Nuclear physics from QCD on lattice

Takashi Inoue @Nihon University

for

HAL QCD Collaboration

S. Aoki	YITP Kyoto Univ.
T. Doi	RIKEN Nishina
T. Hatsuda	RIKEN Nishina
Y. Ikeda	RIKEN Nishina
T. I.	Nihon Univ.
N. Ishii	RCNP Osaka Univ.
K. Murano	RCNP Osaka Univ.
H. Nemura	Univ. Tsukuba
K. Sasaki	Univ. Tsukuba
F. Etminan	Univ. Birjand
T. Miyamoto	Univ. Tsukuba

CD2015, June 29 to July 3, Pisa

Introduction

- Nuclear physics
  - Theories have been developed extensively from 1930's
    - Liquid-drop model and semi-empirical mass formula.
    - Shell models supported by mean-field theory and Brueckner theory.
    - Variational methods w/ advanced technique for light nuclei.
    - Several sophisticated theories for heavy nuclei in these days,
      - eg. the coupled cluster theory, self consistent greens function method etc.

Introduction

✯ Nuclear physics

- Theories have been developed extensively from 1930's
  - Liquid-drop model and semi-empirical mass formula.
  - Shell models supported by mean-field theory and Brueckner theory.
  - Variational methods w/ advanced technique for light nuclei.
  - Several sophisticated theories for heavy nuclei in these days,
    - eg. the coupled cluster theory, self consistent greens function method etc.
- Properties of nuclei are explained and even predicted.
- But, we need input data from experiment.

Introduction

- Nuclear physics
  - Theories have been developed extensively from 1930's
    - Liquid-drop model and semi-empirical mass formula.
    - Shell models supported by mean-field theory and Brueckner theory.
    - Variational methods w/ advanced technique for light nuclei.
    - Several sophisticated theories for heavy nuclei in these days,
      - eg. the coupled cluster theory, self consistent greens function method etc.
  - Properties of nuclei are explained and even predicted.
  - But, we need input data from experiment.

- Quantum Chromodynamics

  No free parameter almost

  - is the fundamental theory of the strong interaction,
  - must explain everything, e.g. hadron spectrum, mass of nuclei.

Introduction

- **Nuclear physics**
  - Theories have been developed extensively from 1930's
    - Liquid-drop model and semi-empirical mass formula.
    - Shell models supported by mean-field theory and Brueckner theory.
    - Variational methods w/ advanced technique for light nuclei.
    - Several sophisticated theories for heavy nuclei in these days,
      - eg. the coupled cluster theory, self consistent greens function method etc.
  - Properties of nuclei are explained and even predicted.
  - But, we need input data from experiment.

- **Quantum Chromodynamics**

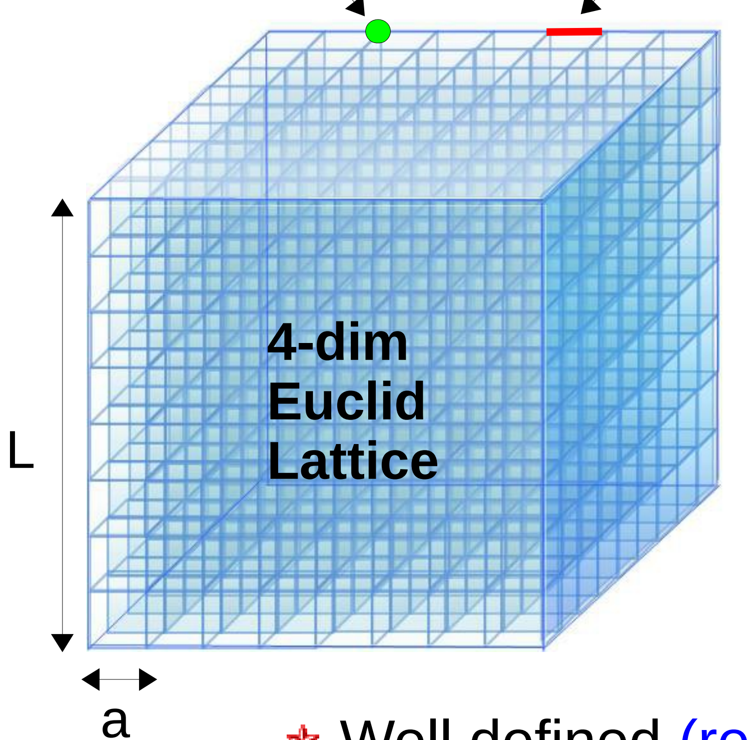
  No free parameter almost
  - is the fundamental theory of the strong interaction,
  - must explain everything, e.g. hadron spectrum, mass of nuclei.
  - But, that is difficult due to the non-perturbative nature of QCD.

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m \bar{q} q$$

quarks q
on the sites

gluons $U = e^{i a A_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU \, d\bar{q} \, dq \, e^{-S(\bar{q},q,U)} \, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)} \, O(D^{-1}(U))$$

quark propagator

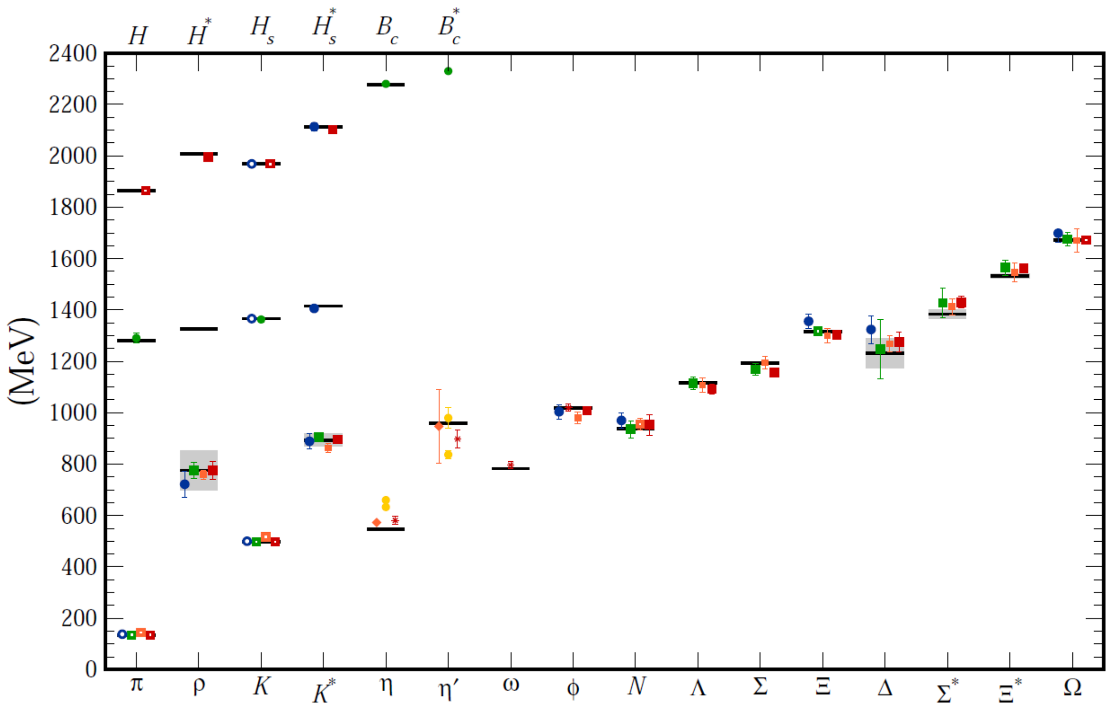
$$= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U) \, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturbative
- Highly predictive

Lattice QCD

- LQCD simulations w/ the physical quark mass ware done.
  - PACS-CS, Phys. Rev. D81 (2010) 075403
  - BMW, JHEP 1108 (2011) 14

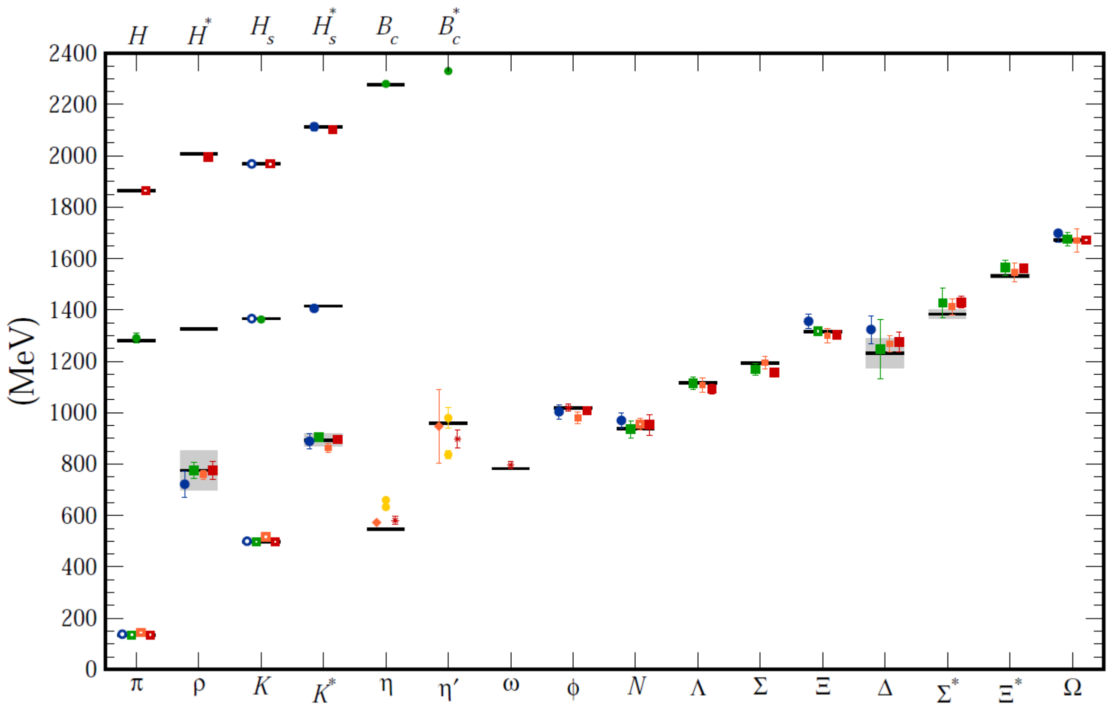


Summary by Kronfeld, arXive 1203.1204

- Mass of hadrons (ground state) are well reproduced!

Lattice QCD

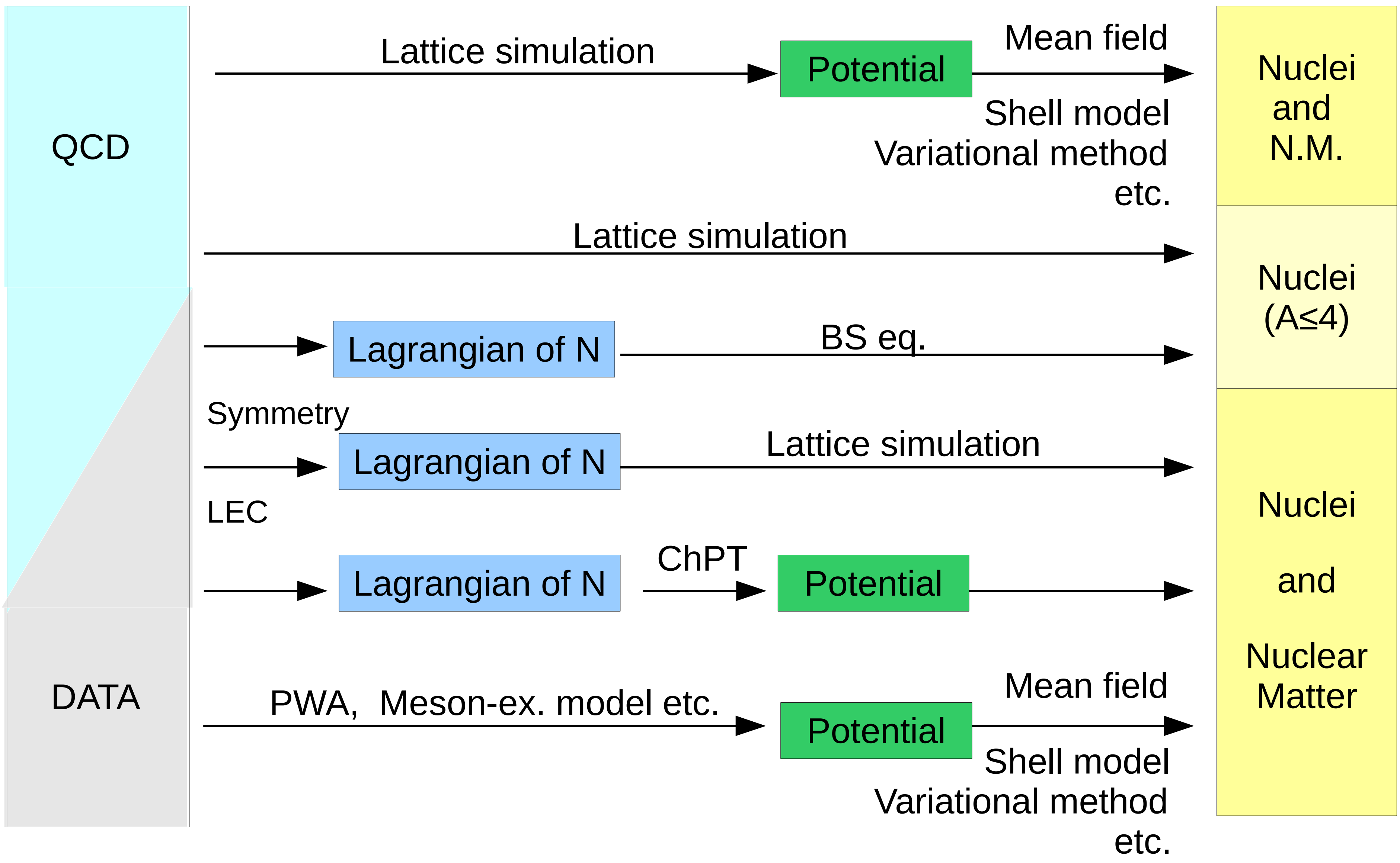
- LQCD simulations w/ the physical quark mass ware done.
  - PACS-CS, Phys. Rev. D81 (2010) 075403
  - BMW,   JHEP 1108 (2011) 14



