Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern
University of Manchester


Prog. Nucl. Part. Phys. 67 841 (2012)

(1) Compton Scattering and polarisabilities
(2) Quick review of EFT calculations
(3) State of current calculations and fits and future directions
Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern
University of Manchester


Prog. Nucl. Part. Phys. 67 841 (2012)

(1) Compton Scattering and polarisabilities

(2) Quick review of EFT calculations

(3) State of current calculations and fits and future directions
Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern
University of Manchester


Prog. Nucl. Part. Phys. 67 841 (2012)

(1) Compton Scattering and polarisabilities

(2) Quick review of EFT calculations

(3) State of current calculations and fits and future directions
Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern
University of Manchester


Prog. Nucl. Part. Phys. 67 841 (2012)

(1) Compton Scattering and polarisabilities

(2) Quick review of EFT calculations

(3) State of current calculations and fits and future directions
Compton Scattering

For large wavelengths, only sensitive to overall charge: Thomson scattering

\[ \lambda \gg d \]

But for smaller wavelengths, the target is polarised by the electric and magnetic fields

\[ \lambda \sim d \]
Compton Scattering

For large wavelengths, only sensitive to overall charge: Thomson scattering

But for smaller wavelengths, the target is polarised by the electric and magnetic fields

To leading order

\[ H_{eff} = \frac{(p - QA)^2}{2m} + Q\phi \frac{1}{2} 4\pi \left( \alpha \vec{E}^2 + \beta \vec{H}^2 \right) + \gamma E_1 \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \gamma M_1 \vec{\sigma} \cdot \vec{H} \times \dot{\vec{H}} - 2\gamma E_2 E_{ij} \sigma_i H_j + 2\gamma M_2 H_{ij} \sigma_i E_j \]

where \( E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i) \) and \( H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i) \)
Compton Scattering from the nucleon

The scattering amplitude has Born and non-Born pieces. The latter probe the structure of the nucleon; polarisabilities are leading signs of non-pointlike nucleons as we increase the photon energy.
Compton Scattering from the nucleon

The scattering amplitude has Born and non-Born pieces. The latter probe the structure of the nucleon; polarisabilities are leading signs of non-pointlike nucleons as we increase the photon energy.
Why $\beta$ matters

Muonic H

EM mass splitting

Subtracted DR

$T_1(v, Q^2) = -v^2 \int_{v_{th}^2}^{\infty} \frac{dv'}{v'^2} \frac{W_1(v', Q^2)}{v'^2 - v^2} + 4\pi \beta Q^2 + O(Q^4)$
Proton radius puzzle

Hydrogen etc: $r_p = 0.8775(51)$ fm, CODATA 2010
Muonic hydrogen: $r_p = 0.84087 \pm 0.00039$ fm


$7\sigma$ deviation!
Lamb shift

\[ \begin{align*}
\text{hydrogen} & : \quad ^2P_{3/2} & \quad ^2S_{1/2} \\
& \quad \quad \uparrow 40 \mu\text{eV} \\
& \quad \quad \downarrow \text{proton size} \\
& \quad \quad \downarrow 4 \text{ neV}
\end{align*} \]

\[ \begin{align*}
\text{muonic hydrogen} & : \quad ^2S_{1/2} & \quad ^2P_{1/2} \\
& \quad \quad \uparrow 23 \text{ meV} \\
& \quad \quad \downarrow \text{proton size} \\
& \quad \quad \downarrow 4 \text{ meV}
\end{align*} \]

\[ \begin{align*}
\text{other transitions} & : \quad ^2P_{3/2} & \quad ^2P_{1/2} \\
& \quad \quad \uparrow 8 \text{ meV} \\
& \quad \quad \downarrow 0.2 \text{ eV}
\end{align*} \]

Not to scale!
Lamb shift

\[ \begin{align*}
^{2}\Sigma_{1/2} & \quad \leftrightarrow \quad ^{2}\Pi_{3/2} \quad 40 \, \mu\text{eV} \\
^{2}\pi_{1/2} & \quad \leftrightarrow \quad ^{2}\pi_{3/2} \\
^{2}\Sigma_{3/2} & \quad \leftrightarrow \quad ^{2}\Sigma_{1/2} \quad \text{proton size} \quad 4 \, \text{neV} \\
^{2}\pi_{1/2} & \quad \leftrightarrow \quad ^{2}\pi_{3/2} \quad 8 \, \text{meV} \\
^{2}\Sigma_{1/2} & \quad \leftrightarrow \quad ^{2}\Sigma_{3/2} \quad 0.2 \, \text{eV} \\
^{2}\Sigma_{1/2} & \quad \leftrightarrow \quad ^{2}\Sigma_{3/2} \quad 23 \, \text{meV} \quad \text{proton size} \quad 4 \, \text{meV}
\end{align*} \]

hydrogen

muonic hydrogen

Not to scale!
Lamb shift

\[ \begin{align*}
^2P_{3/2} & \quad 40 \mu\text{eV} \\
^2S_{1/2} & \quad \text{proton size} \\
^2P_{1/2} & \quad 4 \text{ neV}
\end{align*} \]

hydrogen

\[ \begin{align*}
^2P_{3/2} & \quad 8 \text{ meV} \\
^2P_{1/2} & \quad 0.2 \text{ eV} \\
^2S_{1/2} & \quad 23 \text{ meV} \\
\end{align*} \]

muonic hydrogen

Main contributions to the $\mu_p$ Lamb shift:

- discrepancy: $0.31 \text{ meV}$
- polarizability: $\approx 0.01 \text{ meV}$
- finite size
- recoil
- self-energy + muon VP
- Källen-Sabry
- one-loop VP

Not to scale!
Lamb shift

\( \beta = 3.1 \pm 0.5 \implies \Delta E_{\text{pol}} = -0.0085(11) \text{meV} \)

Main contributions to the \( \mu p \) Lamb shift:

- discrepancy
- polarizability
- finite size
- recoil
- self-energy + muon VP
- Källen-Sabry
- one-loop VP

\( \Delta E_{\text{pol}} = -0.0085(11) \text{meV} \)

At a hadronic level, we consider Compton scattering from the nucleon as probing its excitations and particularly its pionic cloud.
At a hadronic level, we consider Compton scattering from the nucleon as probing its excitations and particularly its pionic cloud.

Optical theorem leads to sum rules for forward scattering

\[ \sum_{X} \left| \frac{X}{X} \right|^2 = \sum_{X} \int_{\omega_{th}}^{\infty} \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} d\omega \]

Baldin SR: \[ \alpha + \beta = \frac{1}{2\pi^2} \int_{\omega_{th}}^{\infty} \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} d\omega \quad \text{and} \quad \gamma_0 = \frac{1}{4\pi^2} \int_{\omega_{th}}^{\infty} \frac{\sigma_{1/2}(\omega) - \sigma_{3/2}(\omega)}{\omega^3} d\omega \]

Both quite accurately evaluated for the proton:
\[ \alpha^p + \beta^p = (13.8 \pm 0.4) \times 10^{-4} \text{ fm}^3 \quad \text{Olmos de Léon et al. EPJA 10 207 (2001);} \]
\[ \gamma_0 = (-0.90 \pm 0.08 \text{(stat)} \pm 0.11 \text{(sys)}) \times 10^{-4} \text{ fm}^4 \quad \text{as byproduct of GDH expt. at MAMI and ELSA. Pasquini et al. Phys. Lett. B 687 160 (2010)} \]
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.

Two common methods: Dispersion relations and Chiral Perturbation Theory
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.

Two common methods: Dispersion relations and Chiral Perturbation Theory

Both consider pions as crucial source of energy-dependence in amplitudes (Delta resonance also captured)
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.

Two common methods: Dispersion relations and Chiral Perturbation Theory

Both consider pions as crucial source of energy-dependence in amplitudes (Delta resonance also captured)

DR uses partial wave analysis of $\gamma N \rightarrow \pi N$ data as input
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.

Two common methods: Dispersion relations and Chiral Perturbation Theory

Both consider pions as crucial source of energy-dependence in amplitudes (Delta resonance also captured)

DR uses partial wave analysis of $\gamma N \rightarrow \pi N$ data as input

Chiral Perturbation Theory is a field theory which treats pions and nucleons as basic degrees of freedom
Non-forward scattering

For the full non-Born contribution (6 independent amplitudes) need different approach.

Two common methods: Dispersion relations and Chiral Perturbation Theory

Both consider pions as crucial source of energy-dependence in amplitudes (Delta resonance also captured)

DR uses partial wave analysis of $\gamma N \rightarrow \pi N$ data as input

Chiral Perturbation Theory is a field theory which treats pions and nucleons as basic degrees of freedom

Both have difficulties with parameter-free predictions; both can be used to fit Compton scattering data and extract polarisabilities.
Chiral Perturbation theory

Effective field theory of QCD—relies on separation of scales

• pions are light \((m_\pi \ll m_\rho)\)
• low-energy pions interact weakly with other matter \((L_{\pi NN} \propto \bar{N}\partial_\mu \pi N)\).

Thus pion loops are suppressed by \(\approx m_\pi^2/\Lambda^2\) where \(\Lambda \approx m_\rho\). The Lagrangian contains infinitely many terms:

\[
\mathcal{L} = \sum_n \mathcal{L}^{(n)}(c_i^{(n)})
\]

Non-pionic nucleon structure shows up in low energy constants \(c_i^{(n)}\), but is suppressed by power of momentum: \((k/\Lambda)^n\):

\[
N^* \quad \rightarrow \quad L^{(4)}
\]

Calculations to \(n\)th order involve vertices from \(\mathcal{L}^{(n)}\) and pion loops with vertices from \(\mathcal{L}^{(n-2)}\); truncation errors are \(\sim (k/\Lambda)^{n+1}\).
χPT for Compton Scattering from the nucleon

We include nucleons, pions and the Delta in our Lagrangian.

\[ \mathcal{L}_{\pi N}^{(4), CT} = 2\pi e^2 H^\dagger \left[ \left( \delta \beta_{M1}^{(s)} + \delta \beta_{M1}^{(v)} \tau_3 \right) \left( \frac{1}{2} g_{\mu\nu} - v_{\mu} v_{\nu} \right) - \left( \delta \alpha_{E1}^{(s)} + \delta \alpha_{E1}^{(v)} \tau_3 \right) v_{\mu} v_{\nu} \right] F^{\mu\rho} F^{\nu\rho} H. \]

Counterterms shift \( \alpha_{E1} \) and \( \beta_{M1} \) at 4th order. Counterterms for spin pols at 5th order.
χPT for Compton Scattering from the nucleon

We include nucleons, pions and the Delta in our Lagrangian.

