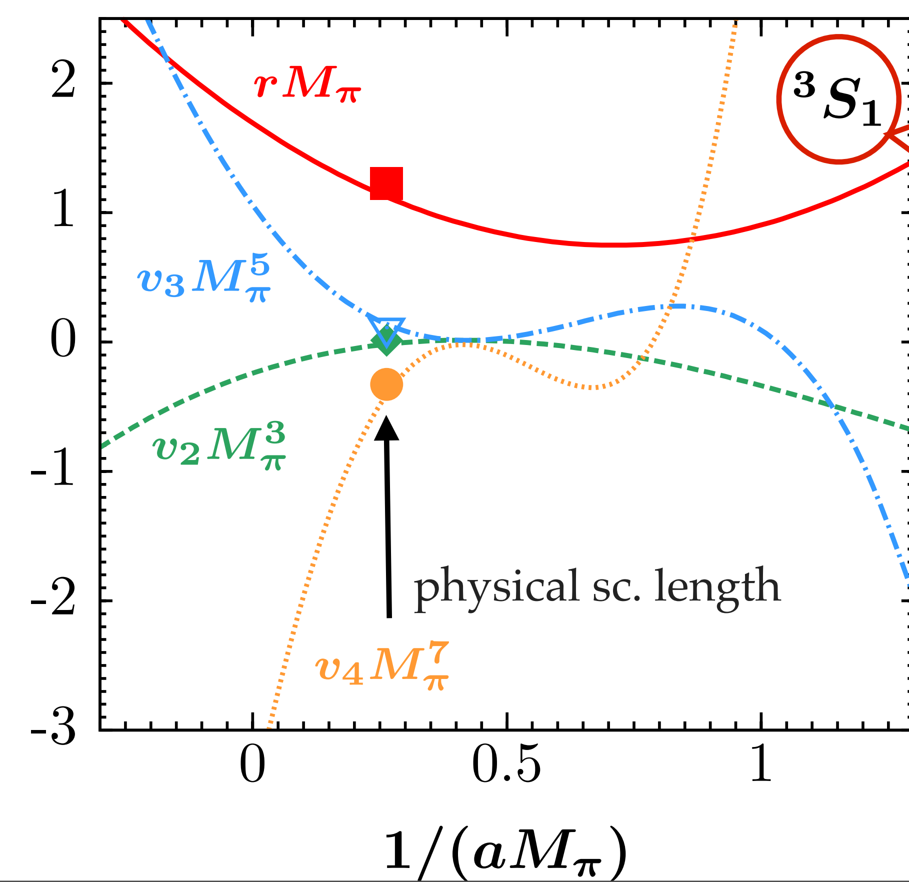
OPE is very weak

Gasparyan et al., EPJ (2015)





Strong tensor part of OPE governs coeffs. in ERE

LETs at NLO

- Include higher order short-range interactions, adjust to a_M and r_M

we want

⇒ non-perturbative + renormalizable theory ⇒ resonance saturation

$$V_{\rm NLO} = V_{1\pi}(\vec{q}\,) + C_0 + \beta\,\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M^2}$$

β: adjust to r_M; M: 700 MeV

once β is adjusted to r_M ⇒ very weak sensitivity to the form of the short-range force

Neutron-proton $^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
LO Epelbaum, Gegelia, PLB716, 2012	fit	1.60	-0.05	0.82	-5.0
NLO, this work	fit	fit	0.06	0.70	-4.0
Empirical values, de Swart et al., nucl-th/9509032	5.42	1.75	0.04	0.67	-4.0
NLO KSW, Cohen, Hansen, PRC59, 1999	fit	fit	-0.95	4.6	-25

