

Muon g-2 and Hadronic Vacuum Polarization: Recent Developments

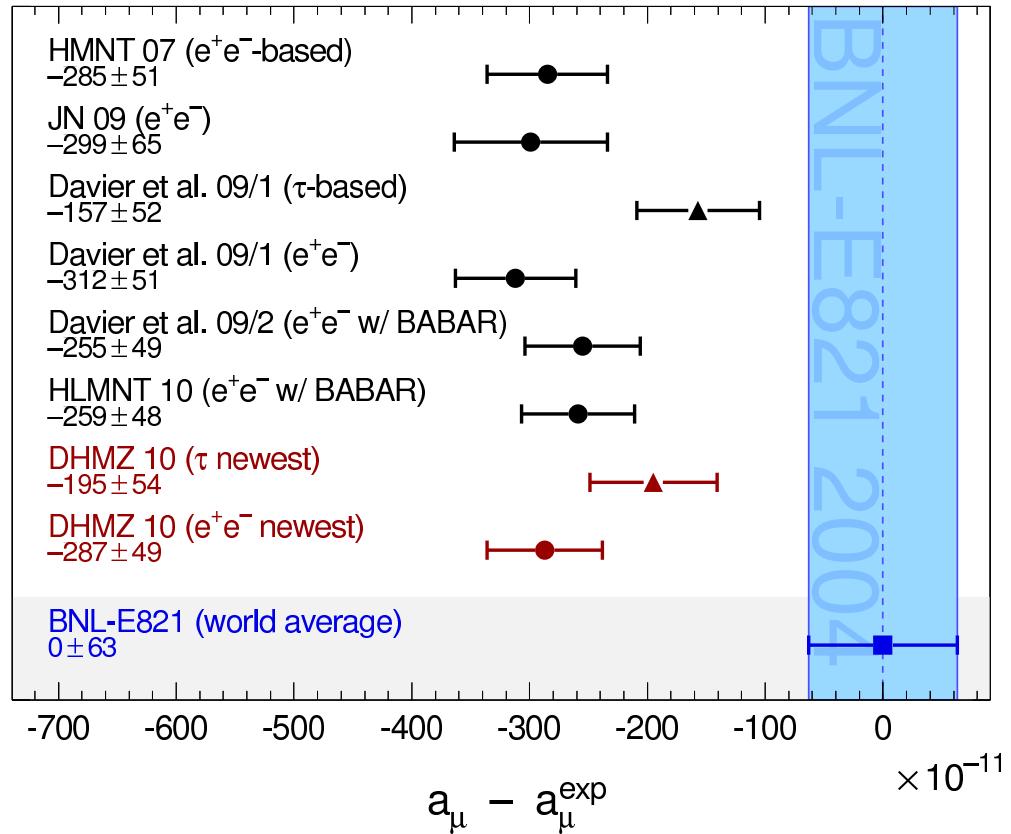
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Novosibirsk, Russia

Outline

1. Status of a_μ
2. Hadronic vacuum polarization from $e^+e^- \rightarrow$ hadrons
3. Transition form factors from $e^+e^- \rightarrow$ hadrons
4. Future and prospects

Experiment vs. Theory

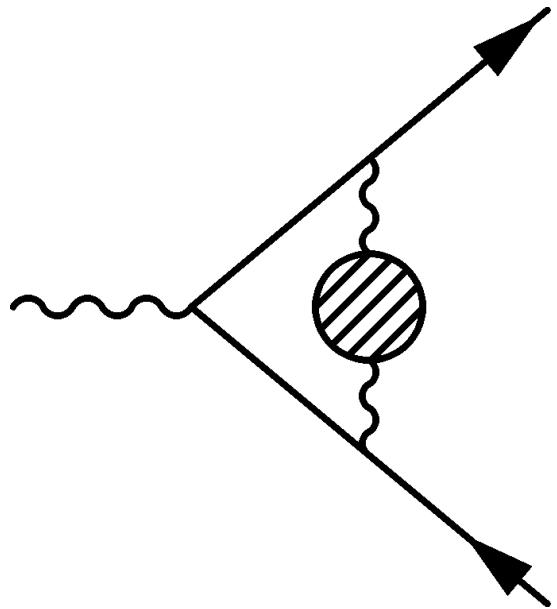


Experiment is higher than theory by ~ 3.5 standard deviations

The error is dominated by Had LO (4.2×10^{-10}) and Had LbL (2.6×10^{-10})

Hadronic contribution a_μ^{had}

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$



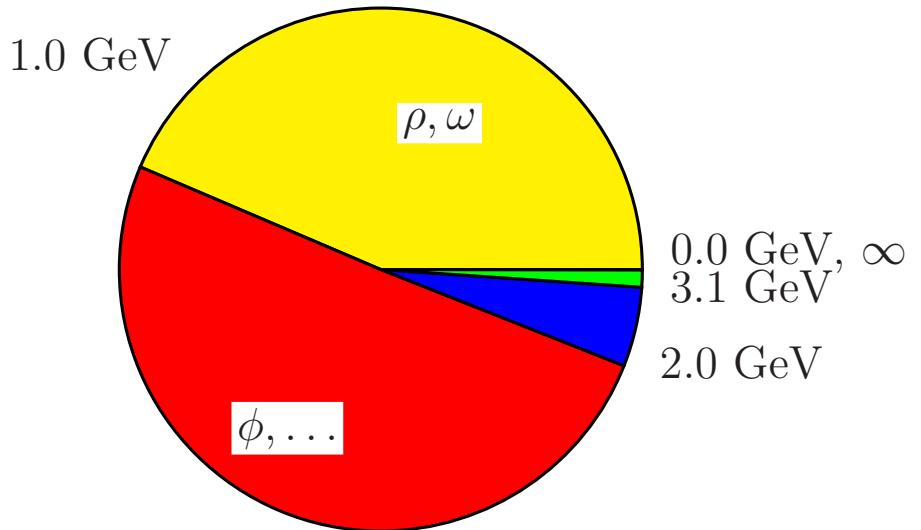
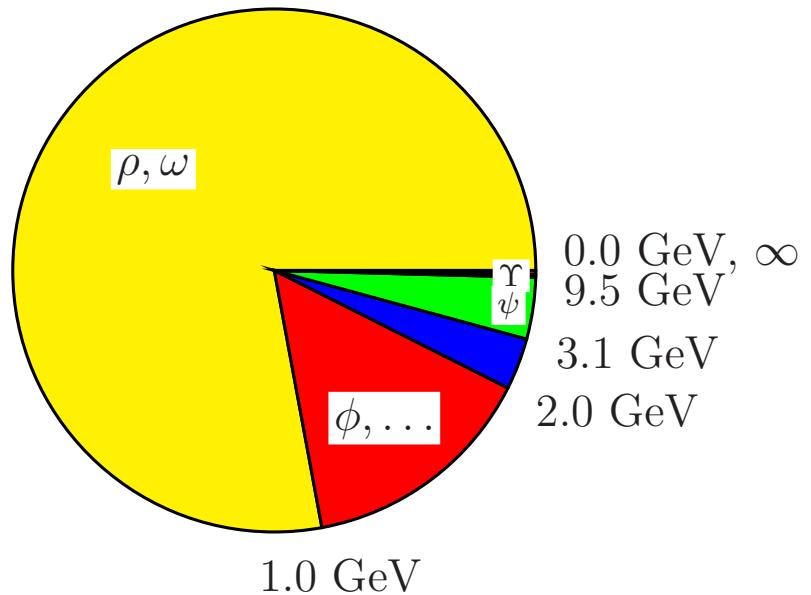
$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^\infty ds \frac{R(s) \hat{K}(s)}{s^2},$$

C. Bouchiat, L. Michel, Bouchiat, 1961;
M. Gourdin, E. de Rafael, 1969

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

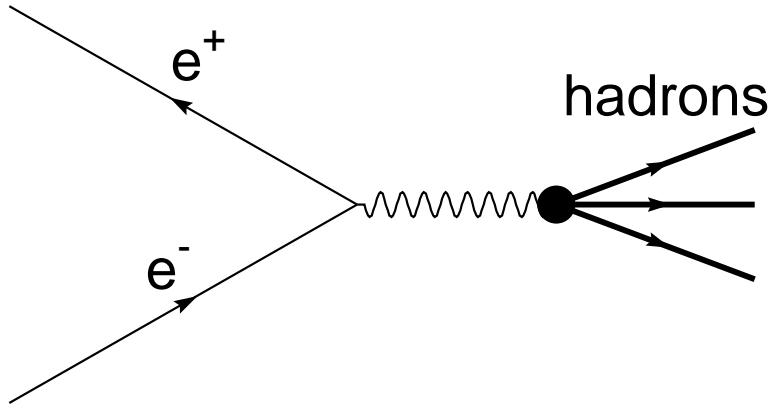
$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \rightarrow \infty$,
 $1/s^2$ emphasizes low energies, particularly $e^+e^- \rightarrow \pi^+\pi^-$.
 $a_\mu^{\text{had,LO}} \sim 700 \cdot 10^{-10} \Rightarrow$ accuracy better than 1% needed

Contributions of Various Energy Ranges to $a_\mu^{\text{had,LO}}$

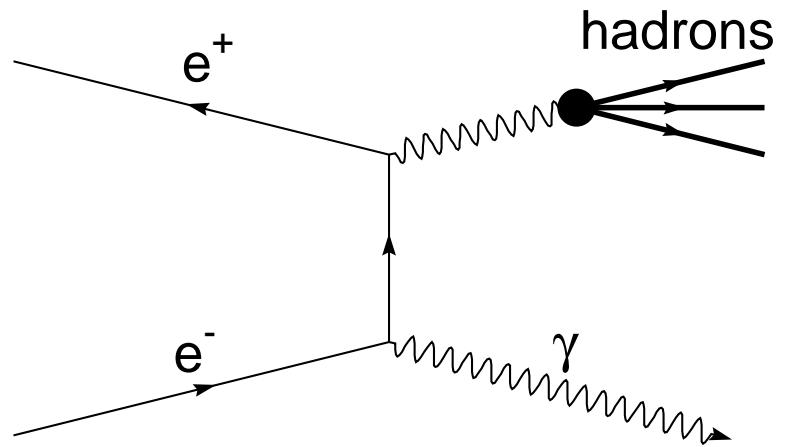


More than 72% of $a_\mu^{\text{had,LO}}$ come from $e^+e^- \rightarrow \pi^+\pi^-$ and
more than 90% from the energy range below 2 GeV

Scan and ISR



Scan

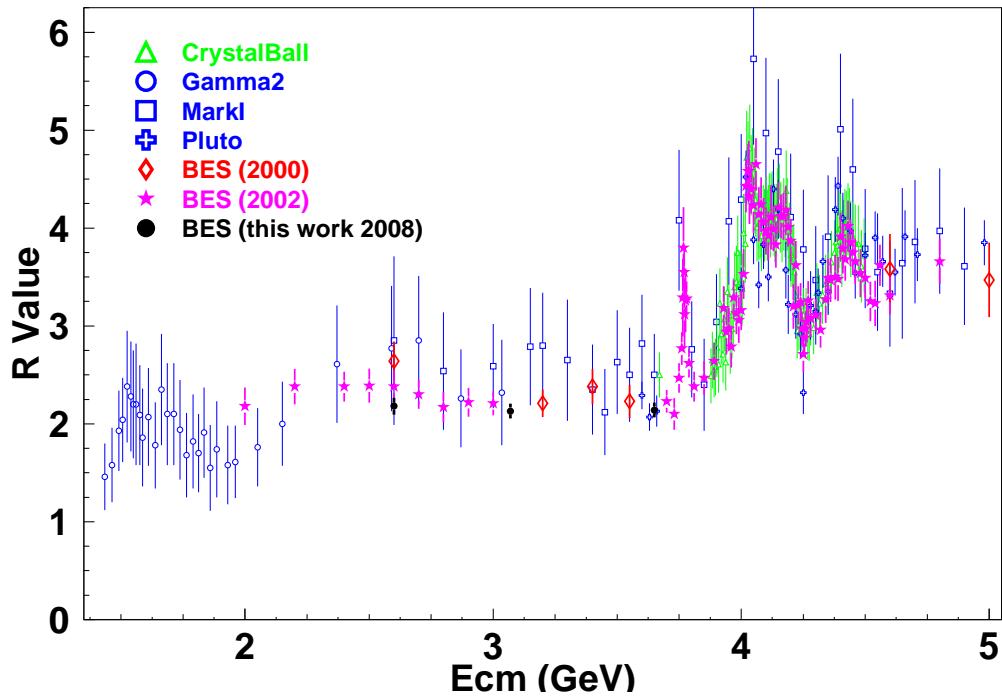


ISR

Scan can provide larger data samples at fixed energy

ISR benefits from the same systematics and flat acceptance,
but may suffer from more complicated radiative effects,
a broad range of collision energies

R Measurement Below 5 GeV

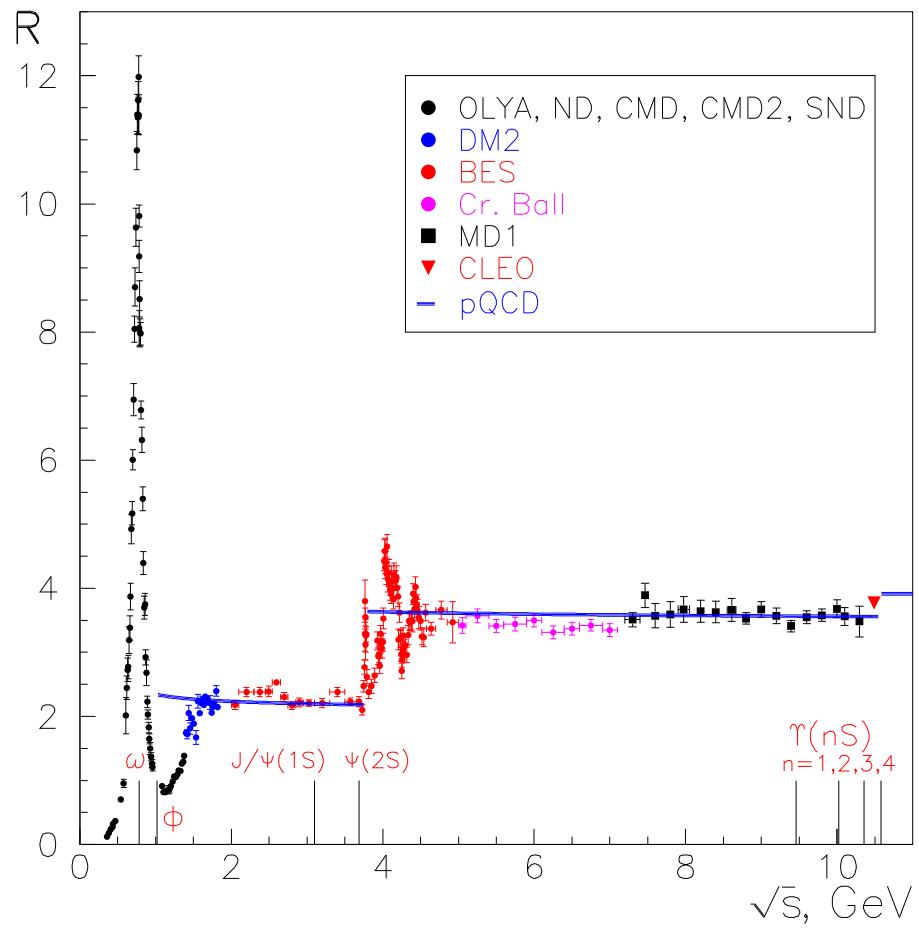


Dominated by BES: stat. errors (3-5)% , syst. errors (5-8)%

J.Z. Bai et al., Phys.Rev.Lett. 84 (2000) 594, Phys.Rev.Lett. 88 (2002) 101802;
M. Ablikim et al., Phys.Rev.Lett. 97 (2006) 262001, Phys.Lett. B677 (2009) 239

Improvement to about 3% in total can be expected from BESIII and KEDR

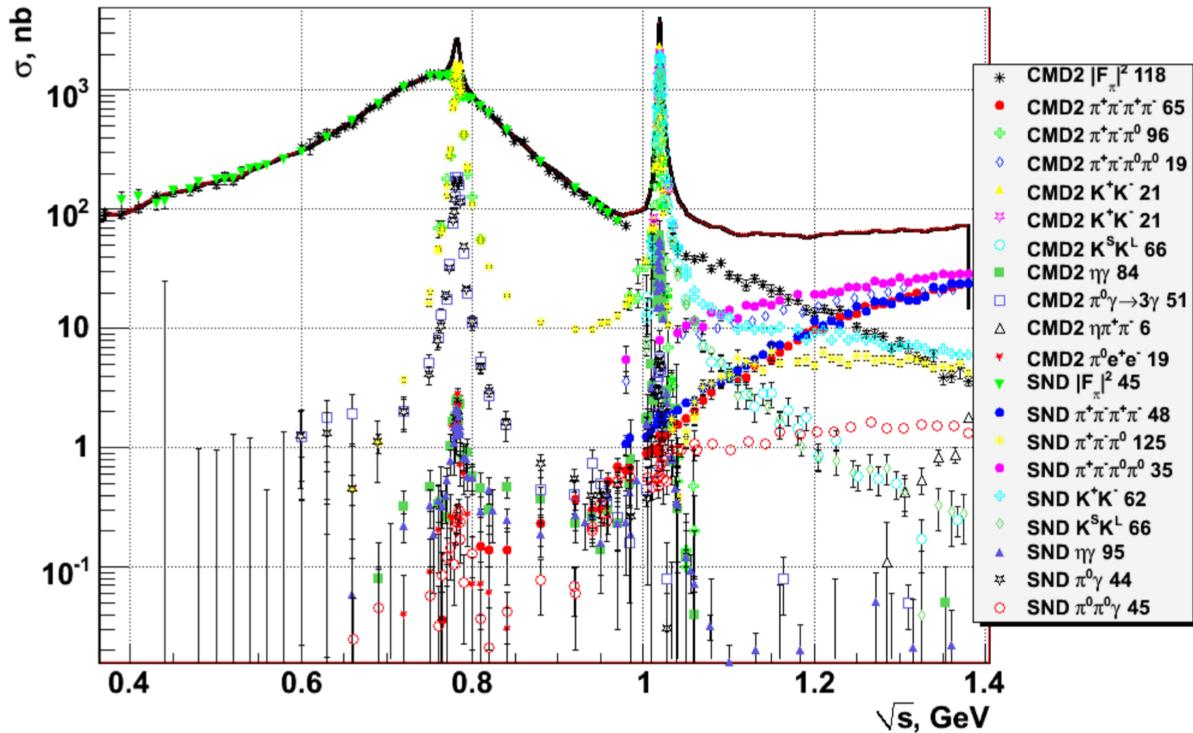
R Measurements below 10 GeV



How Is R Measured at Low Energy ($\sqrt{s} < 2$ GeV)?

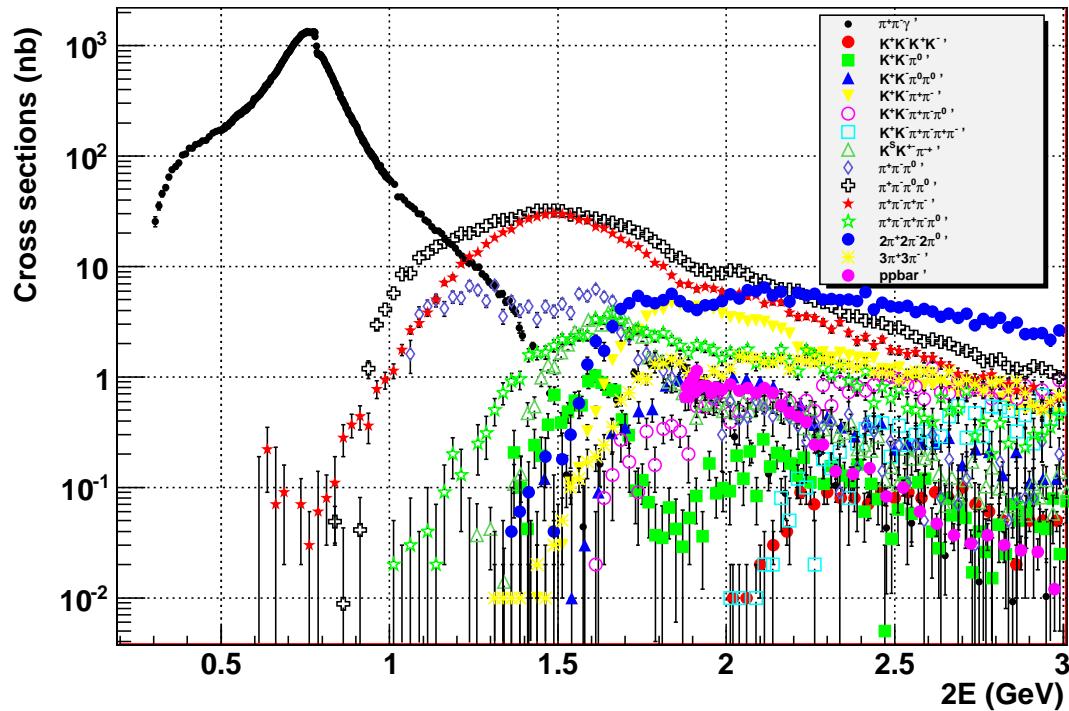
- The cross section rapidly changes with energy because one is far from asymptotics and there are many resonances
- There is no good theoretical model to find ϵ
- Exclusive approach: specific final states studied separately ($\pi^+\pi^-$, K^+K^- , K_SK_L , $n\pi$, $K\bar{K}m\pi$, $p\bar{p}$, $n\bar{n}$, ...)
- The cross sections measured are summed, remember about correlated uncertainties!
- Important not to miss some final states
- Large radiative corrections