\[
\mathcal{L}_{\pi N}^{(4),CT} = 2\pi e^2 H^\dagger \left[ \left( \delta \beta_{M1}^{(s)} + \delta \beta_{M1}^{(v)} \tau_3 \right) \left( \frac{1}{2} g_{\mu\nu} - v_{\mu} v_{\nu} \right) - \left( \delta \alpha_{E1}^{(s)} + \delta \alpha_{E1}^{(v)} \tau_3 \right) v_{\mu} v_{\nu} \right] F^{\mu\rho} F^{\nu\rho} H.
\]

Counterterms shift \( \alpha_{E1} \) and \( \beta_{M1} \) at 4th order. Counterterms for spin pols at 5th order.

\[
\mathcal{L}_{\gamma N\Delta}^{(2)} = \frac{3e}{2M_N(M_N + M_\Delta)} \left[ \bar{\psi} \left( i g_M \tilde{F}^{\mu\nu} - g_E \gamma_5 F^{\mu\nu} \right) \partial_\mu \Psi^3 \right]
\bar{\Psi}^3 \left( i g_M \tilde{F}^{\mu\nu} - g_E \gamma_5 F^{\mu\nu} \right) \psi,
\]
\( \chi PT \) for Compton Scattering from the nucleon

We include nucleons, pions and the Delta in our Lagrangian.

\[
L^{(4),\text{CT}}_{\pi N} = 2\pi e^2 H \left[ \left( \delta \beta^{(s)}_{M_1} + \delta \beta^{(v)}_{M_1} \tau_3 \right) \left( \frac{1}{2} g_{\mu \nu} - v_{\mu} v_{\nu} \right) - \left( \delta \alpha^{(s)}_{E_1} + \delta \alpha^{(v)}_{E_1} \tau_3 \right) v_{\mu} v_{\nu} \right] F^{\mu \rho} F^{\nu \rho} H.
\]

Counterterms shift \( \alpha_{E_1} \) and \( \beta_{M_1} \) at 4th order. Counterterms for spin pols at 5th order.

\[
L^{\text{PP},(2)}_{\pi N \Delta} = \frac{3e}{2M_N(M_N + M_\Delta)} \left[ \bar{\psi} (i g_M F^{\mu \nu} - g E \gamma_5 F^{\mu \nu}) \partial_\mu \Psi^3 - \bar{\Psi}^3 \gamma_5 \partial_\mu (i g_M F^{\mu \nu} - g E \gamma_5 F^{\mu \nu}) \psi \right],
\]

\( \Delta \equiv M_\Delta - M_N \approx 271 \text{ MeV} \) is a rather small scale. Traditionally it is counted as \( \Delta / \Lambda_\chi \sim m_\pi / \Lambda_\chi \) ("SSE"). But in Compton scattering the pion is clearly important at lower energies than the Delta.

Alternative: count \( \frac{m_\pi}{\Delta} \sim \frac{\Lambda}{\Lambda_\chi} \) \( \Rightarrow \delta^2 \equiv \left( \frac{\Delta}{\Lambda_\chi} \right)^2 \sim \frac{m_\pi}{\Lambda_\chi} \)

Then graphs with one \( \Delta \) propagator are one order of \( \delta \) higher than the corresponding nucleon graphs in low energy region.

χPT for Compton Scattering from the nucleon

We include nucleons, pions and the Delta in our Lagrangian.

\[
L^{(4),CT}_{\pi N} = 2\pi e^2 H^\dagger \left[ (\delta \beta_{M_1}^{(s)} + \delta \beta_{M_1}^{(v)} \tau_3) \left( \frac{1}{2} g_{\mu \nu} - v_{\mu} v_{\nu} \right) - (\delta \alpha_{E_1}^{(s)} + \delta \alpha_{E_1}^{(v)} \tau_3) v_{\mu} v_{\nu} \right] F^{\mu \rho} F^{\nu \rho} H.
\]

Counterterms shift \( \alpha_{E_1} \) and \( \beta_{M_1} \) at 4th order. Counterterms for spin pols at 5th order.

\[
L^{PP,(2)}_{\gamma N\Delta} = \frac{3e}{2M_N(M_N + M_\Delta)} \left[ \bar{\psi} \left( i g_M \tilde{F}^{\mu \nu} - g_E \gamma_5 F^{\mu \nu} \right) \partial_\mu \Psi^3 - \bar{\Psi}^3 \gamma_5 \partial_\mu \left( i g_M \tilde{F}^{\mu \nu} - g_E \gamma_5 F^{\mu \nu} \right) \psi \right],
\]

\( \Delta \equiv M_\Delta - M_N \approx 271 \) MeV is a rather small scale. Traditionally it is counted as \( \Delta / \Lambda_\chi \sim m_\pi / \Lambda_\chi \) (“SSE”). But in Compton scattering the pion is clearly important at lower energies than the Delta.

Alternative: count \( \frac{m_\pi}{\Delta} \sim \frac{\Delta}{\Lambda_\chi} \) \( \Rightarrow \delta^2 \equiv \left( \frac{\Delta}{\Lambda_\chi} \right)^2 \sim \frac{m_\pi}{\Lambda_\chi} \)

Then graphs with one \( \Delta \) propagator are one order of \( \delta \) higher than the corresponding nucleon graphs in low energy region.