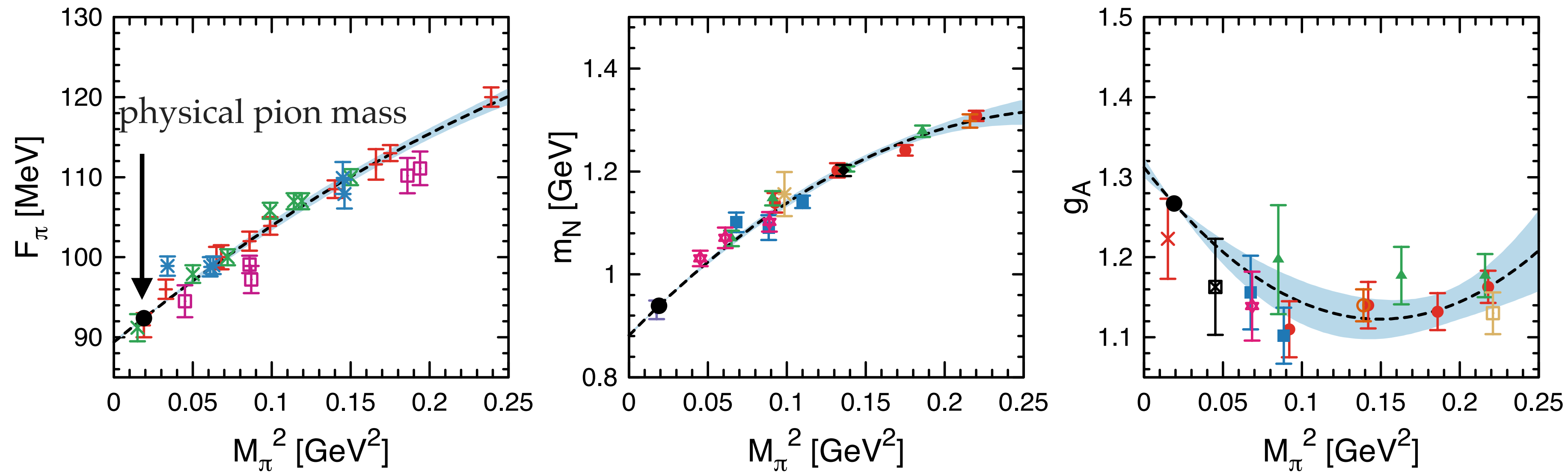
LETs for $m_\pi \neq m_\pi^{\rm ph}$: Strategy

LO

$$V_{\rm LO} = V_{1\pi}(\vec{q},\, m_\pi) + C_0$$

- Shift of the branch point of the left-hand cut from OPE
- Include m_π-dependence of g_A, F_π, m_N