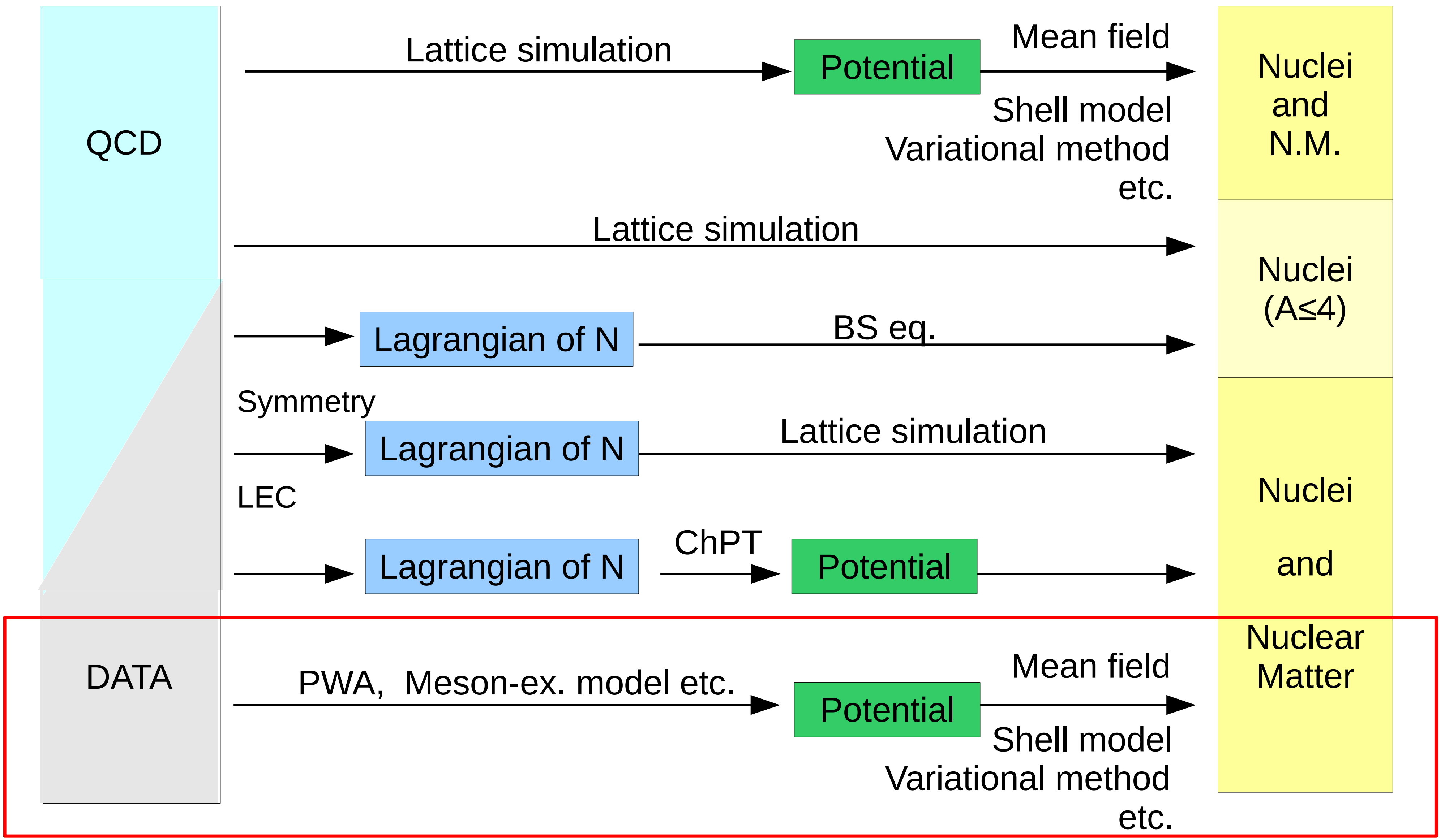
Summary by Kronfeld,
arXive 1203.1204

- Mass of hadrons (ground state) are well reproduced!
- What about atomic nuclei from QCD?

Various approaches in nuclear phys.

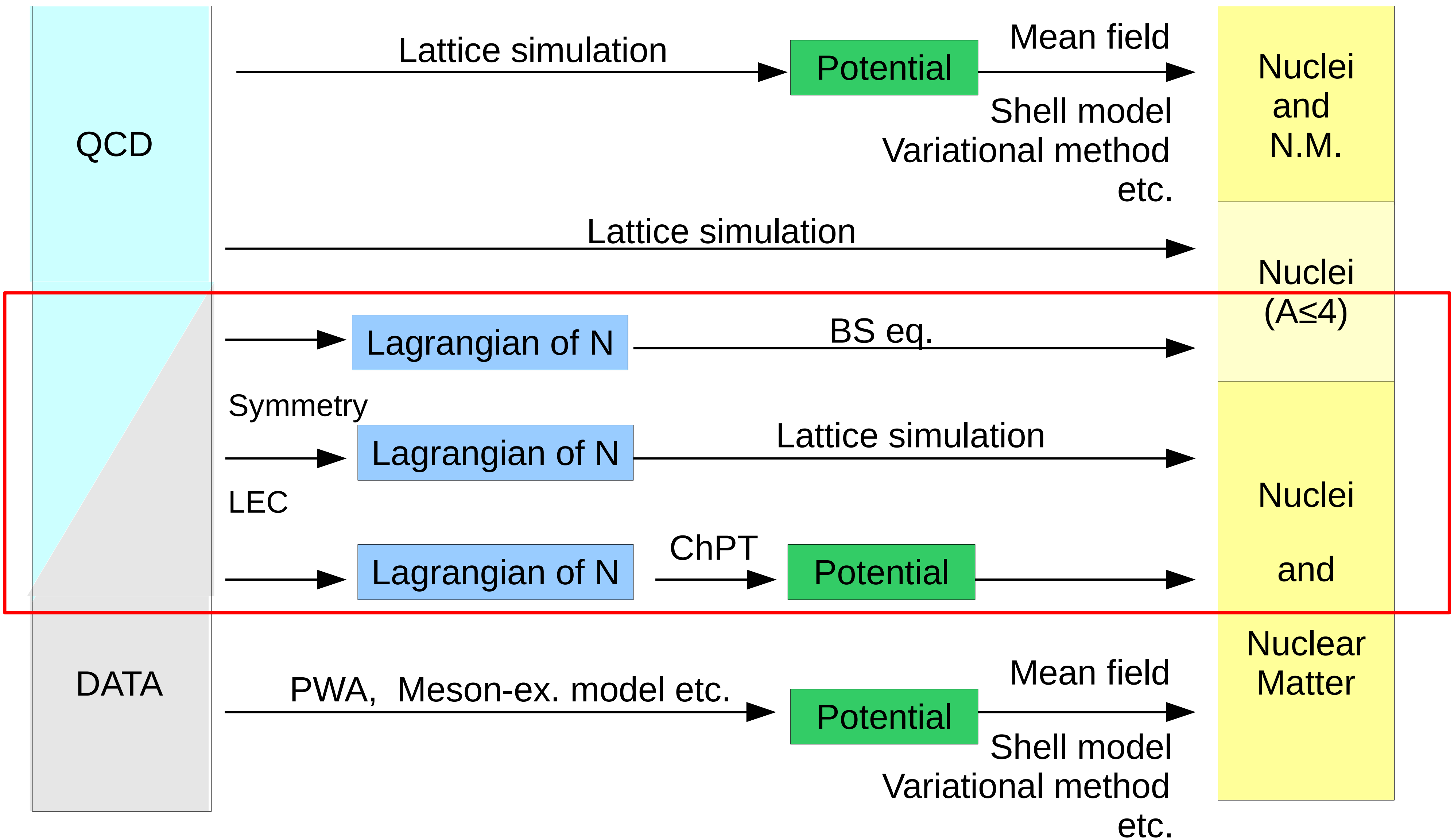


Various approaches in nuclear phys.



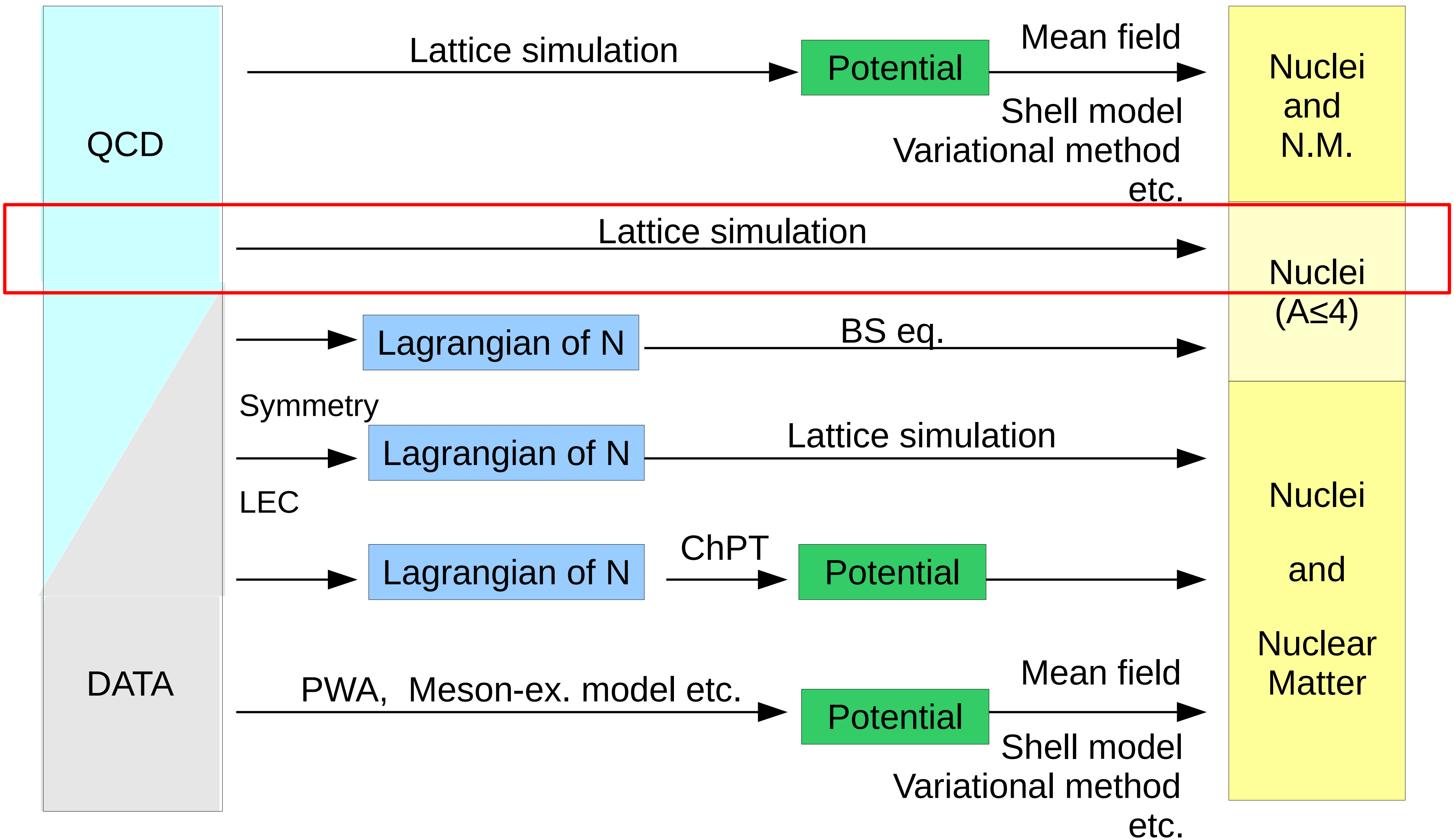
Most traditional. Many success.

Various approaches in nuclear phys.



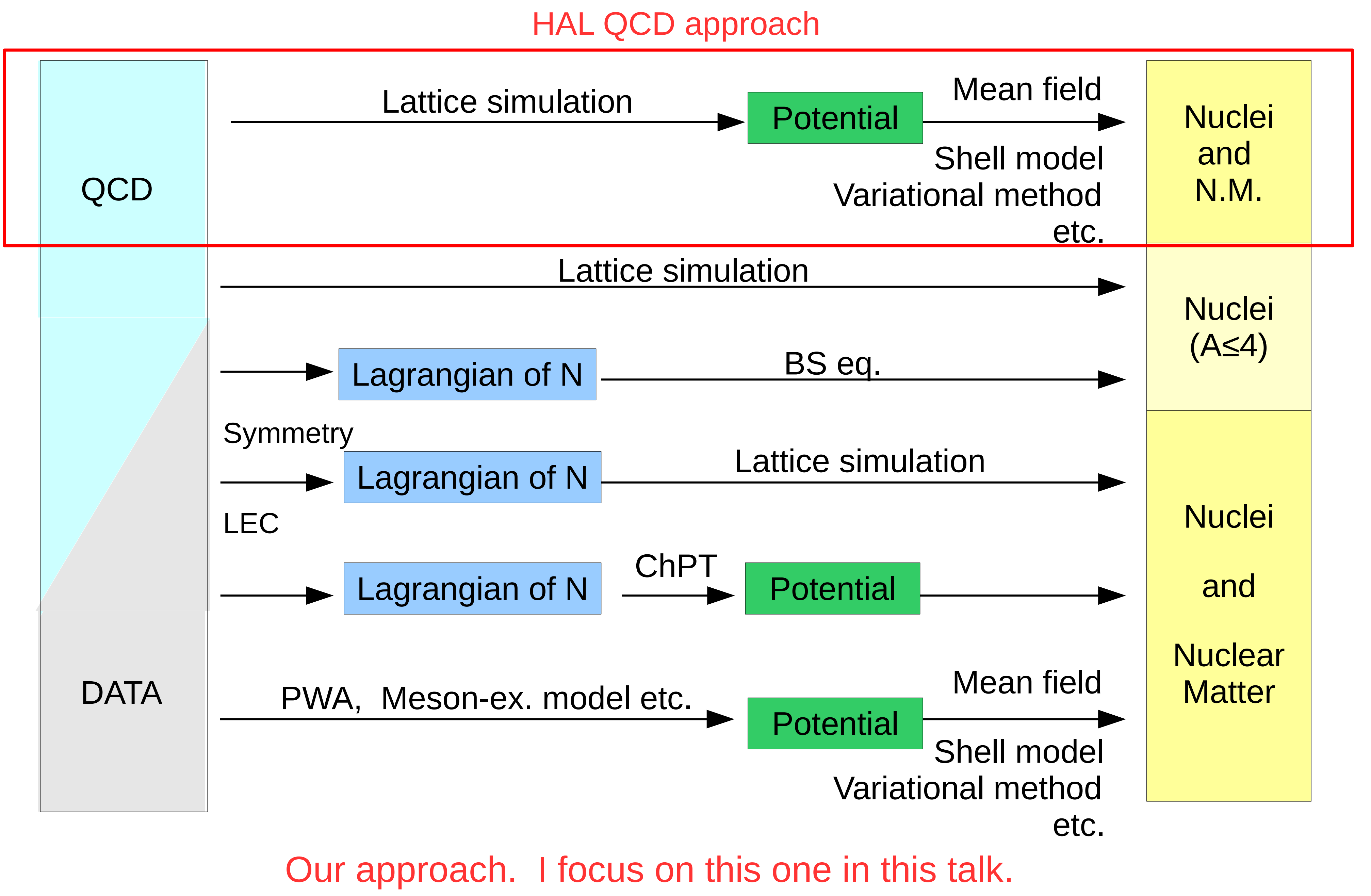
Very popular today. Let's say chiral approach.

Various approaches in nuclear phys.



Very challenging. Let's say LQCD direct approach.

Various approaches in nuclear phys.



HAL QCD Approach

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provide information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector and heavy flavor sector.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provide information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector and heavy flavor sector.

- Disappointing points at this moment
  - Demand long time and huge money at the 1st stage.
    - We had to deal with un-realistic QCD world so far.
    - Un-physically heavy u,d quark. Far from chiral symmetry.
  - Depend on method/approximation used at the 2nd stage.

HAL QCD Approach

- Good points
  - Based on the fundamental theory QCD, hence provide information independent of experiments and models.
  - Feasible. ↔ Direct one must be very difficult for large nuclei.
  - Can utilize established nuclear theories at the 2nd stage.
  - Easy to extend to strange sector and heavy flavor sector.

- Disappointing points at this moment

Only temporal.
We can overcome.

  - Demand long time and huge money at the 1st stage.
    - We had to deal with un-realistic QCD world so far.
    - Un-physically heavy u,d quark. Far from chiral symmetry.
  - Depend on method/approximation used at the 2nd stage.

- Today, I want
  - to show some results of HALQCD approach to nuclei, and to demonstrate that this approach is promising.

Outline

1. Introduction
2. HAL QCD method (the 1st stage)
3. Simulation setup and NN potentials
4. Helium nucleus
5. Medium-heavy nuclei
6. Infinite nuclear matter
7. Summary and Outlook

HAL QCD method

Multi-hadron in LQCD

- Direct：utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL：utilize a potential $V(r)+\ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

  - Advantages
    - No need to separate E eigenstate. Just need to measure
    - Then, potential can be extracted.
    - Demand a minimal lattice volume. No need to extrapolate to V=∞.
    - Can output many observables.

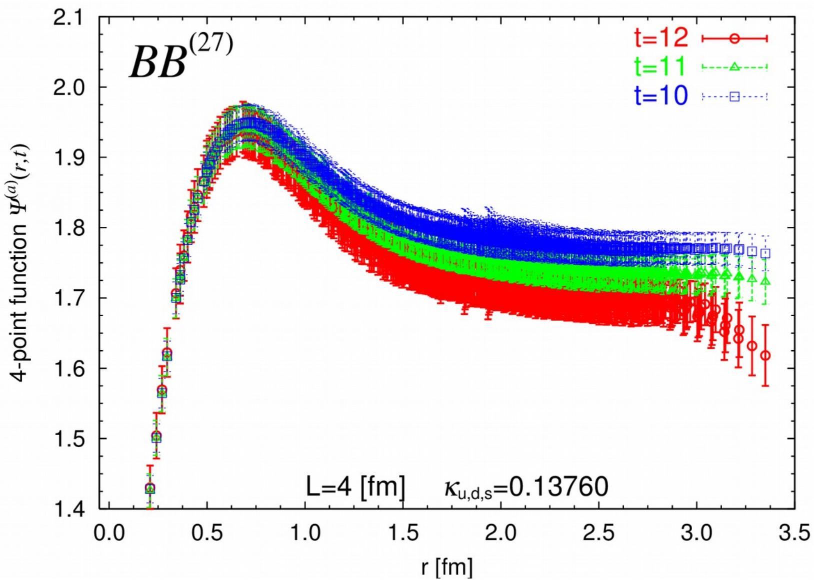
Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL : utilize a potential $V(r) + \ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



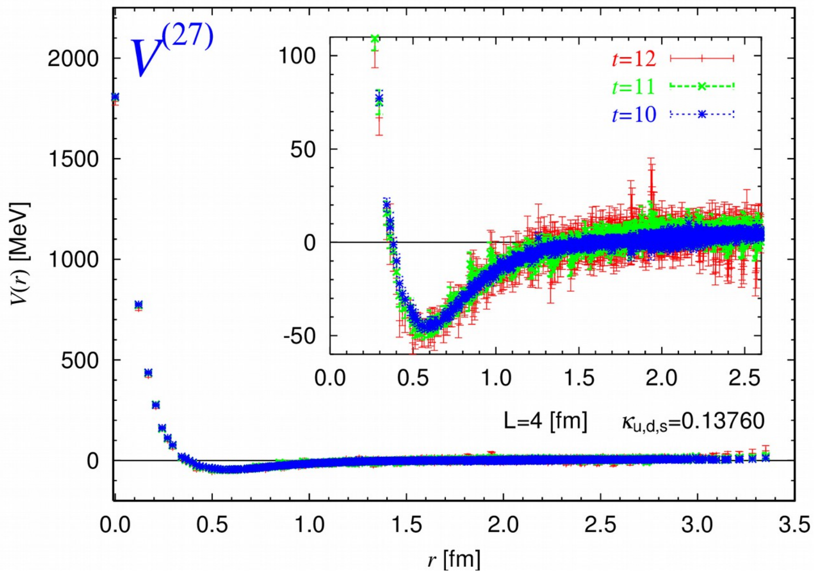
Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL : utilize a potential $V(r) + \ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



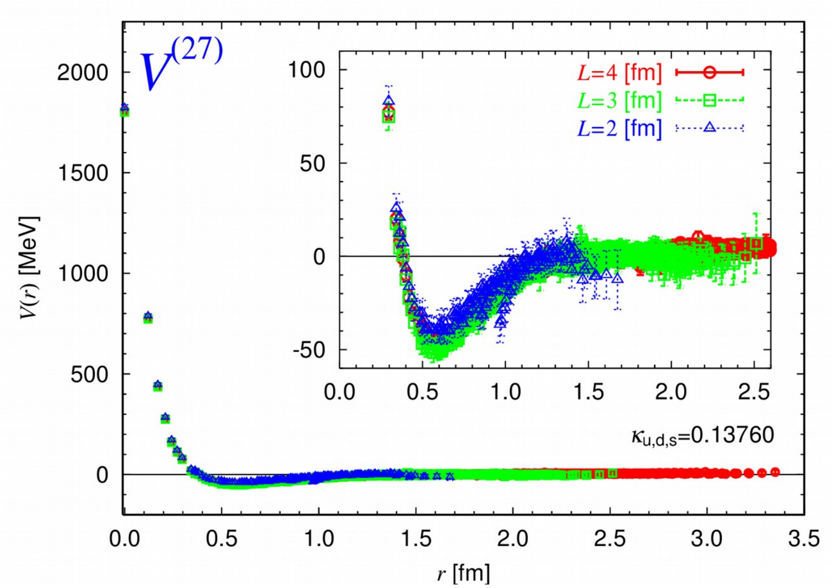
Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL : utilize a potential $V(r) + \ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output many observables.



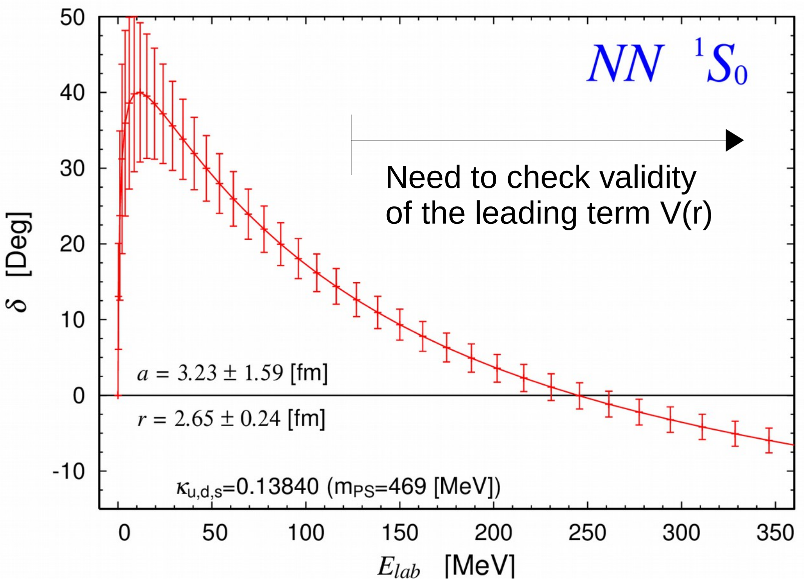
Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL : utilize a potential $V(r) + \ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output more observables.



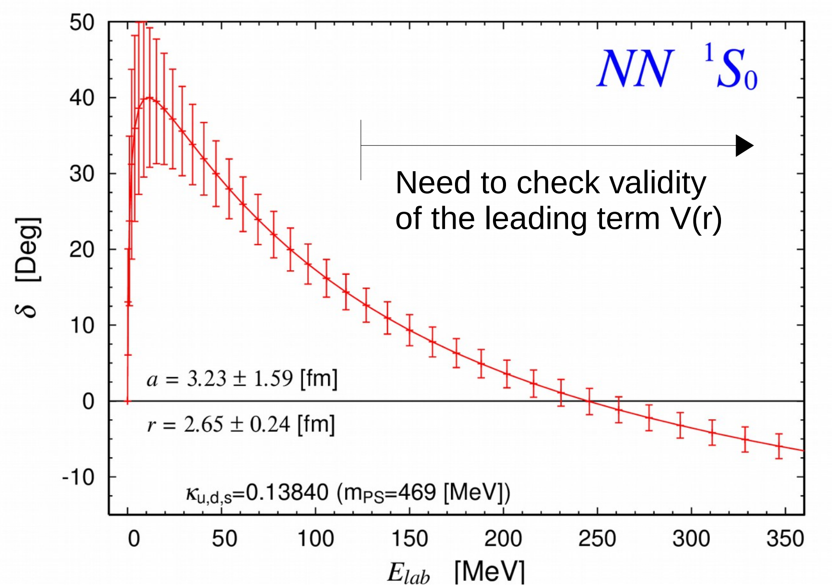
Multi-hadron in LQCD

- Direct : utilize energy eigenstates (eigenvalues)
  - Lüscher's finite volume method for a phase-shift
  - Infinite volume extrapolation for a bound state
- HAL : utilize a potential $V(r) + \ldots$ of interaction

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to V=∞.
  - Can output more observables.



☆ We can attack large nuclei too!!

HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\varphi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

Define a unique potential U for all E eigenstates by a “Schrödinger” eq.

$$\left[-\frac{\nabla^2}{2\mu} \right] \varphi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \varphi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \varphi_{\vec{k}}(\vec{r})$$

Non-local but energy independent

Measure 4-point function in LQCD

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t)\; J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \varphi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion & truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$$

Therefor, in the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

FAQ

1. Does your potential depend on the choice of source?

➔ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?

➔ Yes. It can be regarded as the “scheme” to define a potential. Note that a potential itself is not physical observable. We'll obtain unique result for physical observables irrespective to the choice, as long as the potential U(r,r') is deduced exactly.

FAQ

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

➔ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.

Lattice simulation setup
and
NN potentials from QCD

Simulation setup

T. Inoue etal, Nucl. Phys. A881, 28 (2012)

- Physical-point gauge confs. of the PACS-CS/BMW studies, lattice volume ware small even for NN system. $L \simeq 2$ [fm]
- I've generated gauge confs. on relatively large volume.

size	β	c_{SW}	a [fm]	L [fm]
$32^3 \times 32$	1.83	1.761	0.121(2)	3.87

Kuds	M_{PS} [MeV]	M_B [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1161(2)

limitation

- Iwasaki gauge & Wilson quark.
- Thanks to PACS-CS collaboration for their DDHMC/PHMC code.
- $SU(3)_F$ limit is useful to capture essential feature of BB interaction

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

- We use these five sets of gauge conf. w/ $m_u = m_d = m_s$.

Simulation setup

T. Inoue etal, Nucl. Phys. A881, 28 (2012)

- Physical-point gauge confs. of the PACS-CS/BMW studies, lattice volume ware small even for NN system. $L \simeq 2$ [fm]
- I've generated gauge confs. on relatively large volume.

size	β	C_{SW}	a [fm]	L [fm]
32^3 x 32	1.83	1.761	0.121(2)	3.87

Kuds	M_{PS} [MeV]	M_B [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1161(2)

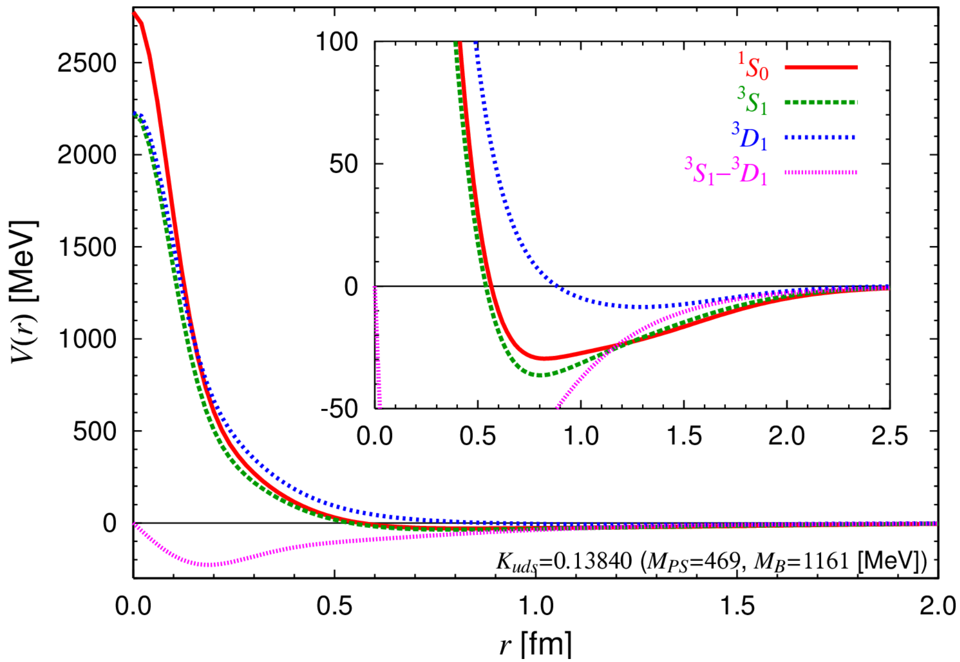
limitation

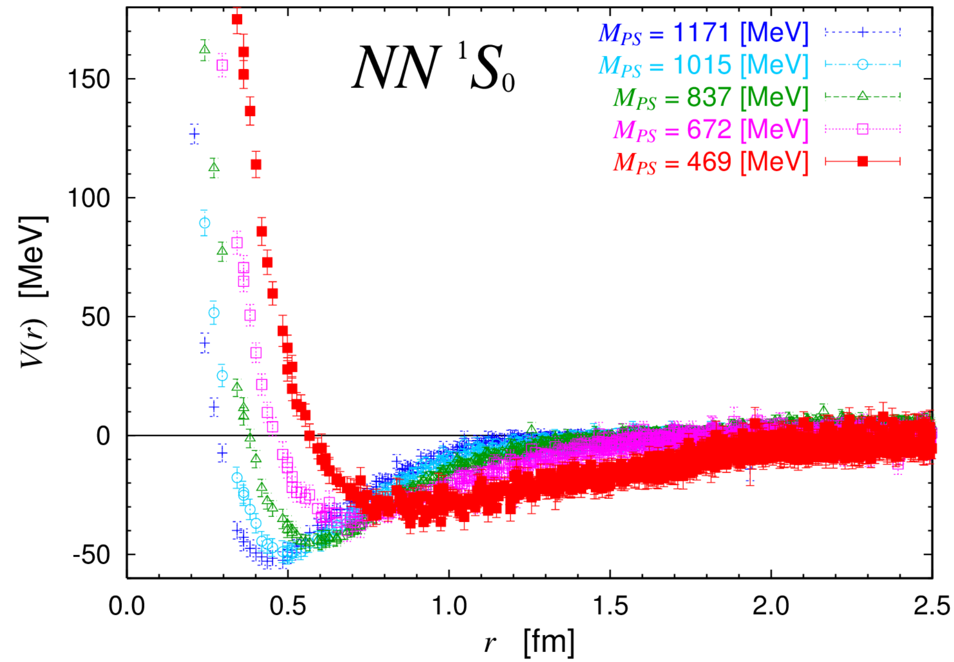
- Iwasaki gauge & Wilson quark.
- Thanks to PACS-CS collaboration for their DDHMC/PHMC code.
- $SU(3)_F$ limit is useful to capture essential feature of BB interaction

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

- We use these five sets of gauge conf. w/ $m_u = m_d = m_s$.
- BTW, gauge confs. on large volume $(L > 8 \text{ [fm]})$ at an almost physical point are generated in 2014 at RIKEN AICS. We are doing simulation now.

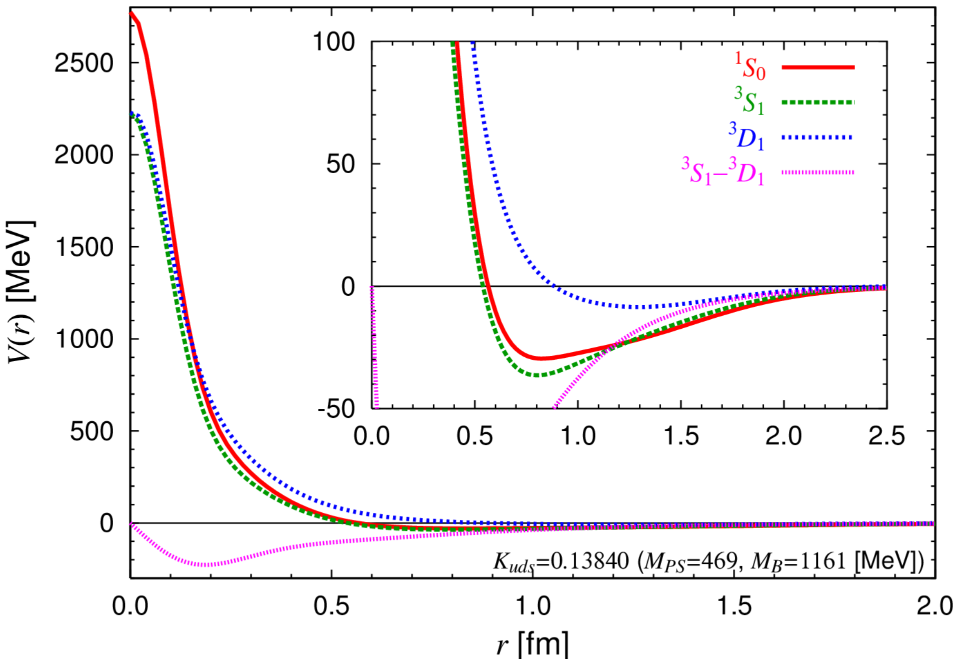
NN potentials from QCD

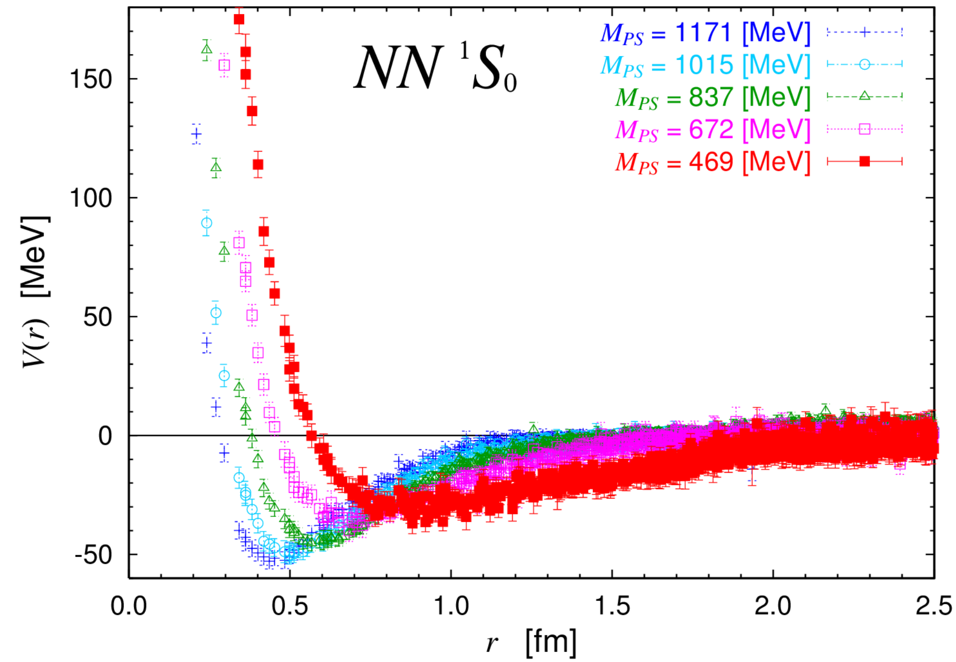




- Left: NN potentials in partial waves at the lightest m_q.
  - Repulsive core & attractive pocket & strong tensor force.
  - Similar to phenomenological potentials qualitatively. e.g. AV18
  - Least χ^2 fit of data which give central value of observable.
  - Higher orders in velocity expansions are not available yet. We restrict us to these leading order potentials.

NN potentials from QCD

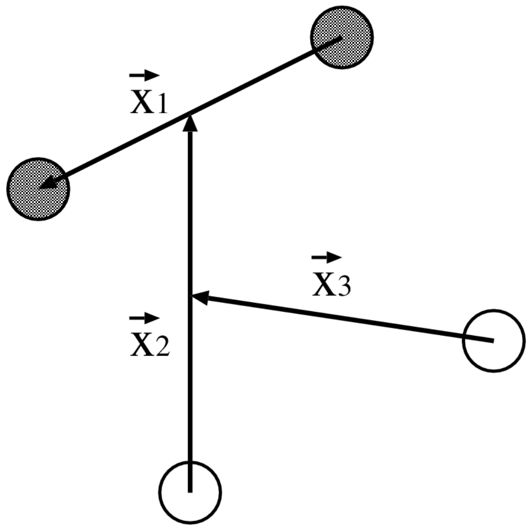




- Left: NN potentials in partial waves at the lightest m_q.
  - Repulsive core & attractive pocket & strong tensor force.
  - Similar to phenomenological potentials qualitatively. e.g. AV18
  - Least χ^2 fit of data which give central value of observable.
  - Higher orders in velocity expansions are not available yet.
    We restrict us to these leading order potentials.
- Right: Quark mass dependence of $V(r)$ of NN $^1S_0$.
  - Potentials become stronger as m_q decrease.

Helium nucleus from QCD

^{4}He nucleus



Schrodinger equation

$$\left[K + V\right]\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = E\ \Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

Correlated Gaussian basis ($L=0$)

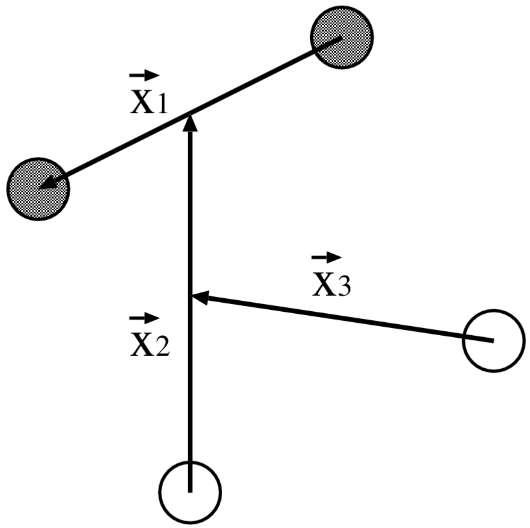
$$f_A\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \exp\left[-\frac{1}{2} X \cdot A\, X^t\right]$$

$$w/\ \ X = \left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right),\ \ A = 3\times 3 \text{ matrix}$$

$$\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \sum_{i=1}^{N} C_i\, f_{A_i}\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

- One can solve the eq. of $4N$ system exactly w/ some method.

^{4}He nucleus



Schrodinger equation

$$\left[K + V\right]\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = E\ \Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

Correlated Gaussian basis (L=0)

$$f_A\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \exp\left[-\frac{1}{2} X \cdot A\, X^t\right]$$

$$w/\ \ X = \left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right),\ \ A = 3\times 3 \text{ matrix}$$

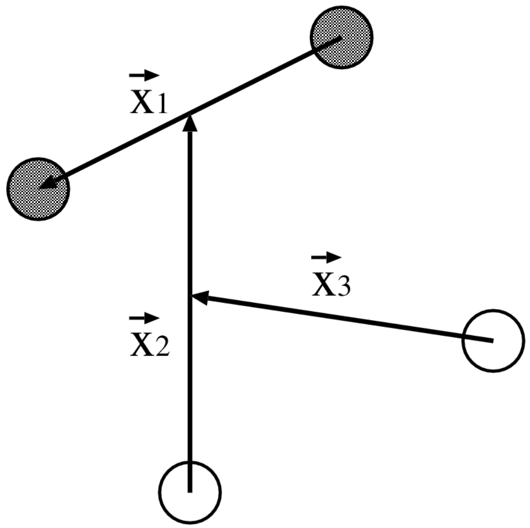
$$\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \sum_{i=1}^{N} C_i\, f_{A_i}\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

- One can solve the eq. of $4N$ system exactly w/ some method.
- Here, we employ the Stochastic Variational Method.

K. Varga and Y. Suzuki, Comp. Phys. Comm. 106 (1997) 157-168

  - By generating matrix A randomly, many function f_A are examined.
  - Most efficient f_A is add to the basis set. = Competing selection.
  - Number of basis gradually increases but remains small.
    No need to prepare a huge basis set. It is easy to solve the eq.

^{4}He nucleus



Schrodinger equation

$$\left[K + V\right]\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = E\ \Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

HALQCD

Correlated Gaussian basis ($L=0$)

$$f_A\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \exp\left[-\frac{1}{2} X \cdot A X^t\right]$$

$$w/\quad X = \left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right), \quad A = 3\times 3 \text{ matrix}$$

$$\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \sum_{i=1}^{N} C_i\, f_{A_i}\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

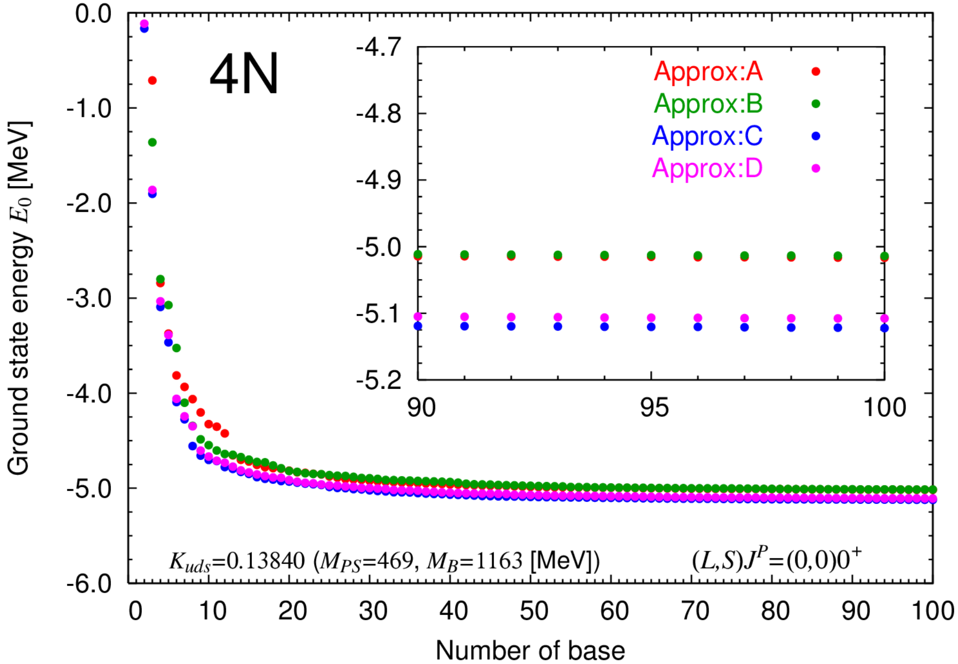
- One can solve the eq. of $4N$ system exactly w/ some method.
- Here, we employ the Stochastic Variational Method.

K. Varga and Y. Suzuki, Comp. Phys. Comm. 106 (1997) 157-168

  - By generating matrix A randomly, many function f_A are examined.
  - Most efficient f_A is add to the basis set. = Competing selection.
  - Number of basis gradually increases but remains small.
    No need to prepare a huge basis set. It is easy to solve the eq.

Ground state of ^{4}He

@SU(3)$_F$ limit
Mps = 469 MeV



	E_0 [MeV]
Approx A	−5.0167
Approx B	−5.0140
Approx C	−5.1222
Approx D	−5.1078

Approx A, B: Wigner wo/ spin-dep. Serber

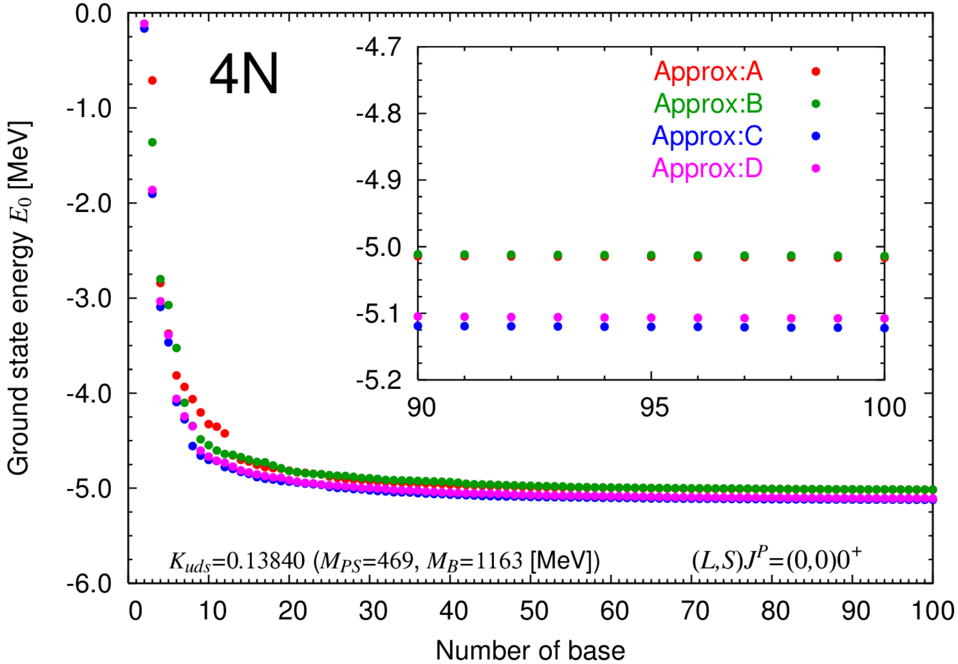
Approx C, D: Wigner w/ spin-dep. Serber

- Number of basis v.s. energy of ^{4}He g.s. at the lightest quark mass.
- We tested 4 approx for unknown odd parity force. But, no diff.

^{4}He = S-shell

Ground state of ^{4}He

@SU(3)$_F$ limit
Mps = 469 MeV



	E_0 [MeV]	
Approx A	−5.0167	Wigner wo/ spin-dep. Serber
Approx B	−5.0140	
Approx C	−5.1222	Wigner w/ spin-dep. Serber
Approx D	−5.1078	

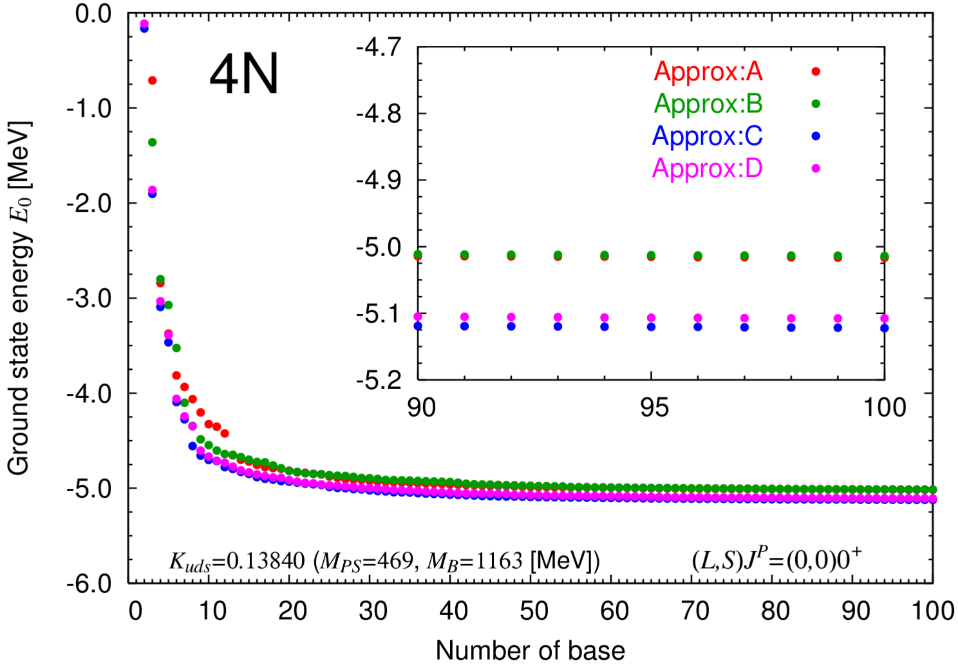
- Number of basis v.s. energy of ^{4}He g.s. at the lightest quark mass.
- We tested 4 approx for unknown odd parity force. But, no diff.

^{4}He = S-shell

- There definitely exists ^{4}He nucleus at M_{PS} = 469 MeV!!

Ground state of ^{4}He

@SU(3)$_F$ limit
Mps = 469 MeV



	E_0 [MeV]
Approx A	−5.0167
Approx B	−5.0140
Approx C	−5.1222
Approx D	−5.1078

Approx A, B: Wigner wo/ spin-dep. Serber

Approx C, D: Wigner w/ spin-dep. Serber

- Number of basis v.s. energy of ^{4}He g.s. at the lightest quark mass.
- We tested 4 approx for unknown odd parity force. But, no diff.

^{4}He = S-shell

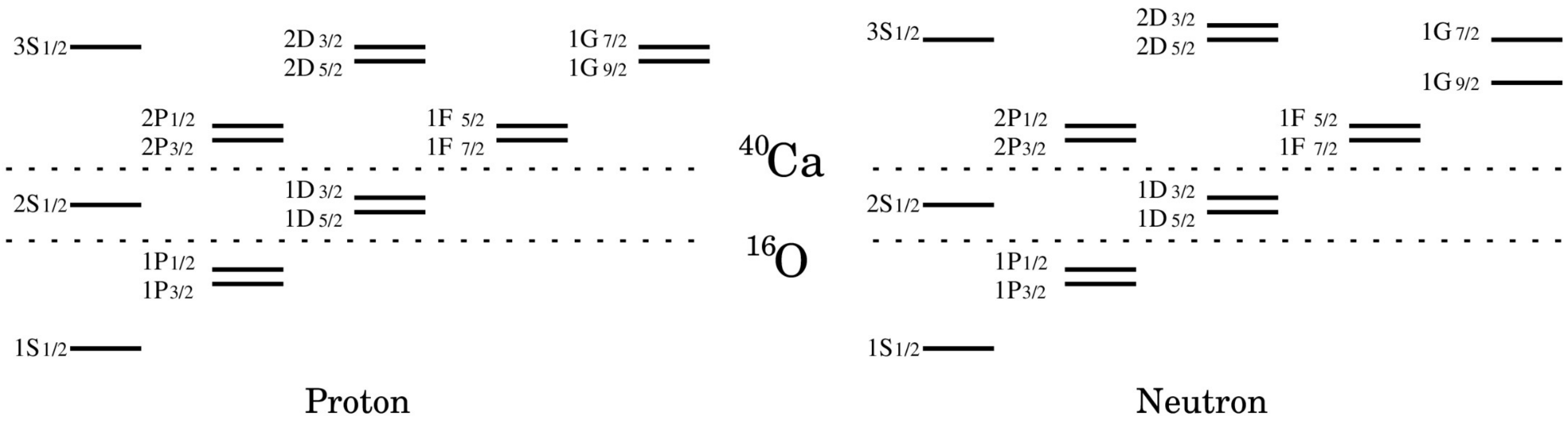
- There definitely exists ^{4}He nucleus at M$_{PS}$ = 469 MeV!!
- No ^{4}He nucleus at quark mass of M$_{PS}$ = 632, 837 MeV.
- No 2N, 3N nuclei at all our five values of quark mass.

in contrast to other groups

T. Inoue etal [HAL QCD Colla.] Nucl. Phys. A881 (2012) 28

Medium-heavy nuclei from QCD

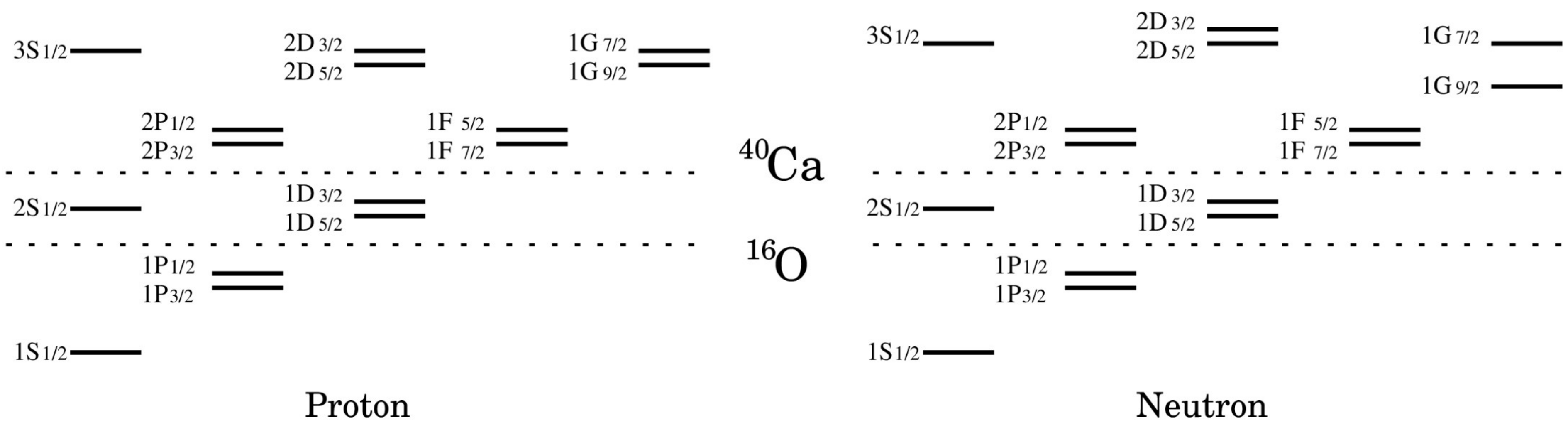
Mean field picture of nuclei



quasi-nucleon

- Schematic diagram of single particle levels in nuclei.
  - Similar to 3D-Harmonic-Oscillator at lower.

Mean field picture of nuclei



quasi-nucleon

- Schematic diagram of single particle levels in nuclei.
  - Similar to 3D-Harmonic-Oscillator at lower.
- Nuclei ^{16}O and ^{40}Ca are called doubly closed nuclei,
  - where their ground state can be assumed safely as iso-symmetric, spin-saturated, and spherical.
  - used as "core" in traditional shell-model calculations.
- Let us study ^{16}O and ^{40}Ca in the HALQCD approach.

Ideal for theoretical study.

Brueckner-Hartree-Fock for nuclei

- Brueckner G-matrix in a single-particle-orbit base

$$G(\omega)_{ij,kl} = V^{LQCD}_{ij,kl} + \frac{1}{2} \sum_{m,n}^{>e_F} V^{LQCD}_{ij,mn} \frac{1}{\omega - e_m - e_n + i\epsilon} G(\omega)_{mn,kl}$$

← Pauli

- Hartree-Fock mean field $U_{ab} = \sum_{c,d} G(\tilde{\omega})_{ac,bd}\, \rho_{dc}$

- New single-particle orbit $[K + U]\, \Psi^i = e_i\, \Psi^i$

iterate until converge

- Hartree-Fock ground state energy

$$E_0 = \sum_{a,b} \left(K_{ab} + \frac{1}{2} U_{ab}\right) \rho_{ba} - K_{c.m.}$$

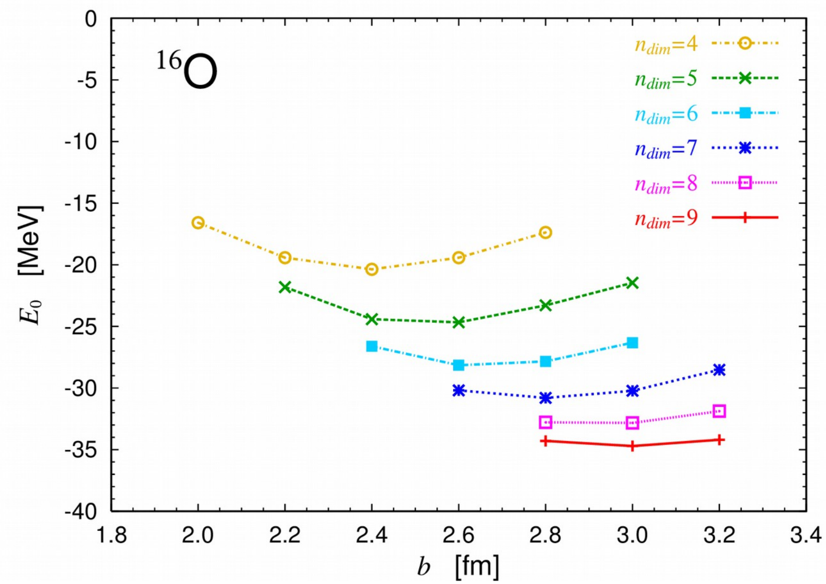
limitation of V^{LQCD}

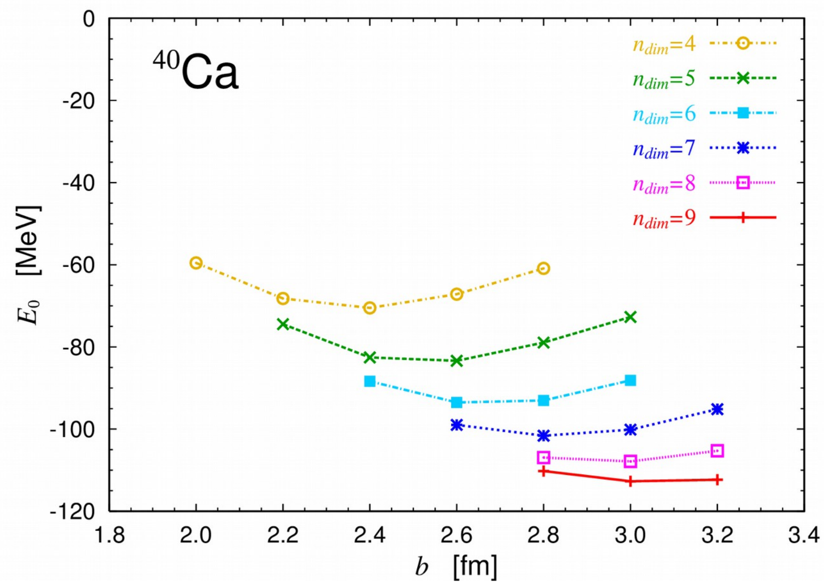
- p.w. decomposition and truncation ${}^{2S+1}L_J = {}^1S_0, {}^3S_1, {}^3D_1, {}^1P_1, {}^3P_J \cdots$

P. Ring and P. Schuck, “The Nuclear Many-Body Problems”, Springer (1980)
K.T.R. Davies, M. Baranger, R.M. Tarbutton T.T.S. Kuo, Phys. Rev. 177 1519 (1969)

Existence of nuclei in LQCD

@SU(3)$_F$ limit
Mps = 469 MeV



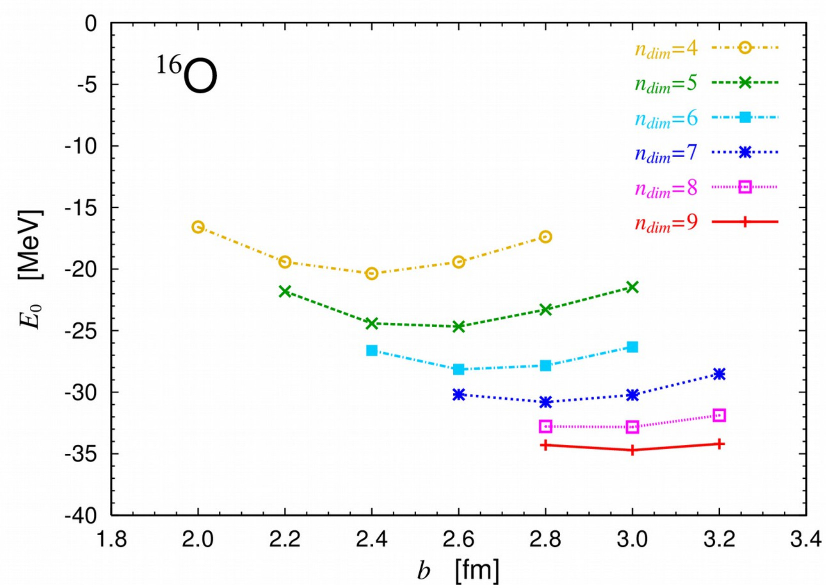


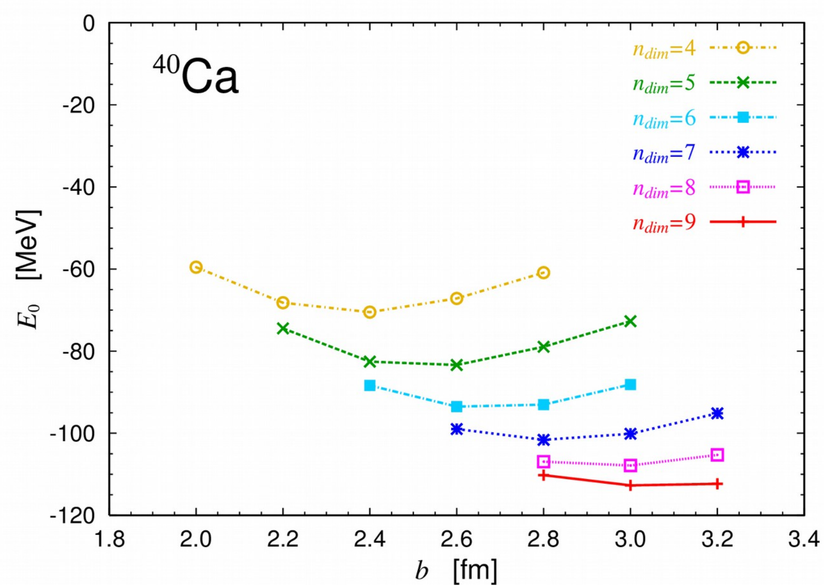
- Ground state energy E_0 at the lightest quark (M_{PS} = 469 MeV)
- Harmonic-Oscillator basis set used to expand s.p. states.

$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+3/2)}} \left(\frac{r}{b}\right)^l e^{-\frac{1}{2}(r/b)^2} \sum_{m=0}^{n} C_{n-m}^{n+l+1/2} \frac{\left(-r^2/b^2\right)^m}{m!} \quad , \quad n = 0, 1 \ldots n_{dim}-1$$

Existence of nuclei in LQCD

@SU(3)$_F$ limit
Mps = 469 MeV





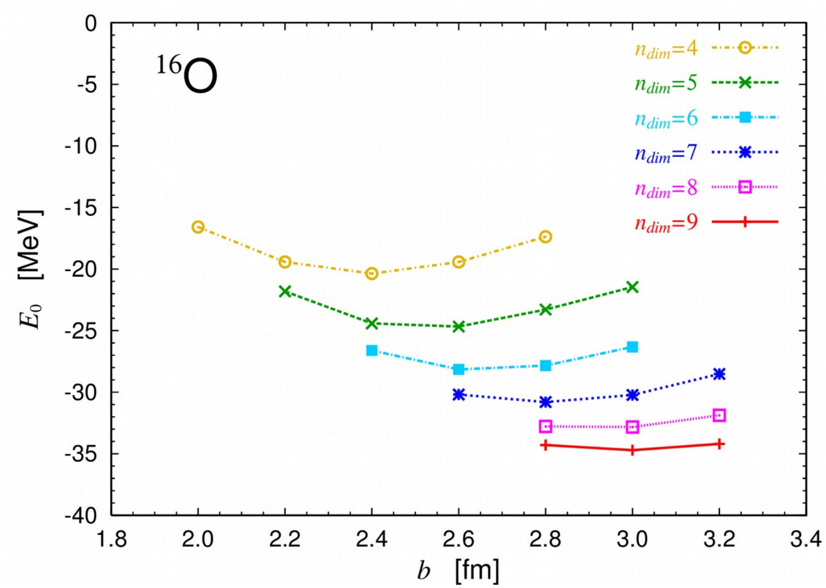
- Ground state energy E_0 at the lightest quark (M_{PS} = 469 MeV)
- Harmonic-Oscillator basis set used to expand s.p. states.

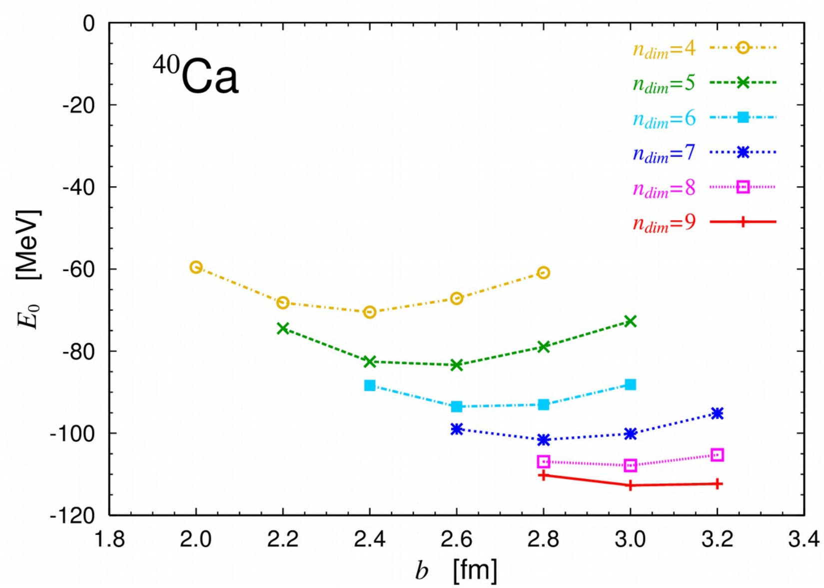
$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+3/2)}} \left(\frac{r}{b}\right)^l e^{-\frac{1}{2}(r/b)^2} \sum_{m=0}^{n} C_{n-m}^{n+l+1/2} \frac{\left(-r^2/b^2\right)^m}{m!} \quad , \quad n = 0, 1 \ldots n_{dim}-1$$

- There definitely exist ^{16}O and ^{40}Ca nuclei at this quark mass!

Existence of nuclei in LQCD

@SU(3)$_F$ limit
Mps = 469 MeV





- Ground state energy E_0 at the lightest quark (M_{PS} = 469 MeV)
- Harmonic-Oscillator basis set used to expand s.p. states.

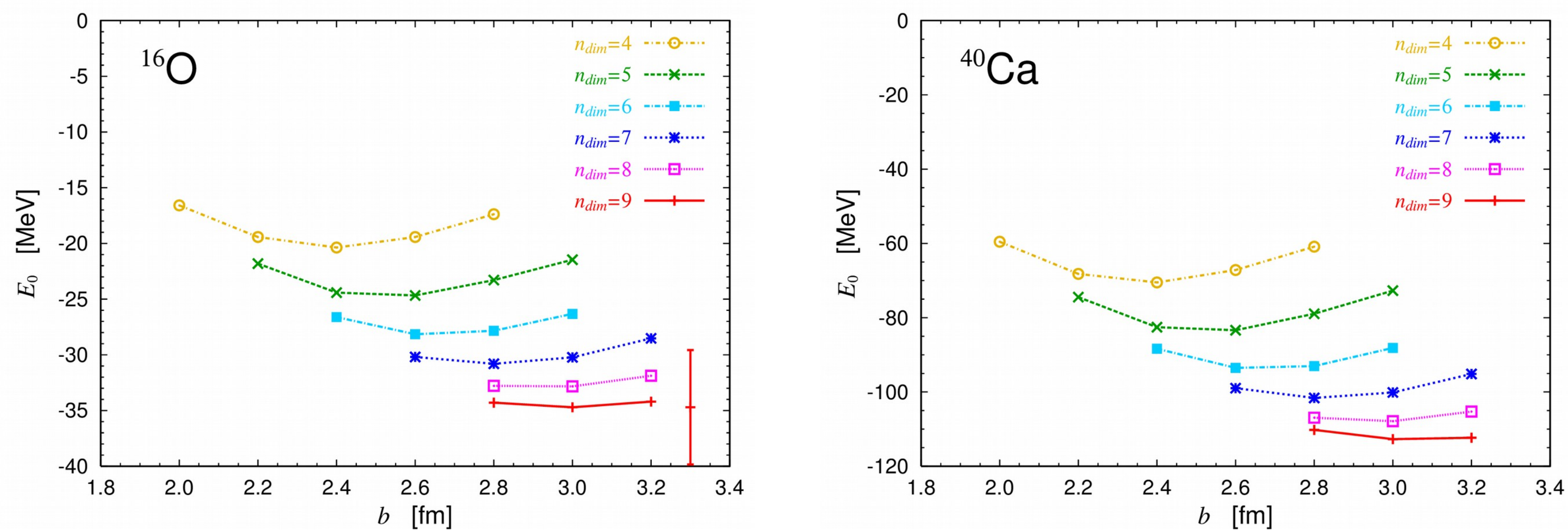
$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+3/2)}} \left(\frac{r}{b}\right)^l e^{-\frac{1}{2}(r/b)^2} \sum_{m=0}^{n} C_{n-m}^{n+l+1/2} \frac{\left(-r^2/b^2\right)^m}{m!} \quad , \quad n = 0, 1 \ldots n_{dim}-1$$

- There definitely exist ^{16}O and ^{40}Ca nuclei at this quark mass!
- We do not find any negative E_0 for the other 4 values of m_q.

= No tightly bound nucleus at those m_q .

Existence of nuclei in LQCD

@SU(3)$_F$ limit
Mps = 469 MeV



- Ground state energy E_0 at the lightest quark (M_{PS} = 469 MeV)
- Harmonic-Oscillator basis set used to expand s.p. states.

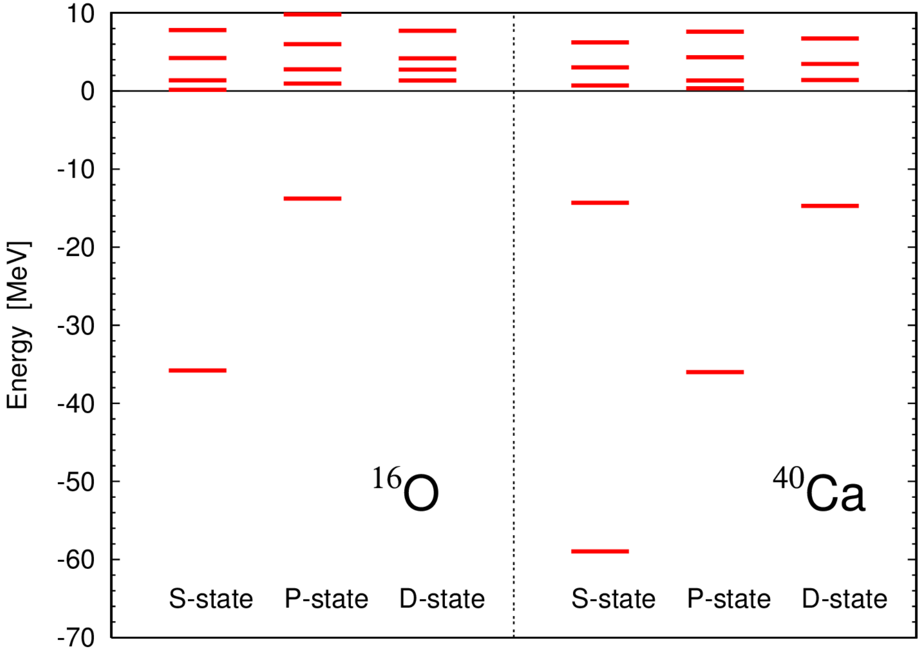
$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+3/2)}} \left(\frac{r}{b}\right)^l e^{-\frac{1}{2}(r/b)^2} \sum_{m=0}^{n} C_{n-m}^{n+l+1/2} \frac{\left(-r^2/b^2\right)^m}{m!} \quad , \quad n = 0, 1 \ldots n_{dim}-1$$

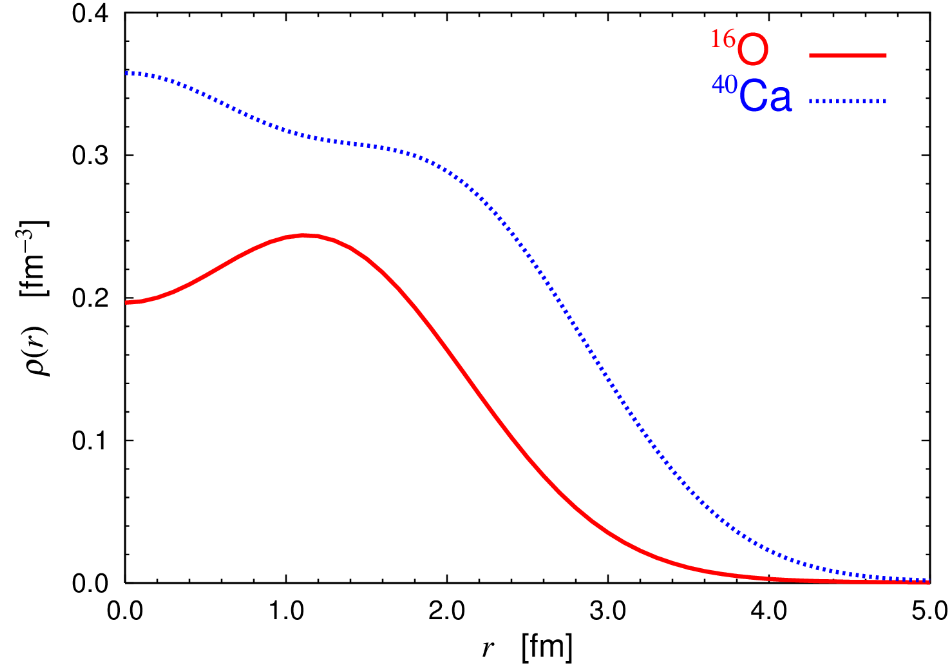
- There definitely exist ^{16}O and ^{40}Ca nuclei at this quark mass!
- We do not find any negative E_0 for the other 4 values of m_q.

= No tightly bound nucleus at those m_q .

Structure of nuclei in LQCD

@$SU(3)_F$ limit
Mps = 469 MeV





- Left: Single particle levels in the nuclei
  - Regular shell structure, which can be seen in experimental data such as a nucleon separation energy.
- Right: Nucleon number density in the nuclei
  - Distinct shell effect at short distance, which can be seen in data such as a charge distribution from electron scattering.

Properties of nuclei in LQCD

@SU(3)$_F$ limit
Mps = 469 MeV

- At the lightest quark (M_{PS} = 469 MeV) with n_{dim} = 9 and b = 3.0 fm

^{16}O

Single particle levels [MeV]				Total binding [MeV]		Radius [fm]
E_{1S}	E_{1P}	E_{2S}	E_{1D}	E_0	E_0/A	$\sqrt{\langle r^2 \rangle}$
-35.8	-13.8			-34.7	-2.17	2.35

^{40}Ca

Single particle levels [MeV]				Total binding [MeV]		Radius [fm]
E_{1S}	E_{1P}	E_{2S}	E_{1D}	E_0	E_0/A	$\sqrt{\langle r^2 \rangle}$
-59.0	-36.0	-14.7	-14.3	-112.7	-2.82	2.78

T. Inoue etal. [HAL QCD Colla.] PRC 91 (2015) 011001(R)

- Obtained binding energies are smaller than the expr. data

$$^{16}\mathrm{O}:\ E_0^{\mathrm{expr.}} = -127.6\ \mathrm{MeV}, \quad ^{40}\mathrm{Ca}:\ E_0^{\mathrm{expr.}} = -342.0\ \mathrm{MeV}$$

primarily due to the heavy u, d quark in our calculation.

We expect more compatible results from the physical point LQCD.

Infinite nuclear matter from QCD

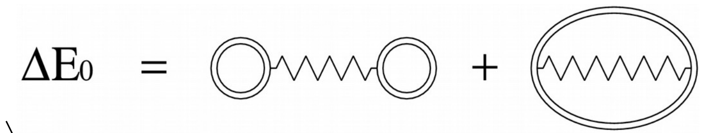
BHF for nuclear matter

K.A. Brueckner and J.L.Gammel
Phys. Rev. 109 (1958) 1023

M.I. Haftel and F. Tabakin,
Nucl. Phys. A158(1970) 1-42

- Ground state energy in BHF framework

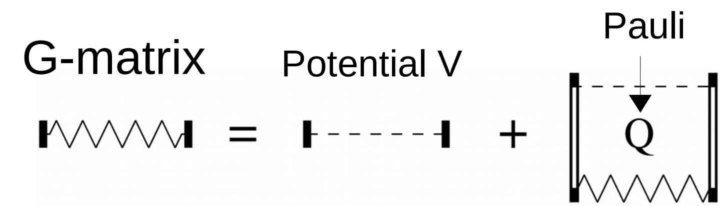
$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \mathrm{Re} \langle\, G_i\big(e(k)+e(k')\big) \,\rangle_A$$



- G-matrix

LQCD V_{NN}

$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$

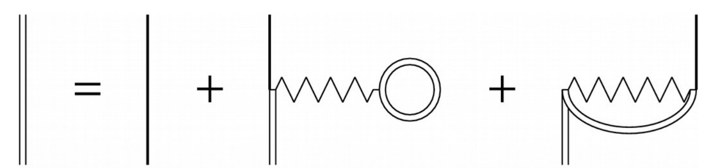


- Single particle spectrum & potential

$$e(k) = \frac{k^2}{2M} + U(k)$$

LQCD M_N

$$U(k) = \sum_{i} \sum_{k' \le k_F} \mathrm{Re} \langle k\, k' | G_i\big(e(k)+e(k')\big) | k\, k' \rangle_A$$

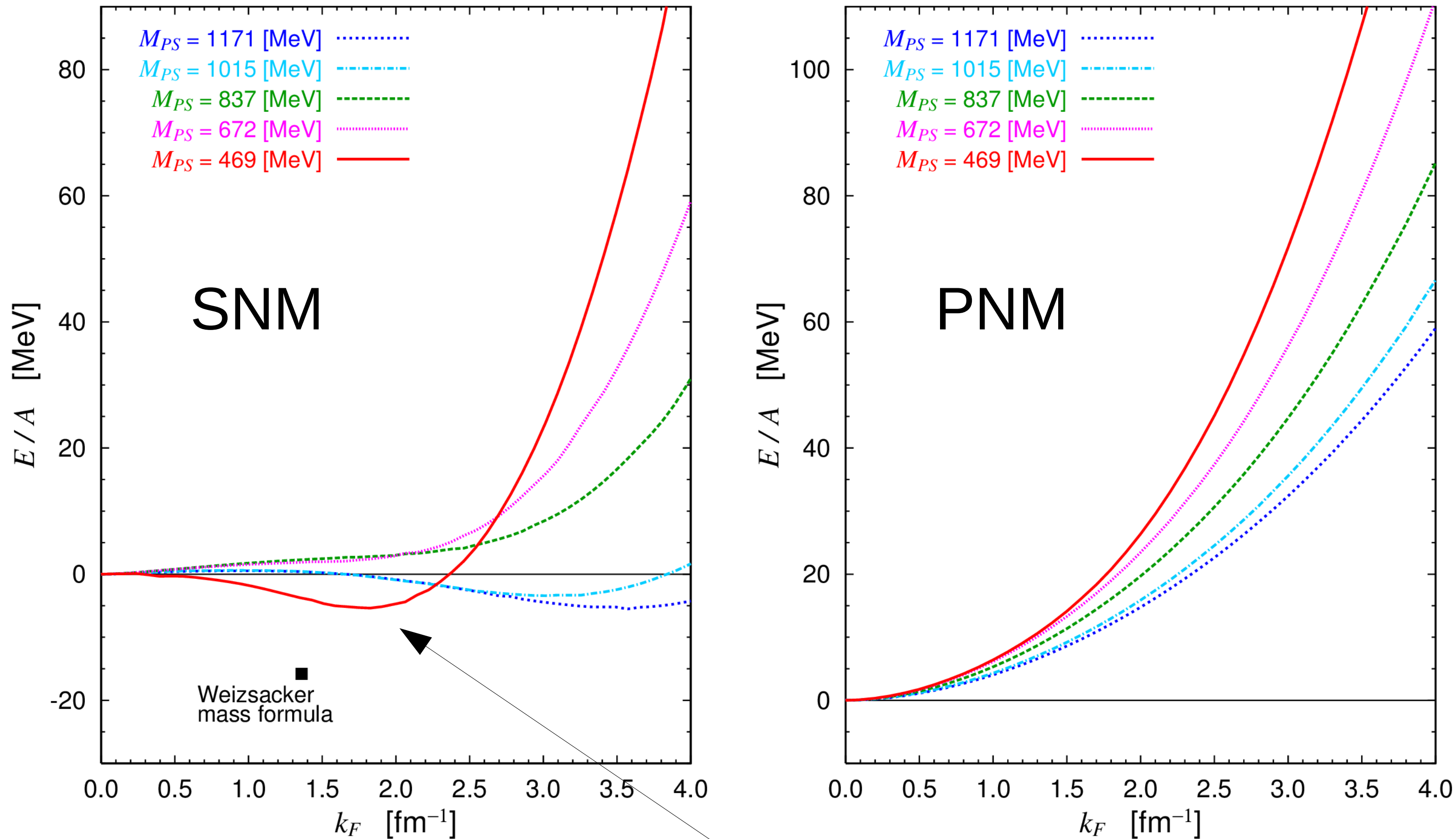


limitation

- p.w. decomposition & truncation $\;{}^{2S+1}L_J = {}^1S_0\,,\ {}^3S_1\,,\ {}^3D_1\,,\ {}^1P_1\,,\ {}^3P_J \cdots$
- Continuous choice w/ parabolic approximation, Angle averaged Q-operator

Matter EoS from QCD

T. Inoue etal. PRL 11, 112503 (2013)



M_N = 2274 MeV
M_N = 2031 MeV
M_N = 1749 MeV
M_N = 1484 MeV
M_N = 1161 MeV

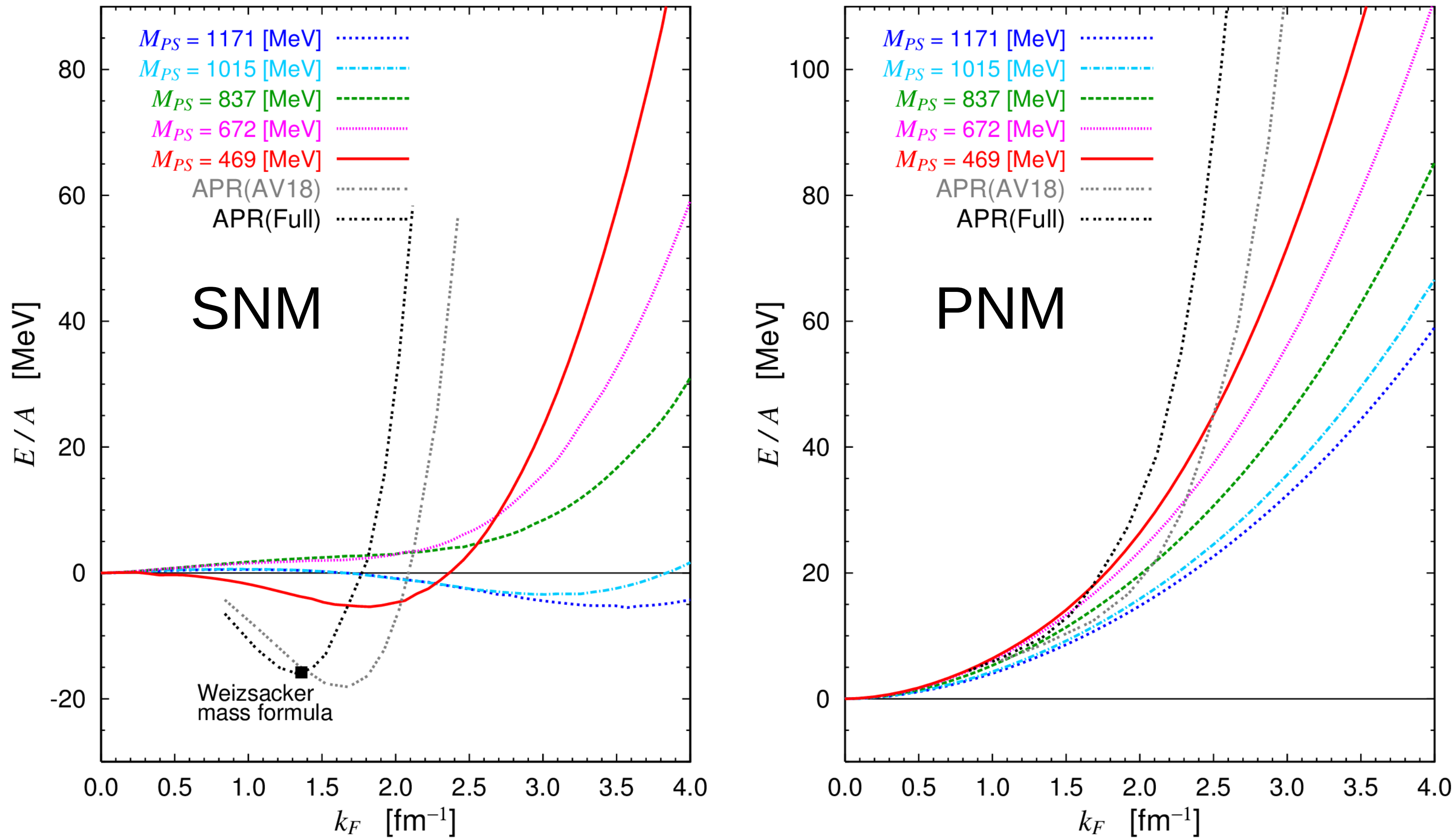
limitation

No NNN force.

- SNM is bound and the saturation occurs at M_{PS} = 469 MeV.
  - Saturation is very delicate against change of quark mass.
- PNM is unbound as normal.
  - PNM become stiff at high density as quark mass decrease.

Matter EoS from QCD

T. Inoue etal. PRL 11, 112503 (2013)



M_N = 2274 MeV
M_N = 2031 MeV
M_N = 1749 MeV
M_N = 1484 MeV
M_N = 1161 MeV

limitation

No NNN force.

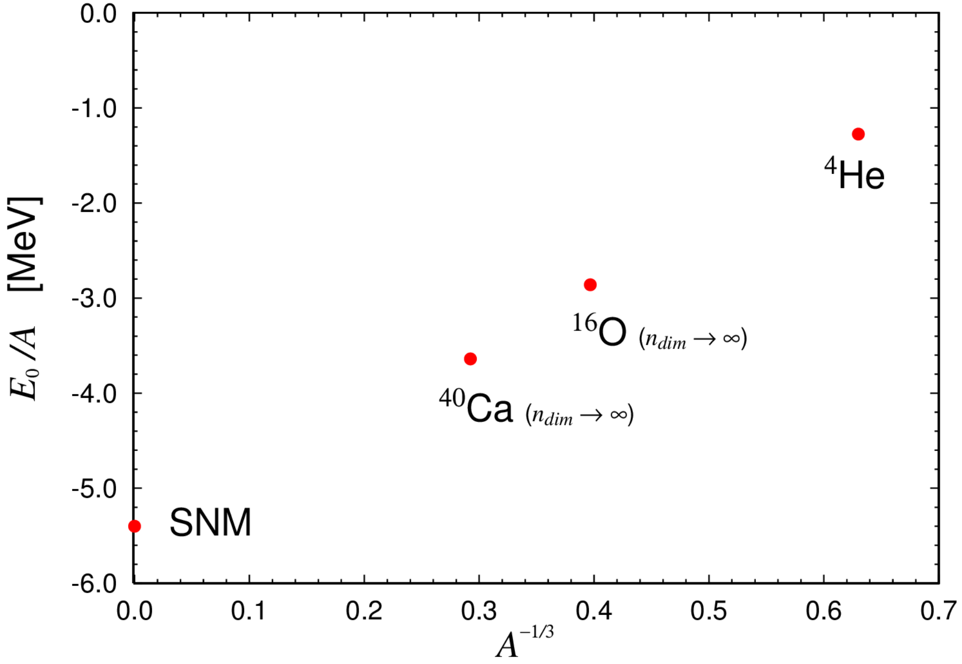
- "APR" curves are EoS of the matter for the physical world, obtained with a variational method and experimental data.
  phenomenological force
- HALQCD EoS largely deviate from the empirical ones, primarily due to the heavy u, d quark in our calculation.

We expect more compatible results from the physical point LQCD.

Summary
and
Outlook

Mass number A dependence

@SU(3)$_F$ limit
Mps = 469 MeV



- E_0 of the nuclei with n_{dim} are extrapolated to $n_{dim} = \infty$ as

$$E_0(A; n_{dim}) = E_0(A; \infty) + c(A)/n_{dim}$$

- E_0/A of SNM was obtained in our BHF calculation

T.I. etal. Phys. Rev. Lett. 111 (2013)

- For the real world, the Bethe-Weizsacker mass formula

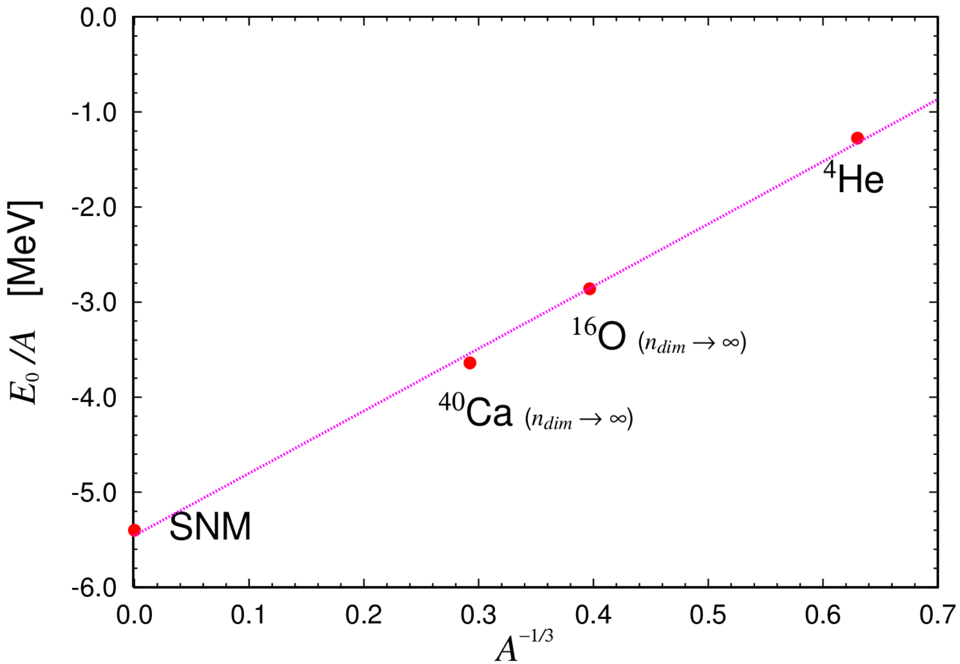
$$E_0(A) = a_V\ A + a_S\ A^{2/3} + \cdots \qquad a_V = -15.7\ [\text{MeV}],\quad a_S = 18.6\ [\text{MeV}]$$

- HALQCD E_0/A at the m_q has a reasonable A dependence which is well described by a BW type mass formula with

$$a_V = -5.46\ [\text{MeV}],\ \ a_S = 6.56\ [\text{MeV}]$$

Mass number A dependence

@SU(3)$_F$ limit
Mps = 469 MeV



- E_0 of the nuclei with n_{dim} are extrapolated to $n_{dim} = \infty$ as
$$E_0(A; n_{dim}) = E_0(A; \infty) + c(A)/n_{dim}$$
- E_0/A of SNM was obtained in our BHF calculation

T.I. etal. Phys. Rev. Lett. 111 (2013)

- For the real world, the Bethe-Weizsacker mass formula
$$E_0(A) = a_V\, A + a_S\, A^{2/3} + \cdots \qquad \begin{aligned} a_V &= -15.7\ [\mathrm{MeV}] \\ a_S &= \quad 18.6\ [\mathrm{MeV}] \end{aligned}$$
- HALQCD E_0/A at the m_q has a reasonable A dependence which is well described by a BW type mass formula with
$$a_V = -5.46\ [\mathrm{MeV}],\ \ a_S = 6.56\ [\mathrm{MeV}]$$

Summary and Outlook

- We've introduced our purpose and strategy.
  - Explain properties of nuclei starting from QCD.
  - Extract NN potentials in lattice QCD numerical simulation.
  - Apply potentials to few-body technique or many-body theory.

Summary and Outlook

- We've introduced our purpose and strategy.
  - Explain properties of nuclei starting from QCD.
  - Extract NN potentials in lattice QCD numerical simulation.
  - Apply potentials to few-body technique or many-body theory.
- Results
  - We confirmed existence of nuclei in QCD at a large m_q. $A \geq 4$ M_{PS} = 469 MeV
  - We deduced mass and structure of ^{4}He, ^{16}O, and ^{40}Ca.
  - We derived a BW type A dependence of mass from QCD.

Summary and Outlook

- We've introduced our purpose and strategy.
  - Explain properties of nuclei starting from QCD.
  - Extract NN potentials in lattice QCD numerical simulation.
  - Apply potentials to few-body technique or many-body theory.

- Results
  - A ≥ 4 M_{PS} = 469 MeV
  - We confirmed existence of nuclei in QCD at a large m_q.
  - We deduced mass and structure of ^{4}He, ^{16}O, and ^{40}Ca.
  - We derived a BW type A dependence of mass from QCD.

- Outlook
  - almost
  - LQCD simulation at the physical point for NN, YN, YY pot.
  - Higher p.w. NN forces and NNN forces form LQCD.
  - I believe, those studies will create new connection between QCD and nuclear physics in near future.

Thank you !!

Approximation for odd parity force

- Approx A: $V_W = V_{WM},\ \ V_M = 0,\ \ V_B = 0,\ \ V_H = 0$ Wigner
  - Ignore V_{BH}. A spin-indep force act on both even and odd parity.
  - In this case, configuration of the 4N ground state is obvious.
  - We can drop spin and iso-spin space and treat nucleon as boson.

- Approx B: $V_W = \frac{V_{WM}}{2},\ \ V_M = \frac{V_{WM}}{2},\ \ V_B = 0,\ \ V_H = 0$ Serber
  - Ignore V_{BH}. For even parity, same as A. No force in odd parity.

- Approx C: $V_W = V_{WM},\ \ V_M = 0,\ \ V_B = V_{BH},\ \ V_H = 0$ "Wigner"
  - Take V_{BH}. A spin-dep force act on both even and odd parity.

- Approx D: $V_W = \frac{V_{WM}}{2},\ \ V_M = \frac{V_{WM}}{2},\ \ V_B = \frac{V_{BH}}{2},\ \ V_H = \frac{V_{BH}}{2}$
  - Take V_{BH}. For even parity, same as C. No force in odd parity. "Serber"
  - This is best for the moment and most honest at least.

NN central potential

Exchange nature of nuclear force $$V_C(r) = V_W(r) + V_M(r)P^r + V_B(r)P^\sigma + V_H(r)P^r P^\sigma$$

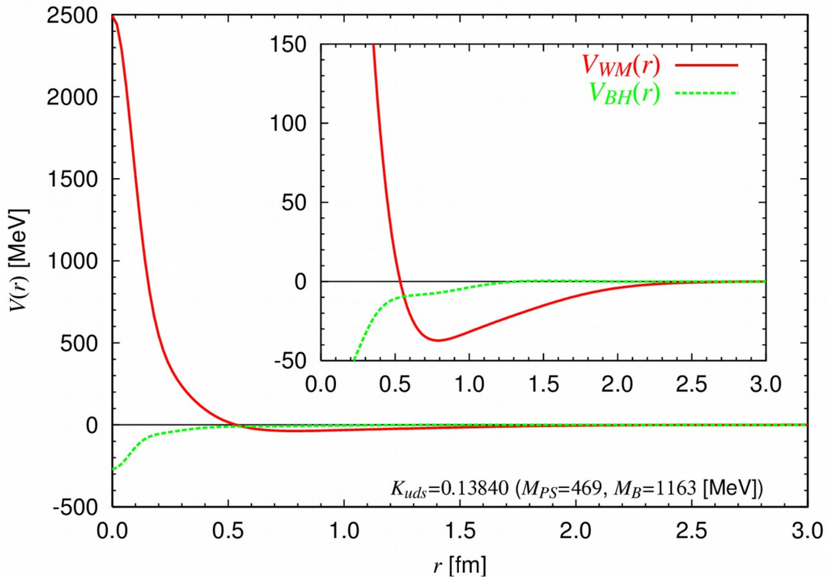
in even parity sector $$= (V_W(r) + V_M(r)) + (V_B(r) + V_H(r))P^\sigma$$

We define $$\equiv V_{WM}(r) + V_{BH}(r)P^\sigma$$

We determine $V_{WM}(r)$ and $V_{BH}(r)$ from data of $V(r)$ in $^1S_0$ and $^3S_1$

$$V_{WM}(r) = \frac{1}{2}\left(V_C(^1S_0;r) + V_C(^3S_1;r)\right)$$

$$V_{BH}(r) = \frac{1}{2}\left(V_C(^3S_1;r) - V_C(^1S_0;r)\right)$$



- We use data at the lightest quark i.e. M_{PS} = 469 MeV K_{uds} =0.13840
- V_{BH} is weak compared to V_{WM} .
- We use the effective central for $V_C(^3S_1)$ so that include V_T partially.