Combined analysis of $\tau \to K_S \pi \nu_\tau$ and $K \eta \nu_\tau$ decays

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CD2015

8th International Workshop on Chiral Dynamics

June 30, 2015

Polo Fibonacci, Pisa (Italy)

Purpose 1: to present a model for the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from K_{l3} decays suited to describe both $\tau \to K\pi\nu_\tau$ and K_{l3} decays simultaneously

Why? because a good knowledge of the $K\pi$ f.f.'s is of fundamental importance for the determination of V_{us} from K_{l3} decays and m_s

Purpose 2: to present a combined analysis of the $\tau \to K_S\pi\nu_\tau$ and $K\eta\nu_\tau$ decays

Why? to further constrain the properties of the $K^*(1410)$ vector resonance

Outline:

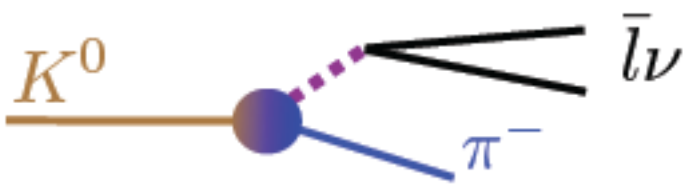
- *Introduction*
- $K\pi$ *form factors*
- *Fit to* $\tau \to K\pi\nu_\tau$
- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* K_{l3}
- *Fit to* $\tau \to K\eta\nu_\tau$
- *Combined analysis of* $\tau \to K_S\pi\nu_\tau$ *and* $K\eta\nu_\tau$ *decays*
- *Summary and Conclusions*

in collab. with D. R. Boito, S. González Solís, M. Jamin and P. Roig,
EPJC 59 (2009) 821, JHEP 09 (2010) 031, JHEP 10 (2013) 039 and JHEP 09 (2014) 042

- *Introduction*

- K_{l3} decays are the main route towards the determination of $|V_{us}|^2$

  H. Leutwyler and M. Roos, ZPC 25 (1984) 91



$$\Gamma_{K_{l3}} \propto |V_{us}|^2 |F_+(0)|^2 I_{K_{l3}}$$

with

$$I_{K_{l3}} = \frac{1}{m_K^8} \int dt\, (\text{p.s.}) \left[\tilde{F}_+(t)^2 + \eta(t, m_l) \tilde{F}_0(t)^2\right]$$

and $\tilde{F}_{+,0}(q^2) \equiv \dfrac{F_{+,0}(q^2)}{F_+(0)}$

- $F_{+,0}(0)$ the normalization from ChPT, Lattice

- $\tilde{F}_{+,0}(q^2)$ the energy dependence from (R)ChPT, dispersion relations

• Kπ *form factors*

Definition

vector f.f.

scalar f.f.

$$\langle \pi^-(p)|\bar{s}\,\gamma^\mu\, u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu\right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

with $F_+(0) = F_0(0)$

Kπ f.f. representation for K_{l3} decays

$$m_l^2 < q^2 < (m_K - m_\pi)^2$$

slope

curvature

$$F_{+,0}(q^2) = F_{+,0}(0)\left[1 + \lambda'_{+,0}\frac{q^2}{m_{\pi^-}^2} + \frac{1}{2}\lambda''_{+,0}\left(\frac{q^2}{m_{\pi^-}^2}\right)^2 + \cdots\right]$$

In this kinematical region the f.f. are real

Kπ f.f. representation for $\tau \to K\pi\nu_\tau$ decays

$$(m_K + m_\pi)^2 < q^2 < m_\tau^2$$

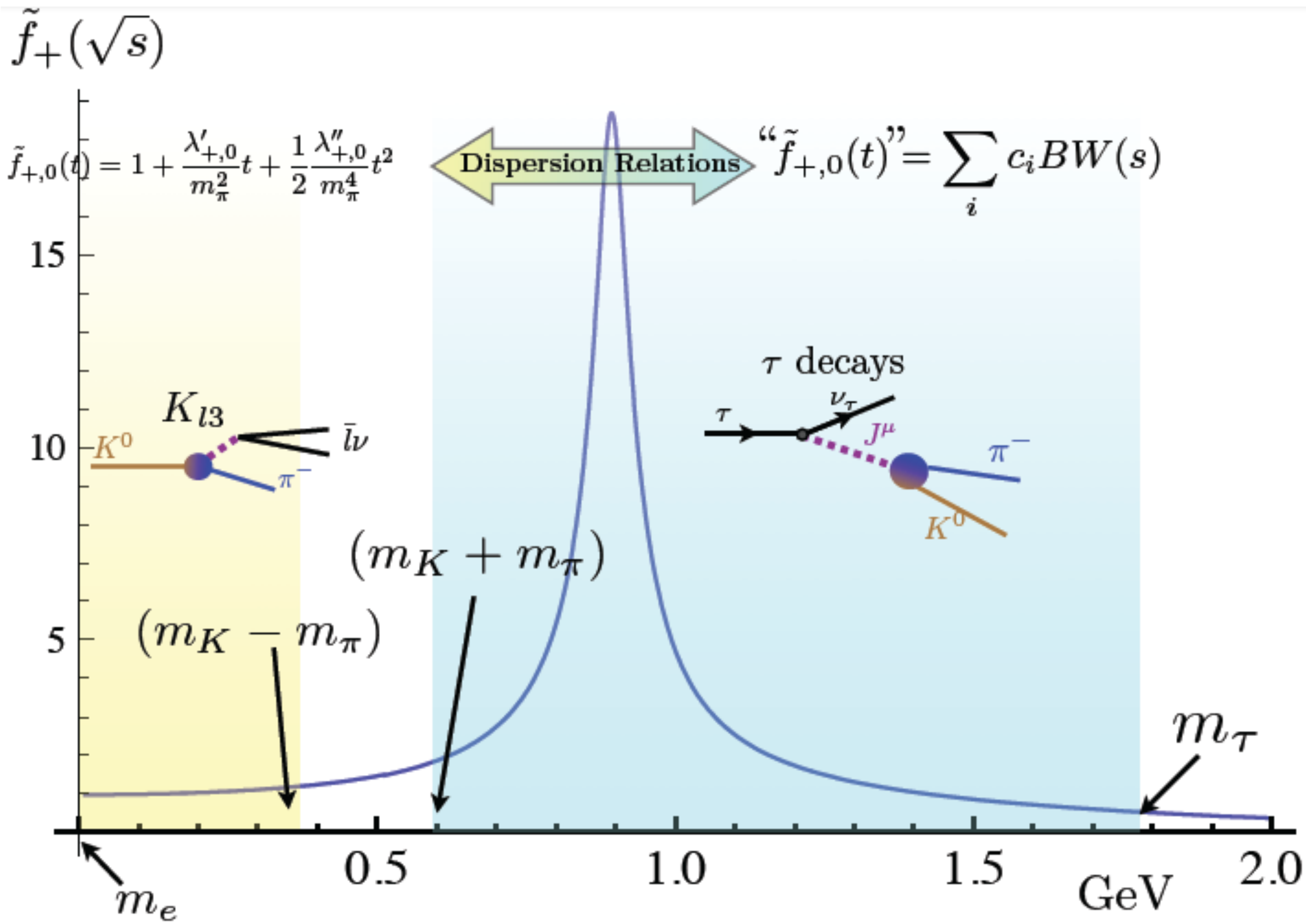
In this kinematical region the f.f. are complex

→ Taylor expansion inadmissible → more sophisticated treatments

- $K\pi$ *form factors*

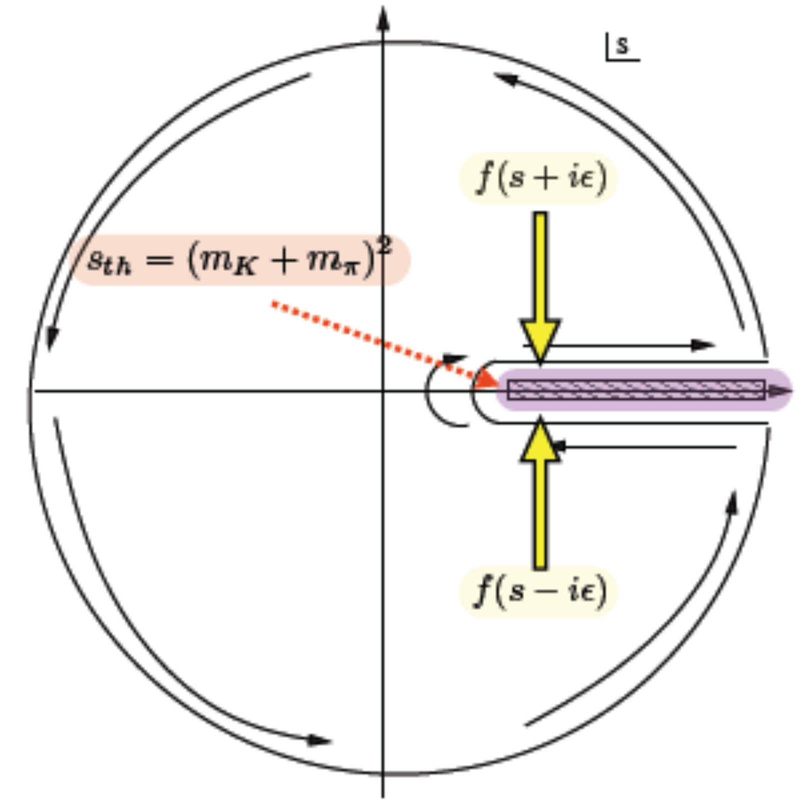
$K\pi$ f.f. dispersive representations

Suited to described both $\tau \to K\pi\nu_\tau$ and K_{l3} decays



• Kπ *form factors*

Kπ f.f. dispersive representations



Analyticity →

$$f(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\mathrm{Im}\, f(s')}{s'-s-i\epsilon}$$

Analyticity + Unitarity

→ Muskelishivili-Omnès equation

$$f(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\tan\delta(s')\mathrm{Re}\, f(s')}{s'-s-i\epsilon}$$

solution

$$f(s)=f(0)\exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

generalized solution (n subtractions at s=0)

$$f(s)=\exp\left[\alpha_1+\alpha_2 s\cdots+\alpha_{n-1}s^{n-1}+\frac{s^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s')^n}\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

Recent dispersive representations:

B. Moussallam, EPJC 53 (2008) 401

V. Bernard *et. al.*, PRD 80 (2009) 034034

D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

V. Bernard *et. al.*, PLB 638 (2006) 480

M. Jamin, J.A. Oller and A. Pich, NPB 587 (2000) 331 & 622 (2002) 279, PRD 74 (2006) 074009

• *Fit to* τ→Kπν$_\tau$

Differential decay distribution

$$|V_{us}|F_+(0) = 0.2163(5)$$

M. Antonelli *et. al.*, Eur. Phys. J. C69 (2010) 399

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2|V_{us}F_+(0)|^2 m_\tau^3}{32\pi^3 s} S_{\rm EW}\left(1-\frac{s}{m_\tau^2}\right)^2 \times$$
$$\times\left[\left(1+2\,\frac{s}{m_\tau^2}\right) q_{K\pi}^3\,|\tilde F_+(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s}\, q_{K\pi}|\tilde F_0(s)|^2\right]$$

with $\tilde F_{+,0}(q^2) \equiv \dfrac{F_{+,0}(q^2)}{F_+(0)}$

normalized vector f.f. normalized scalar f.f.