⇒ Interpolation fits of lattice data up to m_π = 500 MeV



NLO

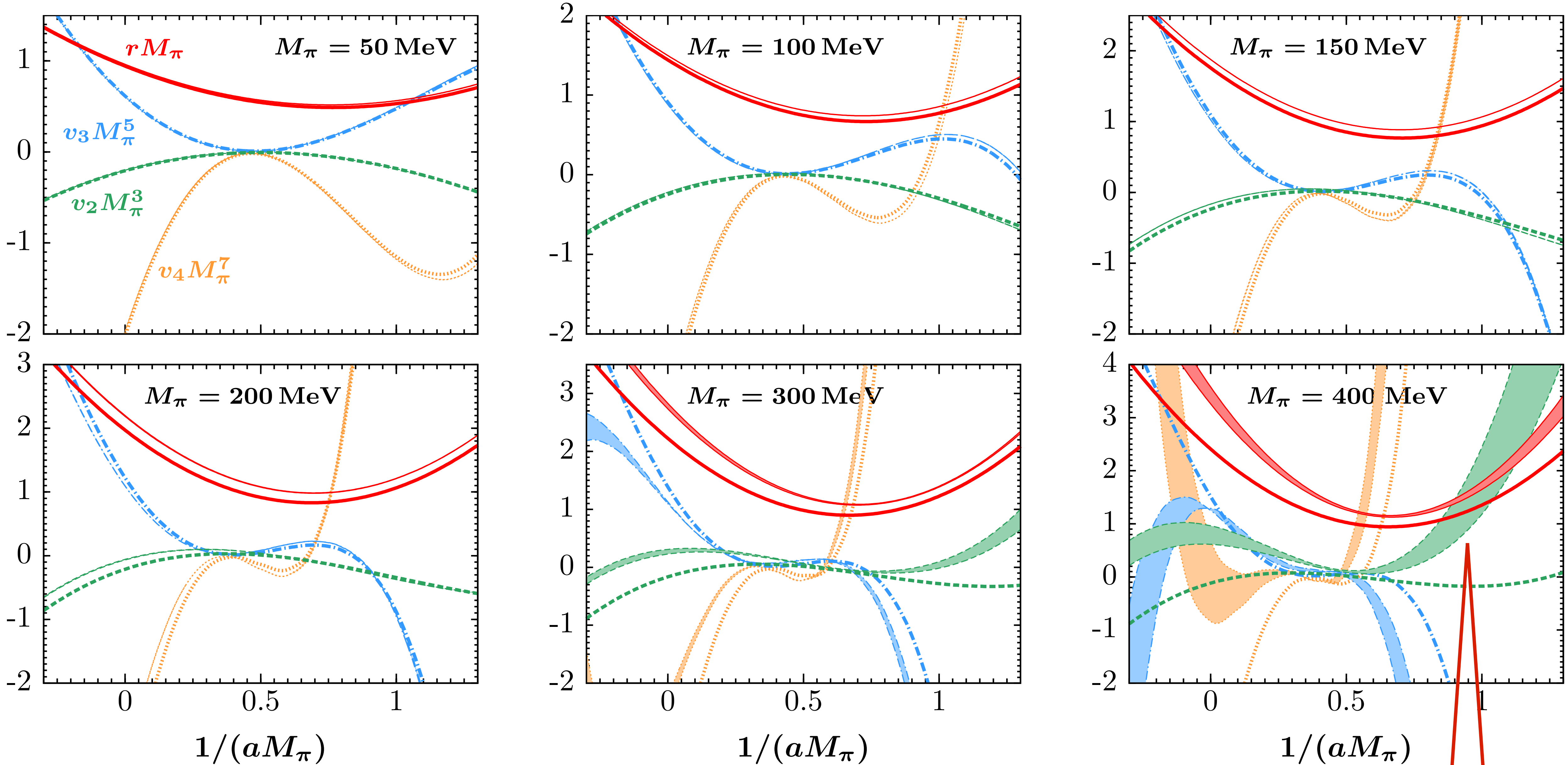
$$V_{\rm NLO} = V_{1\pi}(\vec{q}\,) + C_0 + \beta\,\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M^2}$$

- Include in addition m_π-dependence of subleading short-range term

⇒ Naturalness: β changes by 50% for m_π = 500 MeV

$$1 - 0.5\left|\frac{M_\pi^2-(M_\pi^{\rm ph})^2}{(500\text{ MeV})^2-(M_\pi^{\rm ph})^2}\right| \le \frac{\beta(M_\pi)}{\beta(M_\pi^{\rm ph})} \le 1 + 0.5\left|\frac{M_\pi^2-(M_\pi^{\rm ph})^2}{(500\text{ MeV})^2-(M_\pi^{\rm ph})^2}\right|,$$

LETs for $m_\pi \neq m_\pi^{\rm ph}$. Results



line - LO

band - NLO

Large uncertainty due to $\beta(m_\pi)$

bad separation: $\frac{m_\pi}{\Lambda_\chi} \sim \frac{1}{2}$

Implications of LETs for lattice QCD

- Consistency checks of lattice results if several ERE parameters are extracted!