Current Status of Exclusive Measurements – I



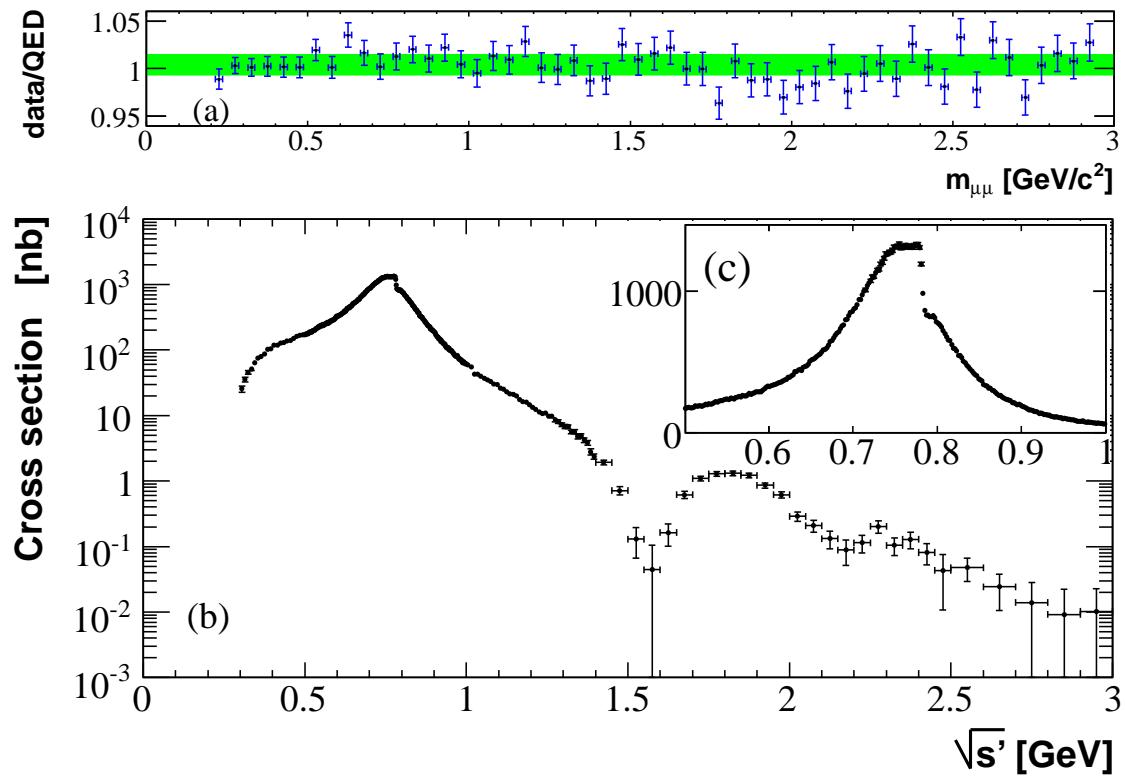
Impressive achievements of CMD-2, SND (scan at $\sqrt{s} < 1.4$ GeV)
and KLOE (ISR at $\sqrt{s} < 1.0$ GeV)

Current Status of Exclusive Measurements – II



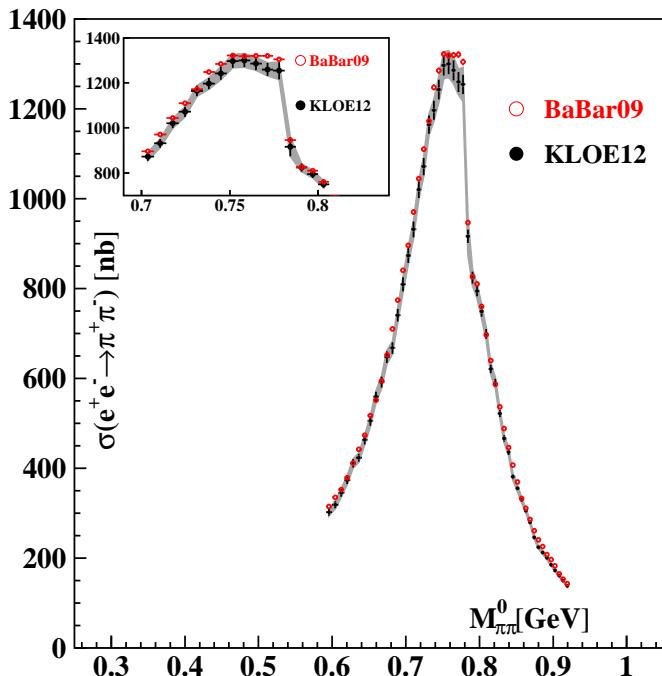
BaBar used ISR to study the energy range $\sqrt{s} < 3.0$ GeV, Belle/BelleII and BESIII can contribute as well to ISR measurements

$e^+e^- \rightarrow \pi^+\pi^-$ at BaBar



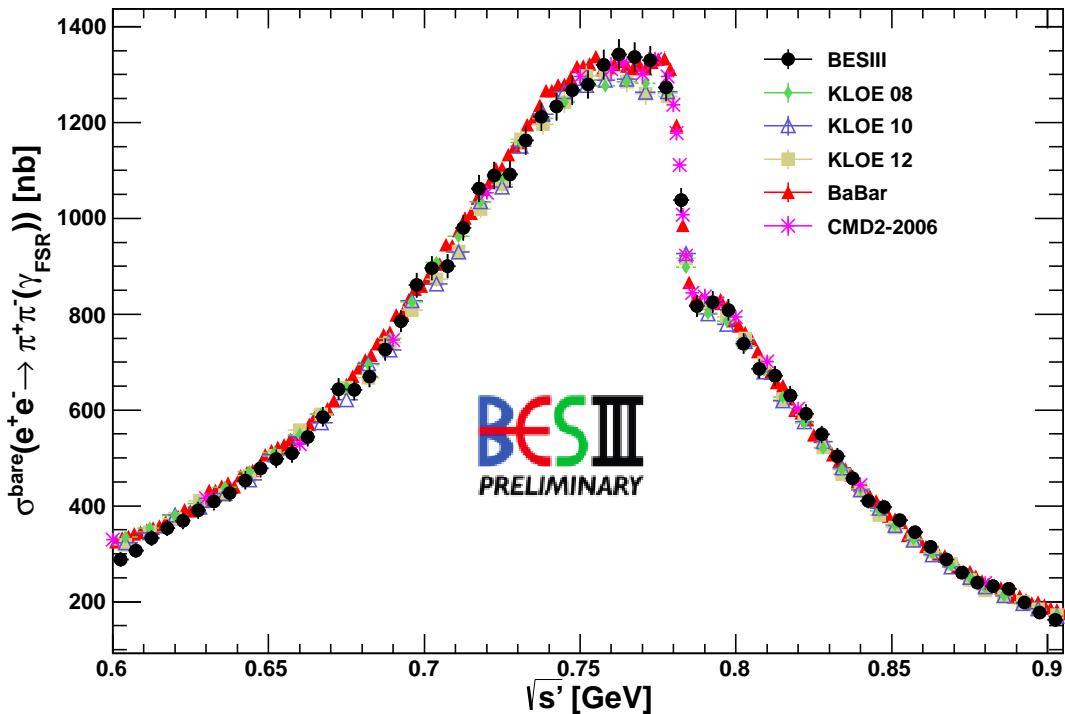
The systematic error near the ρ is 0.5%

J.P. Lees et al., Phys. Rev. D86 (2012) 032013

$e^+e^- \rightarrow \pi^+\pi^-$ at KLOE/KLOE-2

The systematic error is 0.7%

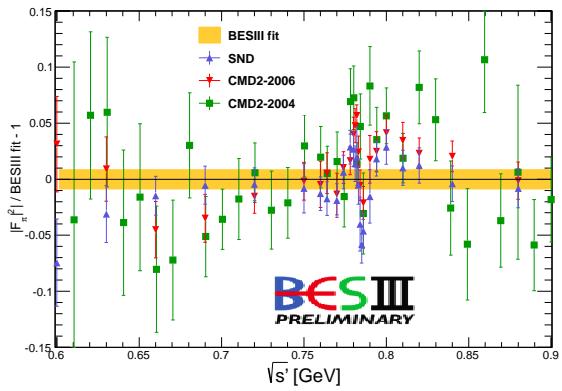
D. Babusci et al., Phys. Lett. B720 (2013) 336

$e^+e^- \rightarrow \pi^+\pi^-$ with ISR at BESIII – I

The achieved systematic error is 1.3%, the goal is (0.9-1.0)%

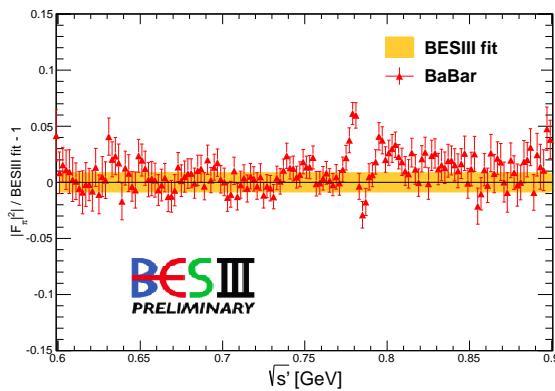
BESIII: B. Kloss, talk at the Photon15, Novosibirsk, June 2015

$e^+e^- \rightarrow \pi^+\pi^-$ with ISR at BESIII – II

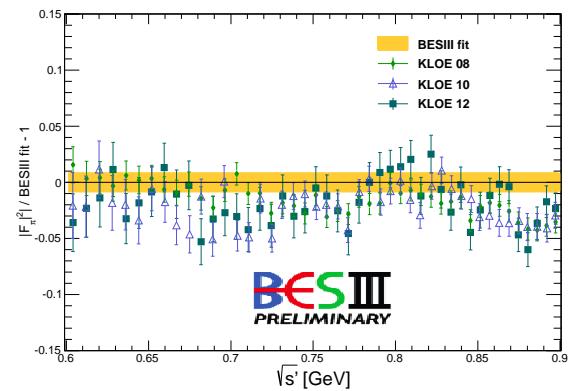


SND: JETP 103 (2006) 380

CMD-2: PLB 648 (2007) 28



BaBar: PRL 103 (2009) 231801

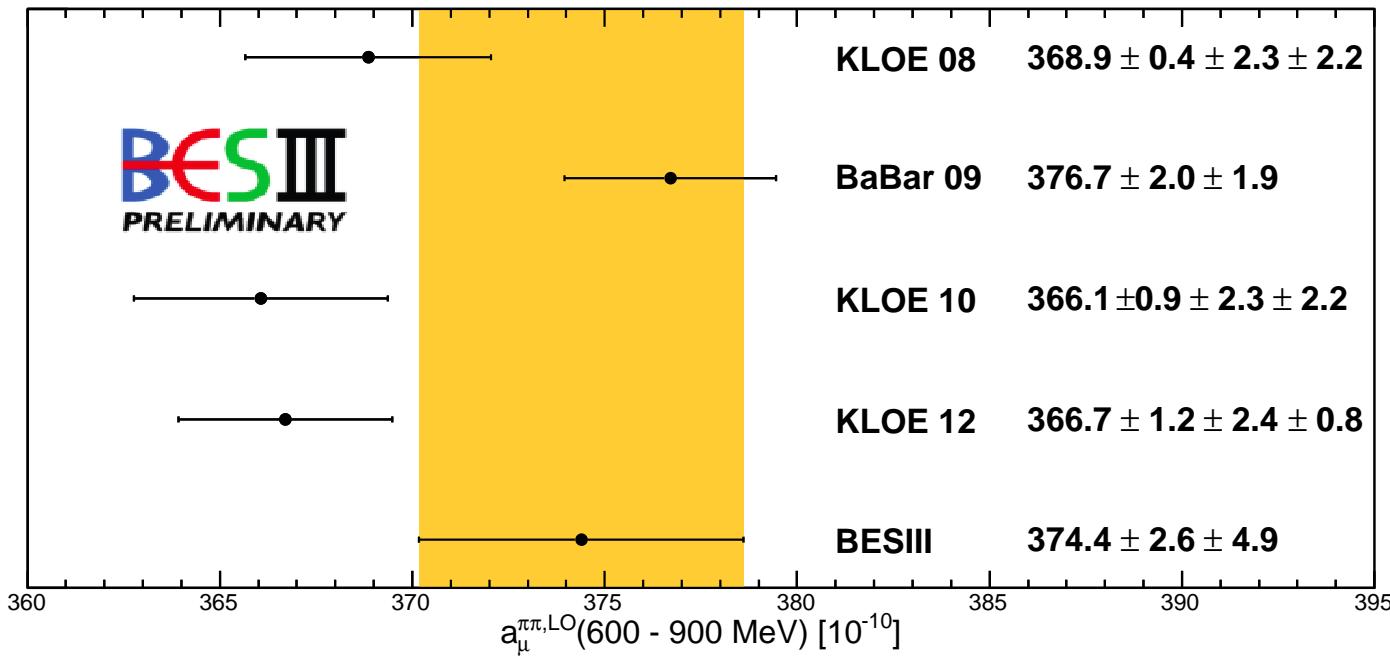


KLOE08: PLB 670 (2009) 285

KLOE10: PLB 700 (2011) 102

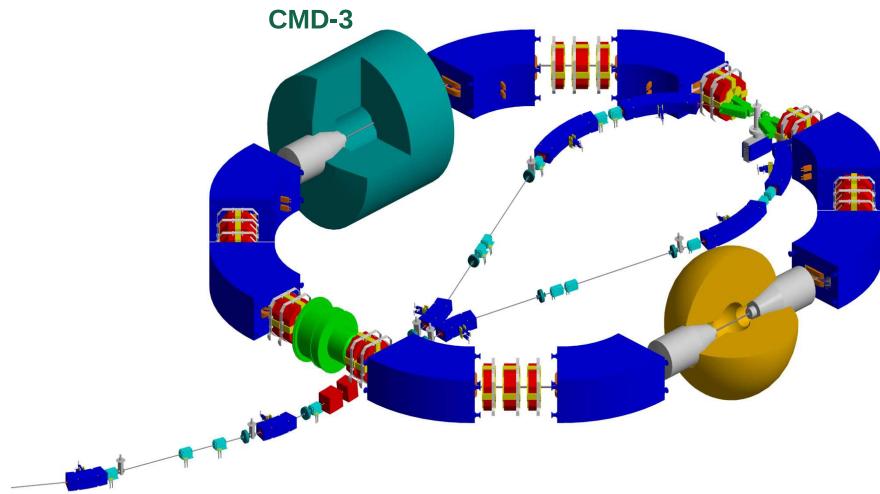
KLOE12: PLB 720 (2013) 336

Agreement between different ISR results is far from perfect
 BESIII: B. Kloss, talk at the Photon15, Novosibirsk, June 2015

$e^+e^- \rightarrow \pi^+\pi^-$ with ISR at BESIII – III


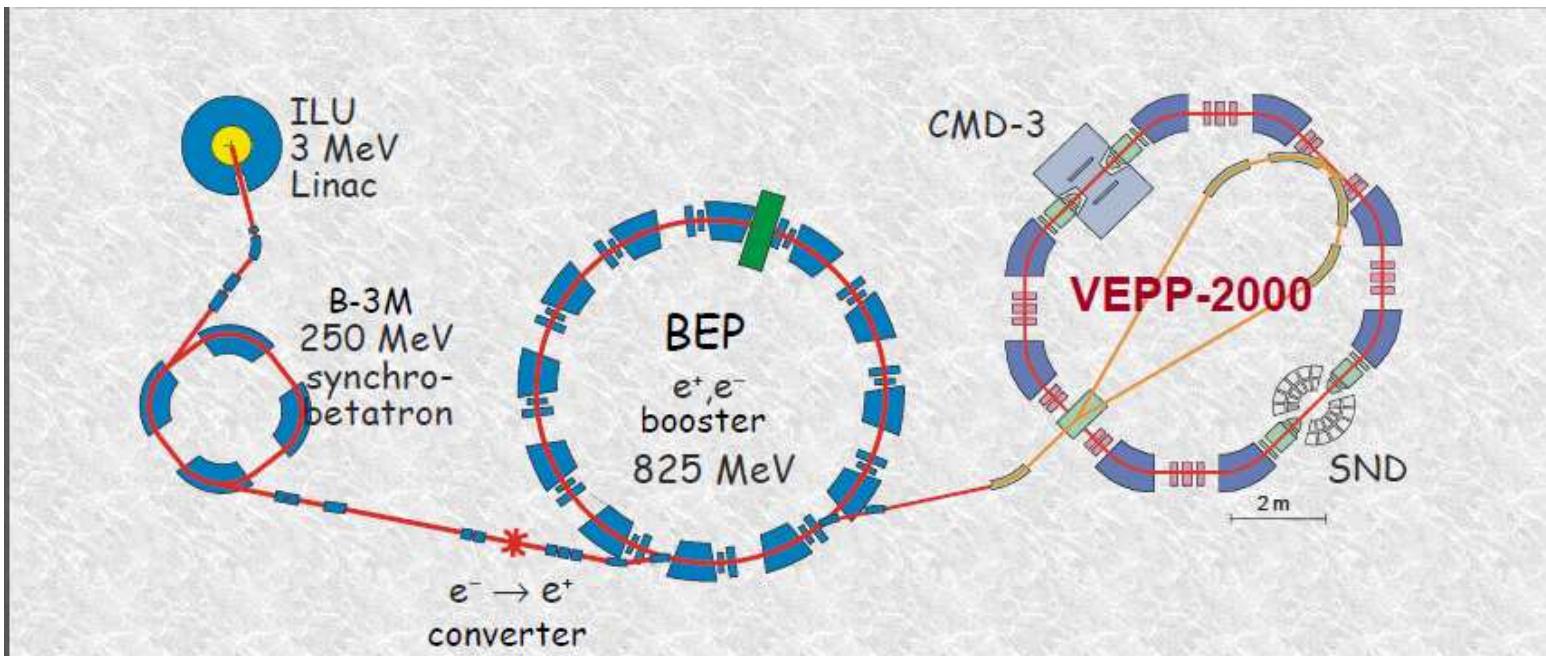
Reevaluation of luminosity should improve systematics to (0.9-1.0)%
and can shift the result a bit down to KLOE
BESIII: B. Kloss, talk at the Photon15, Novosibirsk, June 2015

VEPP-2000 – I



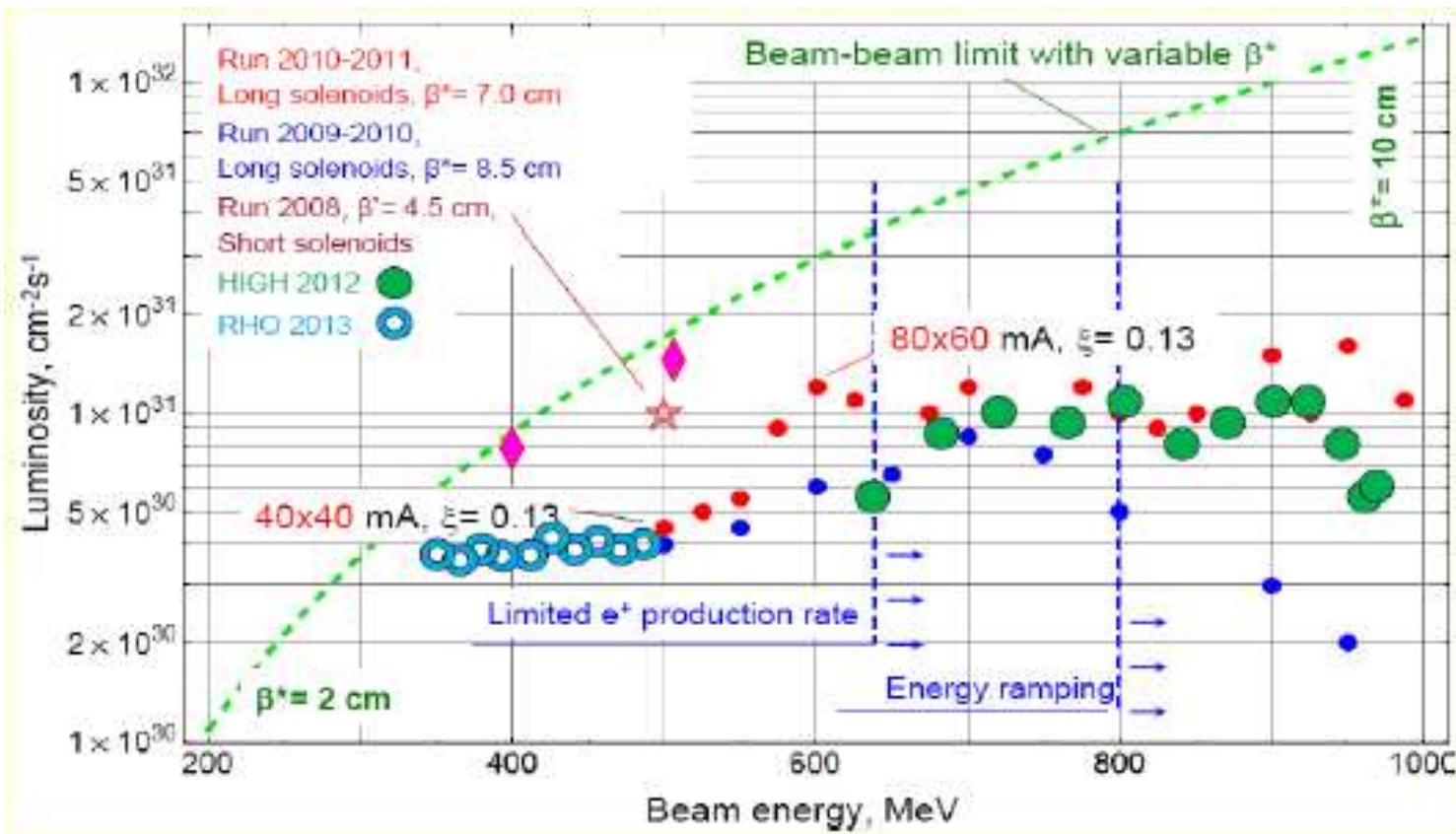
Collider	Operation	\sqrt{s} , MeV	$\mathcal{L}, 10^{30} \text{cm}^{-2}\text{s}^{-1}$
VEPP-2M	1975-2000	[360,1400]	3
VEPP-2000	2010-	[$2m_\pi$, 2000]	100