Different counting in resonance region; we work to at least NLO in both.
Born terms give the Thomson term and spin-dependent LETs (ensured by gauge and Lorentz invariance)

<table>
<thead>
<tr>
<th>Contribution with typical size</th>
<th>$\omega \sim m_\pi$</th>
<th>$\omega \sim \Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$e^2\delta^0$ (LO)</td>
<td>$e^2\delta^0$</td>
</tr>
<tr>
<td>(ii) (a)</td>
<td>$e^2\delta^2$</td>
<td>$e^2\delta^1$</td>
</tr>
<tr>
<td>(iii) (a)</td>
<td>$e^2\delta^4$</td>
<td>$e^2\delta^2$</td>
</tr>
</tbody>
</table>
Tree graphs

Born terms give the Thomson term and spin-dependent LETs (ensured by gauge and Lorentz invariance)

<table>
<thead>
<tr>
<th>contribution with typical size</th>
<th>( \omega \sim m_\pi )</th>
<th>( \omega \sim \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>( e^2 \delta^0 ) (LO)</td>
<td>( e^2 \delta^0 )</td>
</tr>
<tr>
<td>(ii) (a) (b) (c)</td>
<td>( e^2 \delta^2 )</td>
<td>( e^2 \delta^1 )</td>
</tr>
<tr>
<td>(iii) (a) (b)</td>
<td>( e^2 \delta^4 )</td>
<td>( e^2 \delta^2 )</td>
</tr>
</tbody>
</table>

In resonance region Delta-pole graph dominates

| (i)                           | \( e^2 \delta^3 \)   | \( e^2 \delta^{-1} \) (LO) |

Include Delta width by resuming self-energy:
Loops

<table>
<thead>
<tr>
<th>Contribution with typical size</th>
<th>$\omega \sim m_\pi$</th>
<th>$\omega \sim \Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$e^2\delta^2$</td>
<td>$e^2\delta^1$</td>
</tr>
<tr>
<td>(ii) (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (d)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (e)</td>
<td>$e^2\delta^4$</td>
<td>$e^2\delta^2$</td>
</tr>
<tr>
<td>(ii) (f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (g)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (h)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (j)</td>
<td>$e^2\delta^3$</td>
<td>$e^2\delta^1$</td>
</tr>
<tr>
<td>(ii) (k)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (l)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (n)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (o)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (p)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (q)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) (r)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At 4th order we have $1/M$ corrections and $c_i$ contributions

Delta loops are less important in low-energy region

Important: predicts full energy-dependent amplitudes, not just polarisabilities
Running of $\gamma N\Delta$ vertex

<table>
<thead>
<tr>
<th>contribution with typical size</th>
<th>$\omega \sim m_\pi$</th>
<th>$\omega \sim \Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$e\delta^2$</td>
<td>$e\delta^1$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$e\delta^4$</td>
<td>$e\delta^2$</td>
</tr>
<tr>
<td>(iii)</td>
<td>$e\delta^6$</td>
<td>$e\delta^3$</td>
</tr>
</tbody>
</table>

The inclusion of the imaginary part of running vertices satisfies Watson’s theorem
- cancellation of $I = 3/2$ loops at resonance
Anatomy of Compton amplitude—reminder

\[ T(\omega, z) = A_1(\omega, z) (\vec{e}'^* \cdot \vec{e}) + A_2(\omega, z) (\vec{e}'^* \cdot \hat{k}) (\vec{e} \cdot \hat{k}') \\
+ i A_3(\omega, z) \vec{\sigma} \cdot (\vec{e}'^* \times \vec{e}) + i A_4(\omega, z) \vec{\sigma} \cdot (\hat{k}' \times \vec{k}) (\vec{e}'^* \cdot \vec{e}) \\
+ i A_5(\omega, z) \vec{\sigma} \cdot \left[ (\vec{e}'^* \times \hat{k}) (\vec{e} \cdot \hat{k}') - (\vec{e} \times \hat{k}') (\vec{e}'^* \cdot \hat{k}) \right] \\
+ i A_6(\omega, z) \vec{\sigma} \cdot \left[ (\vec{e}'^* \times \hat{k}') (\vec{e} \cdot \hat{k}') - (\vec{e} \times \hat{k}) (\vec{e}'^* \cdot \hat{k}) \right]. \]

\( \omega \) - photon energy, \( z = \cos \theta \); Breit or cm frame

Non-Born pieces:

\[ \tilde{A}_1(\omega, z) = 4\pi [\alpha_{E1} + z \beta_{M1}] \omega^2 + \ldots \]
\[ \tilde{A}_2(\omega, z) = -4\pi \beta_{M1} \omega^2 + \ldots \]
\[ \tilde{A}_3(\omega, z) = -4\pi [\gamma_{E1E1} + z \gamma_{M1M1} + \gamma_{E1M2} + z \gamma_{M1E2}] \omega^3 + \ldots \]
\[ \tilde{A}_4(\omega, z) = 4\pi [-\gamma_{M1M1} + \gamma_{M1E2}] \omega^3 + \ldots \]
\[ \tilde{A}_5(\omega, z) = 4\pi \gamma_{M1M1} \omega^3 + \ldots \]
\[ \tilde{A}_6(\omega, z) = 4\pi \gamma_{E1M2} \omega^3 + \ldots \]

If we write \( \alpha_{E1} \rightarrow \alpha_{E1}(\omega) \) etc we have \( l = 1 \) in a multipole expansion
We can predict the full energy-dependence of the amplitudes, and only the value at the origin for $\alpha$, $\beta$ and $\gamma_{M1M1}$ are fitted.
We can predict the **full energy-dependence** of the amplitudes, and only the value at the origin for \(\alpha, \beta\) and \(\gamma_{M1M1}\) are fitted.