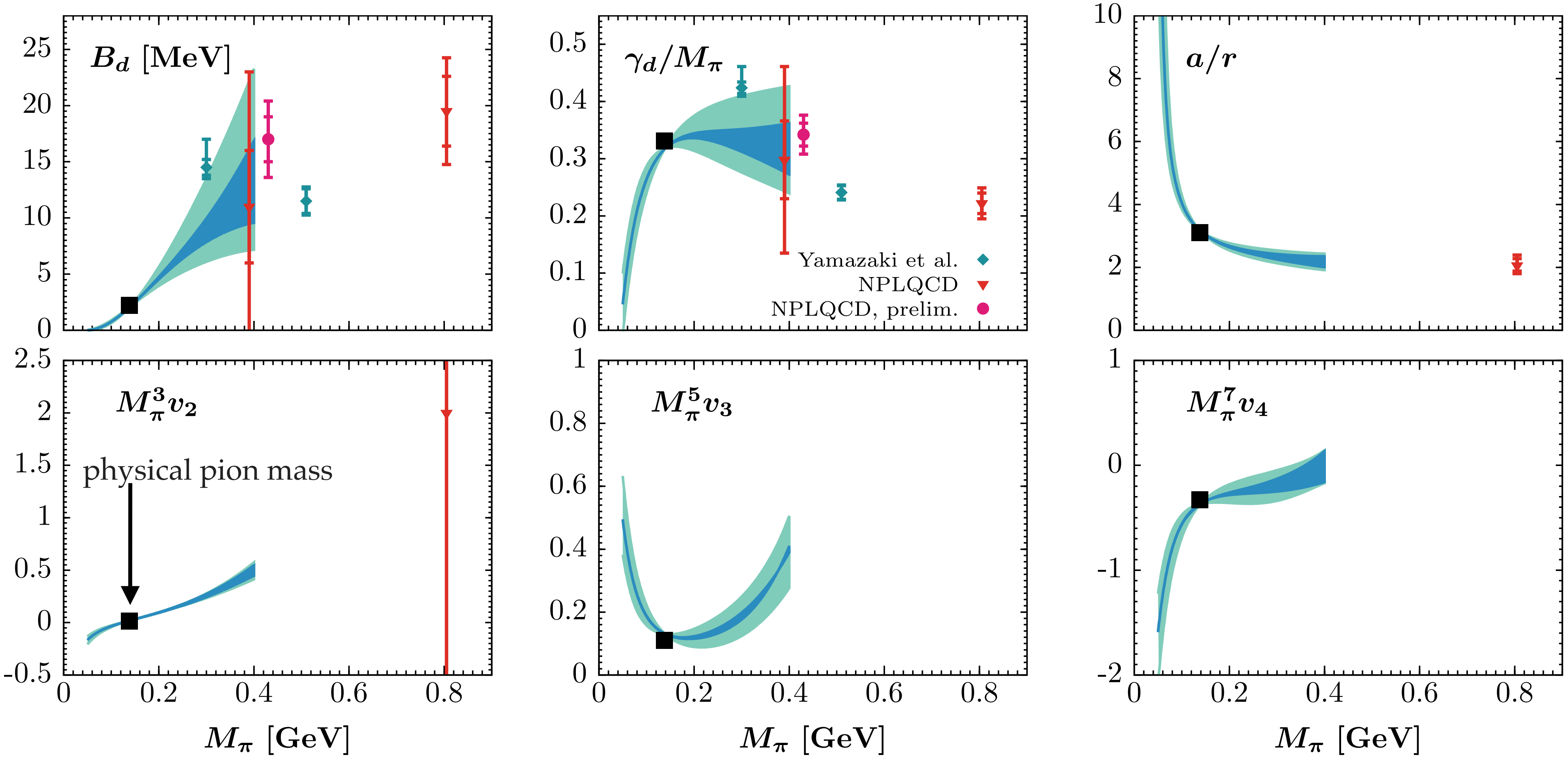
  ☛ NPLQCD extracted B_d and r for $m_\pi \simeq 800$ MeV (Beane et *al.* PRC88 (2013))

  ☛ It was conjectured that

  see Talk by Silas Beane on Tuesday

$$M_\pi r \cong A^{(^3S_1)} + B^{(^3S_1)} M_\pi, \quad \text{where} \quad A^{(^3S_1)} = 0.726^{+0.065+0.072}_{-0.059-0.059}, \quad B^{(^3S_1)} = 3.70^{+0.42+0.42}_{-0.47-0.52}\ \text{GeV}^{-1},$$

Assuming this behavior ⇒ fix short-range interaction C_0 at LO ⇒ predict coeffs. in ERE



⇒ trend of the binding energy is nicely described

Summary

- Correlations between the parameters in the ERE from long-range interactions (low-energy theorems) are investigated for NN scattering

  ▶ *systematically improvable results with* $\chi = \frac{M_L}{M_S}$

☛ Spin-triplet channel: at LO LETs describe empirical data to 25%, at NLO - to a few percents

☛ Spin-singlet channel: only qualitative agreement due to the weakness of OPE

- LETs in the spin-triplet channel are generalized to unphysical pion masses

  ▶ *expected to be valid below* $m_\pi \sim 400$ *MeV*

▶ *effective range and shape parameters are calculated for different* m_π *using scat. length as input*

▶ *should be useful for consistency checks of lattice simulations and for reducing the systematic errors*