Ansatz to analyse the data:

$$N_i^{\rm th} = \mathcal{N}_T\,\frac{1}{2}\,\frac{2}{3}\,\Delta_{\rm b}^i\,\frac{1}{\Gamma_\tau\,\bar B_{K\pi}}\,\frac{d\Gamma_{K\pi}}{d\sqrt{s}}(s_{\rm b}^i)$$

with $\mathcal{N}_T = 53110$ and $\Delta_{\rm b} = 11.5\ {\rm MeV}$

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65

Model for the scalar f.f.

Coupled-channel analysis (analytic and unitary)

M. Jamin, J.A. Oller and A. Pich, NPB 622 (2002) 279

- *Fit to τ→Kπν$_\tau$*

Our model for the vector f.f.

After a detailed analysis in D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

Three-times-subtracted dispersion relation

cut-off to check stability

$$\tilde{F}_+(s) = \exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

with $\lambda'_+ = \alpha_1$ and $\lambda''_+ = \alpha_2 + \alpha_1^2$

Our model for the phase

$$\delta(s) = \tan^{-1}\left[\frac{\mathrm{Im}\ \tilde{f}_+(s)}{\mathrm{Re}\ \tilde{f}_+(s)}\right]$$ where $$\tilde{f}_+(s) = \frac{\tilde{m}_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \gamma\, s}{D(\tilde{m}_{K^*}, \gamma_{K^*})} - \frac{\gamma\, s}{D(\tilde{m}_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

2 vector resonances form inspired by RChPT

M. Jamin, A. Pich and J. Portolés, PLB 640 (2006) 176 & 664 (2008) 78

and

$$D(\tilde{m}_n, \gamma_n) \equiv \tilde{m}_n^2 - s - \kappa_n \,\mathrm{Re}\, \tilde{H}_{K\pi}(s) - i\,\tilde{m}_n \gamma_n(s)$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma(\tilde{m}_n^2)^3}\frac{\gamma_n}{\tilde{m}_n} \qquad \gamma_n(s) = \gamma_n \frac{s}{\tilde{m}_n^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(\tilde{m}_n^2)}$$

$\tilde{H}_{K\pi}(s)$ is the one-loop $K\pi$ bubble integral

Physical masses and widths are obtained from

$$D(\tilde{m}_n, \gamma_n) = 0$$

for $s \to s_R$ with $\sqrt{s_R} = m_R - \frac{i}{2}\Gamma_R$

R. Escribano *et. al.*, EPJC 28 (2003) 107

- *Fit to τ→Kπν$_τ$ with restrictions from K$_{l3}$*

M. Antonelli *et. al.*, Eur. Phys. J. C69 (2010) 399

Results

$B_{K\pi}^{\rm exp} = 0.418(11)\%$

$\lambda_+'^{\,\rm exp} = (24.9 \pm 1.1) \times 10^{-3}$
$\lambda_+''^{\,\rm exp} = (16 \pm 5) \times 10^{-4}$
$\rho_{\lambda_+',\lambda_+''} = -0.95$

$$\chi^2 = \sum_{i=1}^{90}{}' \left(\frac{N_i^{\rm th} - N_i^{\rm exp}}{\sigma_{N_i^{\rm exp}}} \right)^2 + \left(\frac{\bar{B}_{K\pi} - B_{K\pi}^{\rm exp}}{\sigma_{B_{K\pi}^{\rm exp}}} \right)^2 + (\boldsymbol{\lambda}_+^{\rm th} - \boldsymbol{\lambda}_+^{\rm exp})^{\rm T} V^{-1} (\boldsymbol{\lambda}_+^{\rm th} - \boldsymbol{\lambda}_+^{\rm exp})$$

$1.8\,\text{GeV} < \sqrt{s_{\rm cut}} < \infty$

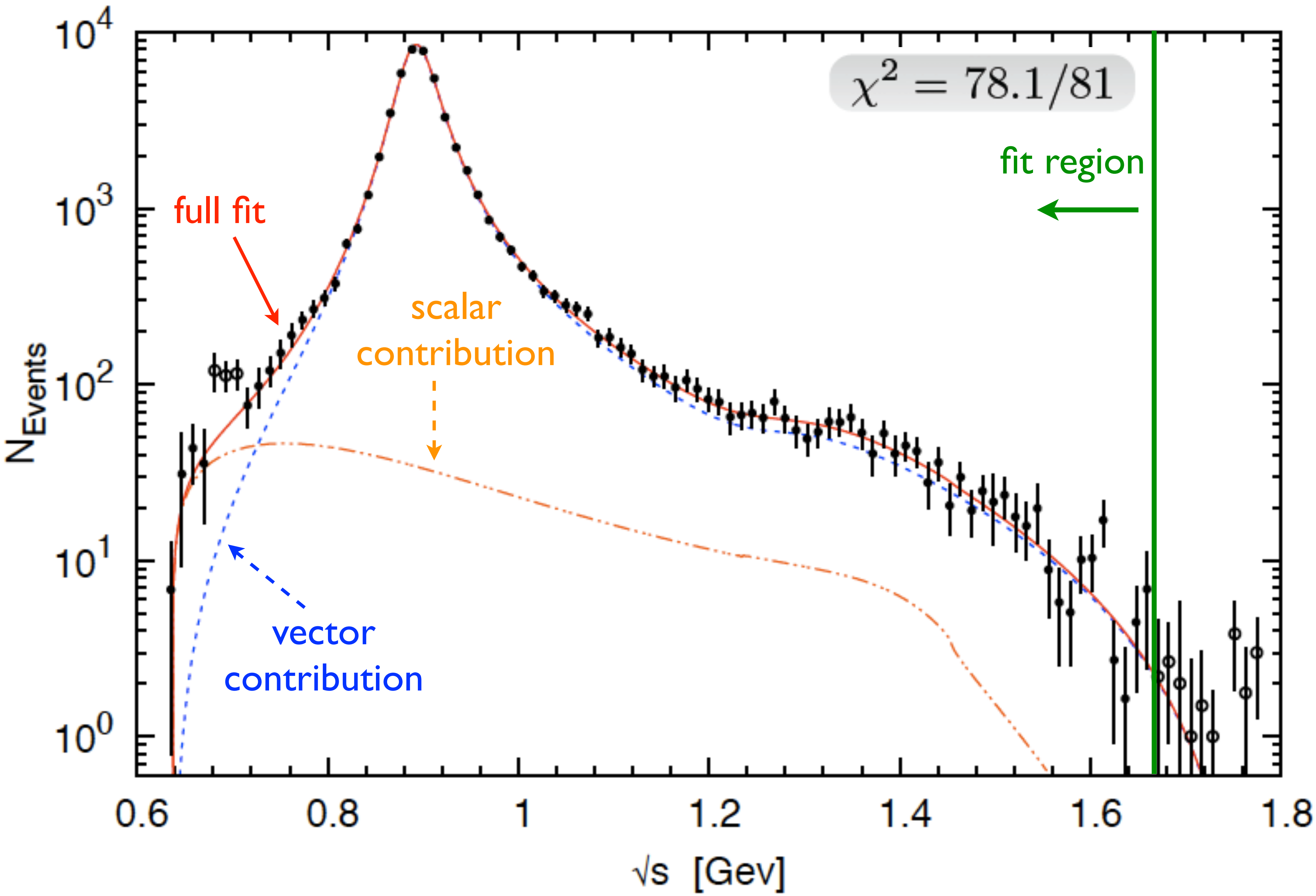
	$s_{\rm cut} = 3.24\ \text{GeV}^2$	$s_{\rm cut} = 4\ \text{GeV}^2$	$s_{\rm cut} = 9\ \text{GeV}^2$	$s_{\rm cut} \to \infty$
$B_{K\pi}$	0.429 ± 0.009	$0.427 \pm 0.008\%$	$0.426 \pm 0.008\%$	$0.426 \pm 0.008\%$
$(B_{K\pi}^{\rm th})$	(0.426%)	(0.425%)	(0.423%)	(0.423%)
m_{K^*} [MeV]	892.04 ± 0.20	892.02 ± 0.20	892.03 ± 0.19	892.03 ± 0.19
Γ_{K^*} [MeV]	46.58 ± 0.38	46.52 ± 0.38	46.48 ± 0.38	46.48 ± 0.38
$m_{K^{*\prime}}$ [MeV]	1257^{+30}_{-45}	1268^{+25}_{-32}	1270^{+24}_{-29}	1271^{+24}_{-29}
$\Gamma_{K^{*\prime}}$ [MeV]	321^{+95}_{-76}	238^{+75}_{-57}	206^{+67}_{-50}	205^{+67}_{-50}
$\gamma \times 10^2$	$-8.2^{+2.2}_{-3.5}$	$-5.4^{+1.4}_{-2.0}$	$-4.4^{+1.2}_{-1.6}$	$-4.4^{+1.2}_{-1.6}$
$\lambda_+' \times 10^3$	25.43 ± 0.30	25.49 ± 0.30	25.55 ± 0.30	25.55 ± 0.30
$\lambda_+'' \times 10^4$	12.31 ± 0.10	12.20 ± 0.10	12.12 ± 0.10	12.12 ± 0.10
χ^2/n.d.f.	$77.9/81$	$78.1\ /81$	$79.0\ /81$	$79.1/81$

- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

Fit to Belle spectrum

D. R. Boito, R. Escribano and M. Jamin, JHEP 09 (2010) 031

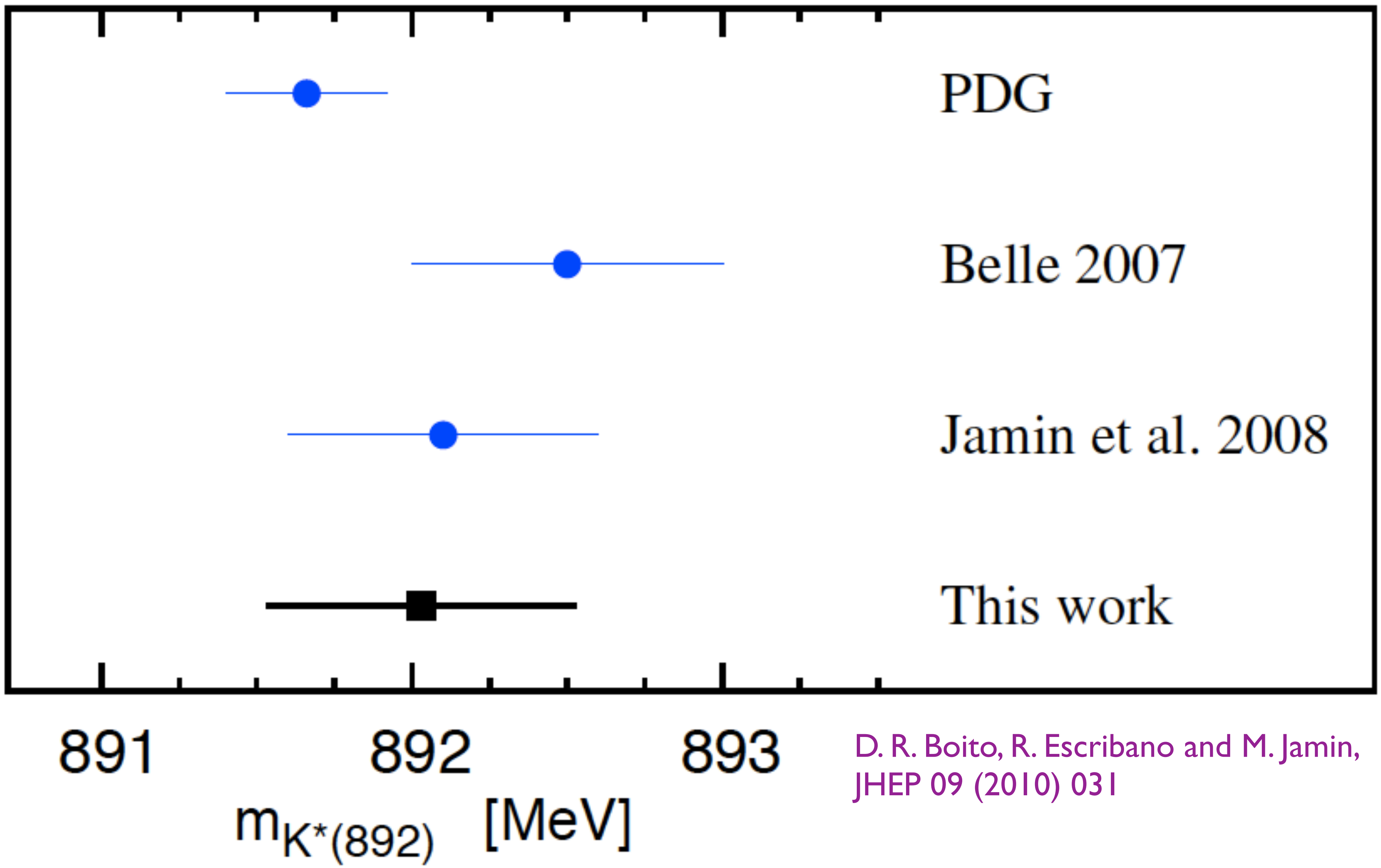
$s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$