Note contribution of Delta, and also of the running of the \(\gamma_{N\Delta}\) vertex.

JMcG *et al.*, in preparation
We can predict the full energy-dependence of the amplitudes, and only the value at the origin for $\alpha$, $\beta$ and $\gamma_{M1M1}$ are fitted.

Note contribution of Delta, and also of the running of the $\gamma_{N\Delta}$ vertex.

JMcG et al., in preparation
Aside: Comparison of Multipoles

Different predictions do not fully agree on the physical origins of the polarisabilities. But Chiral and DR predictions agree very well for the shape of the energy dependence of corresponding multipoles.

Chiral: JMcG et al., V Lensky et al. in preparation
Aside: Comparison of Multipoles

Different predictions do not fully agree on the physical origins of the polarisabilities. But Chiral and DR predictions agree very well for the shape of the energy dependence of corresponding multipoles.


Our strategy: Static polarisabilities best obtained from Compton scattering.
\( \Delta \) significant from around 140 MeV upward—especially at backward angles
Fitting the proton data

\[ \sigma \rightleftharpoons 37 \text{ MeV} \]

\[ \sigma \rightleftharpoons 45 \text{ MeV} \]

\[ \sigma \rightleftharpoons 53 \text{ MeV} \]

\[ \sigma \rightleftharpoons 61 \text{ MeV} \]

\[ \sigma \rightleftharpoons 69 \text{ MeV} \]

\[ \sigma \rightleftharpoons 77 \text{ MeV} \]

\[ \sigma \rightleftharpoons 85 \text{ MeV} \]

\[ \sigma \rightleftharpoons 93 \text{ MeV} \]

\[ \sigma \rightleftharpoons 101 \text{ MeV} \]

\[ \sigma \rightleftharpoons 109 \text{ MeV} \]

\[ \sigma \rightleftharpoons 117 \text{ MeV} \]

\[ \sigma \rightleftharpoons 125 \text{ MeV} \]

\[ \sigma \rightleftharpoons 133 \text{ MeV} \]

\[ \sigma \rightleftharpoons 141 \text{ MeV} \]

\[ \sigma \rightleftharpoons 149 \text{ MeV} \]

\[ \sigma \rightleftharpoons 157 \text{ MeV} \]

\[ \sigma \rightleftharpoons 165 \text{ MeV} \]

\[ \sigma \rightleftharpoons 173 \text{ MeV} \]

\[ \sigma \rightleftharpoons 181 \text{ MeV} \]

\[ \sigma \rightleftharpoons 189 \text{ MeV} \]
\[ \frac{d\sigma}{d\Omega} \]

- \( \omega = 213 \text{ MeV} \)
- \( \omega = 221 \text{ MeV} \)
- \( \omega = 229 \text{ MeV} \)
- \( \omega = 237 \text{ MeV} \)
- \( \omega = 245 \text{ MeV} \)
- \( \omega = 253 \text{ MeV} \)
- \( \omega = 261 \text{ MeV} \)
- \( \omega = 269 \text{ MeV} \)
- \( \omega = 277 \text{ MeV} \)
- \( \omega = 285 \text{ MeV} \)
- \( \omega = 293 \text{ MeV} \)
- \( \omega = 301 \text{ MeV} \)
- \( \omega = 309 \text{ MeV} \)
- \( \omega = 317 \text{ MeV} \)
- \( \omega = 325 \text{ MeV} \)
- \( \omega = 333 \text{ MeV} \)

\[ \theta_{cm} \]

- Triangle: Illinois 60, 67
- Square: Cornell 61
- Green triangle: Moscow 60s
- Blue square: Bonn 76
- Black square: SAL 93
- Diamond: Mainz 92, 96, 99, 02
- Purple diamond: Mainz 01
- Star: Brookhaven 01 (LEGS)

The band gives the spread of the theory curve due to energy binning.
Constraining $\alpha + \beta$ with Baldin Sum rule and fitting consistent data set up to 170 MeV:

$\alpha_p = (10.65 \pm 0.35 \text{(stat)} \pm 0.2 \text{(Bald)} \pm 0.3 \text{(theory)}) \times 10^{-4} \text{fm}^3$

$\beta_p = (3.15 \pm 0.35 \text{(stat)} \pm 0.2 \text{(Bald)} \pm 0.3 \text{(theory)}) \times 10^{-4} \text{fm}^3$
figure courtesy of H. Grießhammer
Details of fit

Resonance region—very sensitive to magnetic $\gamma\eta\Delta$ coupling ($\sim g_M^4$). We iteratively fit $g_M$; value 10% lower than fit to photo production.
Details of fit

Resonance region—very sensitive to magnetic $\gamma_N\Delta$ coupling ($\sim g_M^4$). We iteratively fit $g_M$; value 10% lower than fit to photo production.

We cannot get an acceptable fit with the predicted value of $\gamma_{M1M1} = 6.4$ (large contributions both from $\Delta$ and $O(Q^4)\pi N$ loops).

We FIT it to give $\gamma_{M1M1} = 2.2 \pm 0.5$ (stat). Final fit good: $\chi^2 = 113.2$ for 135 d.o.f.

4th-order statistical errors on $\alpha - \beta$ are larger than 3rd order.

Deduce theory error from convergence:

$\alpha - \beta = 11.25$

$\alpha - \beta = 7.5$
Details of fit

Resonance region—very sensitive to magnetic $\gamma N\Delta$ coupling ($\sim g_M^4$). We iteratively fit $g_M$; value 10% lower than fit to photo production.

