Progress on the muon anomalous magnetic moment from lattice QCD

Tom Blum (UCONN / RBRC)

Chiral Dynamics, Pisa, July 3, 2015

Collaborators

HVP	HLbL
Christopher Aubin (Fordham U)	
Maarten Golterman (SFSU)	Norman Christ (Columbia)
Santiago Peris (Barcelona)	Masashi Hayakawa (Nagoya)
Cheng Tu (UConn)	Taku Izubuchi (BNL/RBRC)
	Luchang Jin (Columbia)
RBC/UKQCD Collaboration	Christoph Lehner (BNL)

Outline I

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) = -\vec{\mu} \cdot \vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin $(c = \hbar = 1)$

$$\vec{\mu} = g \left(\frac{e}{2m}\right) \vec{S}$$

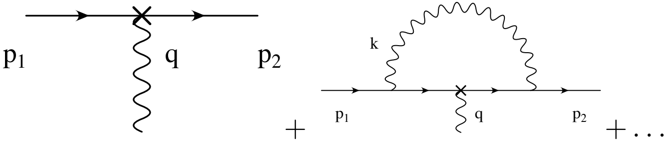
The Landé g-factor is predicted from the free Dirac eq. to be

$$g = 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



$$\gamma^\mu \quad \rightarrow \quad \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + i\,\frac{\gamma^\mu\gamma^\nu\, q^\nu}{2}\,\frac{F_2(q^2)}{2m}\right)$$

which results from Lorentz and gauge invariance when the muon is on-mass-shell.

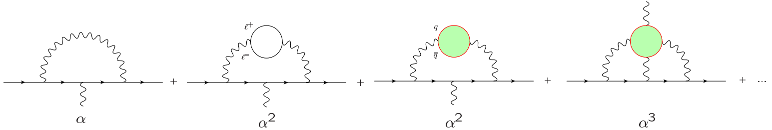
$$F_2(0) \quad = \quad \frac{g-2}{2} \equiv a_\mu \qquad\qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$



Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\dots$

hadronic contributions $\sim 6 \times 10^{-5}$ smaller, dominate theory error.

Experiment - Standard Model Theory = difference

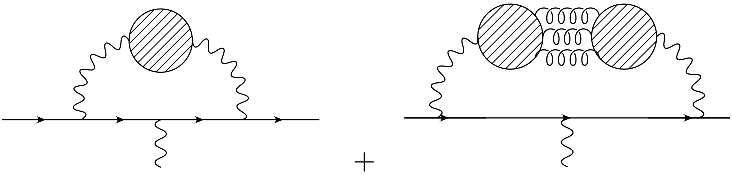
SM Contribution	Value $\pm$ Error $(\times 10^{11})$	Ref
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]
HVP LO	6923 ± 42	[Davier et al., 2011]
	6949 ± 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]
HLbL	105 ± 26	[Prades et al., 2009]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592089 ± 63	[Bennett et al., 2006]
Diff (Exp $-$ SM)	287 ± 80	[Davier et al., 2011]
	261 ± 78	[Hagiwara et al., 2011]
	249 ± 87	[Aoyama et al., 2012]

New experiments+new theory=new physics

- Fermilab E989, begins in early 2017, aims for 0.14 ppm
- J-PARC E34, "late 2010's", aims for 0.1 ppm
- Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\rightarrow\ \sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow\ \sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow\ \sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations important to trust theory errors

Outline I

Hadronic vacuum polarization (HVP)



Using lattice QCD and continuum, ∞-volume QED

[Blum, 2003, Lautrup et al., 1971]

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$f(q^2)$ is known, $\hat{\Pi}(q^2)$ is subtracted HVP, $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

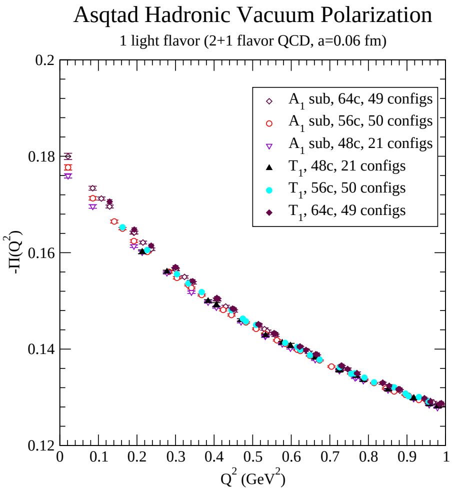
$$\Pi^{\mu\nu}(q) = \int e^{iqx} \langle j^\mu(x) j^\nu(0) \rangle \qquad j^\mu(x) = \sum_i Q_i \bar{\psi}(x)\gamma^\mu\psi(x)$$

$$= \Pi(q^2)(q^\mu q^\nu - q^2\delta^{\mu\nu})$$

Lattice setup (K. Wilson)

- Compute correlation functions (*e.g.* $\langle j^\mu(x) j^\nu(y)\rangle$, $j^\mu = \bar\psi\gamma_\mu\psi$) in Feynman path integral formalism
- 4(5)D hypercubic lattice regularization, non-zero lattice spacing a and finite volume V
- Handle fermion integrals analytically. Propagators inverse of large sparse matrix M, lattice Dirac operator (domain wall, staggered, Wilson, ...)
- Treat path integrals over gauge fields stochastically, using Monte Carlo techniques: generate ensemble of gauge field configurations $\{U\}$ with weight $\det M(U) \exp -S_g$, $\langle\cdots\rangle$ simple average over ensemble
- work entirely in Euclidean space time, analytically continue back to Minkowski at the end (usually trivial)

HVP from lattice QCD calculation



Aubin, Blum, Golterman, and Peris (MILC gauge ensembles)