• *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

K*(892)$^{\pm}$ pole mass

$$m_{K^*(892)^\pm} = 892.03 \pm (0.19)_{\rm stat} \pm (0.44)_{\rm sys}\ \text{MeV}$$

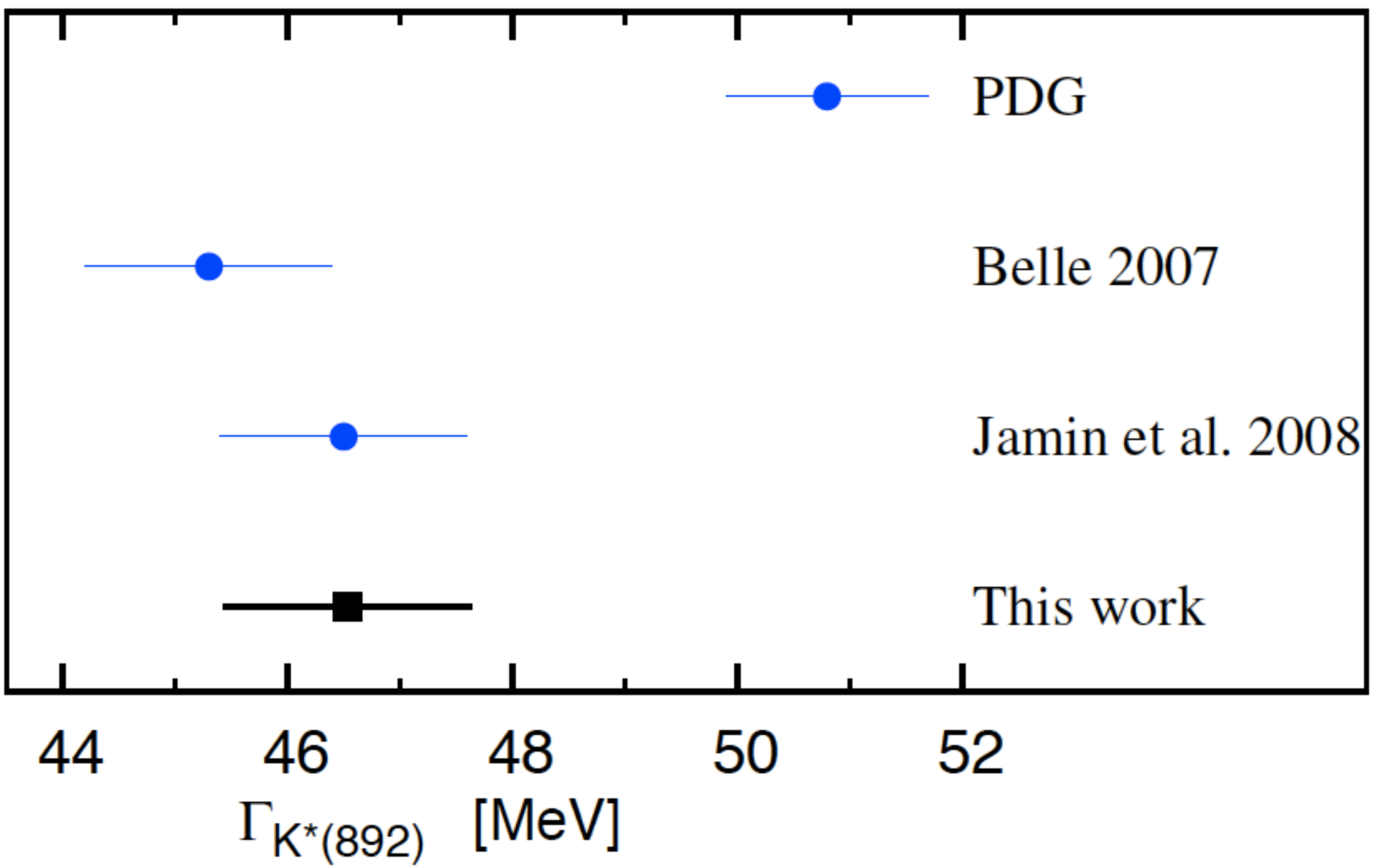


D. R. Boito, R. Escribano and M. Jamin, JHEP 09 (2010) 031

- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

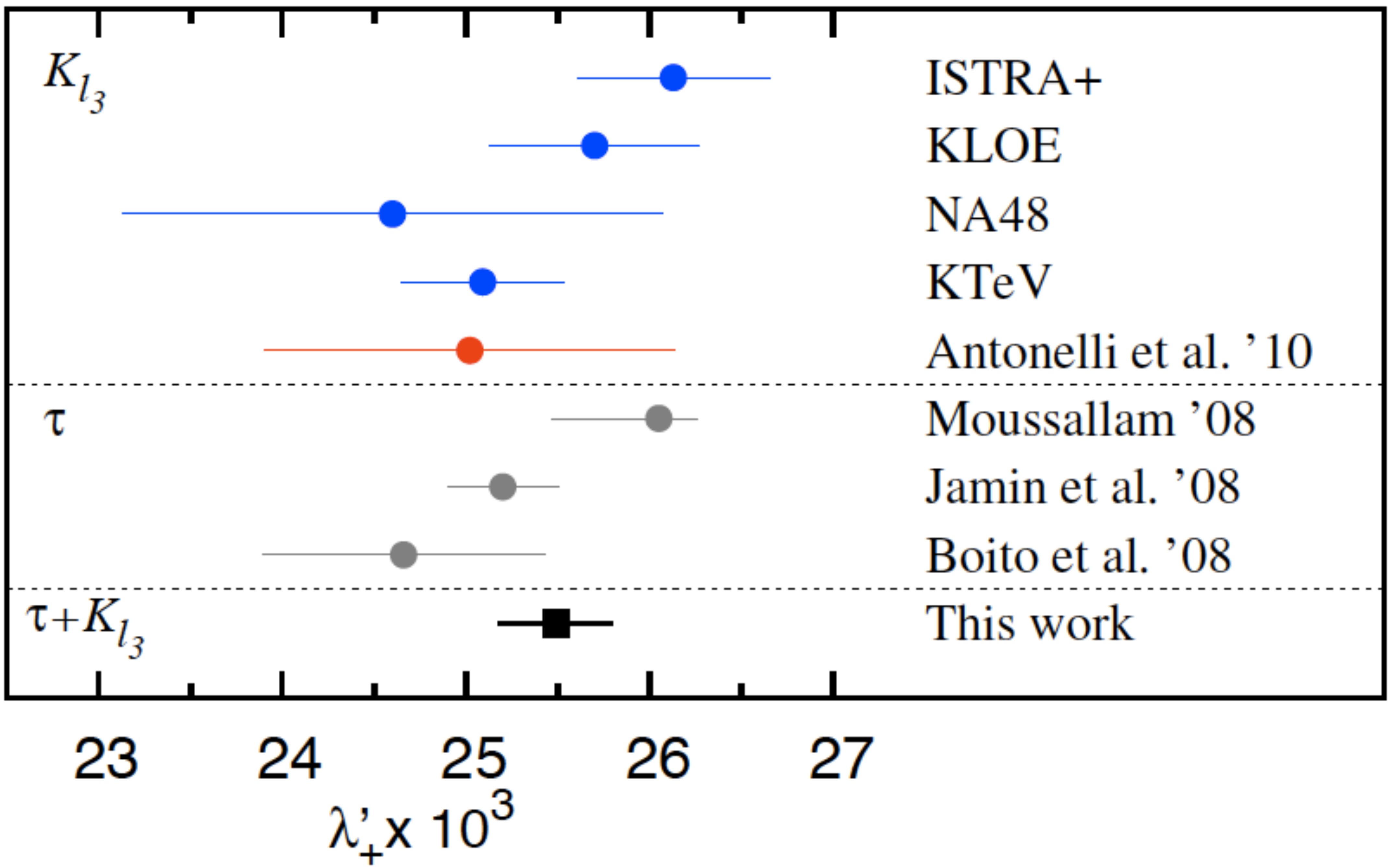
K*(892)$^{\pm}$ pole width

$$\Gamma_{K^*(892)^{\pm}} = 46.53 \pm (0.38)_{\rm stat} \pm (1.0)_{\rm sys}\ \mathrm{MeV}$$



