Causality constraint on bound states and scattering with zero-range force

arXiv:1402.4973 [nucl-th].

do perturbative pions deserve another chance?

Vladimir Pascalutsa

Institut für Kernphysik,
University of Mainz, Germany

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Outline

Motivation
Chiral EFT of few-nucleon systems

Light-by-light scattering sum rules
general principles: unitarity, causality, etc.

Zero-range force:
Bound state, tachyon, K-matrix pole
using the sum rules as consistency (causality) criterion
phi^4 theory

(Relativistic) Wigner’s inequality
positive effective range parameters

Conclusions and outlook
Motivation
**Motivation**

## Motivation

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\( M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \varepsilon_{\lambda_4}^{*\mu_4}(\vec{q}_4) \varepsilon_{\lambda_3}^{*\mu_3}(\vec{q}_3) \varepsilon_{\lambda_2}^{\mu_2}(\vec{q}_2) \varepsilon_{\lambda_1}^{\mu_1}(\vec{q}_1) M_{\mu_1 \mu_2 \mu_3 \mu_4} \)

HELICITY AMPL. 

FEYNMAN AMPL.

**IN THE FORWARD DIRECTION ( \( t = 0, \quad s = 4\omega^2, \quad u = -s \):)**

\[ M_{\mu_1 \mu_2 \mu_3 \mu_4} = A(s) g_{\mu_4 \mu_2} g_{\mu_3 \mu_1} + B(s) g_{\mu_4 \mu_1} g_{\mu_3 \mu_2} + C(s) g_{\mu_4 \mu_3} g_{\mu_2 \mu_1}, \]

\[ M_{++-+}(s) = A(s) + C(s), \]

\[ M_{+-+-}(s) = A(s) + B(s), \]

\[ M_{++--}(s) = B(s) + C(s). \]
Light by light scattering

\[ M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \varepsilon_{\lambda_4}^{* \mu_4} (\vec{q}_4) \varepsilon_{\lambda_3}^{* \mu_3} (\vec{q}_3) \varepsilon_{\lambda_2}^{\mu_2} (\vec{q}_2) \varepsilon_{\lambda_1}^{\mu_1} (\vec{q}_1) M_{\mu_1 \mu_2 \mu_3 \mu_4} \]

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\[ M_{+++-}(s) = A(s) + C(s) , \]

\[ M_{+-+-}(s) = A(s) + B(s) , \]

\[ M_{++++}(s) = B(s) + C(s) . \]

1) **CROSSING SYMMETRY** (1 ↔ 3, 2 ↔ 4):

\[ M_{+-+-}(s) = M_{++++}(-s) , \quad M_{+++-}(s) = M_{+-+-}(-s) \]

**AMPLITUDES WITH DEFINITE PARITY UNDER CROSSING:**

\[ f^{(\pm)}(s) = M_{++++}(s) \pm M_{+-+-}(s) \]

\[ g(s) = M_{+-+-}(s) \]
LbL sum rules

2) CAUSALITY $\Rightarrow$ ANALYTICITY $\Rightarrow$ DISPERSION RELATIONS:

$$\text{Re} \left\{ \frac{f^{(\pm)}(s)}{g(s)} \right\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{ds'}{s' - s} \text{Im} \left\{ \frac{f^{(\pm)}(s')}{g(s')} \right\},$$
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3) OPTICAL THEOREM (UNITARITY):

$$\text{Im} f^{(\pm)}(s) = -\frac{s}{8} \left[ \sigma_0(s) \pm \sigma_2(s) \right],$$

$$\text{Im} g(s) = -\frac{s}{8} \left[ \sigma_{||}(s) - \sigma_{\bot}(s) \right].$$

$\sigma_{0,2}(\sigma_{||,\bot})$ ARE CIRCULARLY (LINEARLY) POLARIZED PHOTON-PHOTON FUSION CROSS-SECTIONS
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$\sigma_{0,2}(\sigma_{||},\perp)$ ARE CIRCULARLY (LINEARLY) POLARIZED PHOTON-PHOTON FUSION CROSS-SECTIONS

$$\text{Re} f^{(+)}(s) = -\frac{1}{2\pi} \int_{0}^{\infty} ds' s'^2 \frac{\sigma(s')}{s'^2 - s^2},$$

$$\text{Re} f^{(-)}(s) = -\frac{s}{4\pi} \int_{0}^{\infty} ds' s' \frac{\Delta \sigma(s')}{s'^2 - s^2},$$

$$\text{Re} g(s) = -\frac{1}{4\pi} \int_{0}^{\infty} ds' s'^2 \frac{\sigma_{||}(s') - \sigma_{\perp}(s')}{s'^2 - s^2},$$

$$\sigma = (\sigma_0 + \sigma_2)/2 = (\sigma_{||} + \sigma_{\perp})/2$$

$$\Delta \sigma = \sigma_2 - \sigma_0$$
Light-by-light scattering sum rules

4) "LOW-ENERGY THEOREM": \[ \mathcal{L}_{EH} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2, \]

\[ f^{(\pm)}(s) = -2(c_1 + c_2) s^2 + O(s^4) \]

LOW-ENERGY EXPANSION

\[ f^{(-)}(s) = O(s^5) \]

\[ g(s) = -2(c_1 - c_2) s^2 + O(s^4) \]

\[ O(s^1) : \quad 0 = \int_{0}^{s} \frac{ds}{s} \left[ \sigma_2(s) - \sigma_0(s) \right] \]

