$B_{\ell 4}$ decay and the extraction of $|V_{ub}|$

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Outline

$B_{\ell 4}$ decay: Motivation

In the Standard Model, CP violation in the quark sector is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The CKM matrix element $|V_{ub}|$ is not well determined yet.

Measurements of $|V_{ub}|$ are available from both *inclusive* charmless semileptonic B decay and *exclusive* $B_{\ell 3}$ decays $B \to \pi(\rho)\ell\bar{\nu}_\ell$.

However, the results from these two sides *do not match well within uncertainties* [PDG(2014)]

$$\begin{aligned} |V_{ub}| &= (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3} \quad \text{inclusive} \\ |V_{ub}| &= (3.28 \pm 0.29) \times 10^{-3} \quad \text{exclusive} \end{aligned}$$

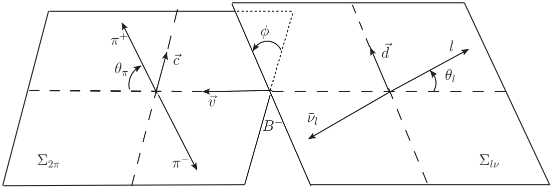
Our strategy

Proposal: investigate the four-body semileptonic B decay mode $B \to \pi\pi\ell\,\bar{\nu}_\ell$, more specifically, taking $B^- \to \pi^+\pi^- e^-\,\bar{\nu}_e$ as example.

Improvement: completely include resonance contributions as well as non-resonant pieces.

Strategy: analyze the hadronic transition form factors in dispersion theory. The resulting free parameters (subtraction constants) and especially the normalization are obtained by matching the dispersive amplitudes to heavy meson chiral perturbation theory.

Kinematics for $B_{\ell 4}$ decay



- five variables
  $s = M_{\pi\pi}^2 = (p_+ + p_-)^2,\ s_\ell = (p_\ell + p_\nu)^2,\ \theta_\pi,\ \theta_\ell,\ \phi$
- form factors F_1, F_2, F_3 (functions of s, s_ℓ, θ_π) for $B^- \to \pi^+\pi^- e^- \bar{\nu}_e$
- partial wave expansion
  $F_1 = \sum_\ell P_\ell(\cos\theta_\pi) f_\ell, \quad F_2 = \sum_\ell P'_\ell(\cos\theta_\pi) g_\ell, \quad F_3 = \sum_\ell P'_\ell(\cos\theta_\pi) h_\ell;$
  $\ell = 0, 1$ up to P-wave, $P_\ell(x)$: Legendre polynomial, "prime": derivative
- differential decay width
  $\frac{d\Gamma}{ds\, ds_l} = |V_{ub}|^2 \left(|f_0(s)|^2,\ |f_1(s)|^2,\ |g_1(s)|^2,\ |h_1(s)|^2 \right)$
  [see also P. Stoffer's talk for $K_{\ell 4}$]

tree-level calculation

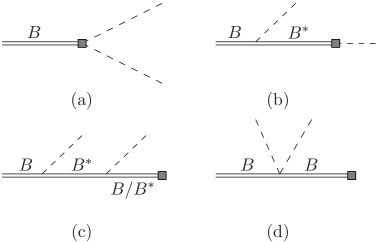
M. B. Wise, PRD(1992)

interaction Lagrangian:

$$\mathcal{L} = -i\,\mathrm{Tr}\bar{H}_a v_\mu \partial^\mu H_a + \frac{1}{2}\mathrm{Tr}\bar{H}_a H_b v^\mu \left(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger\right)_{ba} + \frac{ig}{2}\mathrm{Tr}\bar{H}_a H_b \gamma_\nu \gamma_5 \left(u^\dagger \partial^\nu u - u\partial^\nu u^\dagger\right)_{ba}.$$

Left-handed current for weak interaction:

$$L_{\nu a} = i\sqrt{m_B} f_B (P^*_{b\nu} - v_\nu P_b) u^\dagger_{ba}.$$



- ▷ Identifying the contributions to the individual form factors
- ▷ singling out the B^* pole contributions

Final-state interaction

Muskhelishvili-Omnès equation: a non-perturbative summation

- traditional Omnès problem: [Omnès, Nuovo Cim(1958)]

$$\begin{aligned}
\mathrm{Im} f_\ell(s) &= f_\ell(s) e^{-i\delta_\ell^I(s)} \sin\delta_\ell^I(s) \\
\Longrightarrow f_\ell(s) &= P_n(s)\Omega_\ell^I(s), \\
\Omega_\ell^I(s) &= \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\delta_\ell^I(s')}{s'(s'-s-i\epsilon)}ds'\right).
\end{aligned}$$

$\longrightarrow$ Omnès function

- inhomogeneity relations: [Anisovich and Leutwyler, PLB(1996)]

will be simply called by modified Omnès problem below.

$$\mathrm{Im} M_\ell(s) = \left(M_\ell(s) + \hat{M}_\ell(s)\right) e^{-i\delta_\ell^I(s)} \sin\delta_\ell^I(s)$$

Solution to modified Omnès problem

The solution is given by:

$$M_\ell(s,s_\ell) = \Omega_\ell^I(s)\left\{P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\hat{M}_\ell(s',s_\ell)\sin\delta_\ell^I(s')ds'}{|\Omega_\ell^I(s')|(s'-s-i\epsilon)s'^n}\right\}.$$