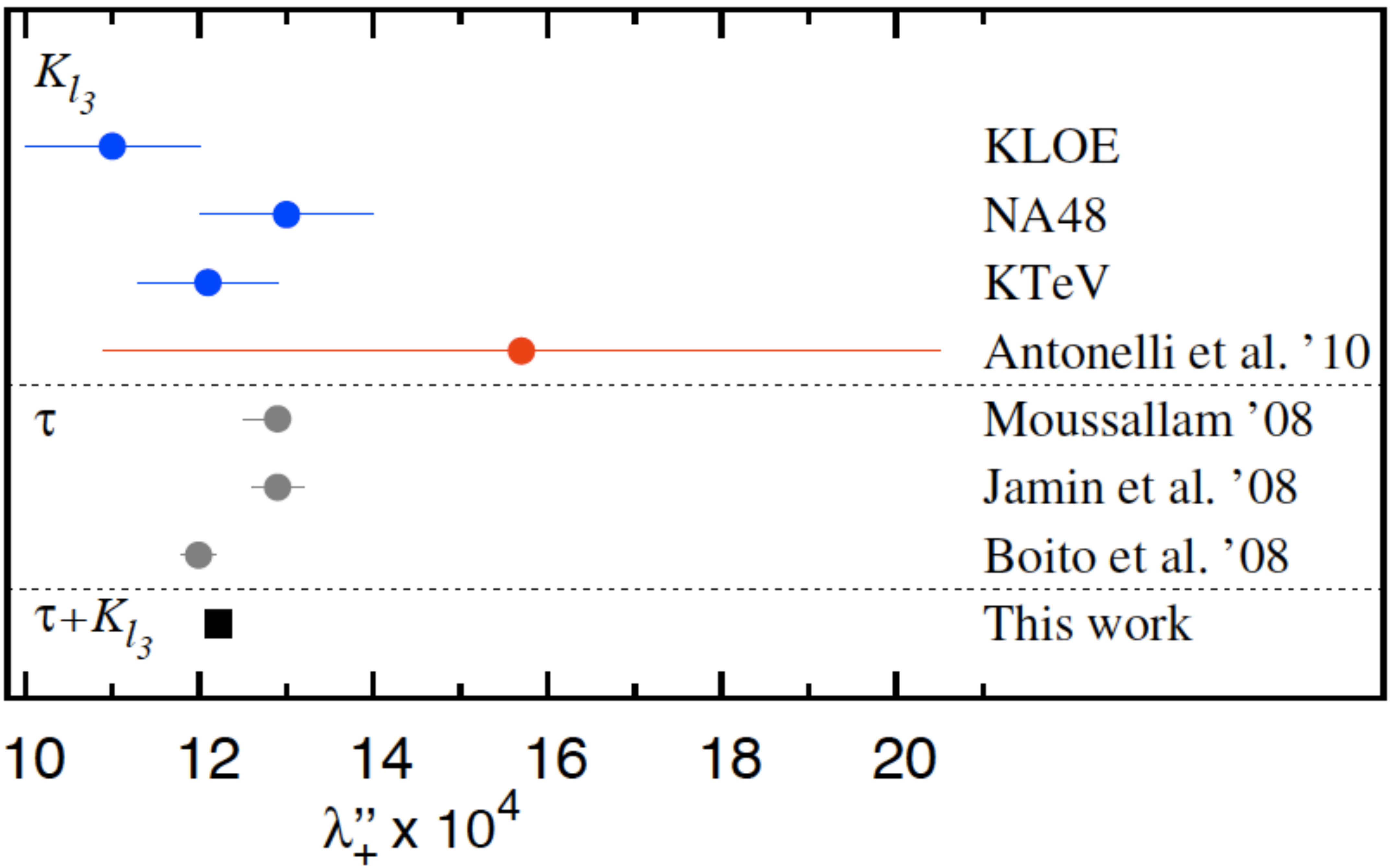
- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

$$\lambda'_+ \times 10^3 = 25.49 \pm (0.30)_{\rm stat} \pm (0.06)_{s_{\rm cut}}$$

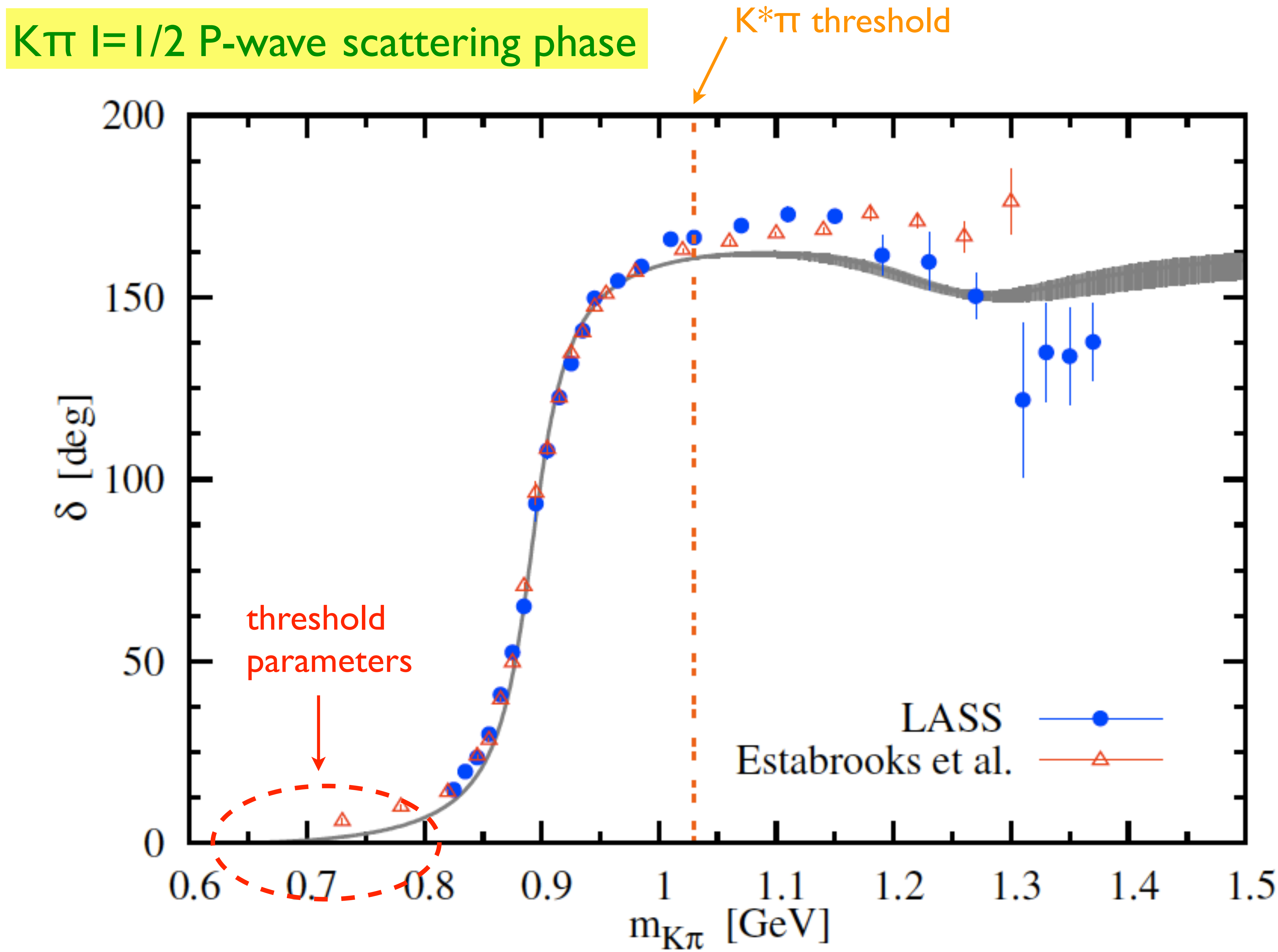


- *Fit to $\tau \to K\pi\nu_\tau$ with restrictions from K_{l3}*

$$\lambda''_+ \times 10^4 = 12.22 \pm (0.10)_{\rm stat} \pm (0.10)_{s_{\rm cut}}$$



- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* K_{l3}



• Conclusions: Intermezzo

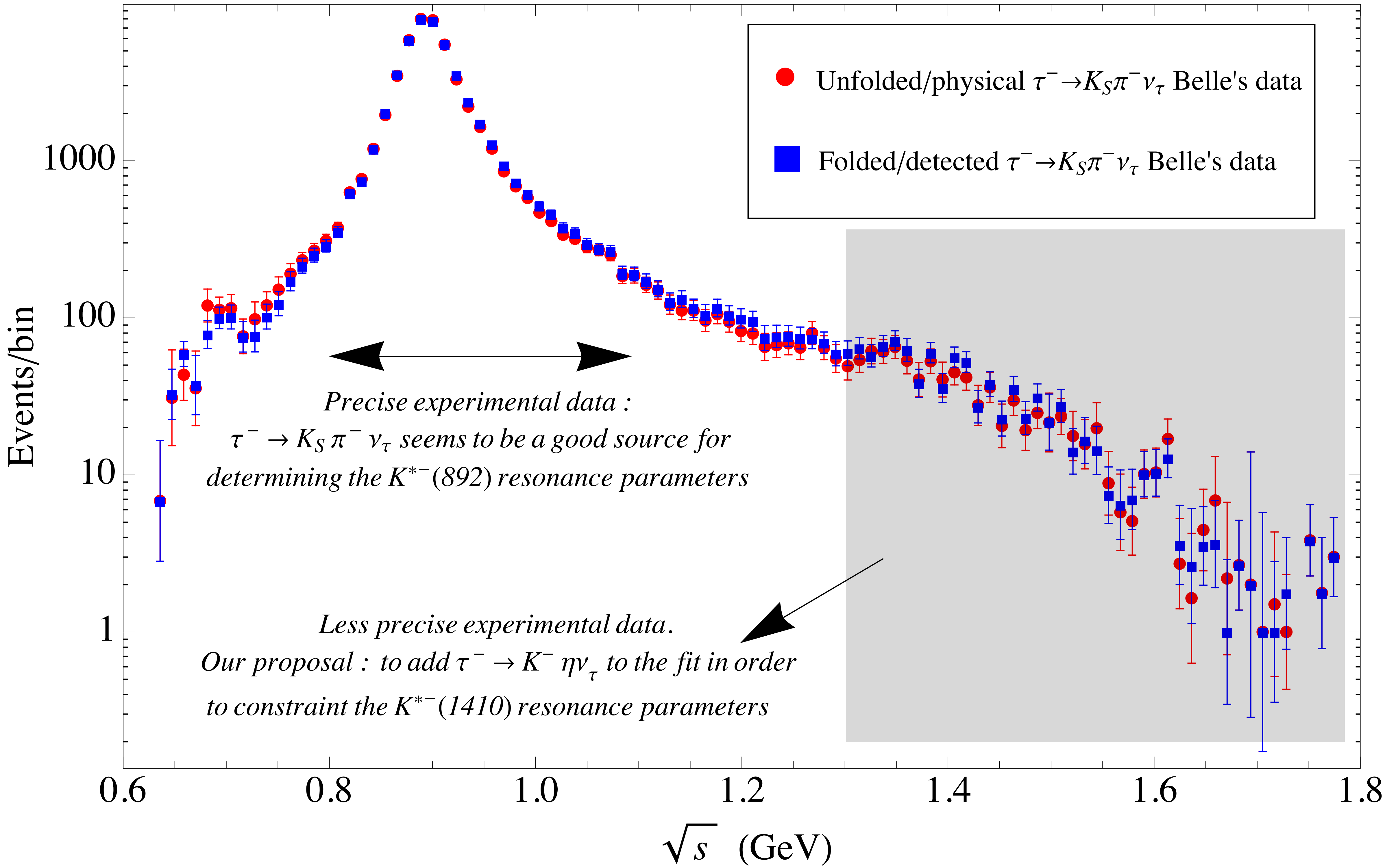
We have presented a model aimed at describing the $K\pi$ vector form factor using a dispersive representation and incorporating constraints from K_{l3} decays suited to describe both $\tau \to K\pi\nu_\tau$ and K_{l3} decays simultaneously

A good detemination of the $K\pi$ vector f.f. and resonance parameters is obtained from a fit of the $\tau \to K\pi\nu_\tau$ spectrum

Competitive results for the $K^*(892)^\pm$ pole mass and width, slope and curvature parameters, K_{l3} phase-space integrals, $K\pi$ I=1/2 P-wave scattering phase and threshold parameters are obtained

A combined fit of the $\tau \to K\pi\nu_\tau$ and K_{l3} spectra should be done in the future