VEPP-2000 – II

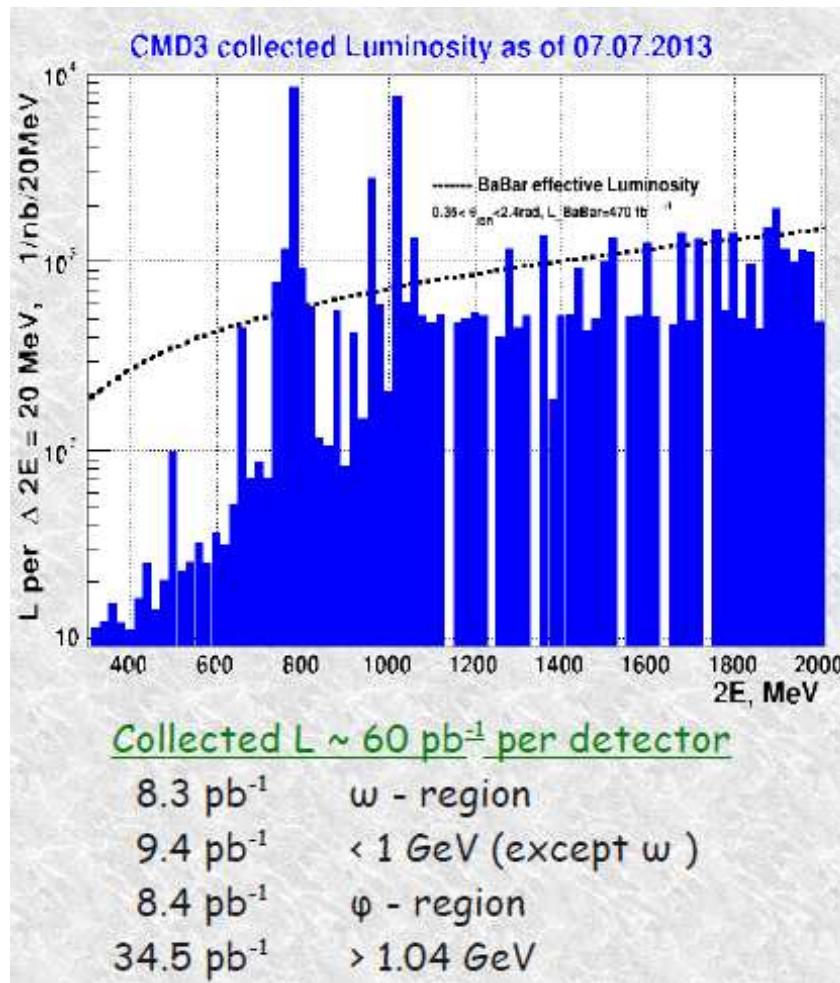


New optics with round beams \Rightarrow higher luminosity,
precise beam energy measurement using LCBS

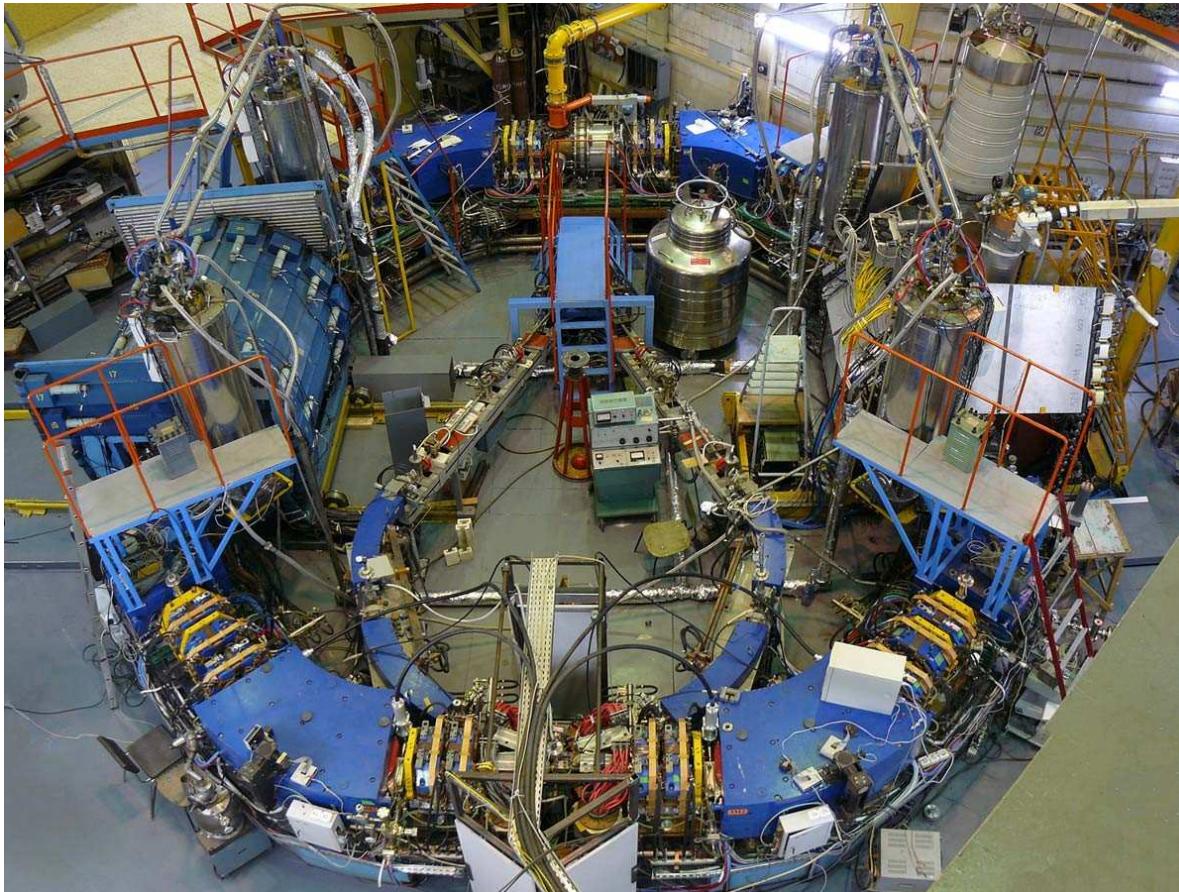
VEPP-2000 – III

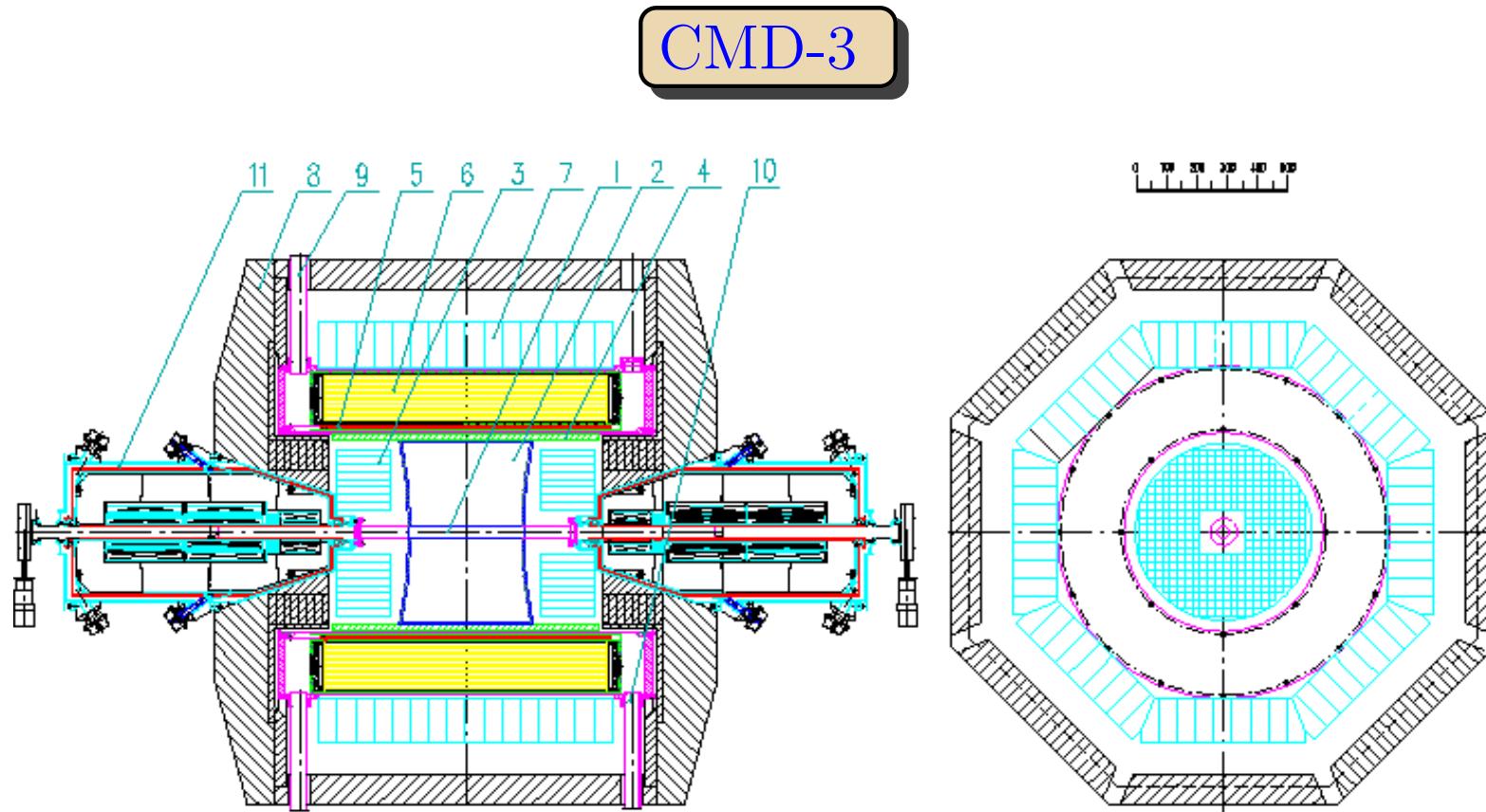


Data Taking at VEPP-2000

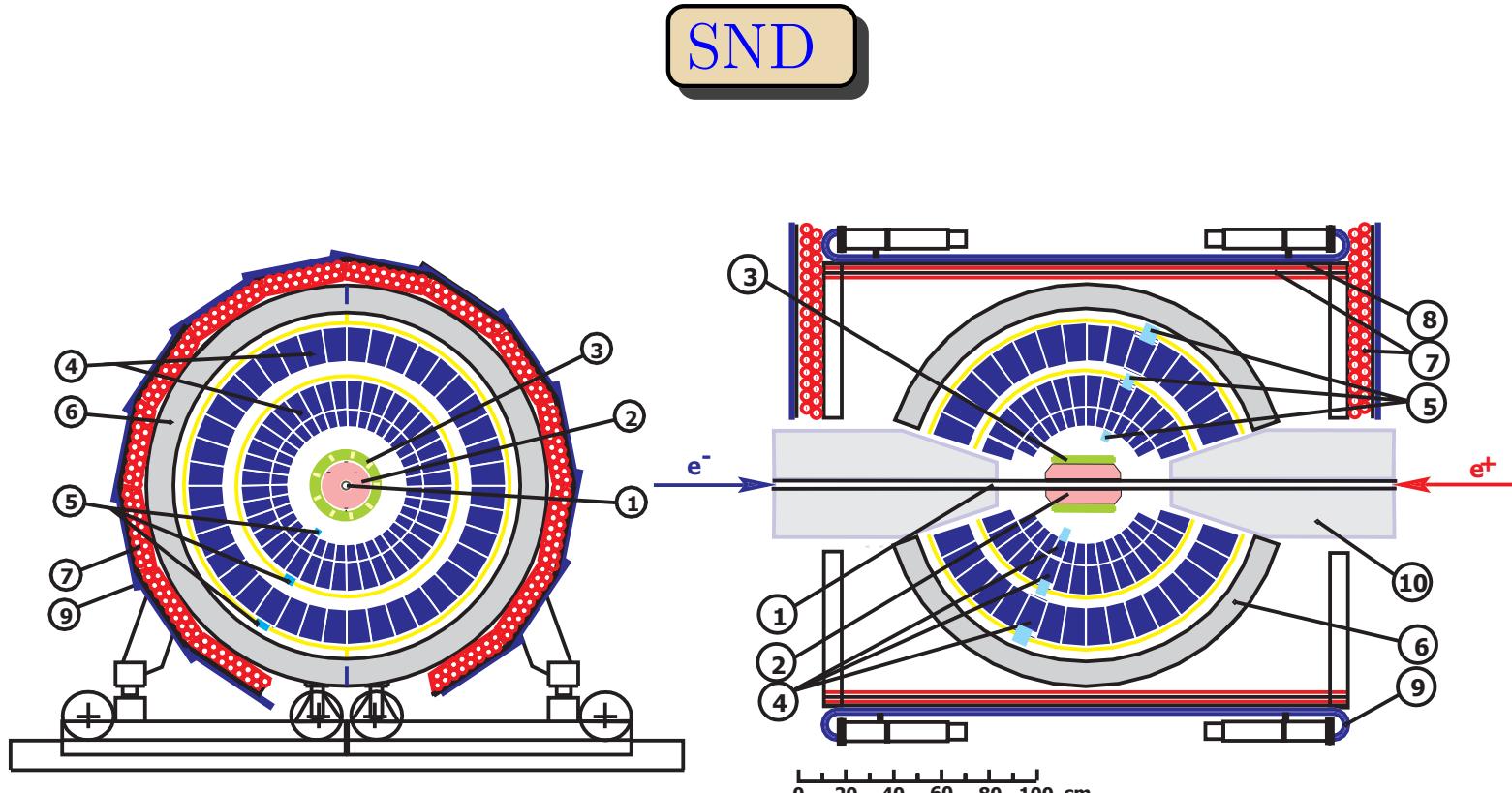


VEPP-2000 and Detectors





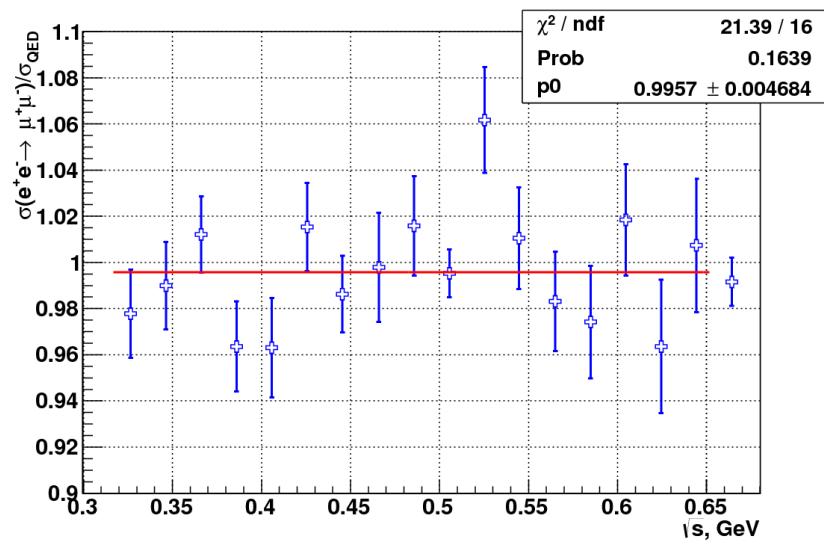
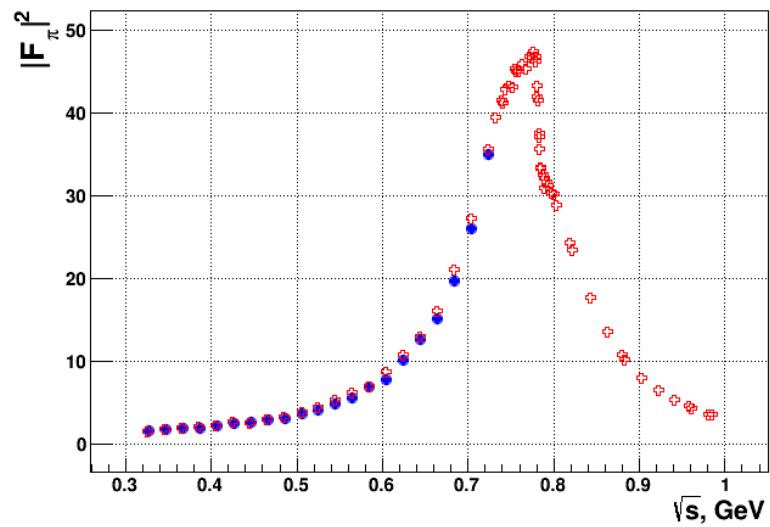
General-purpose magnetic (1.3T) detector with 3 e/m calorimeters (LXe, CsI, BGO)



High-resolution NaI calorimeter with excellent tracking and PID

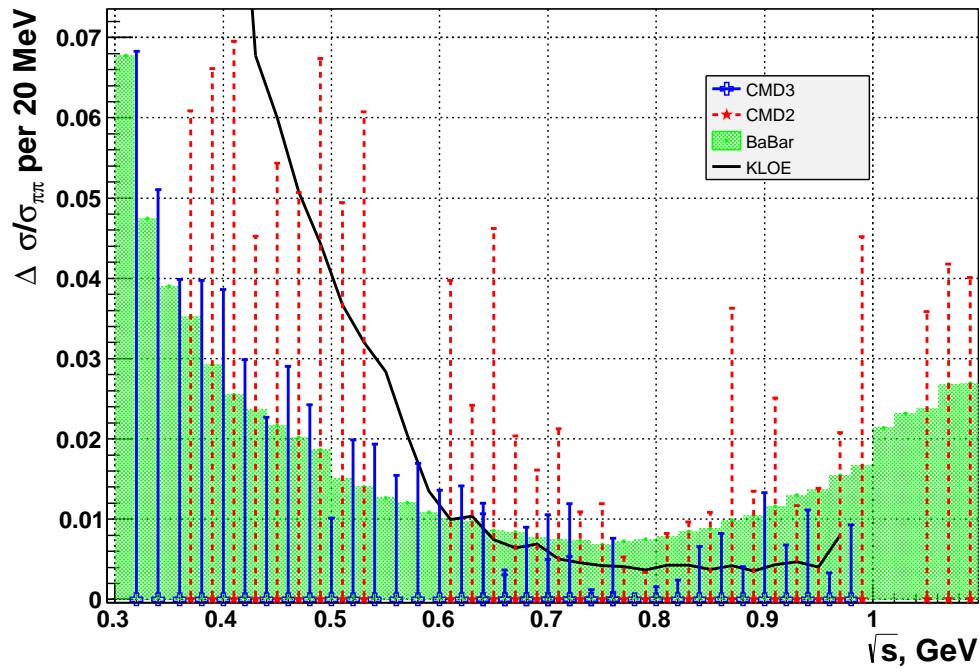
Performance of VEPP-2000 and Detectors

- The maximum luminosity is $2 \cdot 10^{31} \text{ cm}^{-1}\text{s}^{-1}$ at 1.7-1.8 GeV, falling much slower with decreasing energy than before the round beams
- The integrated luminosity is about 60 pb^{-1} per detector, a factor of 6 higher than before from ϕ to 2 GeV, the number of multihadronic events per $1 \text{ pb}^{-1} \sim 50k$
- In 2013 we reached $2 \times 160 \text{ MeV}$, the smallest \sqrt{s} ever
- At high energies lumi is limited by a deficit of positrons and maximum energy of the booster (825 MeV now)
- A long shutdown until 2015 to increase the booster energy to 1 GeV and commission the new injection complex to reach $10^{32} \text{ cm}^{-1}\text{s}^{-1}$
- Both detectors perform reasonably well with reconstruction of both tracks and photons and redundancy ($\eta \rightarrow 2\gamma, \pi^+\pi^-\pi^0, 3\pi^0, \pi^+\pi^-\gamma, \omega \rightarrow \pi^+\pi^-\pi^0, \pi^0\gamma$)

$$e^+e^- \rightarrow \pi^+\pi^- \text{ at CMD-3 - I}$$


Identification at low energy - by DC with separation of $\mu^+\mu^-$

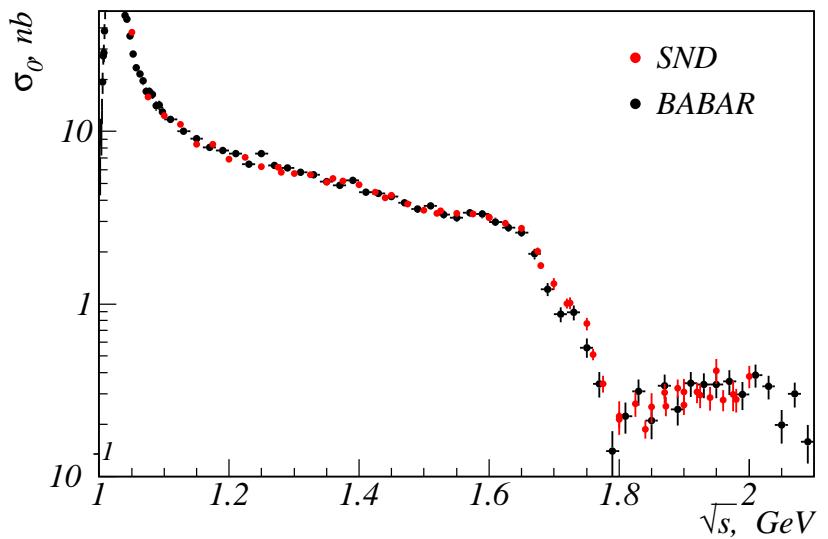
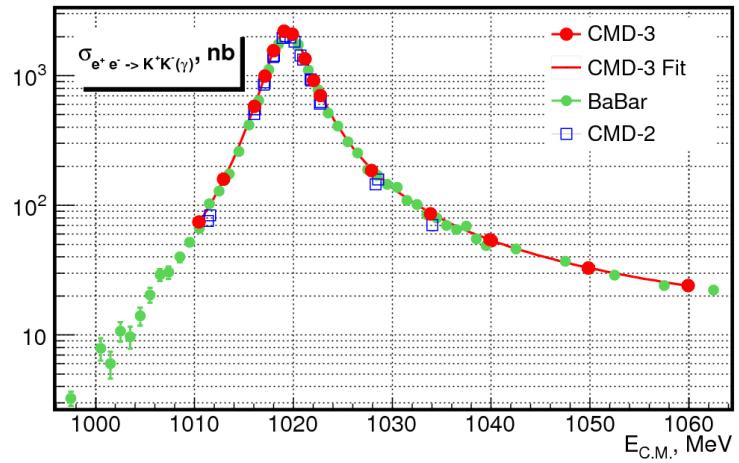
At high energy - by energy deposition in calorimeters