GERASIMOV & MOULIN, NPB (1976)
BRODSKY & SCHMIDT, PLB (1995)

\[ O(s^2) : \quad c_1 = \frac{1}{8\pi} \int_{0}^{\infty} \frac{ds}{s^2} \sigma_{||}(s), \]

\[ c_2 = \frac{1}{8\pi} \int_{0}^{\infty} \frac{ds}{s^2} \sigma_{\perp}(s) \]

V. P. & VANDERHAEGHEN, PRL (2010)
Zero-range force in light of the LbL sum rule

PAUK, V.P. & VANDERHAEGHEN, PLB 2014

\[ T = V + VGT \]

\[ V = \lambda \]

\[ G(s) = -i \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{[(p+\ell)^2 - m^2] (\ell^2 - m^2)} \]

Bubble-chain sum:

\[ T(s) = \frac{1}{\lambda^{-1} - G(s)} \]

with \( p^2 = s \).
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\[ \lambda > 0 : \text{no poles} \]
\[ \lambda < 0 : \text{one pole and one K-matrix pole} \]

\[ T(s) = \frac{1}{K^{-1}(s) - i} \]
Light-by-light sum rule as causality criterion

\[ \int_{s_0}^{\infty} ds \frac{\Delta \sigma(s)}{s} = 0, \]

\[ \mathcal{L} = (D^\mu \phi)^* D_\mu \phi - m^2 \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}, \]

\[ S(\Delta \sigma(0))/s. \]
The results are discussed in Sect. 4, and an outlook is given in Sect. 5. We present a stringer causality criterion for relativistic scattering and bound state solutions.

Abstract

Alternating-sign, since

\[ \int_{s_0}^{\infty} ds \frac{\Delta \sigma(s)}{s} = 0, \]

\[ \mathcal{L} = (D^\mu \phi)^* D_\mu \phi - m^2 \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2 - \frac{1}{4} F^{\mu \nu} F_{\mu \nu}, \]

where

\[ s \]

Keywords:

The bound state appearance, independently of whether it is a tachyon or not, is complemented in this

Though the sum of Eq. (20) is formally undetermined, we can still use a naive resummation at

The tree-level cross section weighted with 1

In going beyond one loop, and in fact beyond perturbation theory, one often relies on a linear

The tree-level cross section in scalar QED, the relative velocity

To cancel the integral one need to introduce the bound state as the asymptotic state i.e., new channel:

Fig. 5 that the sum rule is only valid for positive values of

By Eq. (15) in terms of the renormalized coupling of Eq. (12). In Fig. 5 we show the dependence

which corre-

are the colliding photon four-

which can be easily obtained by a formal resummation of the geometric series of corrections given

potential perturbatively. For example in

in our field-theoretic case we are to consider

is the tree-level cross section in scalar QED, the relative velocity

where

\[ m \]

II

I

III

To cancel the integral one need to introduce the bound state as the asymptotic state i.e., new channel:

instability

bound state

no bound state no tachyon

To cancel the integral one need to introduce the bound state as the asymptotic state i.e., new channel:
Light-by-light sum rule as causality criterion

\[
\int_{s_0}^{\infty} ds \frac{\Delta \sigma(s)}{s} = 0.
\]

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\mathcal{L} = (D^\mu \phi)^* D_\mu \phi - m^2 \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2 - \frac{1}{4} F^{\mu \nu} F_{\mu \nu},
\]

\[
\text{Fig. 5: The amplitude of the bound state production.}
\]

To cancel the integral one need to introduce the bound state as the asymptotic state i.e., new channel:

but not the K-matrix pole...
Phase shifts

Levinson’s theorem:
\[ \delta(0) = \pi N_{\text{bound states}} \]

Figure 7: Phase shift for different values of \( \tilde{\lambda} \).

The 90 degree crossing, i.e. the K-matrix pole does not correspond to any S-matrix pole in this case.
Wigner’s causality bound

\[ r \leq 0 \]

effective range

WIGNER, PHYS REV (1955)

PHILLIPS & COHEN, PLB (1997);
HAMMER & D. LEE, ANN PHYS (2010); ...
Wigner’s causality bound

\[ r \leq 0 \]

effective range

\[ |k| \cot \delta(s) = -\frac{1}{a} + \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n+1} r_n |k|^{2n} \]

WIGNER, PHYS REV (1955)

PHILLIPS & COHEN, PLB (1997);
HAMMER & D. LEE, ANN PHYS (2010); ...

In the tachyon (acausal) regime at least one of the effective range parameters is negative.

Therefore our causality criterion yields:

\[ r_n \geq 0 \]
Non-relativistic limit

\[ T(s) = \frac{1}{\lambda^{-1} - (4\pi)^{-2} B(s)} \]

In non-rel. limi, K-matrix pole disappears and

\[ r = 0 \]

in agreement with Wigner’s bound

Zero eff. range of 2-body force eventually leads to the problems with the 3-body force

[BEDAQUE, HAMMER, VAN KOLCK (1999)]
Conclusion and outlook
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Light-by-light scattering sum rule used as
Light-by-light scattering sum rule used as criterion for consistency of a QFT (with zero-range interaction) truncation
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Positive eff. range(s) emerge, Wigner’s bound violated/NA. :) will help in 3-body problem
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Chiral EFT of NN and few-nucleon systems?
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