• Reason for a $\tau \to K\eta\nu_\tau$ analysis

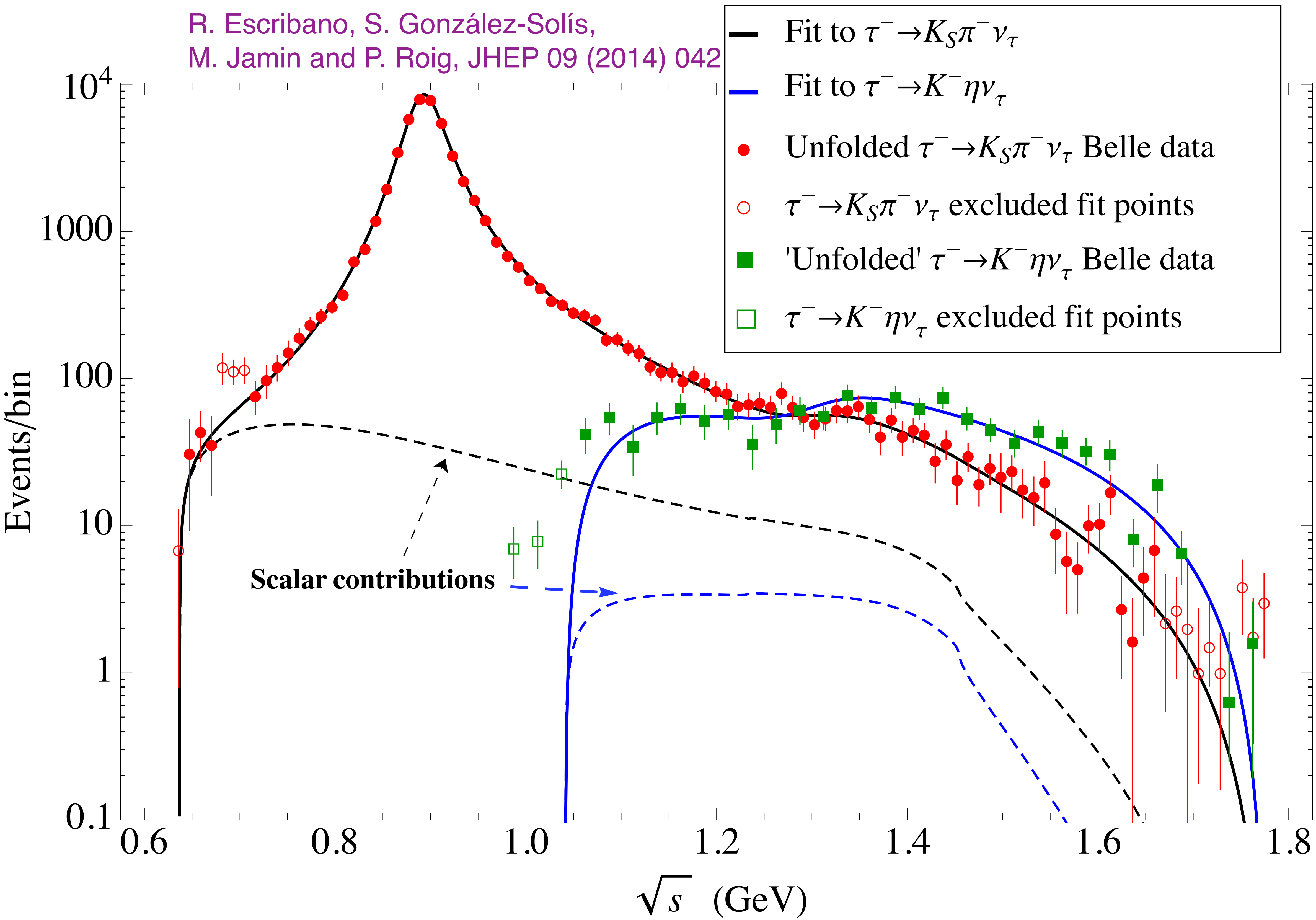


- **Results of the combined analysis**

$N_{events} = 53113$ $\Delta_{bin} = 0.0115$ GeV/bin

$N_{events} = 1271$ $\Delta_{bin} = 0.025$ GeV/bin

R. Escribano, S. González-Solís, M. Jamin and P. Roig, JHEP 09 (2014) 042



• Results of the combined analysis

$$\left.\begin{array}{l} M_{K^{*-}(892)} = 892.03 \pm 0.19\ \text{MeV} \\ \Gamma_{K^{*-}(892)} = 46.18 \pm 0.44\ \text{MeV} \end{array}\right\}\text{no gain}$$

$$\left.\begin{array}{l} M_{K^{*-}(1410)} = 1304 \pm 17\ \text{MeV} \\ \Gamma_{K^{*-}(1410)} = 171 \pm 62\ \text{MeV} \end{array}\right\}\text{improvement}$$

$$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$$

$$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$$

$$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$$

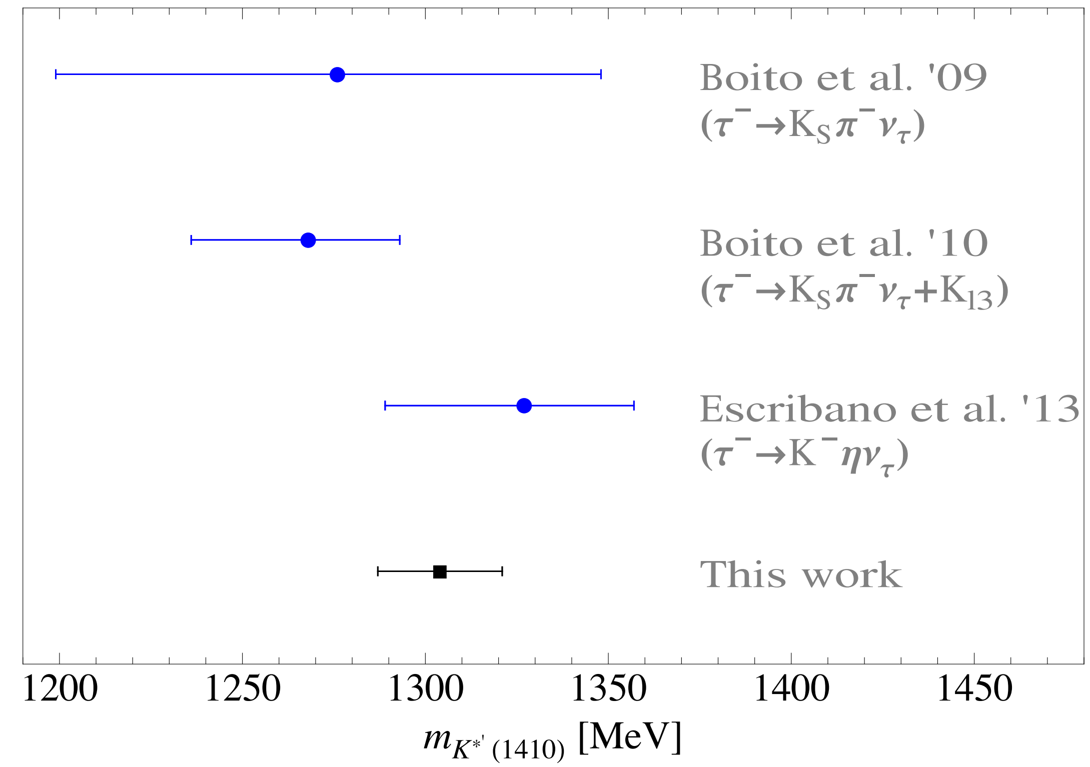
$$\left.\begin{array}{l} \lambda'_{K\pi} = (23.9 \pm 0.9) \cdot 10^{-3} \\ \lambda'_{K\eta} = (20.9 \pm 2.7) \cdot 10^{-3} \end{array}\right\}\text{isospin violation?}$$

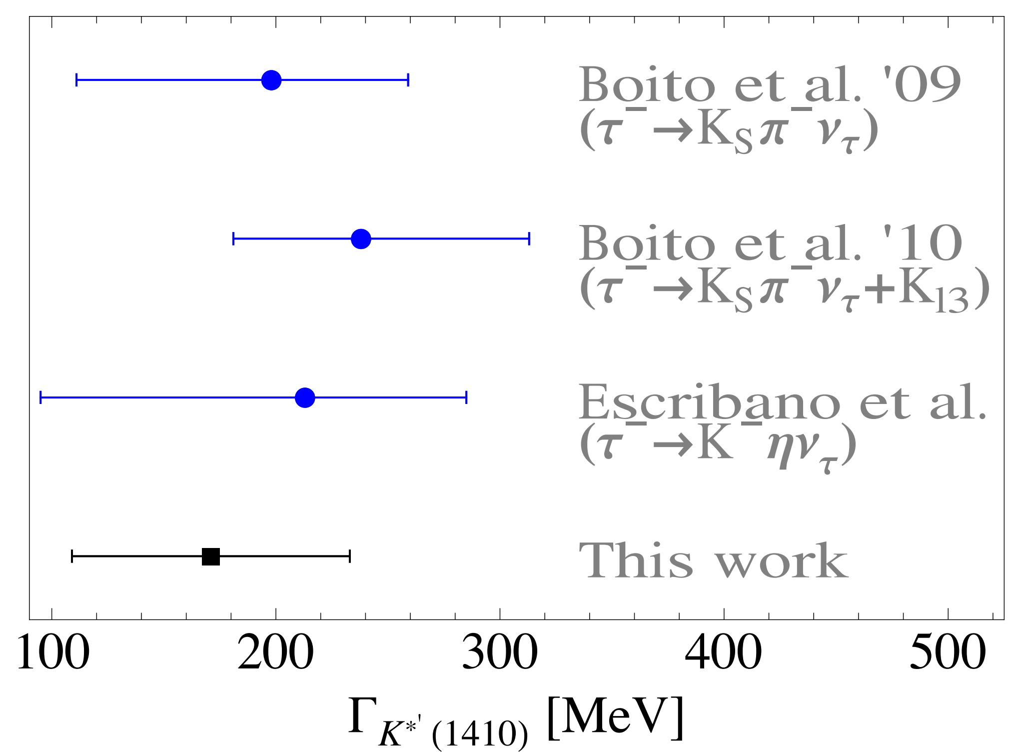
$$\left.\begin{array}{l} \lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4} \\ \lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4} \end{array}\right\}\text{isospin violation?}$$

$$\Uparrow$$

$$\tau^- \to K^-\pi^0\nu_\tau \& K_{\ell 3}$$

$$\chi^2/d.o.f = 108.1/105 = 1.03$$





- **Results of the combined analysis**

Different choices regarding linear slopes and resonance mixing parameters

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{\text{th}}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{\text{th}}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
$\chi^2/\text{n.d.f.}$	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