$e^+e^- \rightarrow \pi^+\pi^-$ at CMD-3 – II

Statistical precision better than that of BaBar

Systematic error: goal 0.35% at the ρ (BaBar achieved 0.5%)

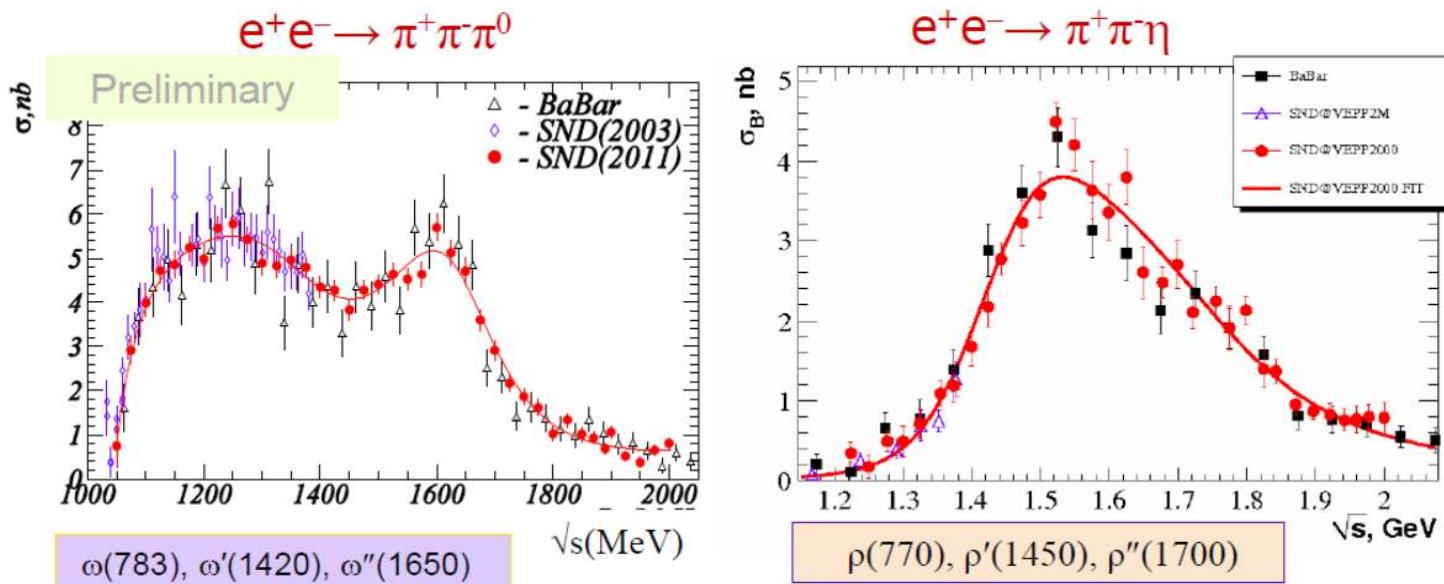
$$e^+e^- \rightarrow K^+K^-$$



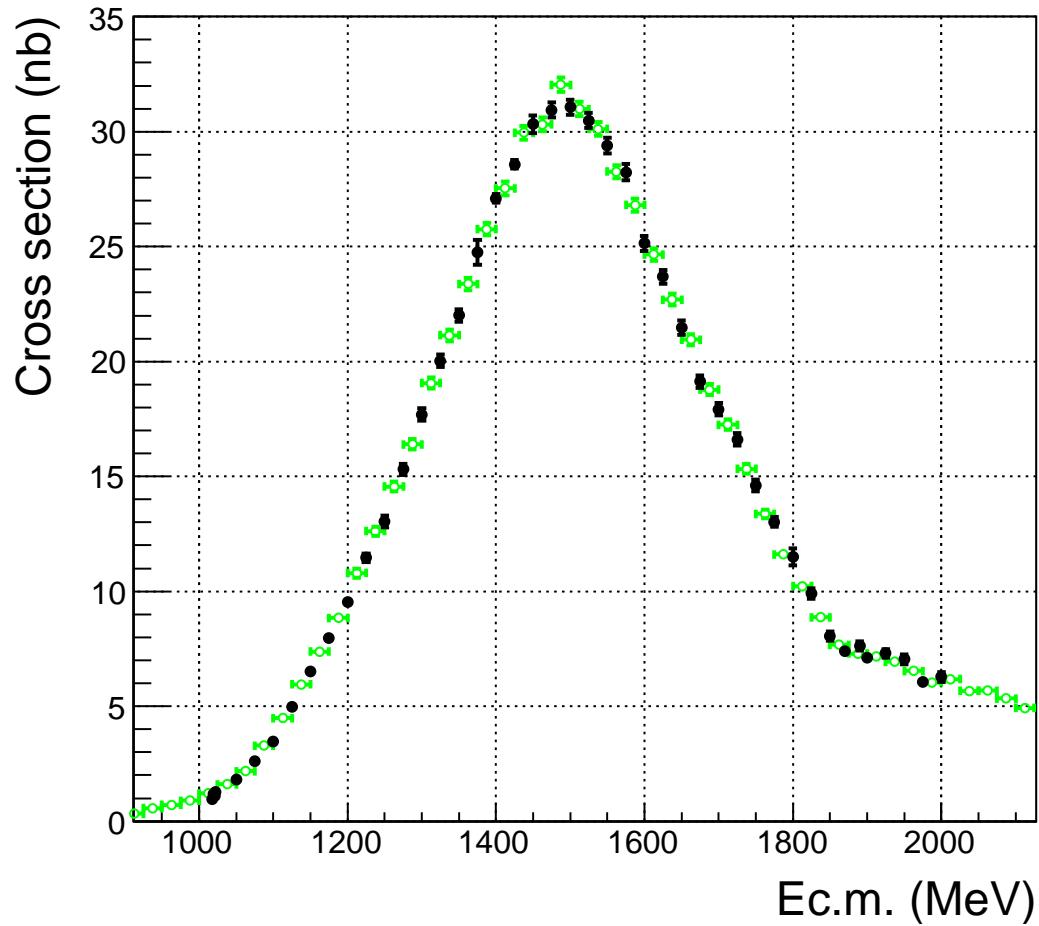
BaBar claims aggressive systematics of 0.72% at the ϕ , increasing to 7% at 2 GeV

CMD-3 hopes to reach (1-2)% at the ϕ and not much worse at higher energy

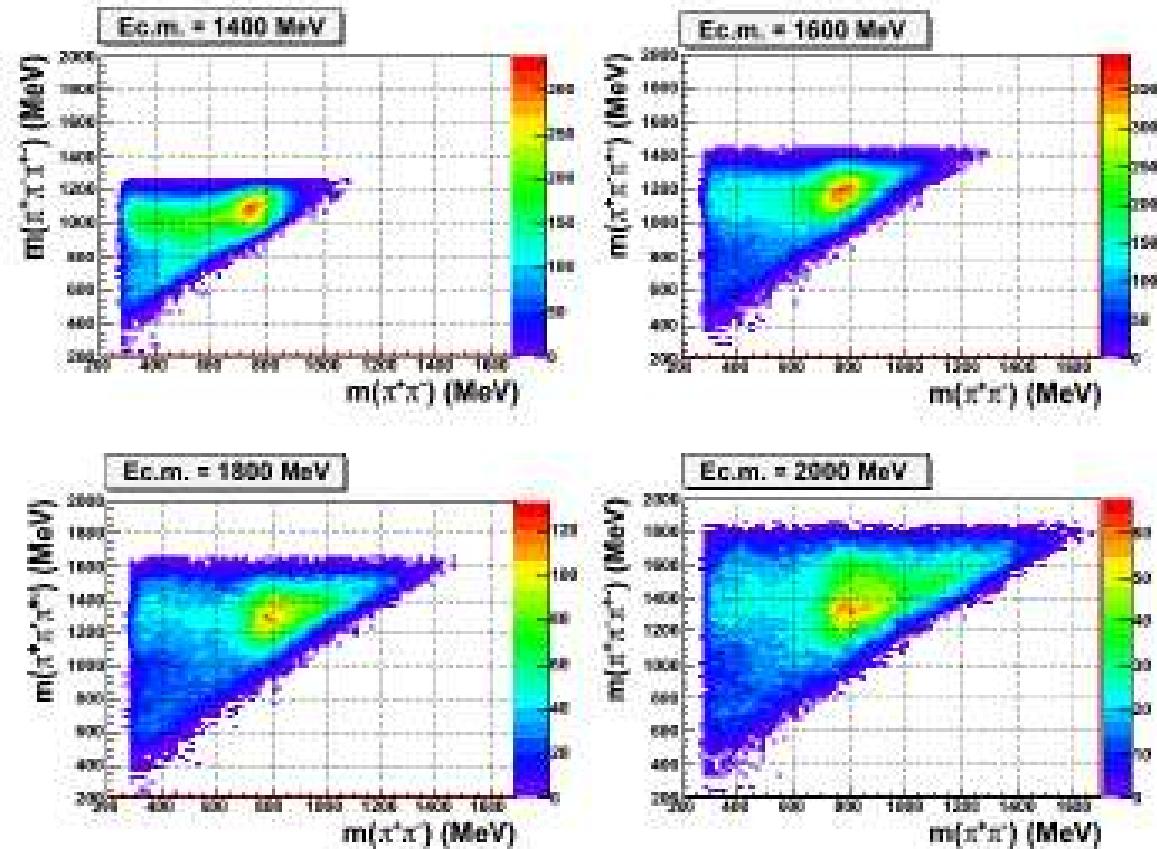
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-\eta$ at SND



At each \sqrt{s} full information on invariant masses
 $\pi^+\pi^-\pi^0$: V. Aulchenko et al., JETP 121 (2015) 34,
 $\pi^+\pi^-\eta$: V. Aulchenko et al., Phys. Rev. D 91 (2015) 052013

$e^+e^- \rightarrow 2\pi^+2\pi^-$ at CMD-3

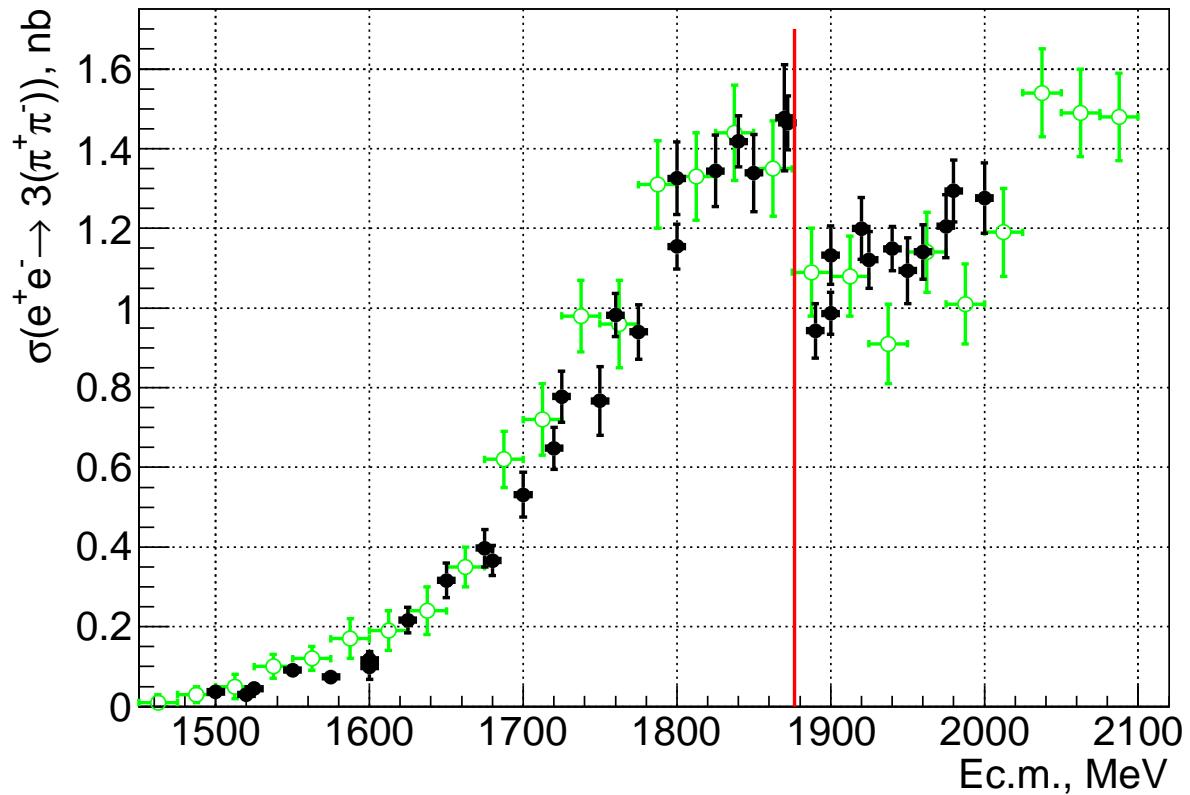
Dynamics of $e^+e^- \rightarrow 2\pi^+2\pi^-$ at CMD-3



A ρ^0 is always present, $a_1^\pm(1260)\pi^\mp$ ($a_2^\pm(1320)\pi^\mp$) significant, at higher \sqrt{s} other mechanisms like $\rho^0 f_0$, $\rho^0 f_2(1270)$ appear

$e^+e^- \rightarrow 3\pi^+3\pi^-$ at CMD-3 – I

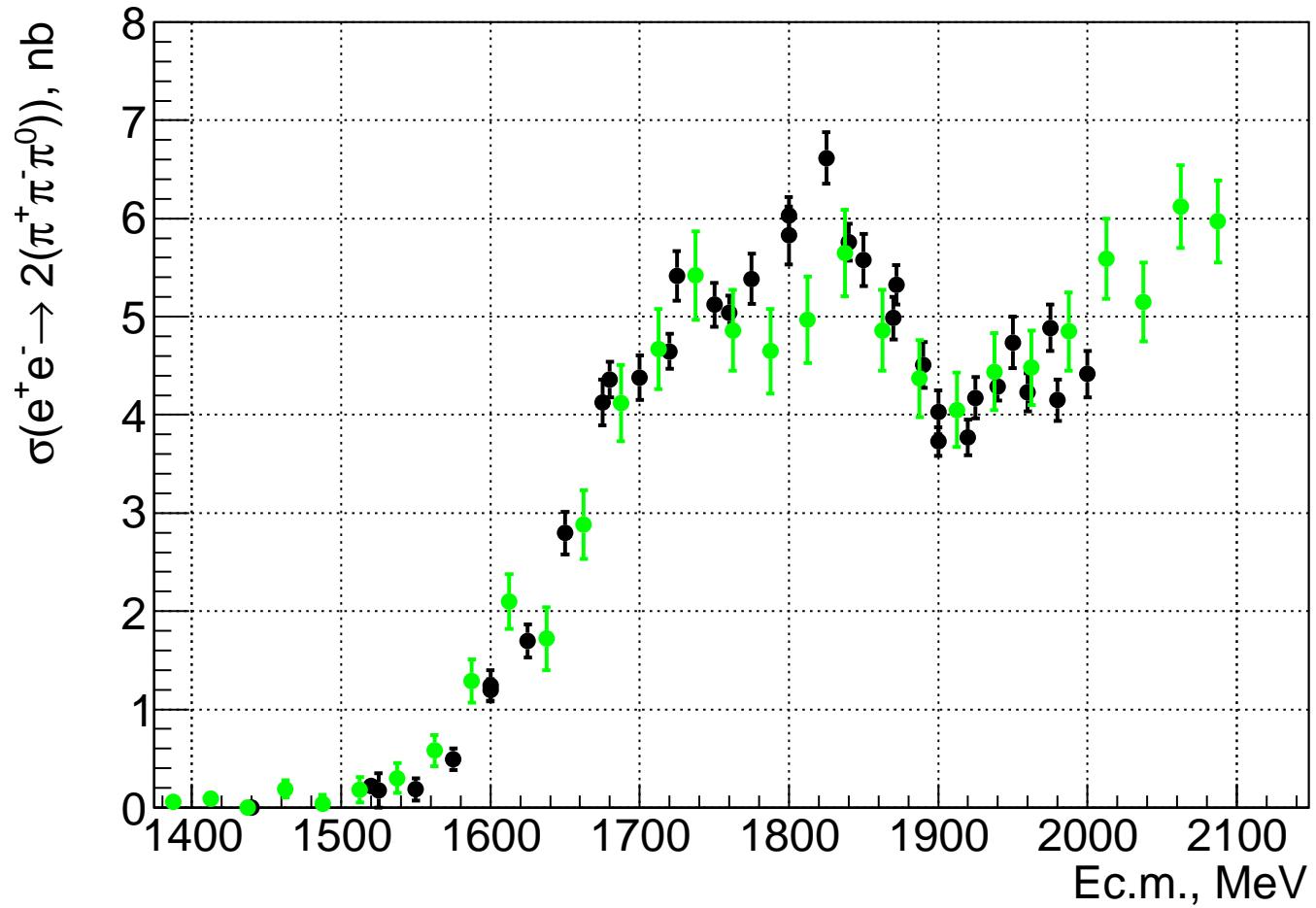
1. $\int Ldt = 22 \text{ pb}^{-1}$ from 1.5 to 2.0 GeV, 25 MeV step
2. About 8k five- (5069) and six-track (2887) events selected
3. We study dynamics, pure phase space doesn't work,
three models with $J^{PC} = 1^{--}$, each with one ρ^0 /event:
 - $\rho(1450)(\pi^+\pi^-)_{S-\text{wave}} \rightarrow a_1(1260)^{\pm}\pi^{\mp}\pi^+\pi^- \rightarrow \rho^0 2(\pi^+\pi^-) \rightarrow 3(\pi^+\pi^-)$
 - $\rho(770)(2\pi^+2\pi^-)_{S-\text{wave}} \rightarrow 3(\pi^+\pi^-)$
3 options for $2\pi^+2\pi^-$: phase space, $f_0(1370)$, $f_0(1500)$
 - $\rho(770)f_2(1270) \rightarrow 3(\pi^+\pi^-)$
 - The best description is with one $\rho(770)$ and 4 pions in S-wave
4. Full analysis of dynamics - common for $3\pi^+3\pi^-$, $2\pi^+2\pi^-2\pi^0$, $\pi^+\pi^-4\pi^0$
5. The systematic uncertainty is 6%, to be improved to 3%

$e^+e^- \rightarrow 3\pi^+3\pi^-$ at CMD-3-II


The dip structure near $N\bar{N}$ threshold is confirmed

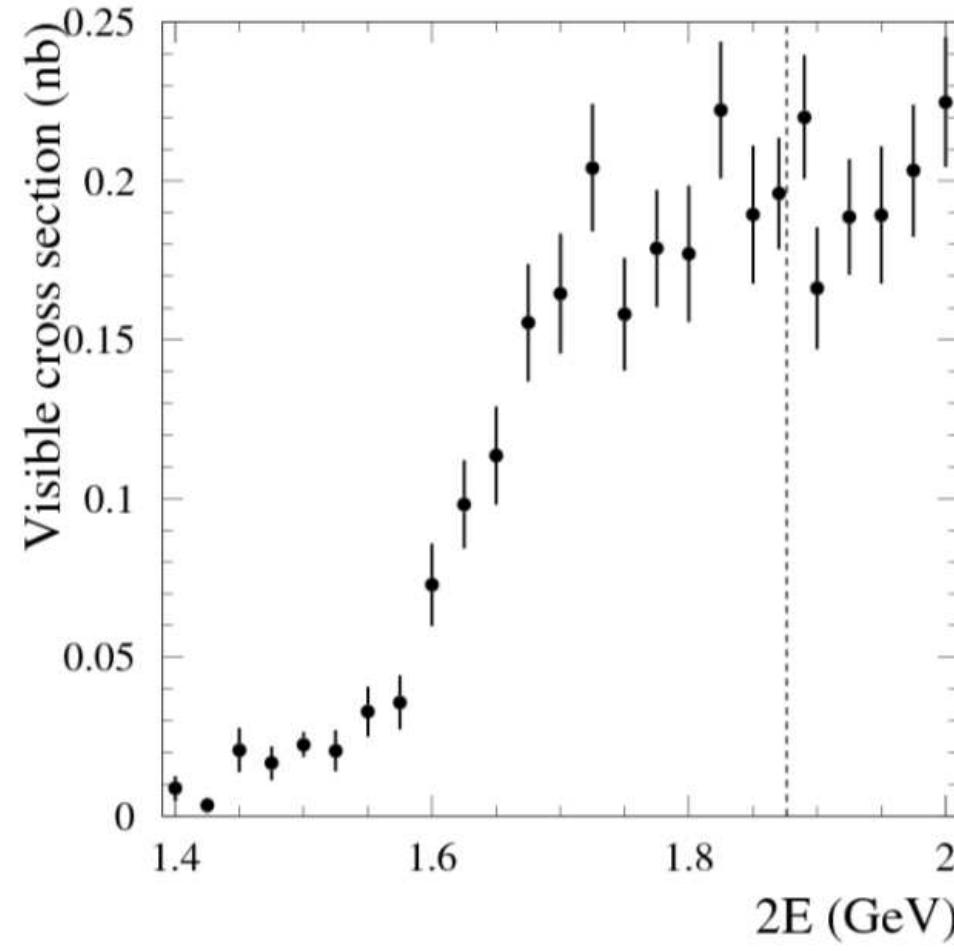
Phys. Lett. B 723 (2013) 82

$e^+e^- \rightarrow 2\pi^+2\pi^-2\pi^0$ at CMD-3



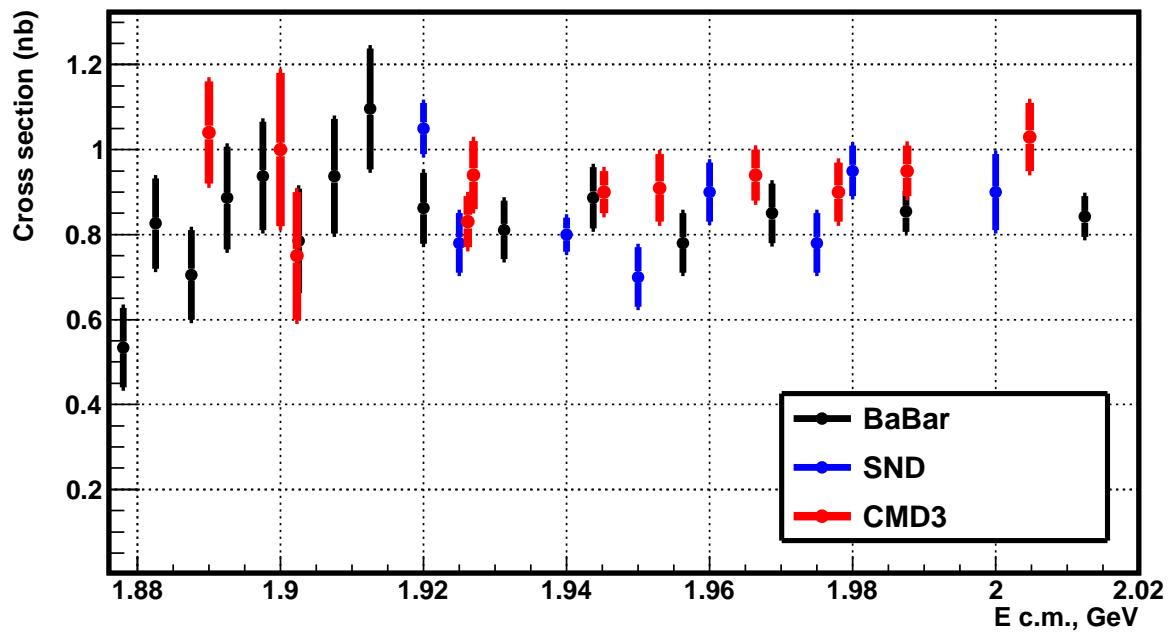
The dip structure near $N\bar{N}$ threshold also seen