We cannot get an acceptable fit with the predicted value of $\gamma_{M1M1} = 6.4$ (large contributions both from $\Delta$ and $O(Q^4) \pi N$ loops).

We fit it to give $\gamma_{M1M1} = 2.2 \pm 0.5$ (stat). Final fit good: $\chi^2 = 113.2$ for 135 d.o.f.

4th-order statistical errors on $\alpha - \beta$ are larger than 3rd order.

Deduce theory error from convergence:

LO ($O(e^2\delta)$, BKM) $\alpha - \beta = 11.25$

$N^2$LO ($O(e^2\delta^4)$ $\alpha - \beta = 7.5$

Also check sensitivity to data: need to be somewhat selective of old data sets to get a good $\chi^2$, can’t fit Hallin data above 150MeV.
Checking in covariant framework (3rd order)

\[ \alpha_p = (10.6 \pm 0.25 \text{(stat)} \pm 0.2 \text{(Bald)} \pm 0.4 \text{(theory)}) \times 10^{-4} \text{fm}^3 \]

\[ \beta_p = (3.2 \pm 0.25 \text{(stat)} \pm 0.2 \text{(Bald)} \pm 0.4 \text{(theory)}) \times 10^{-4} \text{fm}^3 \]

More data needed?

We fit to low-energy data (up to 180-200 MeV), but with constraints from the higher-energy data to ensure the $\Delta$ parameters are sensible.

In spite of the amount of data, the sensitivity to the polarisabilities especially $\beta$ is not very high. Magnetic response varies rapidly with energy and zero-energy value is only a small fraction of the total by 150 MeV.

What would help

- Better data! (Theorist’s view...)
- More data in the region 160-250 MeV
- More data at forward and backward angles
- Data for polarised scattering (beam and target)
Consistent treatment of one- and two-body diagrams

Rescattering diagrams needed for Thomson limit, but fourth order at higher energies.

Where

The $\Delta$ only enters in $\bullet$ at this order.

Ensuring correct Thomson limit for deuteron is important even at 50-60 MeV.
Extraction of isoscalar polarisabilities

So far only $O(Q^3)$; further work required to go above pion threshold.

Older data from Illinois ●, Saskatoon, ◆ and Lund ▲ (29 pts in total)

Extraction of isoscalar polarisabilities

So far only $O(Q^3)$; further work required to go above pion threshold.

Older data from Illinois ●, Saskatoon, ● and Lund ▲ (29 pts in total)


\[ \alpha_s = 11.1 \pm 0.6\text{(stat)} \pm 0.2\text{(BSR)} \pm 0.8\text{(th)} \]

\[ \beta_s = 3.4 \mp 0.6\text{(stat)} \pm 0.2\text{(BSR)} \mp 0.8\text{(th)}. \]

\[ \alpha_n = 11.65 \pm 1.25\text{(stat)} \pm 0.2\text{(BSR)} \pm 0.8\text{(th)} \]

\[ \beta_n = 3.55 \mp 1.25\text{(stat)} \pm 0.2\text{(BSR)} \mp 0.8\text{(th)} \]
Comparison

\[ \beta_{M1} \left( 10^{-4} \text{ fm}^3 \right) \]

\[ \alpha_{E1} \left( 10^{-4} \text{ fm}^3 \right) \]

exp(stat+sys)+theory/model $1\sigma$–error in quadrature

figure courtesy of H. Grießhammer
Lattice and chiral extrapolations

- p,n static $\alpha_{E1}$ fit to exp
- $n\text{HYP} \infty \text{vol}$ prelim. Lujan/Alexandru/... 2014
- Detmold et al. 2010
- $\chi_{EFT \ hg}$/... 2015

$\alpha_{E1}(n) \left[ 10^{-4} \text{fm}^3 \right]$ vs. $m_\pi \left[ \text{MeV} \right]$

$\beta_{M1} \left[ 10^{-4} \text{fm}^3 \right]$ vs. $m_\pi \left[ \text{MeV} \right]$

$\chi_{EFT}$ proton
$\chi_{EFT}$ neutron

$m_\pi \gtrsim \Lambda_{\chi}$

Here Be Dragons

figures courtesy of H. Grießhammer
Spin-dependent Compton scattering

\[ H_{\text{eff}} = \frac{(p - QA)^2}{2m} + Q\phi - \frac{(Q + \kappa)}{2m} \sigma \cdot H - \frac{1}{2} 4\pi \left( \alpha E^2 + \beta \bar{H}^2 \right) + \gamma_E \bar{\sigma} \cdot \bar{E} \times \bar{E} + \gamma_M \bar{\sigma} \cdot \bar{H} \times \bar{H} - 2\gamma_E E_i \sigma_i H_j + 2\gamma_M H_i \sigma_i E_j \right) \]

Spin-polarisabilities have most influence if the beam or target or both are polarised.