• Future prospects for Belle-I and Belle-II

Data / Error	Current	Belle-I	Belle-I $K\pi$	Belle-I $K\eta$	Belle-II	Belle-II $K\pi$	Belle-II $K\eta$
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	± 0.005	± 0.005	± 0.012	$^{\dagger}(0.001)$	$^{\dagger}(0.001)$	± 0.012
M_{K^*}	892.03 ± 0.19	± 0.09	± 0.09	± 0.19	$^{\dagger}(0.02)$	$^{\dagger}(0.02)$	± 0.19
Γ_{K^*}	46.18 ± 0.44	± 0.20	± 0.20	± 0.44	$^{\dagger}(0.02)$	$^{\dagger}(0.03)$	± 0.42
$M_{K^{*\prime}}$	1304 ± 17	$^{\dagger}(7)$	$^{\dagger}(9)$	$^{\dagger}(8)$	$^{\dagger}(1)$	$^{\dagger}(1)$	$^{\dagger}(1)$
$\Gamma_{K^{*\prime}}$	168 ± 62	$^{\dagger}(19)$	$^{\dagger}(24)$	$^{\dagger}(25)$	$^{\dagger}(3)$	$^{\dagger}(4)$	$^{\dagger}(11)$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.9	$^{\dagger}(0.3)$	$^{\dagger}(0.3)$	± 0.8	$^{\dagger}(0.04)$	$^{\dagger}(0.04)$	± 0.8
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	± 0.07	± 0.07	± 0.2	$^{\dagger}(0.01)$	$^{\dagger}(0.01)$	± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	± 0.05	± 0.10	± 0.05	$^{\dagger}(0.01)$	± 0.10	$^{\dagger}(0.01)$
$\gamma_{K\eta}(= \gamma_{K\pi}) \times 10^2$	-3.3 ± 1.3	$^{\dagger}(0.3)$	$^{\dagger}(0.3)$	$^{\dagger}(0.4)$	$^{\dagger}(0.04)$	$^{\dagger}(0.04)$	$^{\circ}(0.3)$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 2.7	$^{\dagger}(0.7)$	± 2.7	$^{\dagger}(0.8)$	$^{\dagger}(0.10)$	± 2.7	$^{\circ}(0.4)$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.5	$^{\dagger}(0.2)$	± 0.5	$^{\dagger}(0.2)$	$^{\dagger}(0.02)$	± 0.5	$^{\dagger}(0.06)$

Table 4. The errors of our final results (3.3) are compared, in turn, to those achievable by analysing the complete Belle-I data sample, and updating only the $K_S\pi^-$ or $K^-\eta$ analyses. The last three columns show the potential of fitting all data collected by Belle-II and the same only for $K_S\pi^-$ or for $K^-\eta$ (assuming the other mode has not been updated to include the complete Belle-I data sample). Current Belle $K_S\pi^-$ ($K^-\eta$) data correspond to 351 (490) fb^{-1} for a complete data set of $\sim 1000\,\mathrm{fb}^{-1} = 1\ \mathrm{ab}^{-1}$. Expectations for Belle-II correspond to 50 ab^{-1}. All errors include both statistical and systematic uncertainties. † means that statistical errors (in brackets) will become negligible, while $^{\circ}$ signals a tension with the current reference best fit values. We thank Denis Epifanov for conversations on these figures and on expected performance of Belle-II at the detector and analysis levels. All errors have been symmetrised for simplicity.

• Conclusions: Finale

A good description of the vector form factor (by analyticity+unitarity arguments) is crucial to unveil the parameters of the intermediate resonances which drive the decays

Limitations: only $\tau \to K_S \pi \nu_\tau$ is published, no access to isospin violations

$\tau \to K \eta \nu_\tau$ not very precise, convoluted with detector effects

Fitting both decay spectra together we have considerable improved the determination of the $K^{*-}(1410)$ mass while we slightly reduced the uncertainty of the width

$$\boxed{M_{K^{*\prime}} = (1304 \pm 17)\ \mathrm{MeV}\,, \quad \Gamma_{K^{*\prime}} = (171 \pm 62)\ \mathrm{MeV}}$$

Call for (an unfolded) analysis of $\tau^- \to K^- \pi^0 \nu_\tau$ for unveiling possible isospin violations on the low-energy parameters $\lambda'^{('')}$

- *Fit to* τ→Kπν$_\tau$

Results

Update of D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

$$B_{K\pi}^{\rm exp} = 0.418(11)\%$$

$$\chi^2 = \sum_{i=1}^{90}{}' \left(\frac{N_i^{\rm th} - N_i^{\rm exp}}{\sigma_{N_i^{\rm exp}}}\right)^2 + \left(\frac{\bar{B}_{K\pi} - B_{K\pi}^{\rm exp}}{\sigma_{B_{K\pi}^{\rm exp}}}\right)^2$$

$1.8\,\mathrm{GeV} < \sqrt{s_{\rm cut}} < \infty$

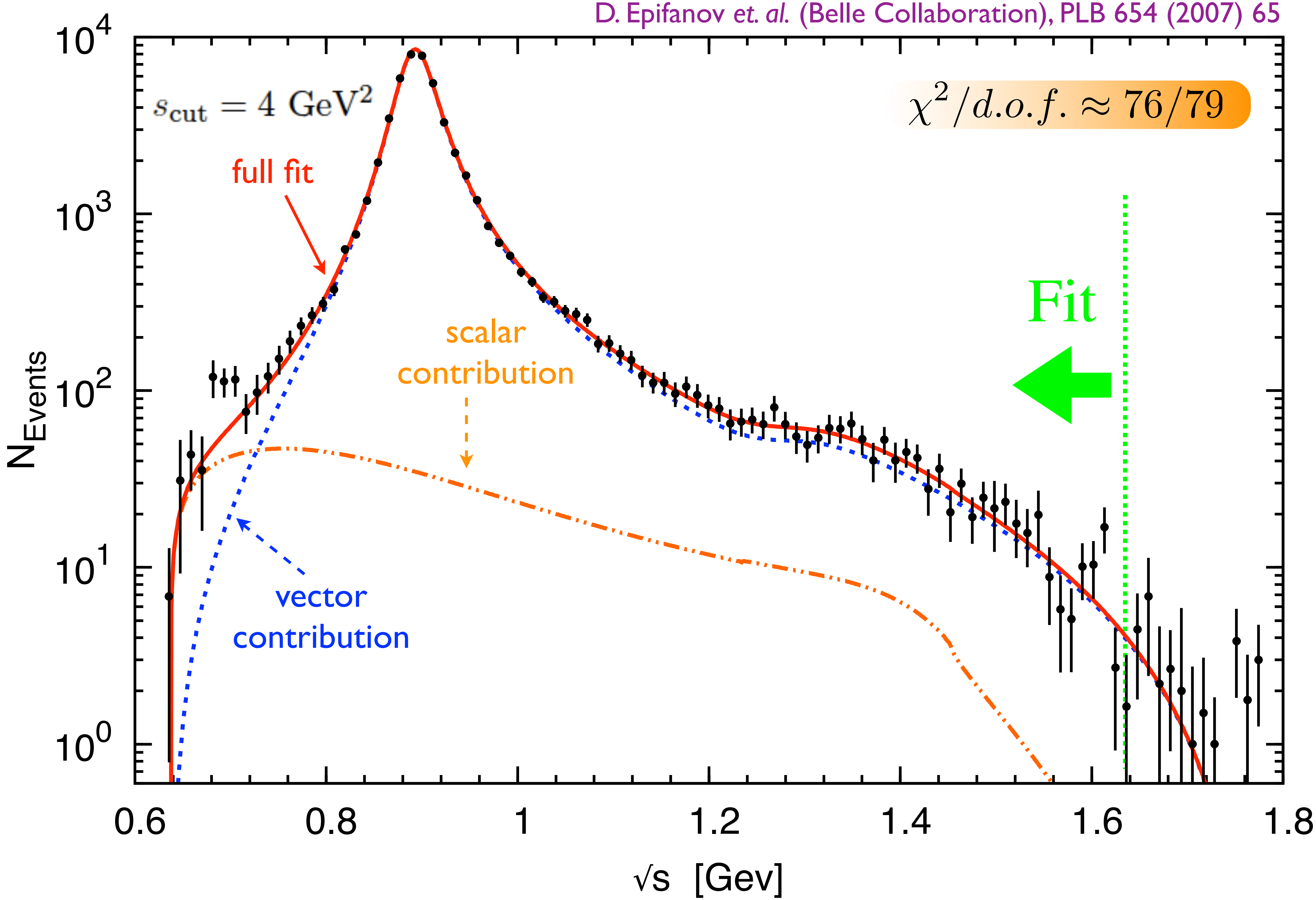
	$s_{\rm cut} = 3.24\ \mathrm{GeV}^2$	$s_{\rm cut} = 4\ \mathrm{GeV}^2$	$s_{\rm cut} = 9\ \mathrm{GeV}^2$	$s_{\rm cut} \to \infty$
$\bar{B}_{K\pi}$	$0.416 \pm 0.011\%$	$0.417 \pm 0.011\%$	$0.418 \pm 0.011\%$	$0.418 \pm 0.011\%$
$(B_{K\pi}^{\rm th})$	(0.414%)	(0.414%)	(0.415%)	(0.415%)
m_{K^*} [MeV]	892.00 ± 0.19	892.02 ± 0.19	892.03 ± 0.19	892.03 ± 0.19
Γ_{K^*} [MeV]	46.14 ± 0.44	46.20 ± 0.43	46.25 ± 0.42	46.25 ± 0.42
$m_{K^{*\prime}}$ [MeV]	1281^{+25}_{-33}	1280^{+25}_{-28}	1278^{+26}_{-27}	1278^{+26}_{-27}
$\Gamma_{K^{*\prime}}$ [MeV]	243^{+92}_{-70}	193^{+72}_{-56}	177^{+66}_{-52}	177^{+66}_{-52}
$\gamma \times 10^2$	$-5.1^{+1.7}_{-2.6}$	$-3.9^{+1.3}_{-1.8}$	$-3.4^{+1.1}_{-1.6}$	$-3.4^{+1.1}_{-1.6}$
$\lambda'_+ \times 10^3$	24.15 ± 0.72	24.55 ± 0.68	24.86 ± 0.66	24.88 ± 0.66
$\lambda''_+ \times 10^4$	11.99 ± 0.19	11.95 ± 0.19	11.93 ± 0.19	11.93 ± 0.19
χ^2/n.d.f.	$74.1/79$	$75.7/79$	$77.2/79$	$77.3/79$

• *Fit to* $\tau \to K\pi\nu_\tau$

Update of D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821

Fit to Belle spectrum

D. Epifanov *et. al.* (Belle Collaboration), PLB 654 (2007) 65



• *Fit to τ→Kπν$_\tau$ with restrictions from K$_{l3}$*

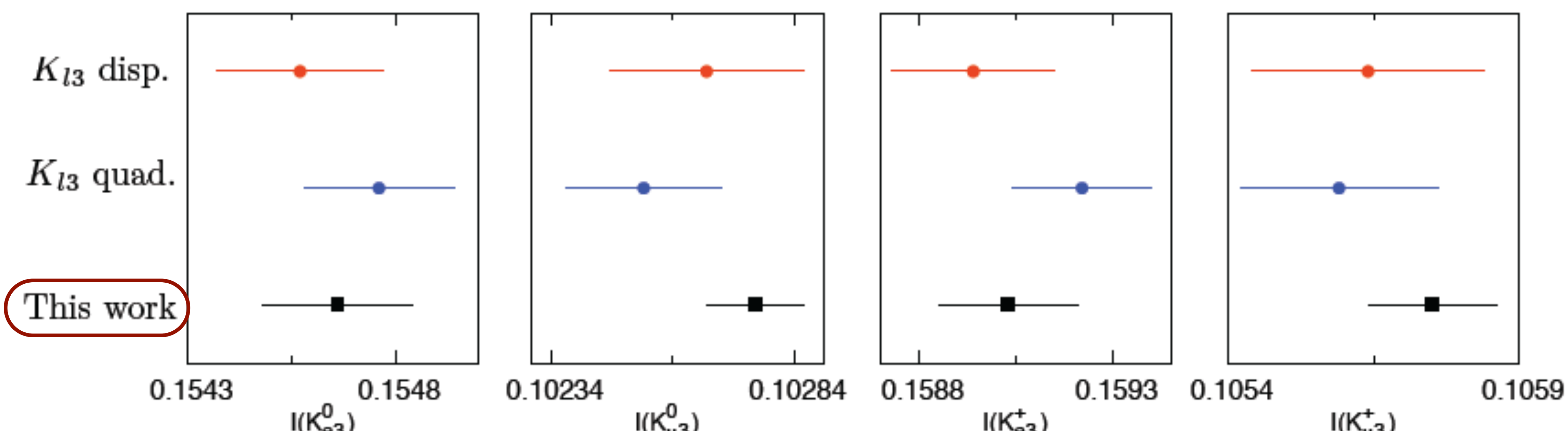
K$_{l3}$ phase-space integrals

$$I_{K_{l_3}} = \frac{1}{m_K^2}\int_{m_l^2}^{(m_K-m_\pi)^2} \mathrm{d}t\,\lambda(t)^{3/2}\left(1+\frac{m_l^2}{2t}\right)\left(1-\frac{m_l^2}{t}\right)^2\left(|\tilde f_+(t)|^2+\frac{3\,m_l^2(m_K^2-m_\pi^2)^2}{(2t+m_l^2)\,m_K^4\,\lambda(t)}|\tilde f_0(t)|^2\right)$$

$$\lambda(t) = 1+t^2/m_K^4+r_\pi^4-2\,r_\pi^2-2\,r_\pi^2\,t/m_K^2-2\,t/m_K^2$$

	This Work	K_{l_3} disp. [9]	K_{l_3} quad. [9]
$I_{K^0_{e3}}$	0.15466(17)	0.15476(18)	0.15457(20)
$I_{K^0_{\mu3}}$	0.10276(10)	0.10253(16)	0.10266(20)
$I_{K^+_{e3}}$	0.15903(17)	0.15922(18)	0.15894(21)
$I_{K^+_{\mu3}}$	0.10575(11)	0.10559(17)	0.10564(20)