$e^+e^- \rightarrow \pi^+\pi^-4\pi^0$ at SND



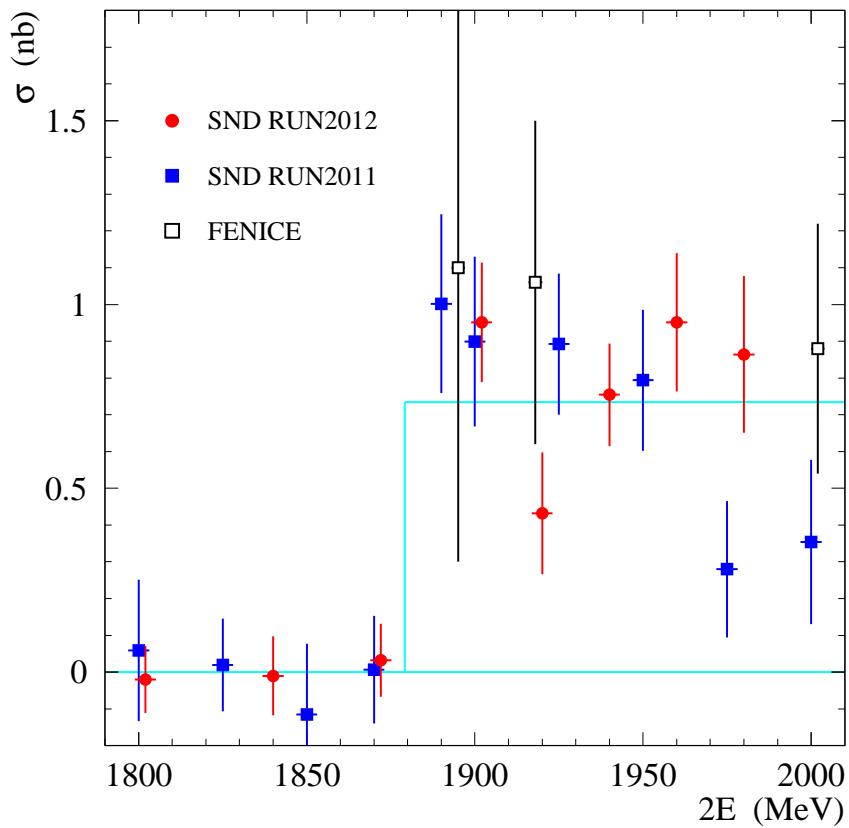
First ever measurement of the process

$p\bar{p}$ Production at VEPP-2000



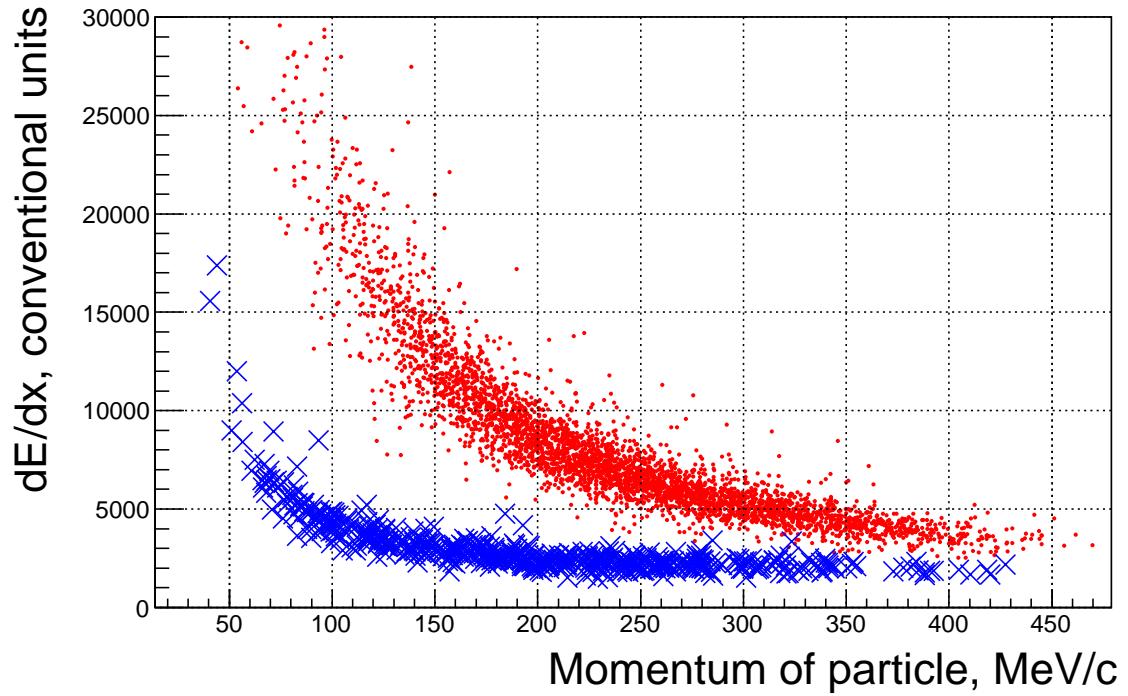
In addition to cross sections, first attempts of measuring f/f made

$e^+e^- \rightarrow n\bar{n}$ at SND

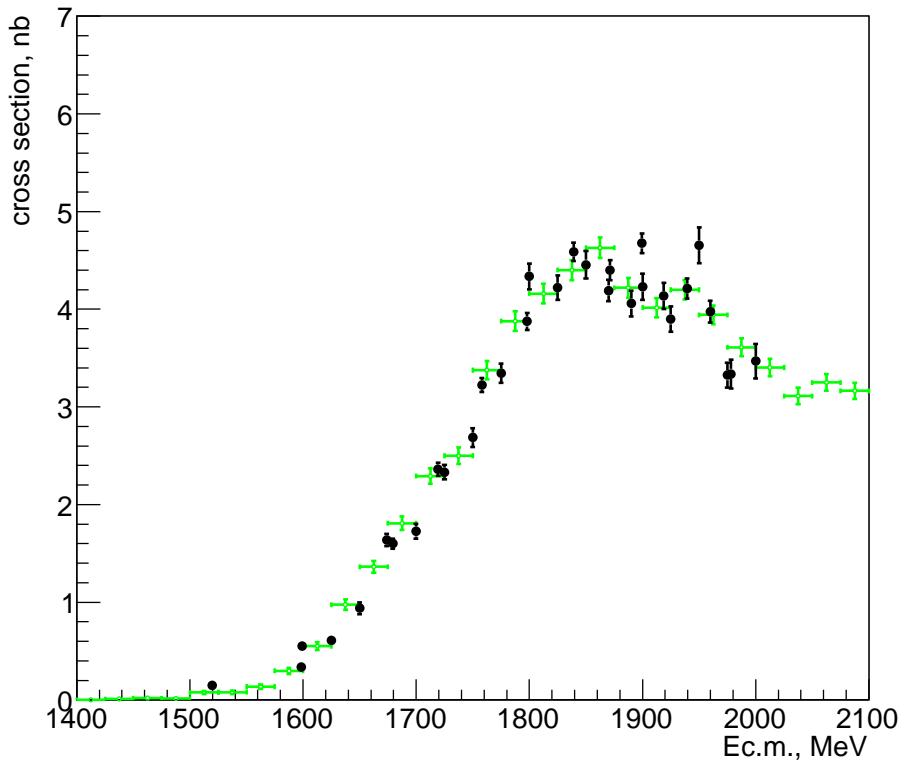


The first and more precise measurement after FENICE
M.N. Achasov et al., Phys. Rev. D 90 (2014) 112007

Multibody Final States with Charged Kaons



Ionization losses in DC (dE/dx) provide good K/π separation

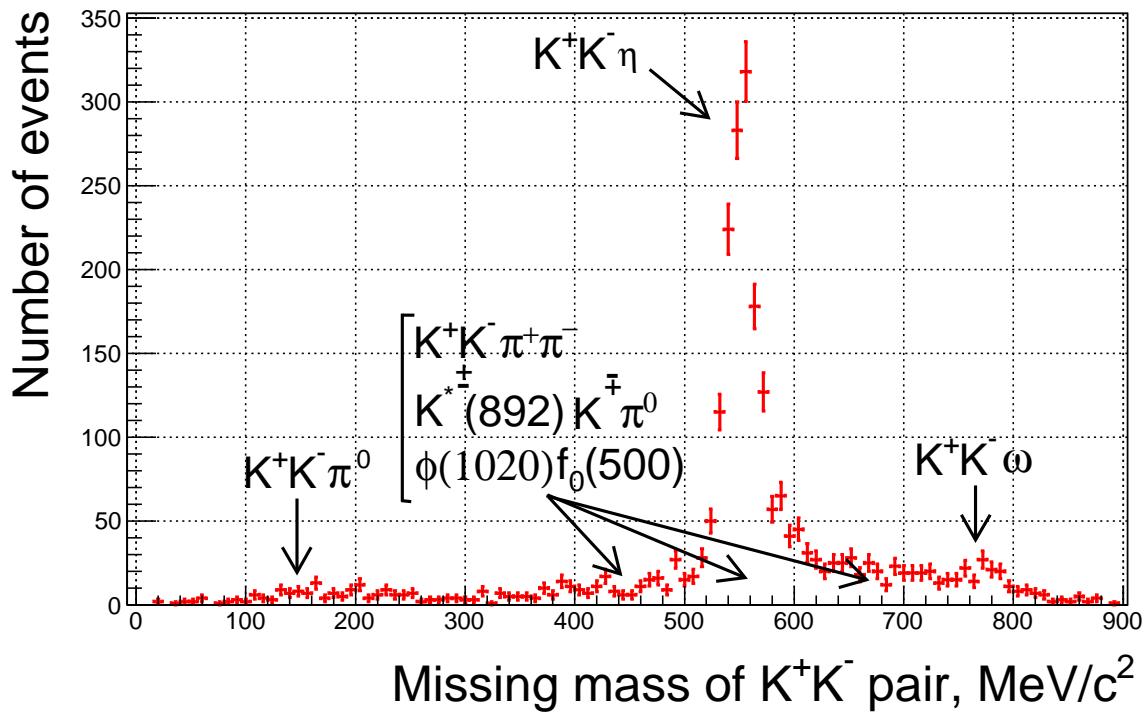
$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ at CMD-3

From more than 10000 events many different mechanisms seen:

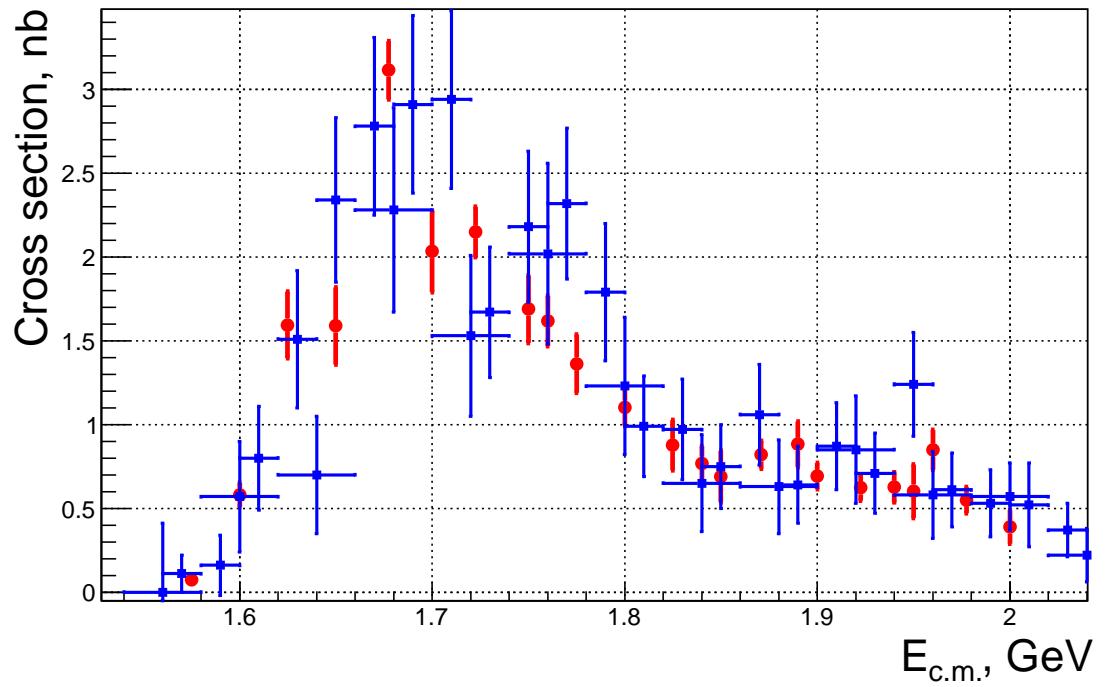
$$K_1(1270)\bar{K} \rightarrow K\bar{K}\rho, K^*(892)\bar{K}\pi,$$

$$K_1(1400)\bar{K} \rightarrow K^*(892)\bar{K}\pi, \phi\pi^+\pi^-$$

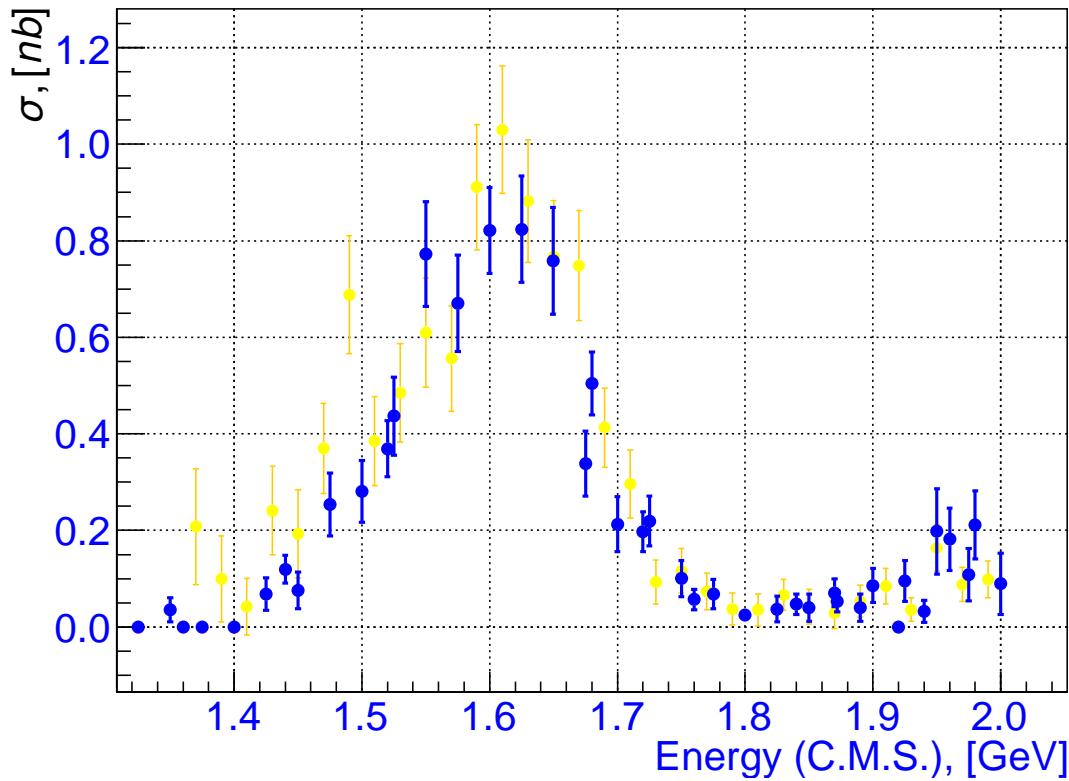
$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – I



Missing mass to K^+K^- clearly shows the dominant signal and BGs

$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – II

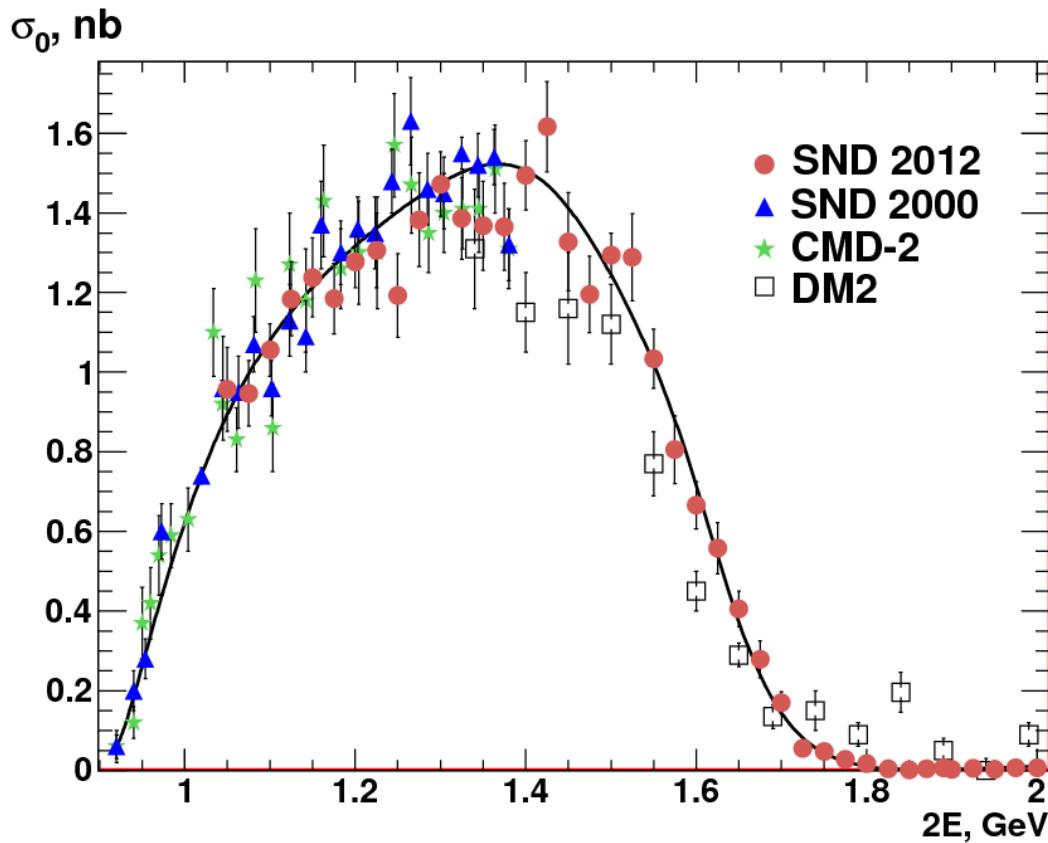
Cross section is consistent with and more precise than BaBar

$e^+e^- \rightarrow K^+K^-\pi^0$ at CMD-3

From 600 events the $\phi\pi^0$ and $K^{*\pm}(892)K^\mp$ mechanisms seen
Cross section is consistent with and more precise than BaBar