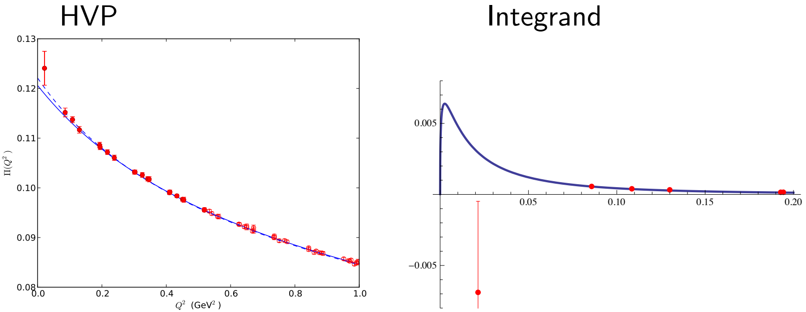
- 2+1f Imp. staggered
- MILC ensembles
- $a = 0.06$ fm, $(3.84\ \text{fm})^3$ box
- $220 \leq m_\pi \leq 315$ MeV
- $m_\pi L \sim 4.3 - 4.5$

Fits

- Need smooth parametrization of lattice HVP
- Integral dominated by low momentum, $m_\mu/2 \lesssim 2\pi/L$
- Fit HVP, plug into integral. Use polynomials [Blum, 2003], VMD [Gockeler et al., 2004], chiral perturbation theory+VMD [Aubin and Blum, 2007]
- Integral sensitive to model dependence because of low Q uncertainties [Aubin and Blum, 2007, Aubin et al., 2012, Golterman et al., 2013]
- VMD does not work [Golterman et al., 2013]
- Use Padé approximants, model independent, based on Stieltjes functions (nice convergence properties) [Aubin et al., 2012].

$$\Pi(Q^2) = \Pi(0) - Q^2\left(a_0 + \sum_{n=1}^{N} \frac{a_n}{b_n + Q^2}\right)$$

Fits circa 2012 [Aubin and Blum, 2007, Aubin et al., 2012]



- 2+1f Imp. staggered (MILC), 220 MeV pion, (3.84 fm)^3
- * [1,1] Padé
- dominated by $q \sim m_\mu/2$ (large box needed for access)
- Fit uncertainty $\leftrightarrow$ large uncertainty in a_μ
- need improved statistical errors and larger box for small q

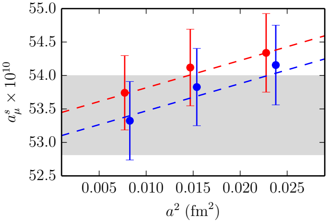
Moments method [Chakraborty et al., 2014] (HPQCD)

- Alternative to fits: compute time moments of two-point correlation function. Coefficients of Taylor exp. about $q^2 = 0$

$$\sum_t \sum_{\vec{x}} t^{2n} \langle j^i(\vec{x}, t) j^i(0) \rangle \quad = \quad (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} \hat{\Pi}(q^2) \Big|_{q^2=0}$$

(Finite difference in FV $\rightarrow$ FVE)

- Use moments to construct Padé approximants for $\hat{\Pi}$,
- Higher moments $\rightarrow$ more statistical noise. OK since Padé's converge rapidly, integral dominated by low Q^2



- all systematics controlled
- $a_\mu^{\text{strange}} = 53.41(59) \times 10^{-10}$
  [Chakraborty et al., 2014] (HPQCD)
- Next, apply to light quark HVP (same difficulty as $q \rightarrow 0$)

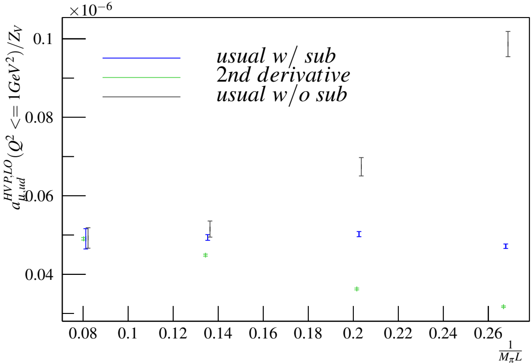
Finite volume HVP [Bernecker and Meyer, 2011]

- Finite volume $\Pi^{\mu\nu}$ transforms under 5 Irreps (1, 1, 2, 3, 3)d: A_1, A_2, E, T_1, T_2 for $L \neq T$
- $\Pi^{\mu\mu}(0) \neq 0$ in FV because Euclidean $O(4)$ symmetry is broken. Terms not constrained by WI, exponentially small
- $\Pi^{\mu\nu}(q)$ is discontinuous at $q = 0$
- $\Pi(q^2)$ depends on irrep
- full $O(4)$ symmetry restored as $L, T \to \infty$

Finite volume effects

- Zero mom subtraction $\Pi_{\nu\nu}(0)$ seen to reduce FV effect

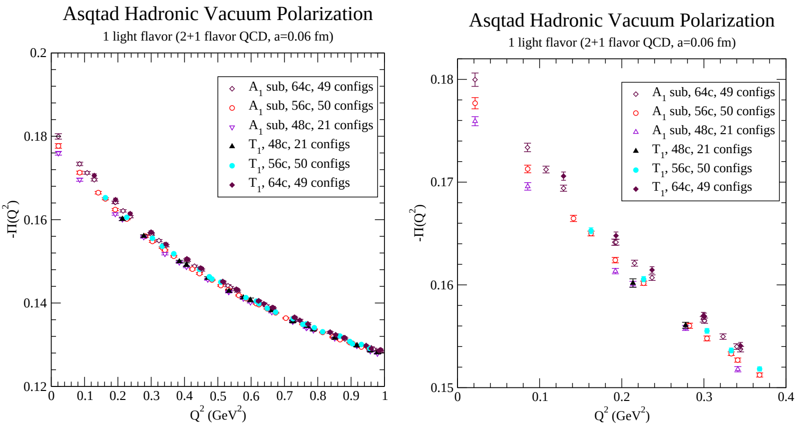
[Malak et al., 2015] (BMWc)