Linearly polarised beam \( \Sigma_3 = \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} \)

Circular beam, polarised target

\( \Sigma_{2x} = \frac{\sigma^{R}_\perp - \sigma^{L}_\perp}{\sigma^{R}_\perp + \sigma^{L}_\perp} \) \quad \Sigma_{2z} = \frac{\sigma^{R}_\parallel - \sigma^{L}_\parallel}{\sigma^{R}_\parallel + \sigma^{L}_\parallel} \)
New programme at A2 experiment using Crystal Ball and TAPS detectors

Large-acceptance detector
Tagged photon beam, circ. or lin. polarised or unpolarised,

Unpolarised (liquid hydrogen)...

or polarised (butanol) protons
First results from MAMI

$\Sigma_{2x}$: Target polarised perpendicular to reaction plane, RH or LH circularly polarised photons

P. Martell, PhD thesis

$\omega_{lab} = 288 \pm 15$ MeV

$\omega_{lab} = 330 \pm 15$ MeV

$\gamma_{M1}$ varied by $\pm 1$
First results from MAMI

$\Sigma_2$: Target polarised perpendicular to reaction plane, RH or LH circularly polarised photons. P. Martell, PhD thesis

$\Sigma_3$: Unpolarised target, photons polarised in or perpendicular to reaction plane

- C. Collicott, PhD thesis (LEGS data ∗)

Judith McGovern  
Polarisabilities  
Pisa July 3rd 2015
Predictions and fits for proton polarisabilities

Chiral prediction ($\delta^3$, BChPT, Lensky) and NLO ($\delta^4$, HBChPT, JMcG) papers pending

<table>
<thead>
<tr>
<th></th>
<th>$\alpha + \beta$</th>
<th>$\alpha - \beta$</th>
<th>$\gamma_0$</th>
<th>$\gamma_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^3$ B</td>
<td>15.1</td>
<td>7.3</td>
<td>-0.9</td>
<td>[−46.4] + 7.2</td>
</tr>
<tr>
<td>$\delta^4$ HB</td>
<td>13.8 ± 0.4</td>
<td>7.5 ± 0.7 ± 0.6</td>
<td>-2.6*</td>
<td>[−46.4] + 5.5*</td>
</tr>
<tr>
<td>SR/DR</td>
<td>13.8 ± 0.4</td>
<td>10.7 ± 0.2</td>
<td>-0.9 ± 0.14</td>
<td>[−46.4] + 7.6 ± 1.8</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{E1E1}$</th>
<th>$\gamma_{M1M1}$</th>
<th>$\gamma_{E1M2}$</th>
<th>$\gamma_{M1E2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^3$ B</td>
<td>-3.4</td>
<td>3.0</td>
<td>0.2</td>
<td>1.1</td>
</tr>
<tr>
<td>$\delta^4$</td>
<td>-1.1 ± 1.8</td>
<td>2.2 ± 0.5 $^{\text{stat}}$ ± 0.7 $^{\text{th}}$</td>
<td>-0.4 ± 0.4</td>
<td>1.9 ± 0.4</td>
</tr>
<tr>
<td>DR</td>
<td>-3.85 ± 0.45</td>
<td>2.8 ± 0.1</td>
<td>-0.15 ± 0.15</td>
<td>2.0 ± 0.1</td>
</tr>
<tr>
<td>MAMI1</td>
<td>-3.5 ± 1.2</td>
<td>3.2 ± 0.9</td>
<td>-0.7 ± 1.2</td>
<td>2.0 ± 0.3</td>
</tr>
<tr>
<td>MAMI2</td>
<td>-5.0 ± 1.5</td>
<td>3.1 ± 0.9</td>
<td>1.7 ± 1.7</td>
<td>1.3 ± 0.4</td>
</tr>
</tbody>
</table>


MAMI1: published extraction from MAMI $\Sigma_{2x}$ and LEGS $\Sigma_3$ Martel
MAMI2: unpublished extraction from $\Sigma_{2x}$ and $\Sigma_3$ Collicott
$\delta^4$: theory errors from convergence. *: $\gamma_{M1M1}$ from fit, otherwise $\gamma_{M1M1} = 6.4$

Note errors mean different things in different lines; DR especially only reflect spread from two databases
Predictions and fits for neutron polarisabilities

Chiral prediction ($\delta^3$, BChPT, Lensky) and NLO ($\delta^4$, HBChPT, JMCG) papers pending

<table>
<thead>
<tr>
<th></th>
<th>$\alpha + \beta$</th>
<th>$\alpha - \beta$</th>
<th>$\gamma_0$</th>
<th>$\gamma_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^3$ B</td>
<td>18.3</td>
<td>9.1</td>
<td>0</td>
<td>[46.4] + 8.9</td>
</tr>
<tr>
<td>$\delta^4$ HB</td>
<td>15.2 ± 0.4</td>
<td>8.1 ± 2.5 ± 0.8</td>
<td>0.5*</td>
<td>[46.4] + 7.7*</td>
</tr>
<tr>
<td>SR/DR</td>
<td>15.2 ± 0.4</td>
<td>11.5</td>
<td>-0.25</td>
<td>[46.4] ± 13.35</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{E1E1}$</th>
<th>$\gamma_{M1M1}$</th>
<th>$\gamma_{E1M2}$</th>
<th>$\gamma_{M1E2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^3$ B</td>
<td>-4.7</td>
<td>2.9</td>
<td>0.2</td>
<td>1.5</td>
</tr>
<tr>
<td>$\delta^4$</td>
<td>-4.0 ± 1.8</td>
<td>1.3 ± 0.5 stat ± 0.7* th</td>
<td>-0.1 ± 0.4</td>
<td>2.4 ± 0.4</td>
</tr>
<tr>
<td>DR</td>
<td>-5.75 ± 0.15</td>
<td>3.8 ± 0.1</td>
<td>-0.8 ± 0.1</td>
<td>3.0 ± 0.1</td>
</tr>
</tbody>
</table>

DR: fixed-t, Holstein et al., Babusci et al.