Transition Form Factors

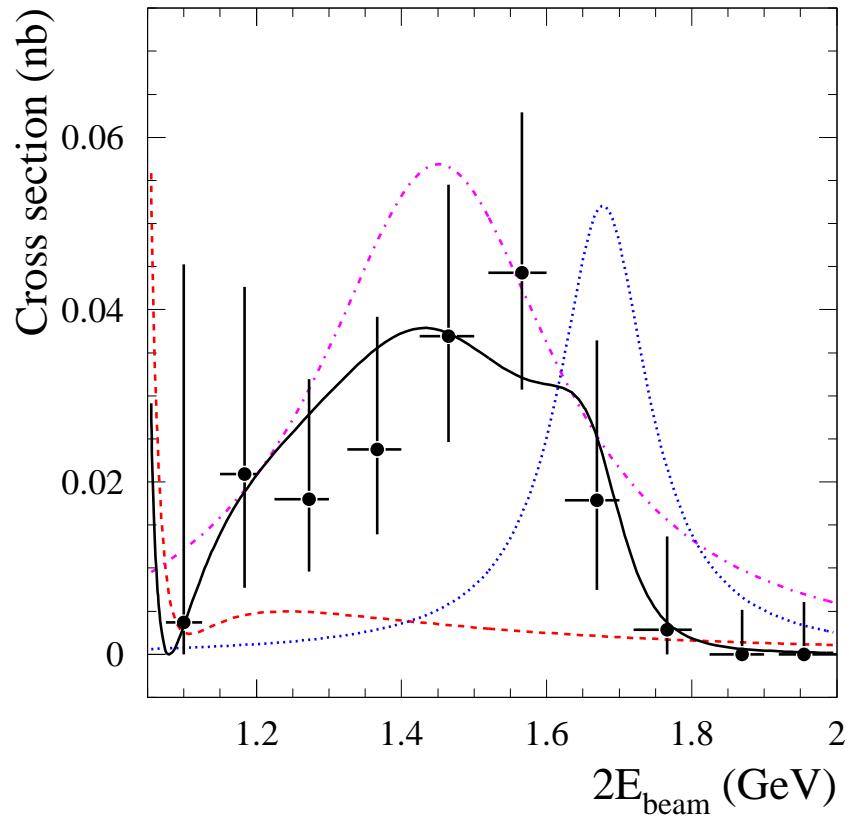
- $\gamma^{(*)}\gamma^{(*)} \rightarrow \pi^0\pi^0$: KLOE-2, BESIII, Belle
- $\gamma^*\gamma \rightarrow \pi^0, \eta, \eta'$: BaBar, Belle, KLOE-2, BESIII
- $e^+e^-(\rho, \omega, \phi) \rightarrow \pi^0\gamma, \eta\gamma, \eta'\gamma$: KLOE-2, CMD-3, SND
- $\pi^0(\eta, \eta') \rightarrow e^+e^-\gamma$: KLOE-2, CMD-3, SND
- $e^+e^-(\rho, \omega, \phi) \rightarrow \pi^0l^+l^-, \eta l^+l^-$: KLOE-2, CMD-3, SND
- $e^+e^- \rightarrow \rho\pi, \omega\pi, \eta\rho$: CMD-3, SND
- $e^+e^- \rightarrow \pi\pi\gamma$: KLOE-2, CMD-3, SND
- $P \rightarrow l^+l^-$, only $\pi^0 \rightarrow e^+e^-$, $\eta \rightarrow \mu^+\mu^-$ observed

$$e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma \text{ at SND}$$


First observation above 1.4 GeV

M.N. Achasov et al., Phys. Rev. D 88 (2013) 054013

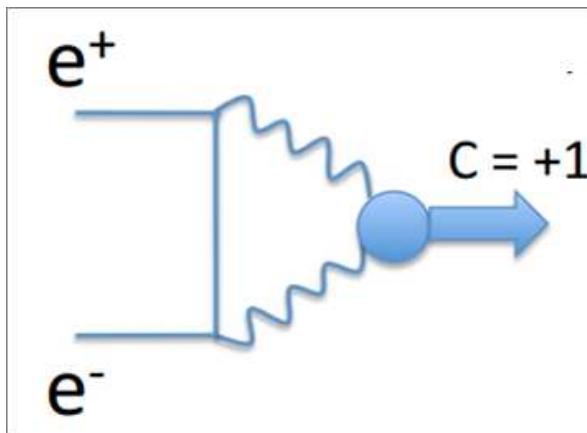
$e^+e^- \rightarrow \eta\gamma$ at SND



The first measurement of radiative decays above 1.4 GeV
M.N. Achasov et al., Phys. Rev. D 90 (2014) 032002

Search for C-even resonances in e^+e^-

Direct production of C-even resonances in e^+e^- collisions is possible
in an inverse reaction via a $\gamma\gamma$ intermediate state



The unitarity bound assuming 2 real photons is

$$\mathcal{B}_{P \rightarrow l^+l^-} = \mathcal{B}_{P \rightarrow \gamma\gamma} \frac{\alpha^2}{2\beta} \left(\frac{m_e}{m_P}\right)^2 \left[\ln\left(\frac{1+\beta}{1-\beta}\right)\right]^2, \quad \beta = \sqrt{1 - 4\left(\frac{m_e}{m_P}\right)^2}.$$

For η' the unitarity bound is $\mathcal{B} = 3.75 \cdot 10^{-11}$

“Standard” mechanism via $e^+e^- \rightarrow e^+e^- P$ involves two almost real photons
and provides $\Gamma(P \rightarrow \gamma\gamma)$ only

Search for $e^+e^- \rightarrow \eta'(958)$ at VEPP-2000

- CMD-3 used 2.69 pb^{-1} at $\sqrt{s} \sim m_{\eta'}$ to look for
 $e^+e^- \rightarrow \eta'(958), \eta' \rightarrow \eta\pi^+\pi^-, \eta \rightarrow 2\gamma,$
 $\Gamma(\eta' \rightarrow e^+e^-) < 0.0024 \text{ eV}$ at 90%CL, Phys. Lett. B740 (2015) 273
- SND used 2.9 pb^{-1} to look for $e^+e^- \rightarrow \eta'(958)$:
 $\eta' \rightarrow \eta\pi^+\pi^-, \eta \rightarrow 2\gamma, 3\pi^0,$
 $\eta' \rightarrow \eta\pi^0\pi^0, \eta \rightarrow 2\gamma, 3\pi^0, \pi^+\pi^-\pi^0,$
 $\Gamma(\eta' \rightarrow e^+e^-) < 0.0020 \text{ eV}$ at 90%CL, Phys. Rev.D91 (2015) 092010
- SND combines their data with CMD-3:
 $\Gamma(\eta' \rightarrow e^+e^-) < 0.0011 \text{ eV}$ at 90%CL,
 $\mathcal{B}(\eta' \rightarrow e^+e^-) < 5.6 \cdot 10^{-9}$ at 90%CL
- The unitarity limit $\mathcal{B}(\eta' \rightarrow e^+e^-) > 3.75 \cdot 10^{-11}$

Search for $e^+e^- \rightarrow \eta$ at VEPP-2000

- SND used 110 nb^{-1} for a feasibility study of $e^+e^- \rightarrow \eta, \eta \rightarrow 3\pi^0, \eta \rightarrow 2\gamma, \pi^+\pi^-\pi^0$ dominated by QED background,
 $\mathcal{B}(\eta \rightarrow e^+e^-) < 3 \cdot 10^{-6}$ at 90%CL
- The best limit is by HADES in Phys. Lett. B731 (2014) 265,
 $\mathcal{B}(\eta \rightarrow e^+e^-) < 2.3 \cdot 10^{-6}$ at 90%CL
- The HADES limit can be improved after a 2-week run
- The unitarity limit is $\mathcal{B}(\eta \rightarrow e^+e^-) > 1.8 \cdot 10^{-9}$

Future

- Two new measurements of a_μ are expected in 3-5 years:
E989 at Fermilab plans to improve the uncertainty from 0.5ppm to 0.14 ppm,
they plan to start running in 2017
J-PARC has the same precision goal, data taking planned in 2019-2021
- What is expected for the theoretical prediction?
Progress in low energy e^+e^- annihilation expected,
improving the LO error from 4.2 to 2.0, so 2.6 from the LbL dominates,
in the new approach 1.0 may be achieved (?) giving 2.2 in total
First principles (lattice) give promising results, but far from final
C.M. Carloni Calame et al., Phys. Lett. B 746 (2015) 325,
 $a_\mu^{\text{had},\text{LO}}$ from $\alpha(t)$ in the spacelike region of Bhabha
- With the same central values of a_μ^{exp} and a_μ^{th}
the today difference will correspond to about $10\sigma!!$

Conclusions

- VEPP-2000 ran smoothly with CMD-3 and SND at $0.32 < \sqrt{s} < 2.00$ GeV, the achieved accuracy is comparable or better than in ISR measurements
- The goals are 0.35%(0.5%) for $\pi^+\pi^-$ and 3% for multibody modes
- Below 2 GeV progress (a factor of 2-3) expected in exclusive σ 's due to scans in Novosibirsk and ISR from KLOE2, BaBar, Belle, BESIII and Belle2
- Various high-statistics experiments will substantially improve the accuracy of vacuum polarization calculations for $(g_\mu - 2)/2$
- Higher statistics ($\sim 1\text{fb}^{-1}$) \Rightarrow a detailed study of dynamics, thus a study of mesons with various quantum numbers, can ChT provide low energy predictions?
- Good prospects for a study of transition form factors and hLbL
- Meanwhile a $\sim 3.5\sigma$ deviation of a_μ^{SM} from a_μ^{exp} persists:
New Physics or various experimental and interpretation errors?

Back-up

$a_\mu^{\text{had,LO}}$ and $\alpha(t)$

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)].$$

Muon Anomalous Magnetic Moment

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \quad a = (g - 2)/2.$$

In Dirac theory for pointlike particles $g = 2$,
higher-order effects or new physics $\Rightarrow g \neq 2$

Any significant difference of a_μ^{exp} from a_μ^{th} indicates
New Physics beyond the Standard Model.

a_μ is much more sensitive to new physics effects than a_e :
the gain is usually $\sim (m_\mu/m_e)^2 \approx 4.3 \cdot 10^4$.

$$a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{NP}}, \quad a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}}.$$

Experimental Status of a_l

$$a_e = 1159652180.73(28) \times 10^{-12} \quad 0.24 \times 10^{-9}$$

D. Hanneke et al., PRL 100, 120801 (2008)
QED test or α determination

$$a_\mu = 116592091(63) \times 10^{-11} \quad 0.54 \times 10^{-6}$$

G.W. Bennett et al. (E821), PRD 73, 072003 (2006)
Sensitive test of the Standard Model

$$a_\tau = -0.018(17) \text{ or } -0.052 < a_\tau < 0.013 \text{ 95%CL}$$

J. Abdallah et al. (DELPHI), EPJ C 35, 159 (2004)
Theory: $117721(5) \times 10^{-8}$, SE, M. Passera, MPL A 22, 159 (2007)

QED Contribution a_μ^{QED}

$$\begin{aligned}
 a_\mu^{\text{QED}} \cdot 10^{10} = \sum C_i \left(\frac{\alpha}{\pi}\right)^i = & \quad 0.5(\alpha/\pi)^1 \quad 1 \text{ diagram} \\
 + & \quad 0.765857426(16)(\alpha/\pi)^2 \quad 9 \\
 + & \quad 24.05050988(28)(\alpha/\pi)^3 \quad > 100 \\
 + & \quad 130.8796(63)(\alpha/\pi)^4 \quad > 1000 \\
 + & \quad 753.29(1.04)(\alpha/\pi)^5 \quad > 20000
 \end{aligned}$$

Analytically – S.Laporta and E.Remiddi, Karlsruhe group, . . . ,

Numerically – group of T. Kinoshita

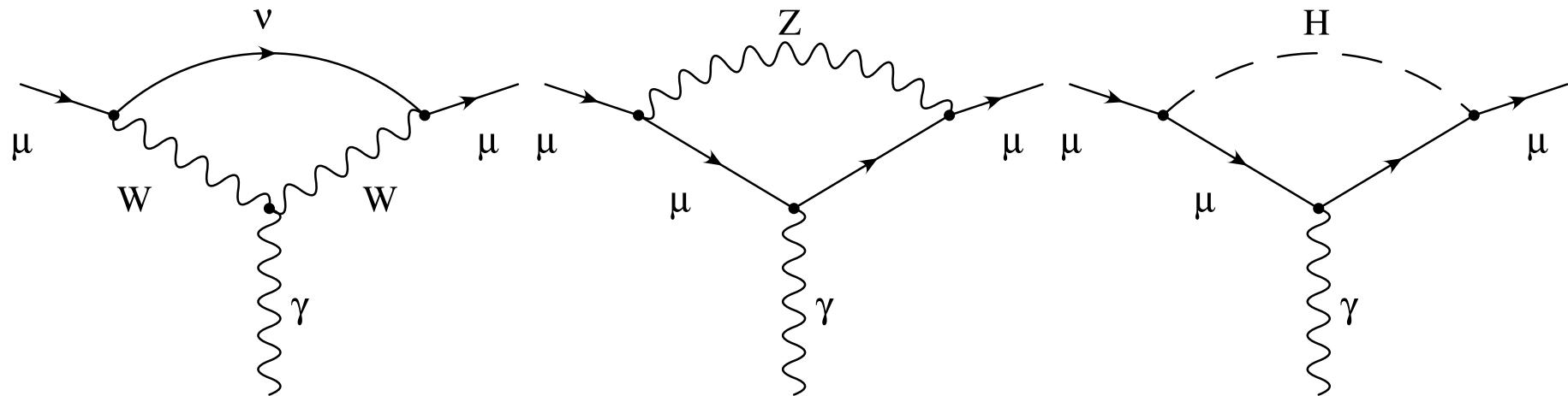
With $\alpha^{-1} = 137.035999049(90)$

$$a_\mu^{\text{QED}} = (116584718.951 \pm 0.022 \pm 0.077) \cdot 10^{-11}.$$

The errors are due to $\mathcal{O}(\alpha^4)$ and α

The $(\alpha/\pi)^4$ term \approx the experimental error!

Electroweak contribution a_μ^{EW}



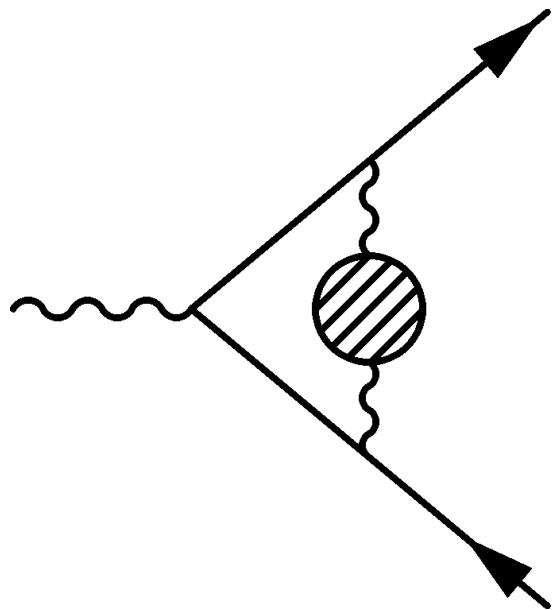
One-loop electroweak contributions

Authors	Year	$a_\mu^{\text{EW}}, 10^{-10}$
..., ..., ...	1972	19.5
A. Czarnecki et al.	1996	15.2 ± 0.4
A. Czarnecki et al.	2002	15.36 ± 0.01

The errors are due to hadr. loops and 3-loop NLO logs

Hadronic contribution a_μ^{had}

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$



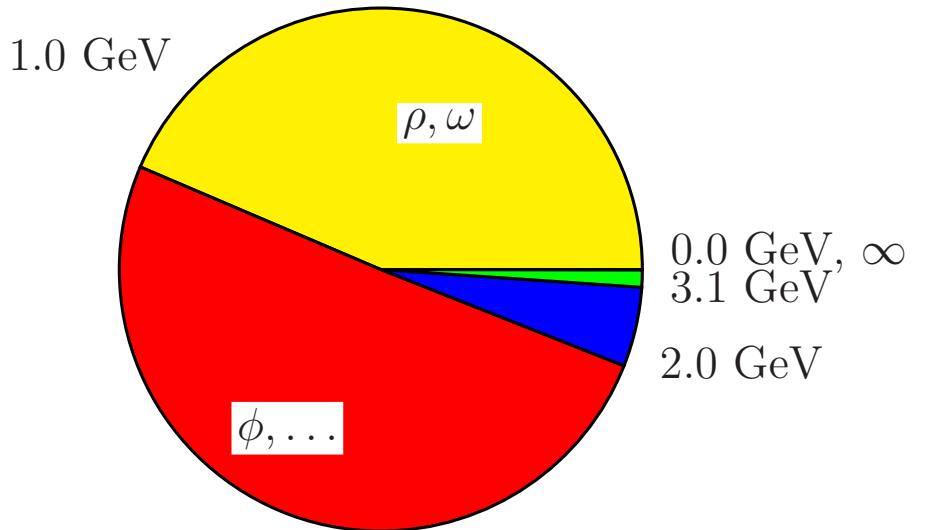
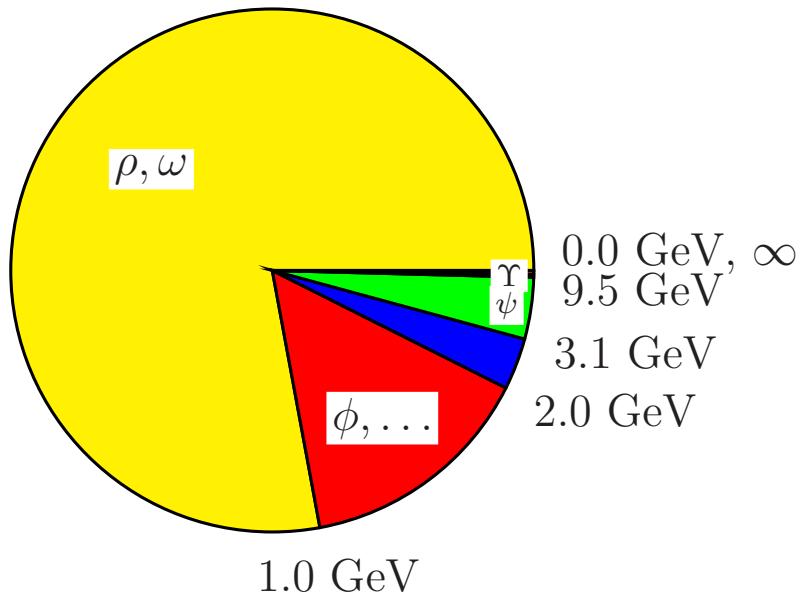
$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^\infty ds \frac{R(s) \hat{K}(s)}{s^2},$$

C. Bouchiat, L. Michel, Bouchiat, 1961;
M. Gourdin, E. de Rafael, 1969

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)},$$

$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \rightarrow \infty$,
 $1/s^2$ emphasizes low energies, particularly $e^+e^- \rightarrow \pi^+\pi^-$.
 $a_\mu^{\text{had,LO}} \sim 700 \cdot 10^{-10} \Rightarrow$ accuracy better than 1% needed

Contributions of Various Energy Ranges to $a_\mu^{\text{had,LO}}$

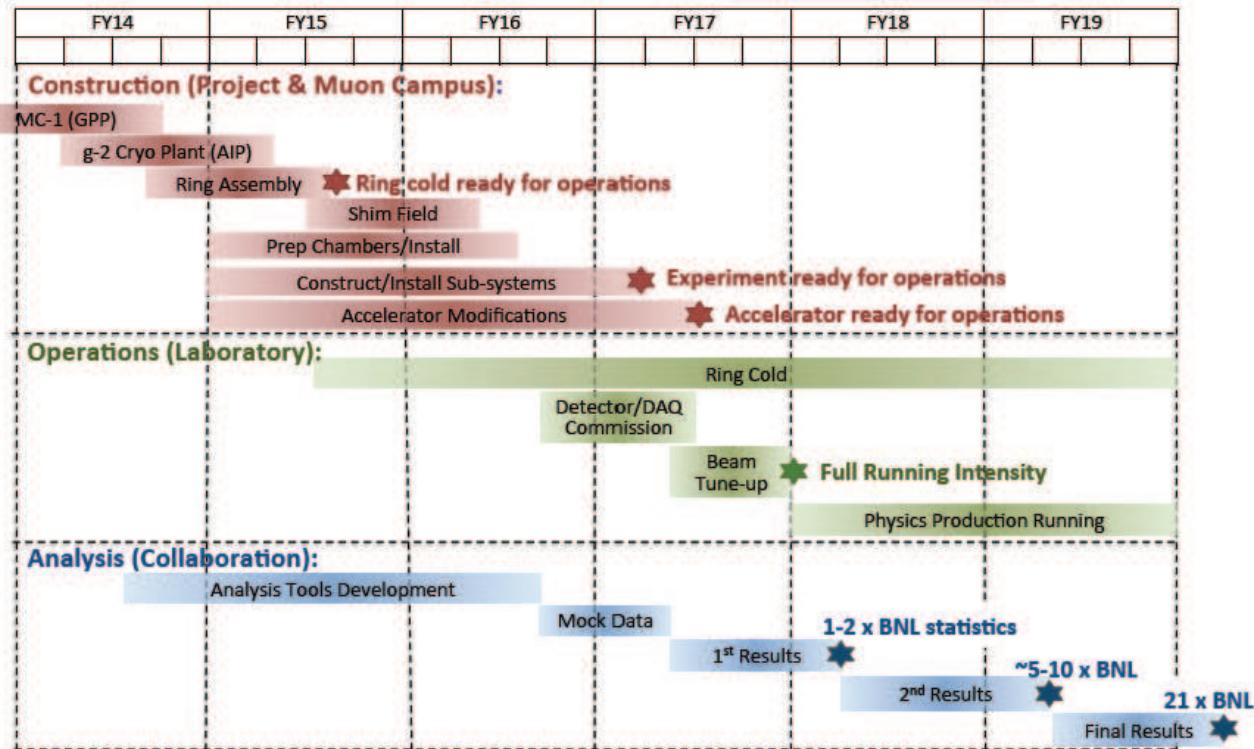


More than 72% of $a_\mu^{\text{had,LO}}$ come from $e^+e^- \rightarrow \pi^+\pi^-$ and
more than 90% from the energy range below 2 GeV