- $2.5 \leq L \leq 8.3$ fm, $5 \leq T \leq 10$ fm,
  $a = 0.104$ fm, $m_\pi = 292$ MeV, $3.7 \leq m_\pi L \leq 12.3$
- 100% error for "small" box, 40% even for $m_\pi L = 4.9$

FV effects: M. Golterman's talk at Lattice 2015 [Aubin et al., 2015]

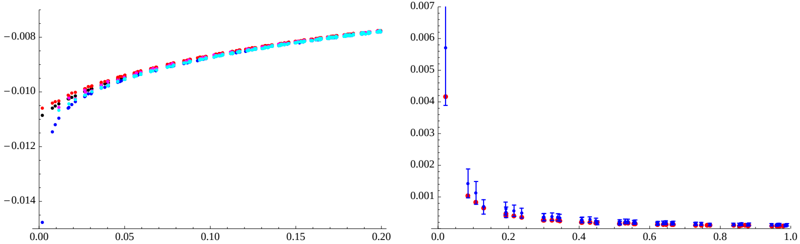
- FVE small, but visible, so fit HVP for each irrep separately



Statistical errors $< 0.4\%$! All mode averaging [Izubuchi et al., 2013]

Finite volume effects [Aubin et al., 2015]

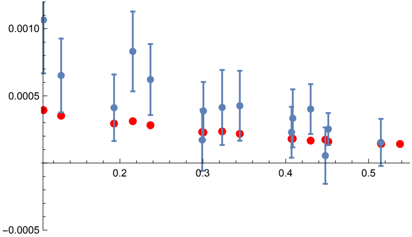
- Use FV SU2 chiral perturbation theory to compute differences between irreps, and same irreps with and without subtraction



- A_1 irrep has lowest Q^2, largest FV effect, $O(\sim 40\,\%)$!, $m_\pi L = 4.2$
- FVE $O(\text{few}\,\%)$ of full HVP, $m_\pi L = 4.2$
- Lattice / χPT show good agreement for *differences*

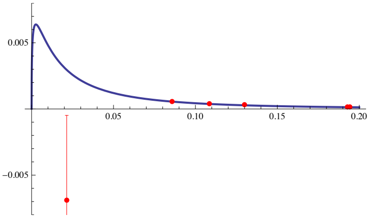
Finite volume effects [Aubin et al., 2015]

Compare lattice and NLO χPT (both in FV)

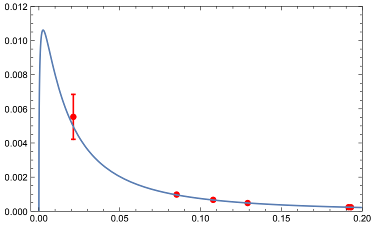


- Difference of A_1 (subtracted) and A_1^{44} irreps
- Differences are $\lesssim 0.5\%$ of total HVP @ $m_\pi L = 4.2$ after zero mom subtraction
- Reasonable assumption: FV effects dominated my pions, negligible for ρ

Fits and the a_μ integrand [Aubin et al., 2012, Aubin et al., 2015]



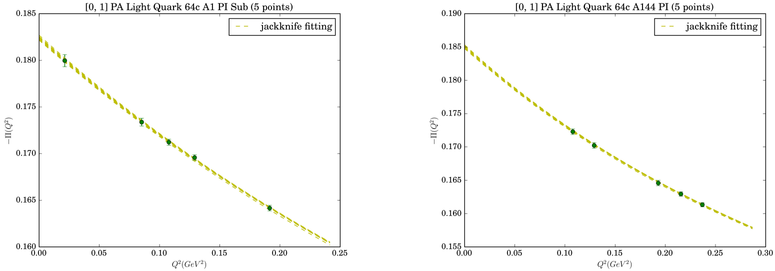
old statistics (2012)



new statistics (AMA)

- dominated by $q \sim m_\mu/2$ (large box needed to access)
- 2+1f Imp. staggered (MILC), 220 MeV pion, $(3.84 \text{ fm})^3$
- A_1 irrep (subtracted)
- better, but still larger box needed

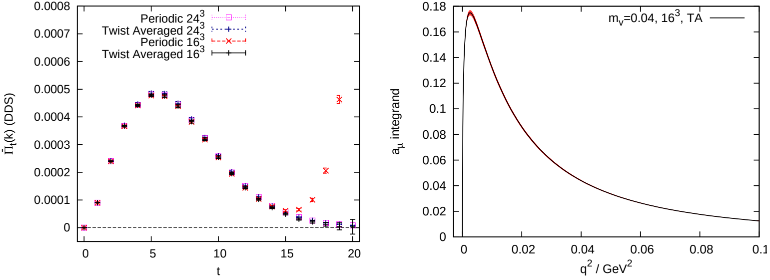
Finite volume errors [Aubin et al., 2015]



- 2+1f Imp. staggered (MILC), 220 MeV pion, $(3.84\text{ fm})^3$
- A_1 irrep (subtracted), $a_\mu = 4.54 \pm 0.25 \times 10^{-8}$
- A_1^{44} irrep, $a_\mu = 5.26 \pm 0.32 \times 10^{-8}$
- Difference $\sim 15\%$
- χPT: irreps straddle ∞ volume result
- FV error $\lesssim 7 - 8\%$ in this case
- Solid understanding of low Q^2 region emerging

Finite volume effects [Lehner and Izubuchi, 2015]