[9] M. Antonelli *et. al.*, Eur. Phys. J. C69 (2010) 399



- *Fit to* $\tau \to K\pi\nu_\tau$ *with restrictions from* K_{l3}

$K\pi$ I=1/2 P-wave threshold parameters

$$\frac{2}{\sqrt{s}}\mathrm{Re}\, t_l^I(s) = \frac{1}{2q}\sin 2\delta_l^I(q) = q^{2l}\left[\underline{a_l^I} + \underline{b_l^I}\, q^2 + \underline{c_l^I}\, q^4 + \mathcal{O}(q^6)\right]$$

	This work	[60]	[61]	[62]	[48]
$m_{\pi^-}^3\, a_1^{1/2} \times 10$	0.166(4)	0.16(3)	0.18	0.18(3)	0.19(1)
$m_{\pi^-}^5\, b_1^{1/2} \times 10^2$	0.258(9)	-	-	-	0.18(2)
$m_{\pi^-}^7\, c_1^{1/2} \times 10^3$	0.90(3)	-	-	-	0.71(11)

[48] P. Büttiker, S. Descotes-Genon and B. Moussallam, EPJC 33 (2004) 209

[60] V. Bernard, N. Kaiser and U. G. Meißner, NPB 357 (1991) 129

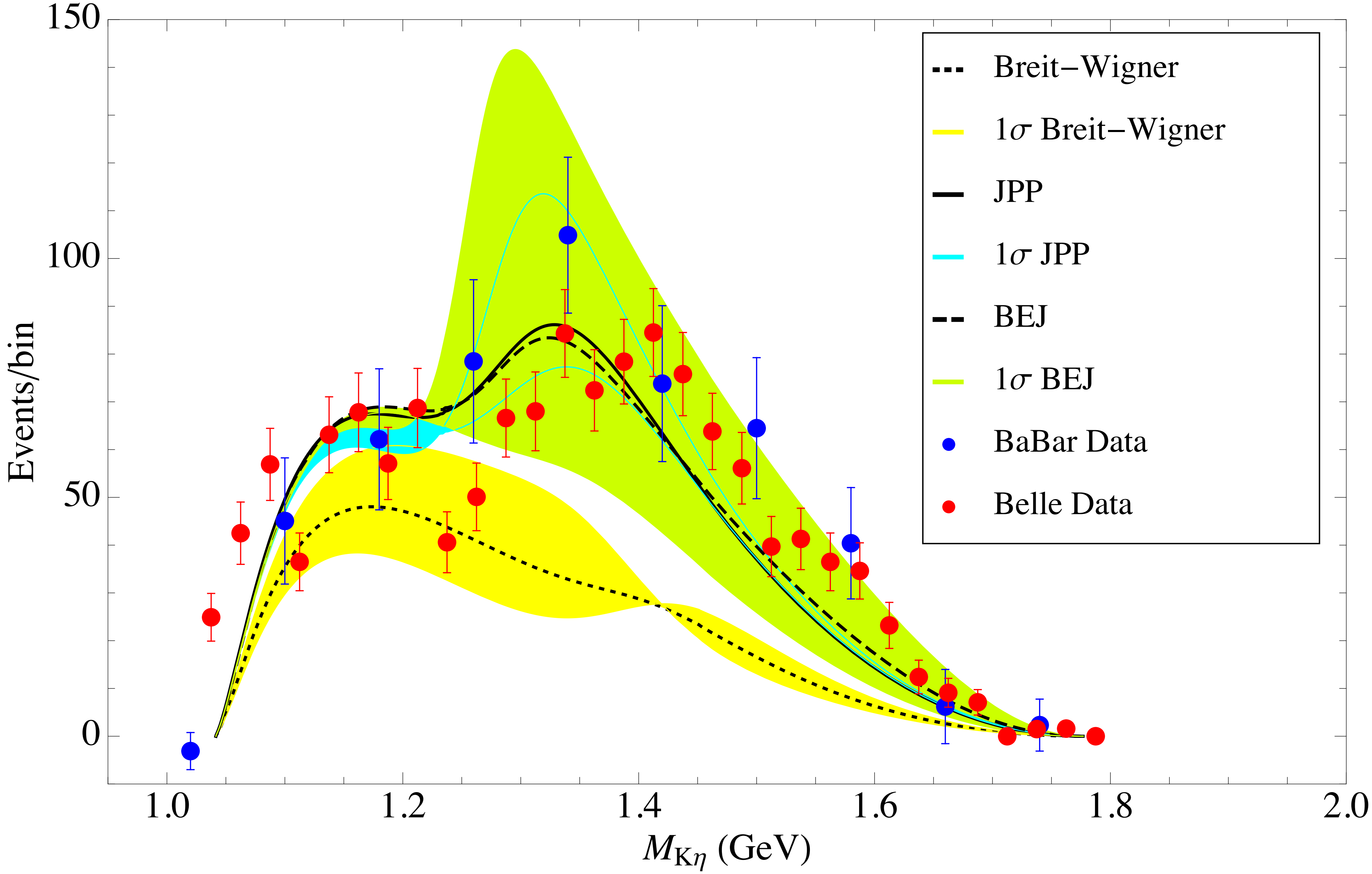
[61] J. Bijnens, P. Dhonte and P. Talavera, JHEP 05 (2004) 036

[62] V. Bernard, N. Kaiser and U. G. Meißner, NPB 364 (1991) 283

• Results of the $\tau \to K\eta\nu_\tau$ analysis

Predictions based on the $\tau \to K\pi\nu_\tau$ analysis

R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039



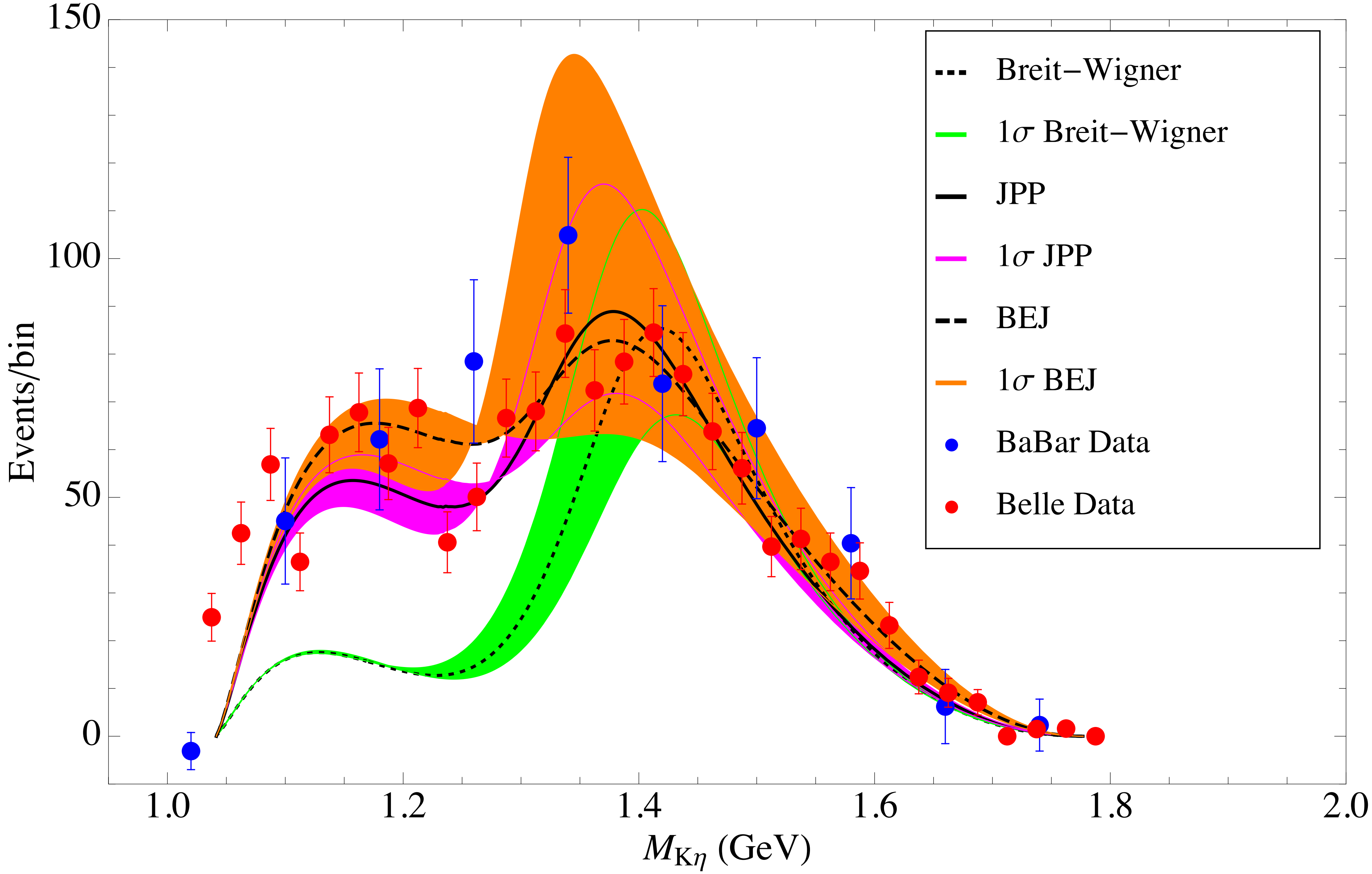
K. Inami *et. al.* (Belle Collaboration), PLB 672 (2009) 109

P. del Amo Sanchez *et. al.* (BaBar Collab.), PRD 83 (2011) 032002

• Results of the $\tau \to K\eta\nu_\tau$ analysis

Fit to the $\tau \to K\eta\nu_\tau$ experimental data

R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039



K. Inami *et. al.* (Belle Collaboration), PLB 672 (2009) 109

P. del Amo Sanchez *et. al.* (BaBar Collab.), PRD 83 (2011) 032002

• Results of the $\tau \to K\eta\nu_\tau$ analysis

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R. Escribano, S. González-Solís and P. Roig, JHEP 10 (2013) 039