Schedule of Fermilab E989



Schedule overview



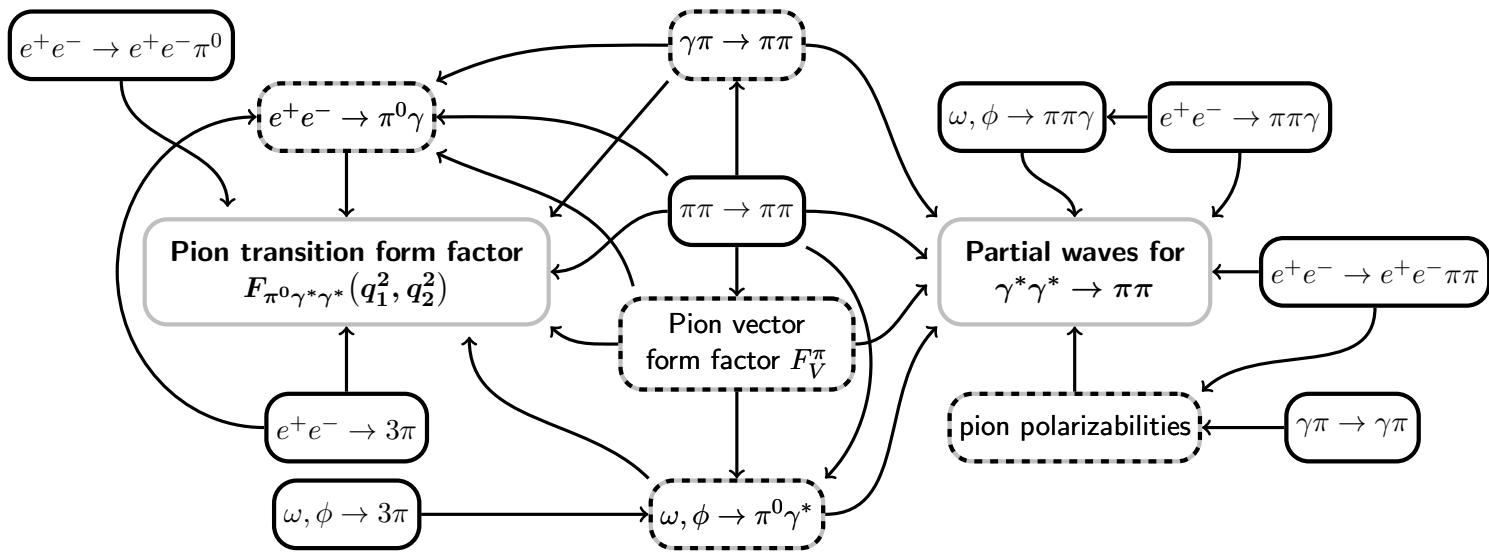
20 March 2014
Chris Polly

Collaboration Meeting

20

Hadronic LbL - New Approach

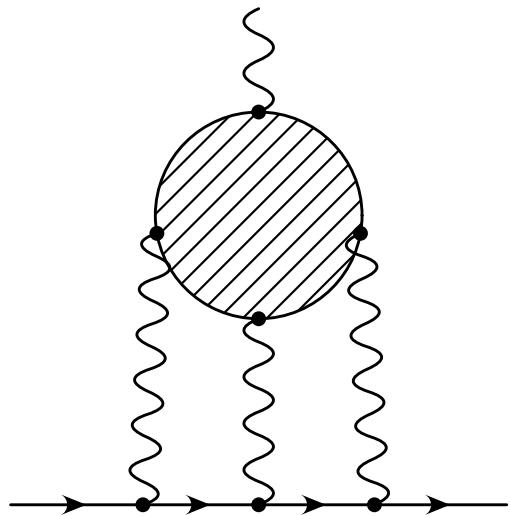
New dispersive approach relates data on transition f/f to the HLbL,
 G. Colangelo et al., JHEP 09 (2014) 091, Phys. Lett. B638 (2014) 6



Measurements of various processes are in order followed by calculations of many integrals with different kernels

The largest contribution to a_μ^{LBL} is expected from
 the pseudoscalars (π^0 , η , η')

Light-by-Light Scattering – I



Various approaches used:

- Vector Dominance and Chiral models
- Data on $\gamma\gamma^* \rightarrow \pi^0, \eta, \eta'$ (single-tag)
- Effective field theory
- Dyson-Schwinger equations

M. Knecht and A. Nyffeler, 2002: the correct sign!

Light-by-Light Scattering – II

Authors	Year	$a_\mu^{\text{lbl}}, 10^{-10}$
J. Bijnens et al.	1996 (2002)	8.3 ± 3.2
M. Hayakawa and T. Kinoshita	1998 (2002)	9.0 ± 1.5
K. Melnikov and A. Vainshtein	2003	13.6 ± 2.5
M. Davier and W. Marciano	2004	12.0 ± 3.5
J. Prades, E. de Rafael, and A. Vainshtein	2009	10.5 ± 2.6
D. Greynat and E. de Rafael	2012	15.0 ± 0.3
T. Goecke, C.S. Fischer and R. Williams	2013	18.8 ± 9.0

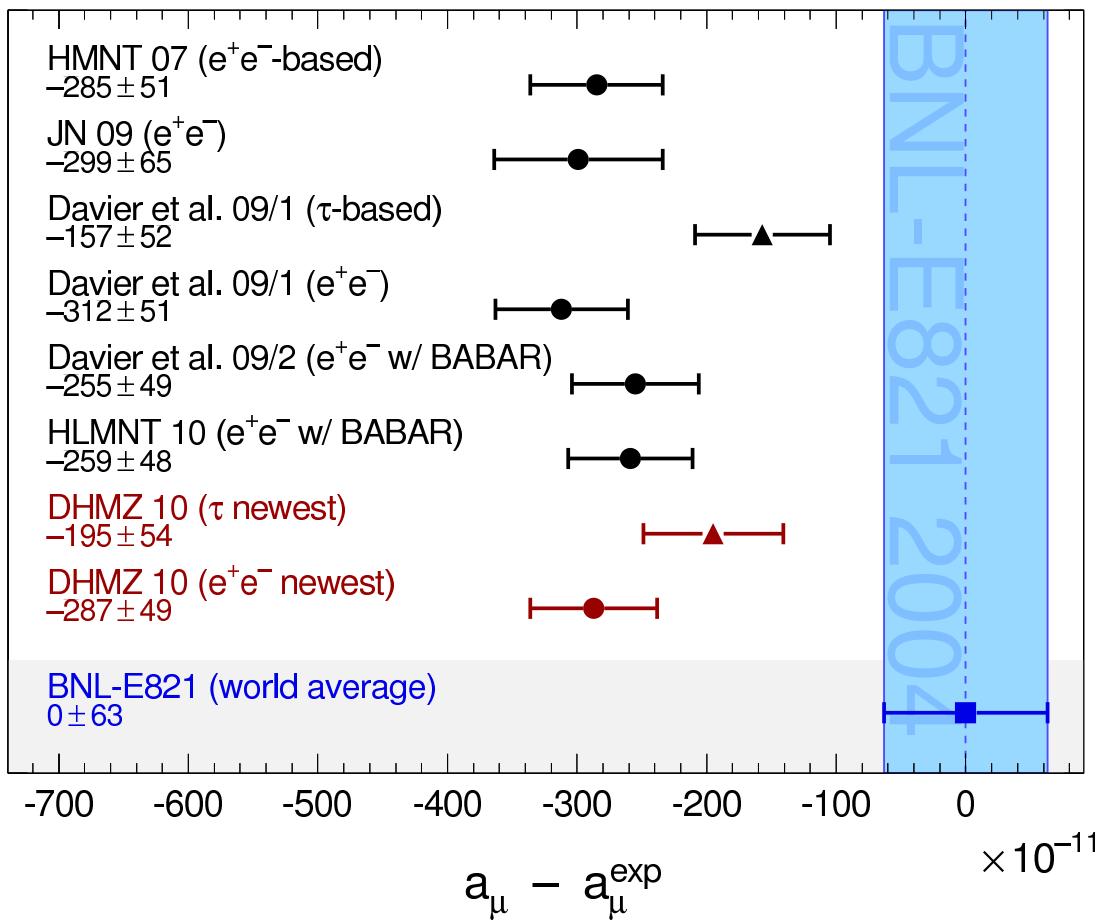
Experiment vs. Theory – I

$$a_\mu = (g_\mu - 2)/2, \ 10^{-10}$$

Experiment	$11659209.1 \pm 5.4 \pm 3.3$
QED	11658471.895 ± 0.008
EW	15.4 ± 0.1
Had LO	692.3 ± 4.2
Had HO	-9.8 ± 0.1
Had LbL	10.5 ± 2.6
Theory	11659180.3 ± 4.9
Exp.-Th.	28.8 ± 8.0

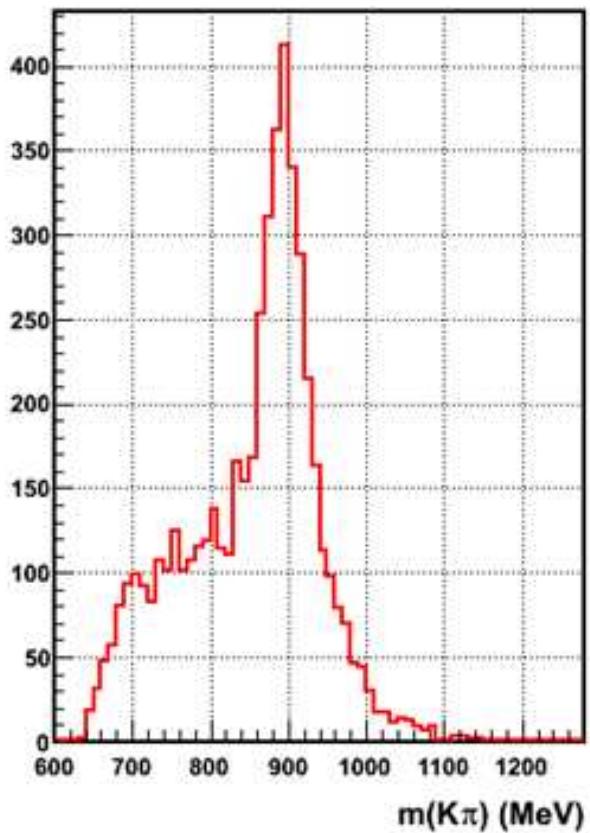
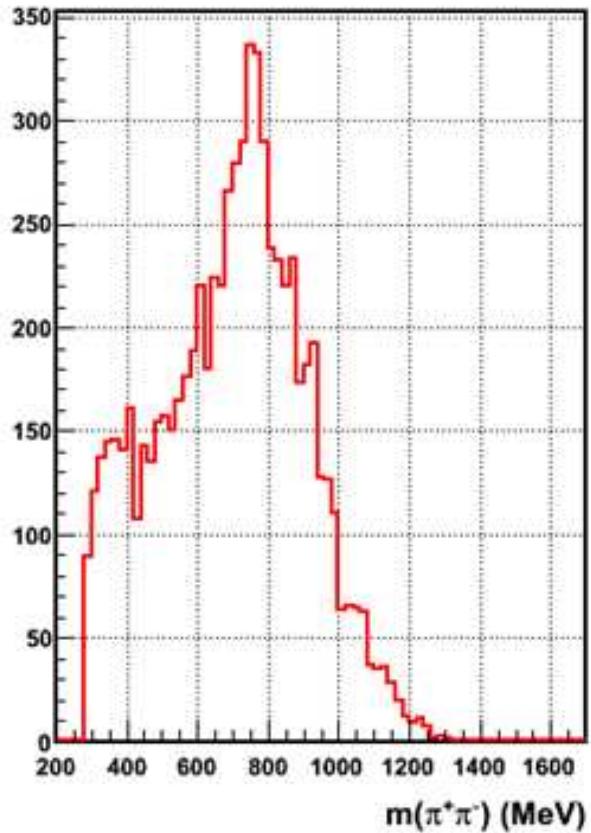
Experiment is higher than theory by 3.6 standard deviations

Experiment vs. Theory – II

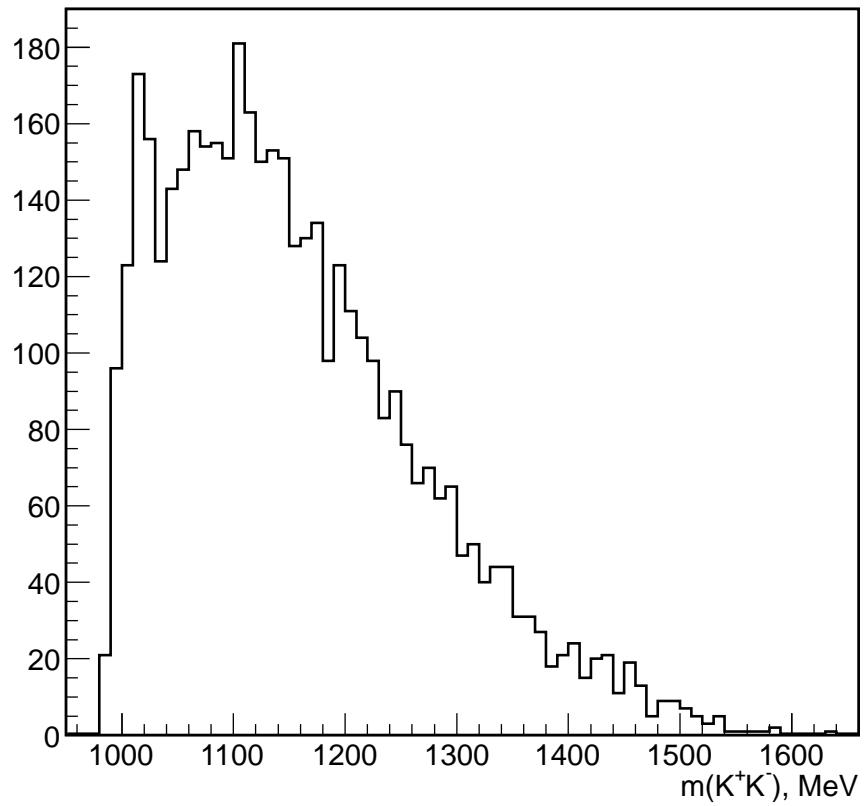


$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ at CMD-3 – I

- CMD-3 studied $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ with 22 pb^{-1} between 1.5 and 2 GeV
- More than 10000 4-track and 3-track events observed
- Analysis of $\pi^+\pi^-$, $K^\pm\pi^\mp$, K^+K^- invariant masses shows clear ρ^0 , $K^{*0}(892)$, ϕ signals
- Many different mechanisms seen: $K_1(1270)\bar{K} \rightarrow K\bar{K}\rho$, $K^*(892)\bar{K}\pi$, $K_1(1400)\bar{K} \rightarrow K^*(892)\bar{K}\pi$, $\phi\pi^+\pi^-$

$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ at CMD-3 – II ρ^0 in $\pi^+\pi^-$ and $K^{*0}(892)$ in $K^\pm\pi^\mp$

$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ at CMD-3 – III

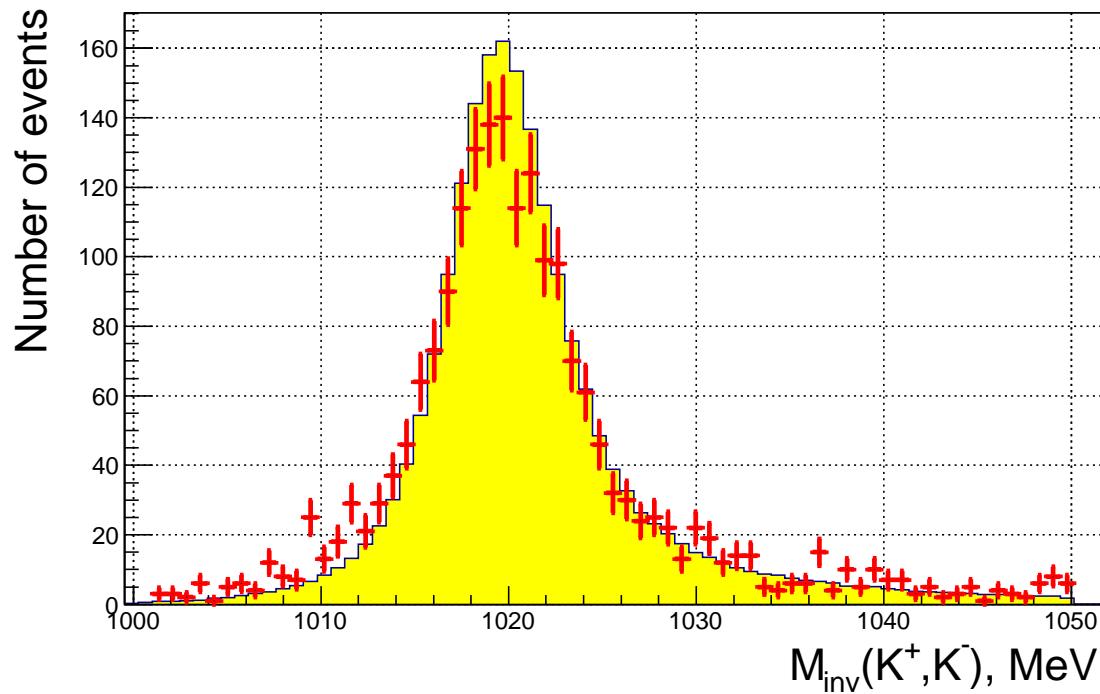


ϕ in K^+K^- combinations

$$e^+e^- \rightarrow K^+K^-\eta \text{ at CMD-3 - I}$$

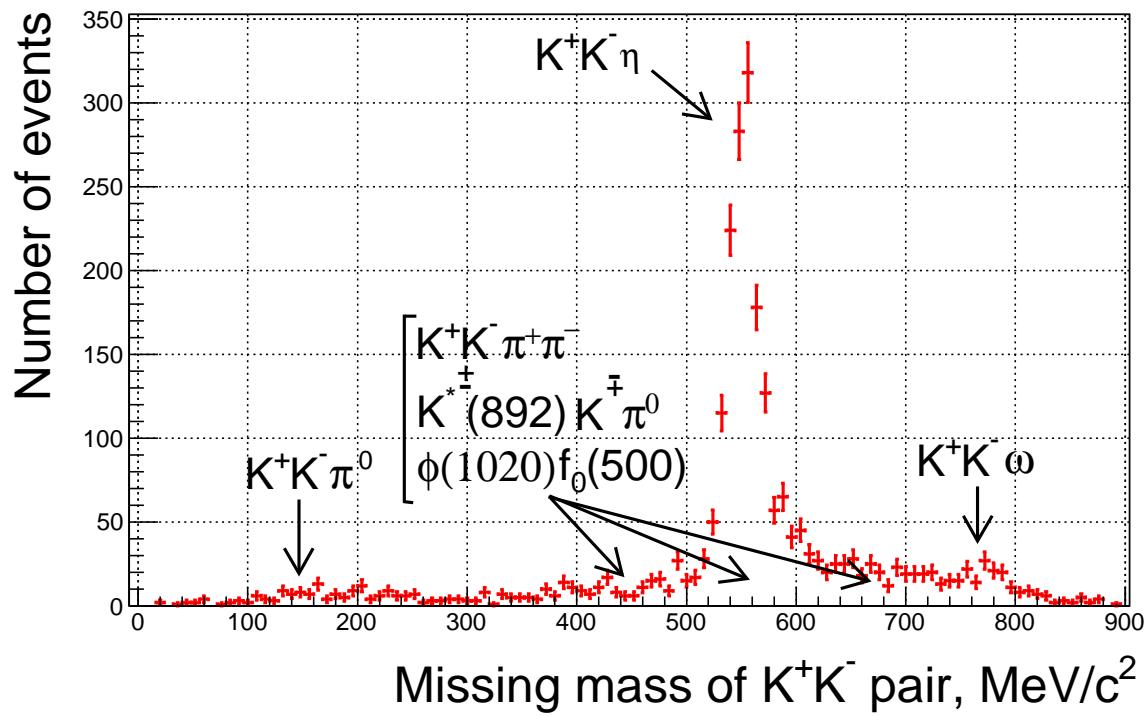
- A data sample of 22 pb^{-1} collected in 2011-2012 is used to study $e^+e^- \rightarrow K^+K^-\eta$
- 23 c.m. energy points between 1.57 and 2.0 GeV
- Analysis method emphasizes the dominant $\phi\eta$ signal, studies of non-resonant $K^+K^-\eta$ needed
- Rich background with numerous components seen
- The data sample includes 1600 events of the signal and ~ 600 background events

$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – II

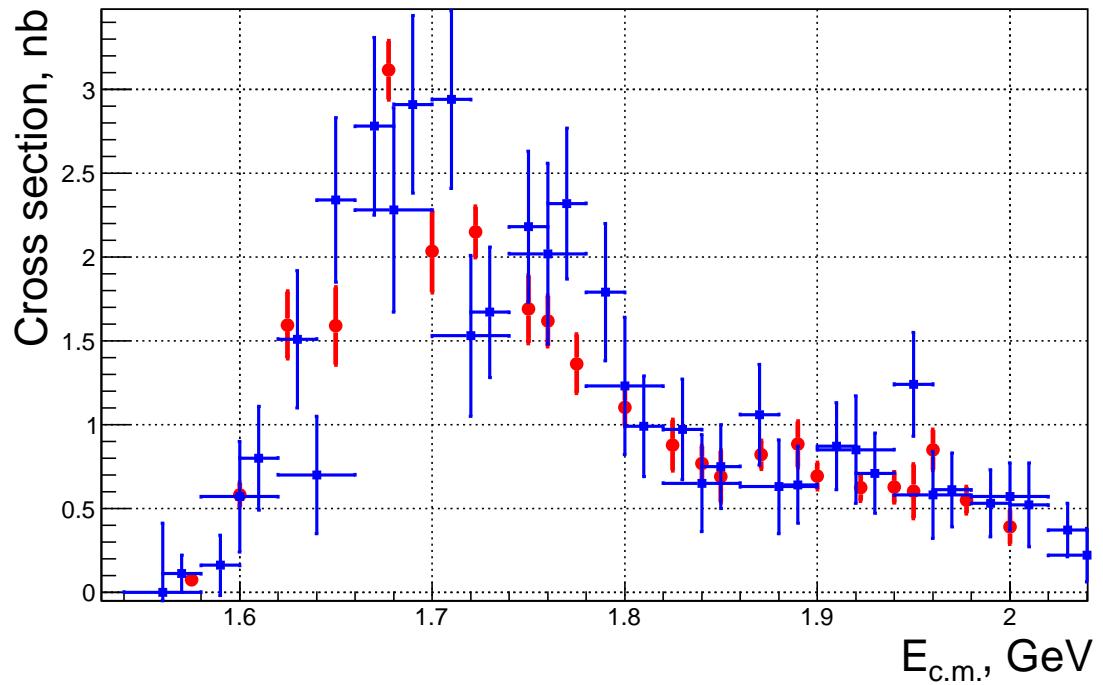


Dynamics dominated by the $\phi\eta$, $\phi \rightarrow K^+K^-$ channel

$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – III



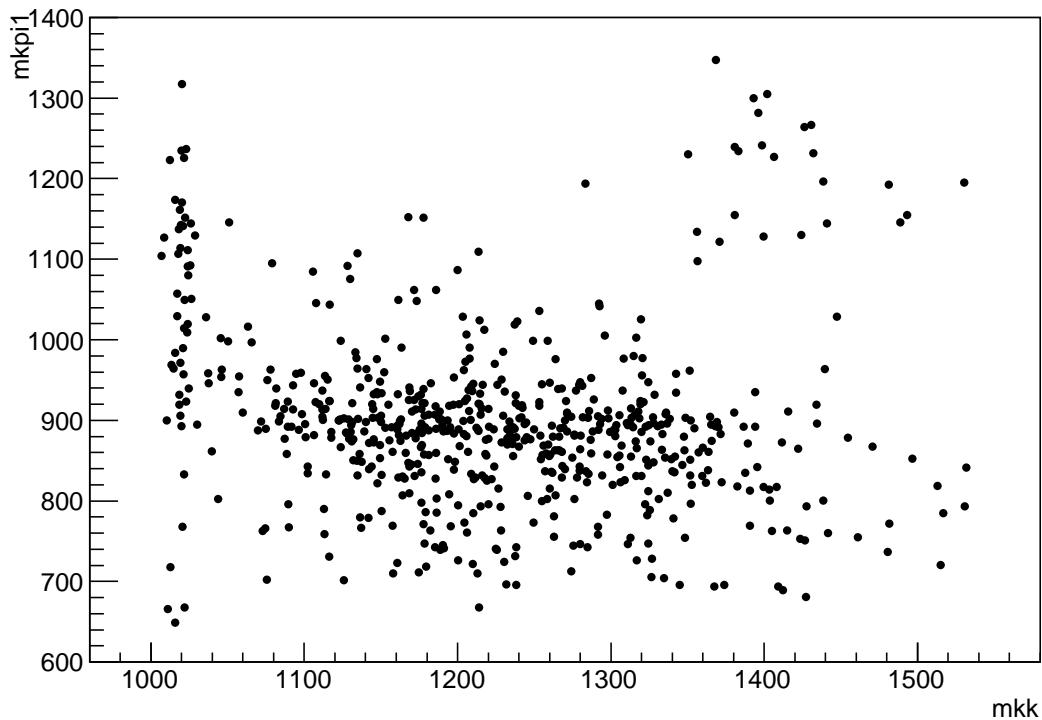
Missing mass to K^+K^- clearly shows the dominant signal and BGs