- ∞-volume for valence quarks by average over twisted bc's



- Allows continuous variation of momentum (avoid fit. also sine-cardinal constr: exp. small errors [del Debbio and Portelli, 2015])
- "direct double subtraction" found ind. of [Bernecker and Meyer, 2011]

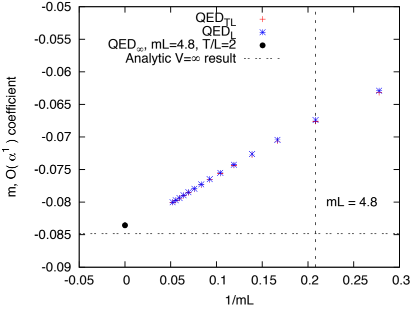
$$\hat{\Pi}(q^2) = \left\langle \sum_t \Re\left(\frac{e^{iqt}-1}{q^2} + \frac{1}{2}t^2\right) \Re C_{\mu\mu}(t) \right\rangle$$

- sub $\Pi^{\mu\mu}(0)$ and $\Pi(0)$ on each config: reduced statistical errors

C. Lehner's talk at Lattice 2015 (Kobe)

Reducing finite volume effects in QCD+QED simulations

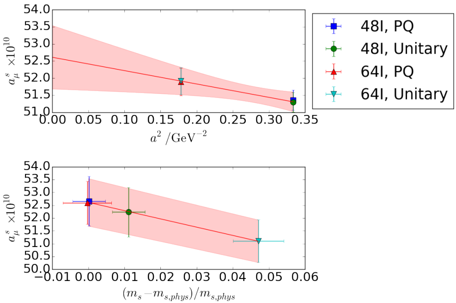
- ∞ volume photon on finite lattice (QED_∞)
- mass correction in simple scalar model



$$
\begin{aligned}
\hat{k}_\mu &= 2\sin k_\mu/2 \\
G(x) &= \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{e^{ikx}}{\hat{k}^2} \\
G_I(k') &= \sum_{x\in V} G(x) e^{-ik'x} \\
k' &= 2\pi n_\mu / L_\mu
\end{aligned}
$$

Strange: Matt Spraggs's talk at Lattice 2015 (Kobe)

- Strange contribution, 2+1 f Möbius DWF, continuum limit



- Physical masses
- $a = 0.114$ and 0.09 fm
- $(5.5 \text{ fm})^3$ boxes

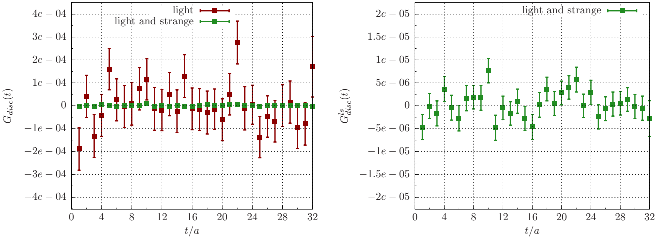
RBC/UKQCD

- results independent of analysis method (fits or moments)
- remarkable agreement with HPQCD 2+1+1 staggered fermion result 53.41 (59) (1% level) [Chakraborty et al., 2014]

Disconnected diagrams

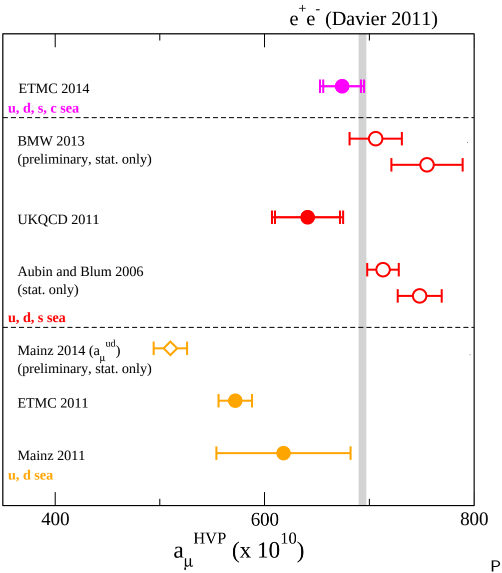
- Zero contribution in the SU3 flavor limit
- 10% of connected in χPT [Della Morte and Juttner, 2010]
- Computed by several groups so far

  [Feng et al., 2011, Gulpers et al., 2014, Burger et al., 2015]
- Compute light-strange to cancel noise (Mainz Group)



Zero within $\sim$ 3% statistical errors for heavier quarks

HVP summary



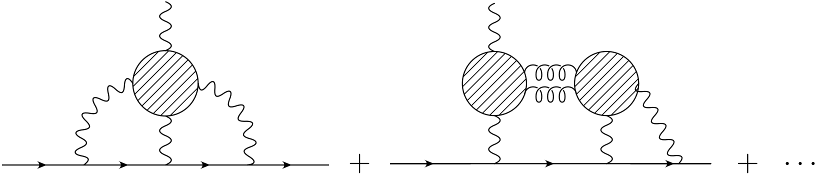
- Systematic errors incomplete, underestimated, or missing
- Connected contribution only
- Some way to go to match precision of dispersive result

S. Eidelman's talk

Plot courtesy of R. Van de Water

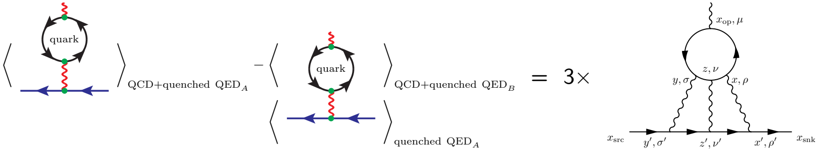
Outline I

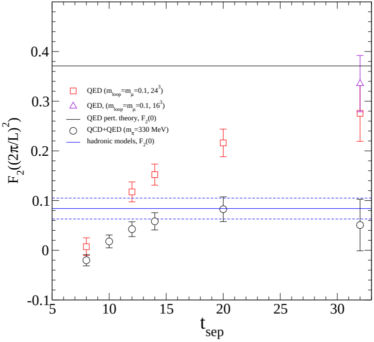
Hadronic light-by-light (HLbL) scattering