- an integral equation [see P. Stoffer's talk]; in our case $\hat{M}_\ell$ is the projection of the πB interaction, and here will be approximated by B^* pole terms.
- the power n is chosen such that the integration converges
- $P_{n-1}(s)$ is a subtraction polynomial of degree $n-1$, needs to be fixed, either by experimental information or by another theoretical model
- phase shifts are known and needed as input

Results: expressions

for a fixed s_ℓ

$$F_1 \sim F_1^{\text{pole}} + M_0(s) + \cos\theta_\pi M_1(s)$$

$$\text{Im}\, f_0(s) = \text{Im}\, M_0(s) \quad \text{[via partial wave expansion]}$$

$$f_0(s) = M_0(s) + \hat{M}_0(s) \quad [\hat{M}_0 \text{ real function}]$$

$$\hat{M}_0 = S\text{-wave projection of pole terms} \longrightarrow \text{diagrams (b) and (c)}$$

$$M_0 = \Omega_0(s)\left\{a_0 + a_1 s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\hat{M}_0(s')\sin\delta_0(s')ds'}{|\Omega_0(s')|(s'-s-i\epsilon)s'^2}\right\}$$

from now on, s_ℓ dependence will be suppressed

Inputs

- $\pi\pi$ scattering phase shifts are known up to $\sqrt{s_0} = 1.4\,\text{GeV}$.

  [Ananthanarayan et al., Phys. Rept(2001)]

  [García-Martín et al., PRD(2011)]

- above $\sqrt{s_0} = 1.4\,\text{GeV}$: a continuation

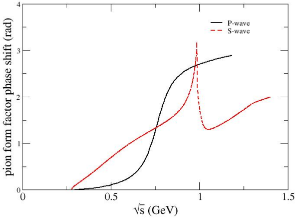
  [Moussallam, EPJC(2000)]

$$\delta_\ell(s \geq s_0) = \pi + (\delta_\ell(s_0) - \pi) f\left(\frac{s}{s_0}\right), \quad f(x) = \frac{2}{1+x^{3/2}}.$$

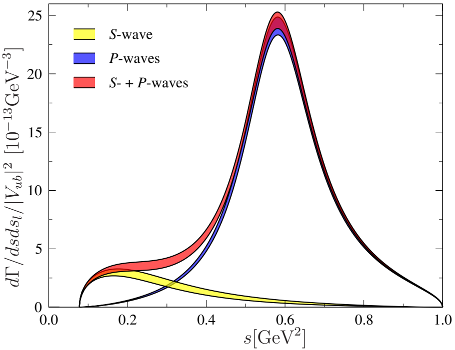
- fix the subtraction constants

  1) taking care of the high-energy behavior

  2) match to the non-pole contribution at leading order



$\pi\pi$ mass spectrum



at $s_\ell = (m_B - 1\,\mathrm{GeV})^2$

▷ in contrast to other approaches, we have a well-defined S-wave "background" to "rho-dominance".

▷ shape fixed by dispersion theory; normalization fixed by HMChPT

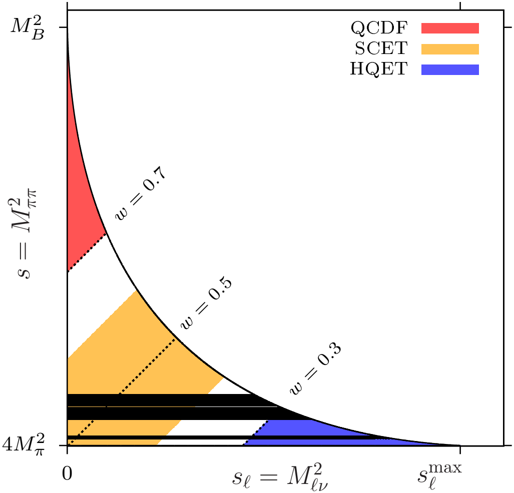
Summary and outlook

- $\pi\pi$ final state interaction is taken into account rigorously in dispersion theory; *S*- and *P*-wave can be treated in the equal footing, no need to refer to a particular resonance $\rho(770)$ or $f_0(500)$.
  [$K\pi$ sector, see Meißner and Wang, JHEP(2014)]
- left-hand structure in πB interaction is approximated as B^* pole.
- a data-driven analysis: the resulting subtraction constants should be finally fitted to experimental data combined with HMChPT to fix the normalization.
- with data $|V_{ub}|$ can be extracted, as a comparison and also supplement with exclusive as well as the inclusive mode.
- given the fact of the dominant pole terms, higher-order contribution is expected to be significant; needs more studies on theoretical uncertainty
- *formalism applicable only in small corner of phase space*

s_ℓ dependence

[Faller et al., PRD(2014)]

current work: blue region



CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \qquad (1)$$

- The down-type mass eigenstates (d, s, b) are transformed into weak eigenstates (d', s', b') by the unitarity CKM matrix.
- The CKM matrix contains all the flavor-changing and *CP*-violating couplings of the Standard Model.
- The phase δ in the standard parametrization is necessary for *CP* violation.