$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – IV

Cross section is consistent with and more precise than BaBar

$e^+e^- \rightarrow K^+K^-\pi^0$ at CMD-3 – I

About 600 signal events selected
mkpi1:mkk {BestChi2N}

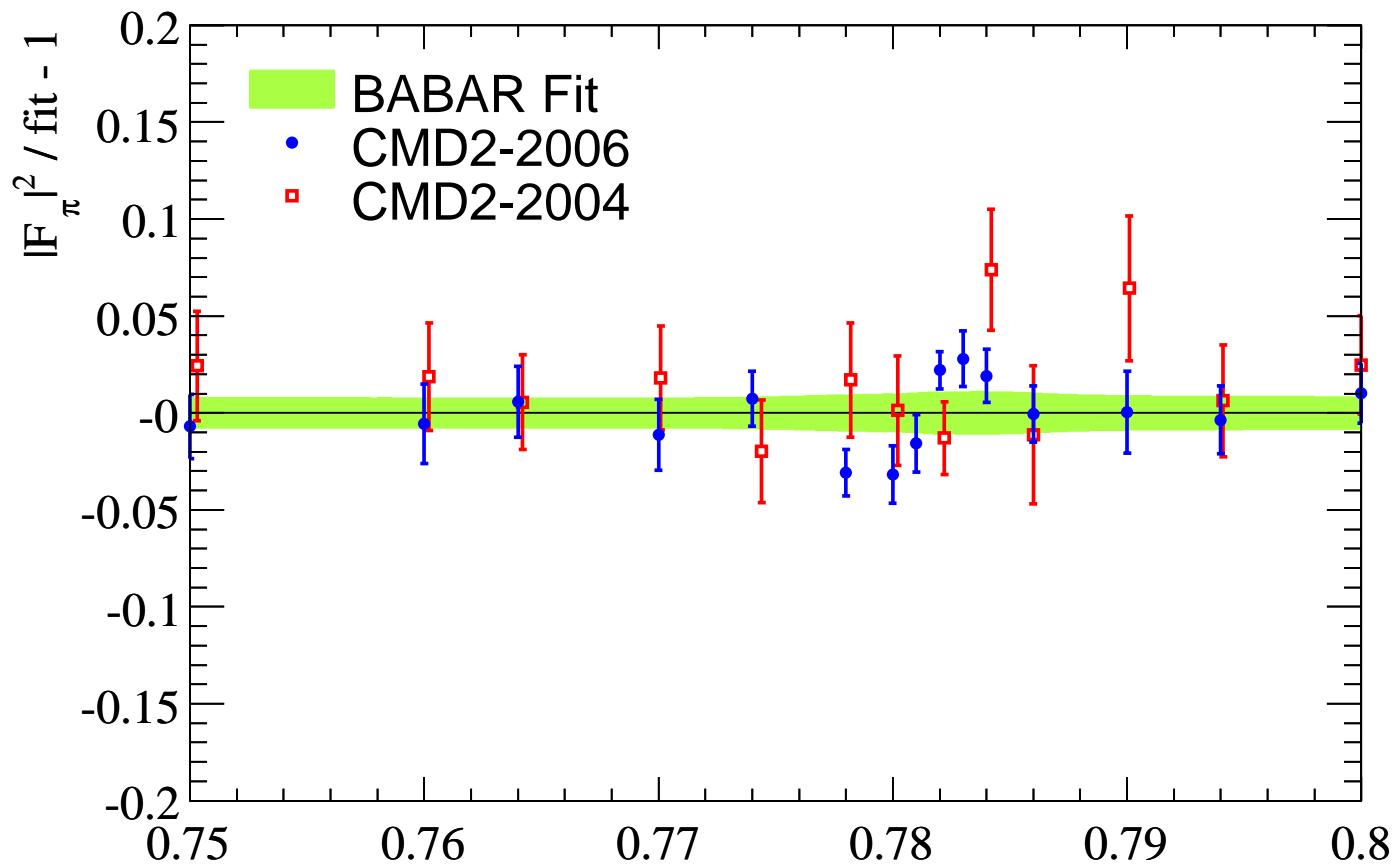


The $K\pi$ vs. K^+K^- plot clearly shows
the $\phi\pi^0$ and $K^{*\pm}(892)K^\mp$ mechanisms

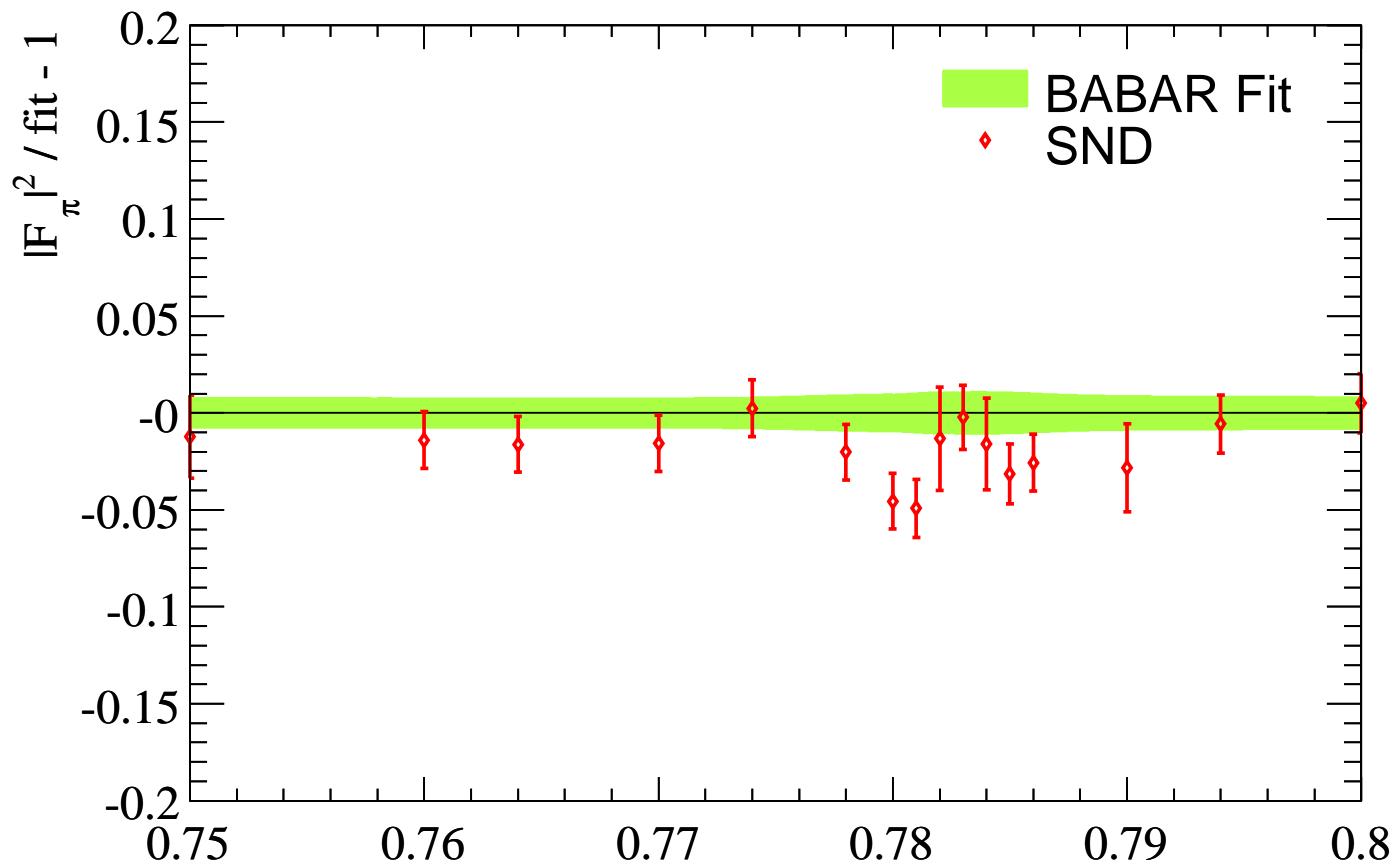
How Real is a_μ^{had} Accuracy?

- Radiative corrections: ISR and HVP probably OK,
FSR demands testing (charge asymmetry, $\pi^+\pi^-\gamma$)
- Scan vs. ISR method
- Missing states: neutrals, $\pi^+\pi^-n\pi^0$, $K\bar{K}n\pi$ - isospin
- Correlations
- Averaging
- Light-by-light term
- Double counting (LO and HO)

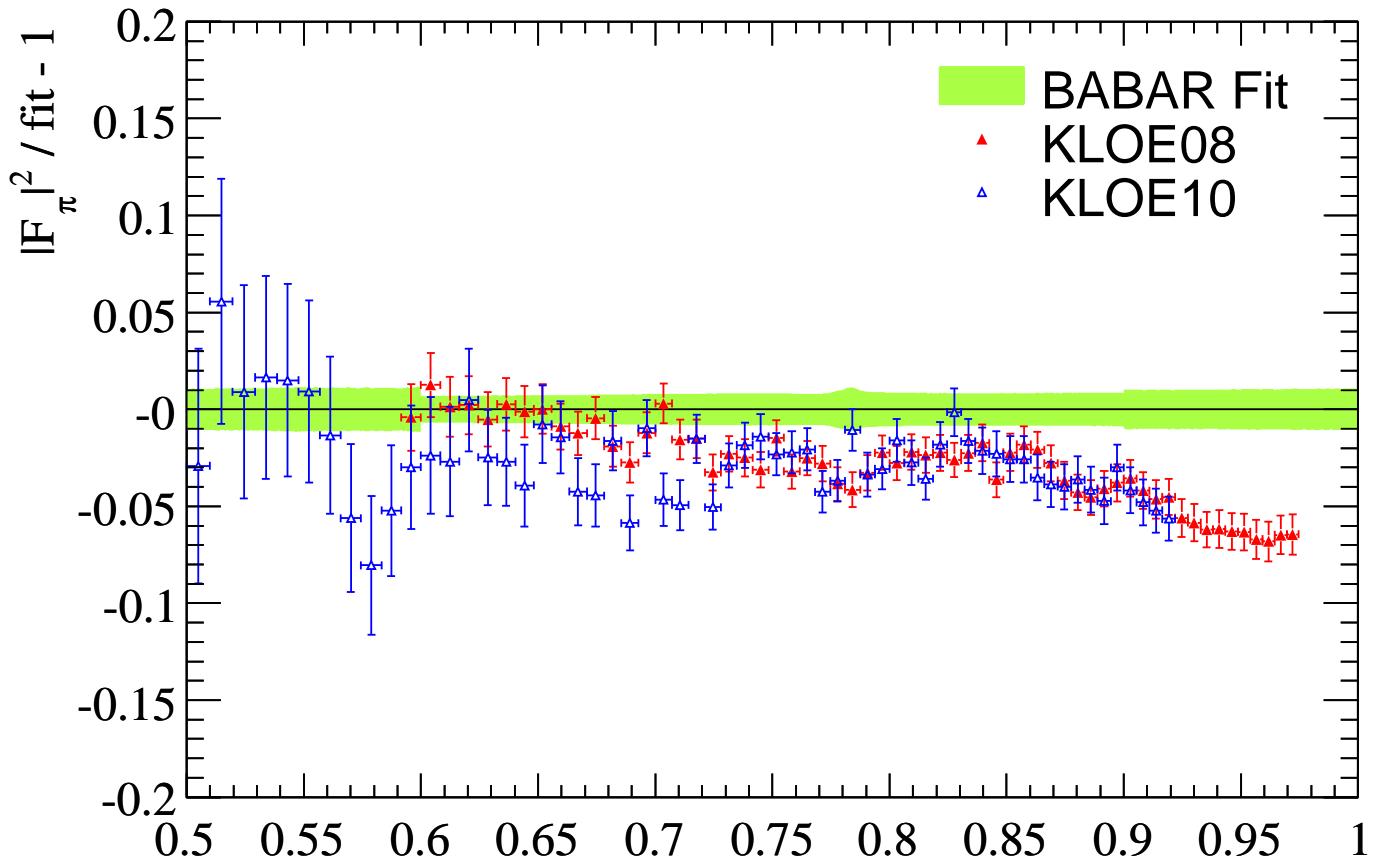
$e^+e^- \rightarrow \pi^+\pi^-$, BaBar vs. CMD-2



$e^+e^- \rightarrow \pi^+\pi^-$, BaBar vs. SND



$e^+e^- \rightarrow \pi^+\pi^-$, BaBar vs. KLOE



Do we have completely correct ISR theory?

Possible Progress for $a_\mu^{\text{LO,had}}$

Three upgraded e^+e^- colliders are running at low energy:

- VEPP-2000 (VEPP-2M upgrade) in Novosibirsk with 2 detectors (CMD-3 and SND), \sqrt{s} = from 0.3 to 2 GeV with $L_{\max} = 10^{32} \text{ cm}^{-2}\text{s}^{-1}$, more than 60 pb^{-1} per detector collected
- DAΦNE in Frascati should resume operation with the KLOE-2 detector at 1.02 GeV and $L \sim (2 - 3) \cdot 10^{33} \text{ cm}^{-2}\text{s}^{-1}$
- BEPCII in Beijing with the BESIII detector from 2 to 4.6 GeV and $L = 7 \cdot 10^{32} \text{ cm}^{-2}\text{s}^{-1}$

BaBar and Belle are continuing ISR analysis

Future

1. Experiment

The new projects at FermiLAB and JPARC expect 4 times better accuracy each

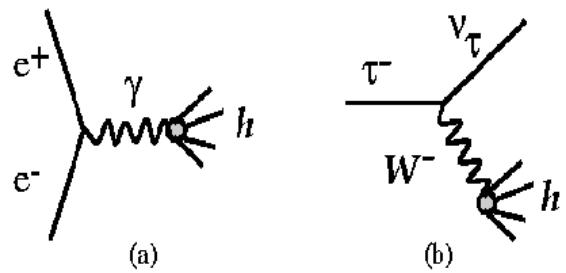
2. Theory (Experiment + Models)

- Such accuracy for $a_\mu^{\text{had,LO}}$ corresponds to 0.2%, hardly ever achievable with absolute $\sigma(e^+e^- \rightarrow \text{hadrons})$
- Additional limitation from $a_\mu^{\text{had,LBL}}$

3. Theory (First principles – QCD, Lattice)

- QCD instanton model (A. Dorokhov, 2003)
- Lattice – T. Blum et al., K. Jansen et al., M. Hayakawa et al.

CVC. $e^+e^- \rightarrow X^0$ and $\tau^- \rightarrow \nu_\tau X^-$



Allowed $I^G J^P = 1^+ 1^-$:
 $X^- = \pi^- \pi^0, (4\pi)^-, \omega \pi^-,$
 $\eta \pi^- \pi^0, K^- K^0, (6\pi)^-, \dots$

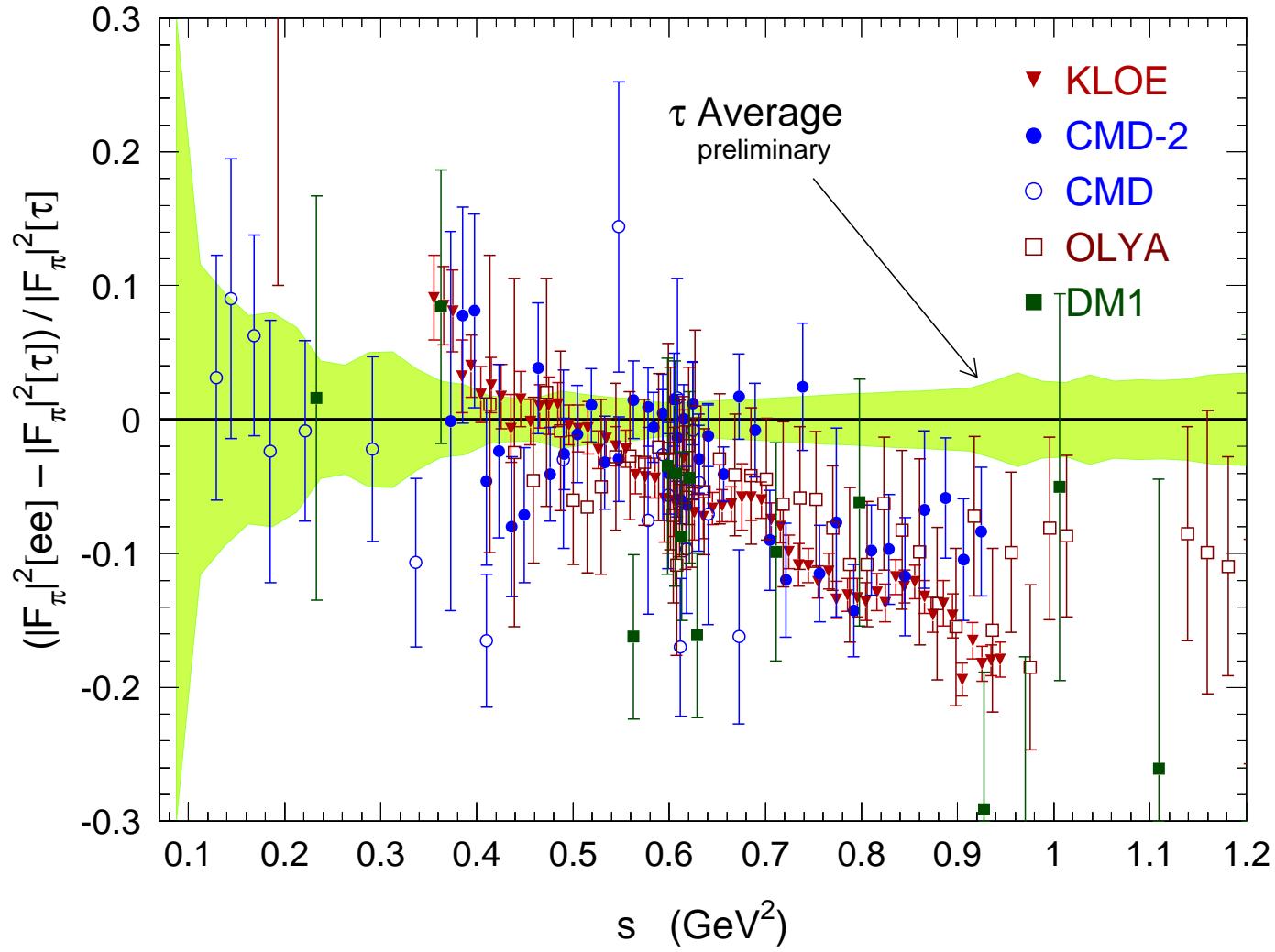
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ud}|^2 S_{EW}}{32\pi^2 m_\tau^3} f_{\text{kin}} v_1(q^2) \text{ with}$$

$$v_1(q^2) = \frac{q^2 \sigma_{e^+e^-}^{I=1}(q^2)}{4\pi\alpha^2}.$$

CVC tests showed good agreement of the τ branchings predicted from e^+e^- with τ data (N. Kawamoto and A. Sanda, 1978, F. Gilman and D. Miller, 1978, S. Eidelman and V. Ivanchenko, 1991, 1997).

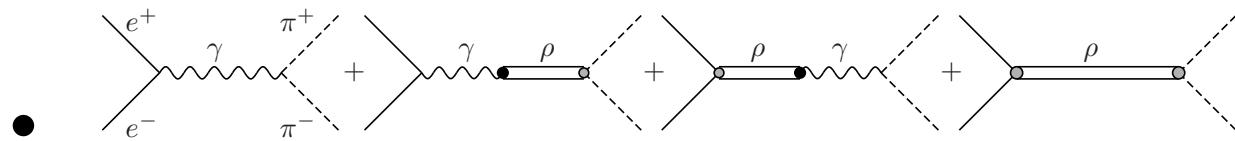
The very first application of τ data to $a_\mu^{\text{had}, \text{LO}}$ improved the accuracy by a factor of 1.5 (R. Alemany, M. Davier, A. Höcker, 1998)!

τ vs. e^+e^-



New Developments in τ vs. e^+e^-

- FJ and RS, EPJ C71, 1632 (2011) reconsidered the $\rho - \gamma$ mixing



- $\mathcal{B}_{\pi\pi}^\tau = (25.34 \pm 0.06 \pm 0.08)\%$
- $\mathcal{B}_{\pi\pi}^{e^+e^-} = (25.20 \pm 0.17 \pm 0.28)\%$
- $a_{e^+e^-}^{\text{had,LO}} = (690.75 \pm 4.72) \times 10^{-10}$
- $a_{e^+e^-+\tau}^{\text{had,LO}} = (690.96 \pm 4.65) \times 10^{-10}$