- Model calculations: $(105 \pm 26) \times 10^{-11}$
  [Prades et al., 2009, Benayoun et al., 2014]
- Model systematic errors difficult to quantify
- Dispersive approach difficult, but progress is being made
  [Colangelo et al., 2014b, Colangelo et al., 2014a, Pauk and Vanderhaeghen, 2014b, Pauk and Vanderhaeghen, 2014a, Colangelo et al., 2015]
- First non-PT QED+QCD calculation [Blum et al., 2015]
- Very rapid progress with Pert. QED+QCD [Jin et al., 2015]

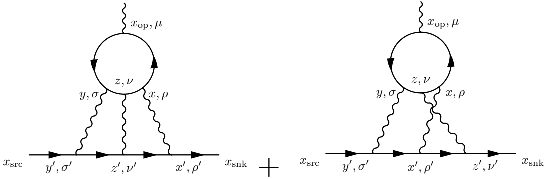
Non-perturbative QED method [Blum et al., 2015]





- quark-connected part of HLbL
- $a^{-1} = 1.7848$ GeV, $(2.7\text{ fm})^3$
- $m_\pi = 330$ MeV, $m_\mu = 190$ MeV
- Consistent with model expectations (J. Bijnens)
- Agreement with models accidental
- $O(\alpha^2)$ noise, $O(\alpha^4)$ corrections

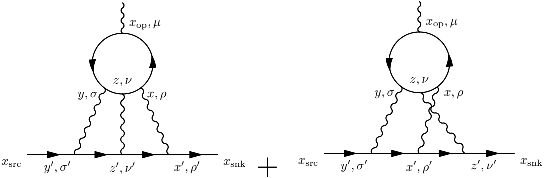
HLbL: Pert. QED, L. Jin's talk, Lattice 2015 [Jin et al., 2015]



+ permutations

- Compute quark loop non-perturbatively
- Photons, muon on lattice, but use (exact) tree-level propagators
- Work in configuration space
- Do QED (two) loop integrals stochastically
- Key insight: quark loop exponentially suppressed with x and y separation. Concentrate on short distance
- Chiral (DW) fermions at finite lattice spacing: UV properties like in continuum, modified by $O(a^2)$

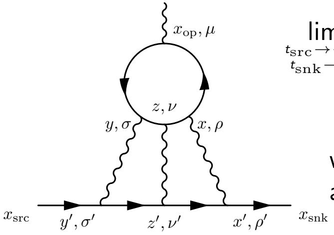
HLbL: Perturbative QED [Jin et al., 2015]



+ permutations

$$
\begin{aligned}
\mathcal{F}_\nu(x, y, z, x_{\rm op}, x_{\rm snk}, x_{\rm src}) \quad = \\
-(-ie)^3 \sum_{q=u,d,s} (ie_q)^4 \left\langle {\rm tr}\left[\gamma_\nu S_q\left(x_{\rm op}, x\right)\gamma_\rho S_q(x,z)\gamma_\kappa S_q(z,y)\gamma_\sigma S_q\left(y, x_{\rm op}\right)\right]\right\rangle_{\rm QCD} \\
\cdot \sum_{x',y',z'} G_{\rho\rho'}(x,x') G_{\sigma\sigma'}(y,y') G_{\kappa\kappa'}(z,z') \\
\cdot \Big[S_\mu\left(x_{\rm snk}, x'\right)\gamma_{\rho'} S_\mu(x',z')\gamma_{\kappa'} S_\mu(z',y')\gamma_{\sigma'} S_\mu\left(y', x_{\rm src}\right) \\
+ S_\mu\left(x_{\rm snk}, z'\right)\gamma_{\kappa'} S_\mu(z',x')\gamma_{\rho'} S_\mu(x',y')\gamma_{\sigma'} S_\mu\left(y', x_{\rm src}\right) \\
+ 4 \text{ other permutations} \Big].
\end{aligned}
$$

HLbL: Perturbative QED, point source method [Jin et al., 2015]



$$\mathcal{F}_\nu(\vec{q}, x, y, z, x_{\rm op}) = \lim_{\substack{t_{\rm src}\to-\infty \\ t_{\rm snk}\to\infty}} e^{E_{q/2}(t_{\rm snk}-t_{\rm src})} \sum_{\vec{x}_{\rm snk}, \vec{x}_{\rm src}} e^{-i\frac{\vec{q}}{2}\cdot(\vec{x}_{\rm snk}+\vec{x}_{\rm src})} e^{i\vec{q}\cdot\vec{x}_{\rm op}} \mathcal{F}_\nu(x, y, z, x_{\rm op}, x_{\rm snk}, x_{\rm src})$$

with momentum transfer $\vec{q} = 2\pi\vec{z}/L$ and translational invariance

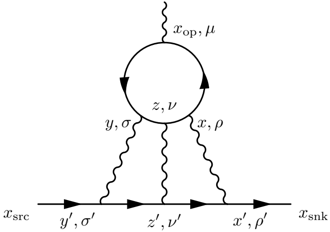
$$\mathcal{M}_\nu(\vec{q}) = \sum_{x,y,z} \mathcal{F}_\nu(\vec{q}, \frac{x-y}{2}, -\frac{x-y}{2}, z-\overline{x}, x_{\rm op}-\overline{x})$$

$$= \sum_r \left\{ \sum_{z', x'_{\rm op}} \mathcal{F}_\nu(\vec{q}, \frac{r}{2}, -\frac{r}{2}, z', x'_{\rm op}) \right\}$$

$$\overline{x} = \frac{x+y}{2}, \quad r = \frac{x-y}{2}, \quad z' = z-\overline{x} \quad \text{and} \quad x'_{\rm op} = x_{\rm op}-\overline{x}$$