$\delta^4$: theory errors as proton. *: including input from proton fit.
But not just polarisabilities
But not just polarisabilities

$\Sigma_3$: Unpolarised target, photons polarised in or perpendicular to reaction plane
But not just polarisabilities

**Σ₃**: Unpolarised target, photons polarised in or perpendicular to reaction plane

- Q_{cm} = 65 ± 5°
- Q_{cm} = 75 ± 5°
- Q_{cm} = 90 ± 5°
- Q_{cm} = 105 ± 5°
- Q_{cm} = 115 ± 5°
- Q_{cm} = 125 ± 5°
- Q_{cm} = 135 ± 5°
- Q_{cm} = 145 ± 5°

- \( \omega_{lab} = 277 ± 10 \text{ MeV} \)
- \( \omega_{lab} = 297 ± 10 \text{ MeV} \)
But not just polarisabilities

**Σ₃**: Unpolarised target, photons polarised in or perpendicular to reaction plane

![Graphs showing Σ₃ as a function of g_M increased for different values of θ_cm.](image)
But not just polarisabilities

\( \Sigma_3 \): Unpolarised target, photons polarised in or perpendicular to reaction plane
Multipoles again

MAMI data is taken well into the resonance region....
Not ideal for extracting zero-energy polarisabilities!
Some PRELIMINARY data on $\Sigma_3$ from MAMI V. Sokhoyan and E. Downie

90 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)

110 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)

130 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)
Lower energy experiments

Some PRELIMINARY data on $\Sigma_3$ from MAMI V. Sokhoyan and E. Downie

- 90 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)
- 110 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)
- 130 MeV, Born, $\beta=3.15$ (full), 3.15 (LEX)

- 90 MeV, Born, full with $\beta=1.15,3.15,5.15$
- 110 MeV, Born with $\beta=1.15,3.15,5.15$
- 130 MeV, Born, full with $\beta=1.15,3.15,5.15$
Some PRELIMINARY data on $\Sigma_3$ from MAMI
V. Sokhoyan and E. Downie

Experiments also planned at HI$\gamma$S @TUNL
low energy—up to about 100 MeV currently, 150 MeV after upgrades.
Polarised scattering from deuterium

$$\Delta_x^{\text{circ}} = \frac{d\sigma}{d\Omega} \uparrow \rightarrow - \frac{d\sigma}{d\Omega} \uparrow \leftarrow$$

$$\omega_{\text{lab}} = 125 \text{ MeV}, \delta \alpha_{E1} = \pm 2$$

$$\omega_{\text{lab}} = 125 \text{ MeV}, \delta \beta_{M1} = \pm 2$$

$$\omega_{\text{lab}} = 125 \text{ MeV}, \delta \gamma_{E1E1} = \pm 2$$

$$\omega_{\text{lab}} = 125 \text{ MeV}, \delta \gamma_{M1M1} = \pm 2$$

$\Delta$ included, 3rd order.

Polarised scattering from $^3\text{He}$

Unpolarised, varying $\beta_n$

$\Delta_z$, varying $\gamma_{1n}$

$\Delta_x$, varying $\gamma_{1n}$

120 MeV, 3rd order, no $\Delta$

Future

Experimental programme at MAMI, MAXlab and HIγS
Future

Experimental programme at MAMI, MAXlab and HIγS

Polarised $\gamma p$ scattering at MAMI: data being analysed; 120 MeV data especially interesting for polarisabilities. Active target being developed for double-polarised experiments at low energies

Plans for $^3$He
Future

Experimental programme at MAMI, MAXlab and H1γS

Polarised $\gamma p$ scattering at MAMI: data being analysed; 120 MeV data especially interesting for polarisabilities. Active target being developed for double-polarised experiments at low energies

Plans for $^3$He

Further data on deuteron at higher energies expected from MAX-lab / MAX-IV
Future

Experimental programme at MAMI, MAXlab and H1γS

Polarised $\gamma p$ scattering at MAMI: data being analysed; 120 MeV data especially interesting for polarisabilities. Active target being developed for double-polarised experiments at low energies

Plans for $^3$He

Further data on deuteron at higher energies expected from MAX-lab / MAX-IV

H1γS up to about 100 MeV: approved experiments on polarised proton, deuteron and $^3$He
Future

Experimental programme at MAMI, MAXlab and HIγS

Polarised $\gamma p$ scattering at MAMI: data being analysed; 120 MeV data especially interesting for polarisabilities. Active target being developed for double-polarised experiments at low energies

Plans for $^3$He

Further data on deuteron at higher energies expected from MAX-lab / MAX-IV

HIγS up to about 100 MeV: approved experiments on polarised proton, deuteron and $^3$He

Should soon know much more about the polarisabilities of the proton and neutron!
See the slides of

**Evie Downie** Compton scattering and the nucleon polarizabilities

**Gerald Feldman**
New results for Compton scattering on deuterium: A better determination of the neutron electromagnetic polarizabilities

**Haiyan Gao** Latest results on few-body physics from HI\(\gamma\)S

**John Annand** Compton scattering from 3He and 4He using an active target

**Harald Grießhammer**
Compton scattering and nucleon polarisabilities in chiral EFT: The next steps
Assessing theory errors using residual cutoff dependence (also Daniel Phillips)

**Berhan Demissie** Extracting neutron polarizabilities from Compton scattering on quasi-free neutron in \(\gamma d \rightarrow \gamma np\)

And also talks by Franziska Hagelstein, Nadiia Krupina, Jose Manuel Alarcon, Kalyan Allada ...