JPP vector form factor

$$M_{K^{\star\prime}} = 1332^{+16}_{-18}, \quad \Gamma_{K^{\star\prime}} = 220^{+26}_{-24}, \quad \gamma = -0.078^{+0.012}_{-0.014}$$

BEJ vector form factor

$$M_{K^{\star\prime}} = 1327^{+30}_{-38}, \quad \Gamma_{K^{\star\prime}} = 213^{+72}_{-118}, \quad \gamma = -0.051^{+0.012}_{-0.036}$$

JPP and BEJ averaged determinations from the $K\pi$ system

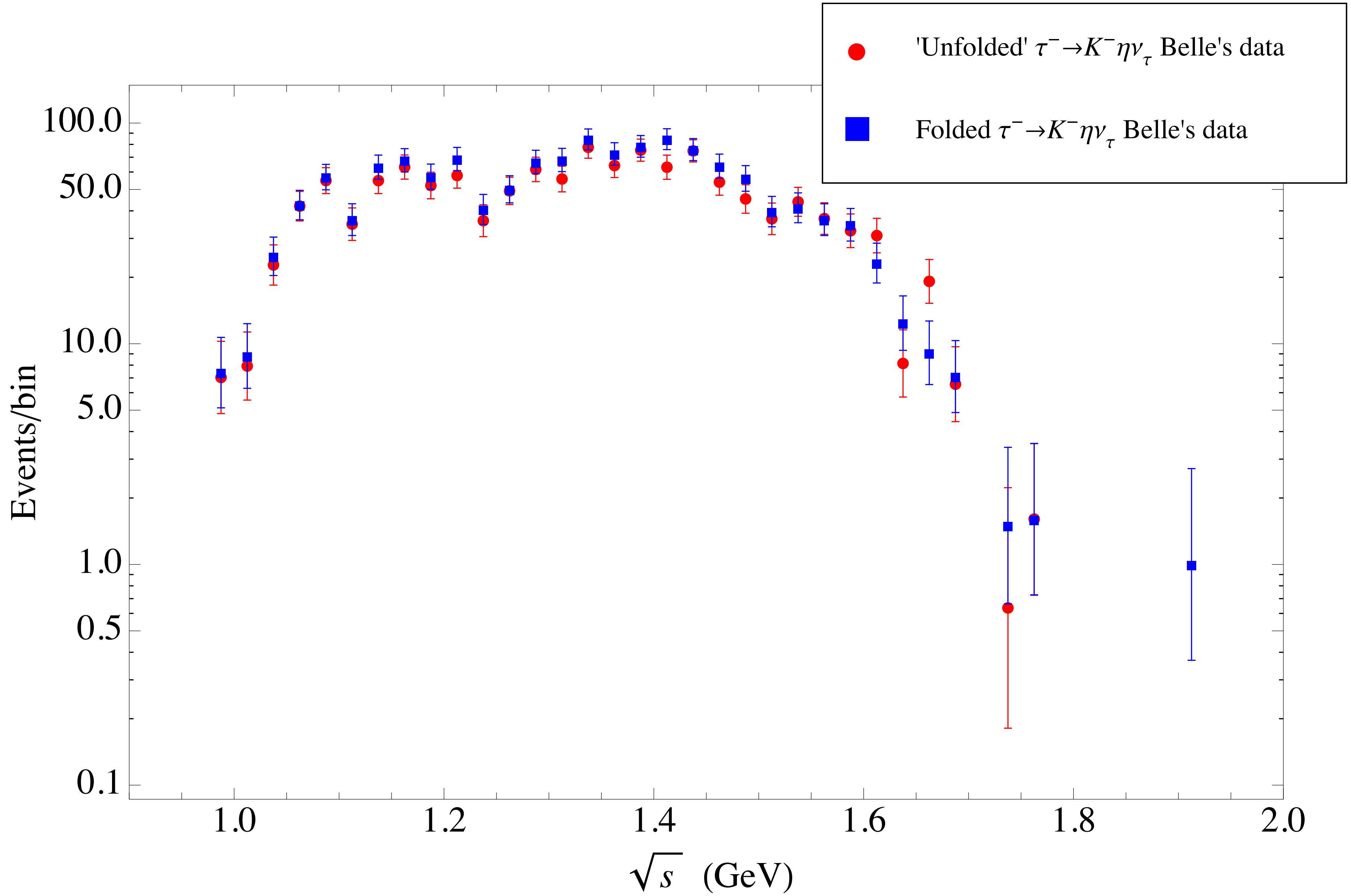
$$M_{K^{\star\prime}} = 1277^{+35}_{-41}, \quad \Gamma_{K^{\star\prime}} = 218^{+95}_{-66}, \quad \gamma = -0.049^{+0.019}_{-0.016}$$

JPP and BEJ averaged determinations from the $K\eta$ system

$$M_{K^{\star\prime}} = 1330^{+27}_{-41}, \quad \Gamma_{K^{\star\prime}} = 217^{+68}_{-122}, \quad \gamma = -0.065^{+0.025}_{-0.050}$$

• Results of the combined analysis

Unfolding $\tau^- \to K^- \eta \nu_\tau$ Belle's data through an "unfolding" function from $\tau^- \to K_S \pi^- \nu_\tau$



- Experimentalist: To provide unfolded data would be really useful ☺
- Theorists: To provide theoretical models to be fitted by experimentalists

- **Results of the $\tau \to K\eta\nu_\tau$ analysis**

JPP vector form factor M. Jamin, A. Pich and J. Portolés, PLB 640 (2006) 176 & 664 (2008) 78

$$f_+^{K\pi}(s) \;=\; \frac{M_{K^\star}^2}{M_{K^\star}^2 - s - iM_{K^\star}\Gamma_{K^\star}(s)} \exp\left\{\frac{3}{2} Re\left[\widetilde{H}_{K\pi}(s) + \widetilde{H}_{K\eta}(s)\right]\right\}$$

BEJ vector form factor D. R. Boito, R. Escribano and M. Jamin, EPJC 59 (2009) 821 & JHEP 09 (2010) 039

$$\widetilde{f}_+(s) \;=\; \exp\left[\alpha_1 \frac{s}{m_\pi^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_\pi^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta(s')}{(s')^3(s'-s-i0)}\right]$$

$$\delta(s) \;=\; \tan^{-1}\left[\frac{\mathrm{Im}\widetilde{f}_+(s)}{\mathrm{Re}\widetilde{f}_+(s)}\right] \qquad \widetilde{f}_+(s) \;=\; \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star},\, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}},\, \gamma_{K^{\star\prime}})}$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n Re\left[H_{K\pi}(s)\right] - i m_n \gamma_n(s)$$

$$\kappa_n \;=\; \frac{192\pi F_K F_\pi}{\sigma^3(m_n^2)} \frac{\gamma_n}{m_n}\,, \qquad \gamma_n(s) \;=\; \gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- **Results of the combined analysis**

$K^*(1410)$ MASS

Value (MeV)		Document ID		TECN	CHG	Comment
$\mathbf{1414 \pm 15}$	**OUR AVERAGE** Error includes scale factor of 1.3.					
$1380 \pm 21 \pm 19$		ASTON	1988	LASS	0	11 $K^- p \to K^- \pi^+ n$
$1420 \pm 7 \pm 10$		ASTON	1987	LASS	0	11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$
*** We do not use the following data for averages, fits, limits, etc ***						
1276^{+72}_{-77}	1, 2	BOITO	2009	RVUE		$\tau^- \to K^0_S \pi^- \nu_\tau$
1367 ± 54		BIRD	1989	LASS	-	11 $K^- p \to \overline{K}^0 \pi^- p$
1474 ± 25		BAUBILLIER	1982B	HBC	0	8.25 $K^- p \to \overline{K}^0 2\pi n$
1500 ± 30		ETKIN	1980	MPS	0	6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$

[1] From the pole position of the $K\pi$ vector form factor in the complex s-plane and using EPIFANOV 2007 data.

[2] Systematic uncertainties not estimated.

- **Results of the combined analysis**

$K^*(1410)$ WIDTH

Value (MeV)		Document ID		TECN	CHG	Comment
$\mathbf{232 \pm 21}$	**OUR AVERAGE** Error includes scale factor of 1.1.					
$176 \pm 52 \pm 22$		ASTON	1988	LASS	0	11 $K^- p \to K^- \pi^+ n$
$240 \pm 18 \pm 12$		ASTON	1987	LASS	0	11 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$
*** We do not use the following data for averages, fits, limits, etc ***						
$198\,^{+61}_{-87}$	1, 2	BOITO	2009	RVUE		$\tau^- \to K^0_S \pi^- \nu_\tau$
114 ± 101		BIRD	1989	LASS	-	11 $K^- p \to \overline{K}^0 \pi^- p$
275 ± 65		BAUBILLIER	1982B	HBC	0	8.25 $K^- p \to \overline{K}^0 2\pi n$
500 ± 100		ETKIN	1980	MPS	0	6 $K^- p \to \overline{K}^0 \pi^+ \pi^- n$

[1] From the pole position of the $K\pi$ vector form factor in the complex s-plane and using EPIFANOV 2007 data.

[2] Systematic uncertainties not estimated.

• Results of the combined analysis

Reference fit results obtained for different values of s_{cut}

$s_{\rm cut}\,(\mathrm{GeV}^2)$ / Fitted value	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{\rm th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$	$=\gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{\rm th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
$\chi^2/\mathrm{n.d.f.}$	$105.8/105$	$108.1/105$	$111.0/105$	$111.1/105$

• Results of the combined analysis

Correlation coefficients

	$\bar{B}_{K\pi}$	M_{K^*}	Γ_{K^*}	$M_{K^{*\prime}}$	$\Gamma_{K^{*\prime}}$	$\lambda'_{K\pi}$	$\lambda''_{K\pi}$	$\bar{B}_{K\eta}$	$\gamma_{K\eta} = \gamma_{K\pi}$	$\lambda'_{K\eta}$	$\lambda''_{K\eta}$
M_{K^*}	-0.163	1									
Γ_{K^*}	0.028	-0.060	1								
$M_{K^{*\prime}}$	-0.063	-0.104	-0.142	1							
$\Gamma_{K^{*\prime}}$	0.126	0.130	0.292	-0.556	1						
$\lambda'_{K\pi}$	0.800	-0.100	0.457	-0.244	0.432	1					
$\lambda''_{K\pi}$	$\boxed{0.928}$	-0.215	0.328	-0.166	0.304	0.942	1				
$\bar{B}_{K\eta}$	-0.003	-0.005	-0.010	0.003	-0.001	-0.015	-0.009	1			
$\gamma_{K\eta} = \gamma_{K\pi}$	-0.155	-0.173	-0.378	0.498	$\boxed{-0.878}$	-0.565	-0.373	0.019	1		
$\lambda'_{K\eta}$	0.058	0.028	0.117	0.050	0.337	0.182	0.128	0.434	-0.340	1	
$\lambda''_{K\eta}$	0.035	-0.017	0.037	0.106	0.218	0.080	0.064	0.561	-0.174	0.971	1

Table 3. Correlation coefficients corresponding to our reference fit with $s_{\rm cut} = 4\,{\rm GeV}^2$, second column of table 1. In the fits where $\gamma_{K\pi} = \gamma_{K\eta}$ is not enforced, their correlation coefficient turns out to be ≈ 0.67.