Sum over r and $\overline{x}$ stochastically, $x'_{\rm op}$ and z' exact

HLbL: Perturbative QED, point source method [Jin et al., 2015]



$$G(x,x')_{\rho\rho'} = \sum_k \frac{1}{(2\sin k/2)^2} e^{ik(x-x')}$$

- QED_L [Hayakawa and Uno, 2008]
- Muon propagators FV (analytic), tree-level DWF with $L_s = \infty$
- Compute 2 point source props in QCD at x, y, connect sink points at $x'_{\rm op}$ and z', do the latter sums exactly
- $t_{\rm src}$, $t_{\rm snk}$ chosen for each $\overline{x} \pm T/2$
- Do sums over r, $\overline{x}$ (x, y) stochastically, average over QCD configurations then yields $\mathcal{M}_\nu(\vec{q})$

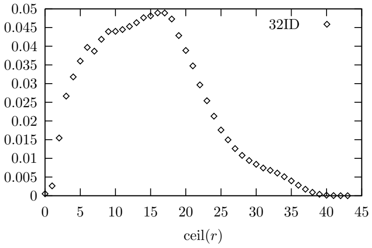
HLbL: Perturbative QED, point source method [Jin et al., 2015]

- Use importance sampling to do sum over r efficiently (sample $|r| \lesssim 1$ fm most frequently)

$$p(|x_i - \overline{x}|) \quad \propto \quad \begin{cases} 1 & (|x_i - \overline{x}| < R) \\ 1/|x_i - \overline{x}|^{3.5} & (|x_i - \overline{x}| \geqslant R) \end{cases} ,$$

The distribution of the relative distance $|r|$ between any two points drawn from this set is:

$$P(r) \quad = \quad \sum_x p(|x - r|)p(|x|).$$



- 2+1f DWF+I-DSDR ensemble RBC/UKQCD
- 171 MeV pion mass
- $R = 4$

HLbL: point source method results [Jin et al., 2015]

Label	size	$m_\pi L$	m_π/GeV	#qcdtraj	t_{sep}	$\frac{F_2 \pm \mathrm{Err}}{(\alpha/\pi)^3}$	$\frac{\mathrm{Cost}}{\mathrm{BG/Q\ rack\ days}}$
16I	$16^3 \times 32$	3.87	0.423	16	16	0.1235 ± 0.0026	0.63
24I	$24^3 \times 64$	5.81	0.423	17	32	0.2186 ± 0.0083	3.0
24IL	$24^3 \times 64$	4.57	0.333	18	32	0.1570 ± 0.0069	3.2
32ID	$32^3 \times 64$	4.00	0.171	47	32	0.0693 ± 0.0218	10

Table 2. Central values and errors. $a^{-1} = 1.747\mathrm{GeV}$ except for 32ID where $a^{-1} = 1.371\mathrm{GeV}$. Muon mass and pion mass ratio is fixed at physical value. For comparison, at physical point, model estimation is 0.08 ± 0.02.

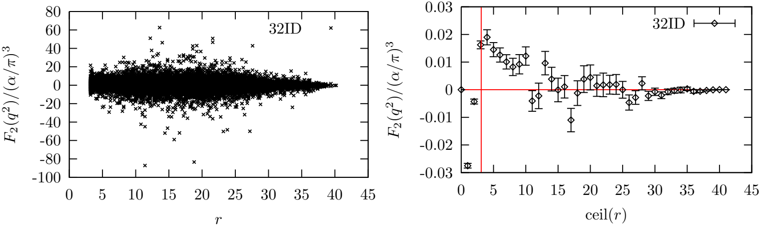


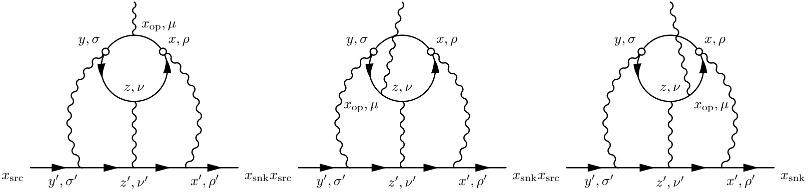
Figure 13. $32^3 \times 64$ lattice, with $a^{-1} = 1.371\mathrm{GeV}$, $m_\pi = 171\mathrm{MeV}$, $m_\mu = 134\mathrm{MeV}$.

HLbL: Current conservation [Jin et al., 2015]

To ensure small statistical errors as $q \to 0$, Ward Identity (conserved current) must be exact on each configuration

$$\partial_\mu \langle j^\mu(x_{\rm op})\bar\psi(x)\gamma^\rho\psi(x)\cdots\rangle = i\delta(x_{\rm op}-x)\langle\bar\psi(x)\gamma_\nu\psi(x)\cdots\rangle - i\delta(x_{\rm op}-x)\langle\bar\psi(x)\gamma_\nu\psi(x)\cdots\rangle + \cdots$$

$$\langle j^\mu(x_{\rm op})\bar\psi(x)\gamma^\rho\psi(x)\bar\psi(z)\gamma^\nu\psi(z)\bar\psi(y)\gamma^\sigma\psi(y)\rangle =$$



- after doing Wick contractions. Compute all 3 diagrams WI exact (to numerical precision) on each configuration
- signal *and* error both vanish as $q \to 0$. Error on $F_2(q^2) \sim$ constant

HLbL: Moment method for $F_2(0)$ in FV [Jin et al., 2015]

- Can do calculation directly at zero momentum for large L

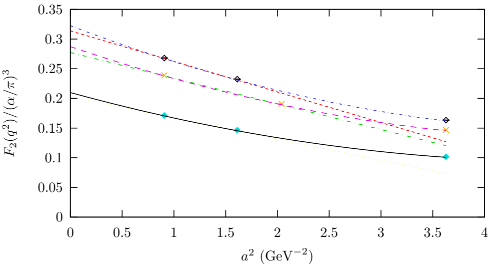
$$\bar{u}(p')\left[i\frac{F_2(q^2)}{4m}[\gamma_\mu,\gamma_\nu]q_\nu\right]u(p) = \sum_{x_{op}} \exp\left(iq\cdot x_{op}\right)\mathcal{M}'_\mu(q,x_{op})$$
$$\approx \sum_{x_{op}} \left(1+iq\cdot x_{op}\right)\mathcal{M}'_\mu(q,x_{op})$$
$$\approx \sum_{x_{op}} iq\cdot x_{op}\mathcal{M}'_\mu(q,x_{op})$$

- The "1" term vanishes in ∞ volume, exponentially small in FV

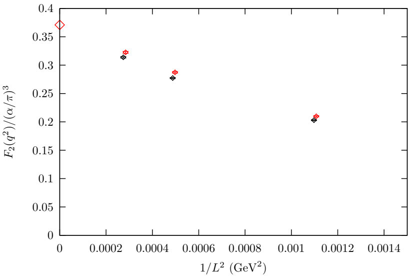
$$\bar{u}(p'=0)\left[i\frac{F_2(q^2)}{4m}[\gamma_\mu,\gamma_\nu]q_\nu\right]u(p=0) = \sum_{x_{op}} iq\cdot x_{op}\mathcal{M}'(q=0,x_{op})$$

- Can use local (not conserved) current for all four currents since $x_{op}=0$ kills contact terms

Continuum and ∞ volume limits in QED [Jin et al., 2015]



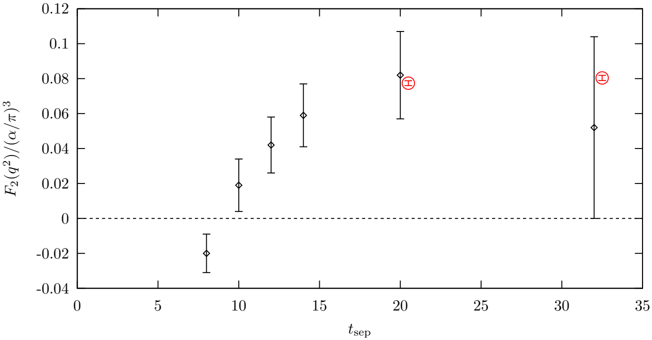
- QED continuum limit
- $a \equiv m_\mu^{\text{lat}} / m_\mu^{\text{phys}}$
- $q = 0$



- QED ∞ volume limit
- $q = 0$

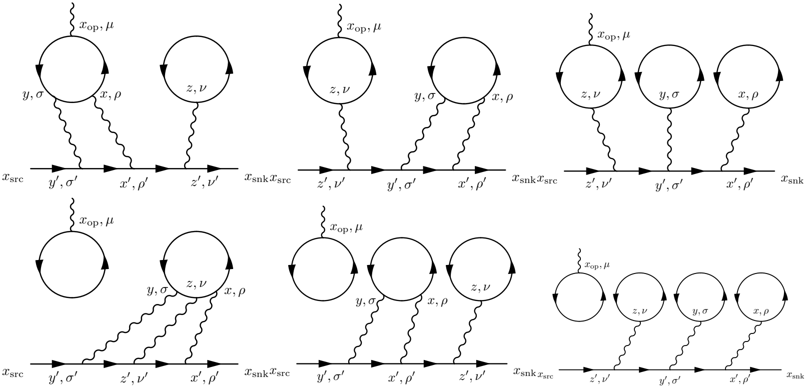
Dramatic improvement [Jin et al., 2015]

- Including all improvements, statistical errors reduced by 10×



- quark-connected part of HLbL
- $a^{-1} = 1.7848$ GeV, $(2.7 \text{ fm})^3$
- $m_\pi = 330$ MeV, $m_\mu = 190$ MeV
- $q = 2\pi/L$

M. Hayakawa's talk at Lattice 2015 [Jin et al., 2015]



- Use dynamical QED+QCD or only valence quarks
- Requires explicit HVP subtraction when any quark loop with two photons is not connected to others by gluons

Outline I

Next calculation

- Applying improved point source method to physical light quark mass 2+1f Möbius DWF ensemble (RBC/UKQCD, ANL ALCF)
- $(5.5\ \mathrm{fm})^3$ QCD box, $a = 0.114$ fm ($a^{-1} = 1.7848$ GeV)
- Different size boxes for QCD and QED
- Parasitic studies: HVP, mass splittings, ...

Summary/Outlook

- HVP
  - Very high statistical precision required
  - Progress in understanding systematics, FV, fits, moments, ...
  - Strange contribution done very well ✓
  - physical quark mass, large volume calcs in progress
  - Disconnected challenging, maybe small
- HLBL
  - First calculations for connected part very promising– calculation within reach of lattice methods
  - FV effects large but controllable. $1/L^2$ dependence in QED, ∞ volume limit consistent with PT. Put QCD and QED in different boxes
  - Applying improved point source method to physical quark mass 2+1f Möbius DWF ensemble RBC/UKQCD
  - Disconnected part challenging, new ideas under investigation
  - Lattice important to compare (SM) with experiment

Acknowledgments

- This research is supported in part by the US DOE
- Computational resources provided by the RIKEN BNL Research Center, RIKEN, and USQCD Collaboration
- Lattice computations done on
  - Ds cluster at FNAL (USQCD)
  - USQCD BQ/Q at BNL

Outline I

Aoyama, T., Hayakawa, M., Kinoshita, T., and Nio, M. (2012).
Complete Tenth-Order QED Contribution to the Muon g-2.
Phys.Rev.Lett., 109:111808.

Aubin, C. and Blum, T. (2007).
Calculating the hadronic vacuum polarization and leading hadronic contribution to the muon anomalous magnetic moment with improved staggered quarks.
Phys.Rev., D75:114502.

Aubin, C., Blum, T., Golterman, M., and Peris, S. (2012).
Model-independent parametrization of the hadronic vacuum polarization and g-2 for the muon on the lattice.
Phys.Rev., D86:054509.

Aubin, C., Blum, T., Golterman, M., Peris, S., and Tu, C. (2015).
Finite volume effects in the hadronic vacuum polarization and the muon g-2.
in preparation.

Benayoun, M., Bijnens, J., Blum, T., Caprini, I., Colangelo, G., et al. (2014).

Hadronic contributions to the muon anomalous magnetic moment Workshop. $(g-2)_\mu$: Quo vadis? Workshop. Mini proceedings.

Bennett, G. et al. (2006).

Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL.

Phys.Rev., D73:072003.

Bernecker, D. and Meyer, H. B. (2011).

Vector Correlators in Lattice QCD: Methods and applications.

Eur.Phys.J., A47:148.

Blum, T. (2003).

Lattice calculation of the lowest order hadronic contribution to the muon anomalous magnetic moment.

Phys.Rev.Lett., 91:052001.

Blum, T., Chowdhury, S., Hayakawa, M., and Izubuchi, T. (2015).

Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD.

Phys.Rev.Lett., 114(1):012001.

Burger, F., Hotzel, G., Jansen, K., and Petschlies, M. (2015).

Leading-order hadronic contributions to the electron and tau anomalous magnetic moments.

Chakraborty, B. et al. (2014).

Strange and charm quark contributions to the anomalous magnetic moment of the muon.

Phys.Rev., D89(11):114501.

Colangelo, G., Hoferichter, M., Kubis, B., Procura, M., and Stoffer, P. (2014a).

Towards a data-driven analysis of hadronic light-by-light scattering.

Phys.Lett., B738:6–12.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2014b).

Dispersive approach to hadronic light-by-light scattering.
JHEP, 1409:091.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2015).
Dispersion relation for hadronic light-by-light scattering: theoretical foundations.

Davier, M., Hoecker, A., Malaescu, B., and Zhang, Z. (2011).
Reevaluation of the Hadronic Contributions to the Muon g-2 and to alpha(MZ).
Eur.Phys.J., C71:1515.

del Debbio, L. and Portelli, A. (2015).
in preparation.

Della Morte, M. and Juttner, A. (2010).
Quark disconnected diagrams in chiral perturbation theory.
JHEP, 1011:154.

Feng, X., Jansen, K., Petschlies, M., and Renner, D. B. (2011).

Two-flavor QCD correction to lepton magnetic moments at leading-order in the electromagnetic coupling.

Phys.Rev.Lett., 107:081802.

Gnendiger, C., Stckinger, D., and Stckinger-Kim, H. (2013).

The electroweak contributions to $(g-2)_\mu$ after the Higgs boson mass measurement.

Phys.Rev., D88:053005.

Gockeler, M. et al. (2004).

Vacuum polarization and hadronic contribution to muon g-2 from lattice QCD.

Nucl.Phys., B688:135–164.

Golterman, M., Maltman, K., and Peris, S. (2013).

Tests of hadronic vacuum polarization fits for the muon anomalous magnetic moment.

Phys.Rev., D88(11):114508.

Gulpers, V., Francis, A., Jger, B., Meyer, H., von Hippel, G., et al. (2014).
The leading disconnected contribution to the anomalous magnetic moment of the muon.
PoS, LATTICE2014:128.

Hagiwara, K., Liao, R., Martin, A. D., Nomura, D., and Teubner, T. (2011).
$(g-2)_\mu$ and alpha(M_Z^2) re-evaluated using new precise data.
J.Phys., G38:085003.

Hayakawa, M. and Uno, S. (2008).
QED in finite volume and finite size scaling effect on electromagnetic properties of hadrons.
Prog.Theor.Phys., 120:413–441.

Izubuchi, T., Blum, T., and Shintani, E. (2013).
New class of variance-reduction techniques using lattice symmetries.
Phys.Rev., D88(9):094503.

Jin, L., Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., and Lehner, C. (2015).

Lattice Calculation of Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment.

in preparation.

Kurz, A., Liu, T., Marquard, P., and Steinhauser, M. (2014).

Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order.

Phys.Lett., B734:144–147.

Lautrup, B., Peterman, A., and De Rafael, E. (1971).

On sixth-order radiative corrections to a(mu)-a(e).

Nuovo Cim., A1:238–242.

Lehner, C. and Izubuchi, T. (2015).

Towards the large volume limit - A method for lattice QCD + QED simulations.

PoS, LATTICE2014:164.

Malak, R. et al. (2015).
Finite-volume corrections to the leading-order hadronic contribution to $g_\mu - 2$.
PoS, LATTICE2014:161.

Pauk, V. and Vanderhaeghen, M. (2014a).
Anomalous magnetic moment of the muon in a dispersive approach.
Phys.Rev., D90(11):113012.

Pauk, V. and Vanderhaeghen, M. (2014b).
Two-loop massive scalar three-point function in a dispersive approach.

Prades, J., de Rafael, E., and Vainshtein, A. (2009).
Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment.