Applying Efimov physics to few-nucleon systems

A. Kievsky

INFN, Sezione di Pisa (Italy)

The 8th International Workshop on Chiral Dynamics
Pisa 29 June-3 July 2015

In collaboration with M. Gattobigio (INLN)

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Preliminaries: low energy $n-d$ scattering

Effective range formula

$$k\cot\delta_{nd} = \frac{-\frac{1}{a_{nd}} + \frac{1}{2}r_s k^2}{1+k^2/k_0^2}$$

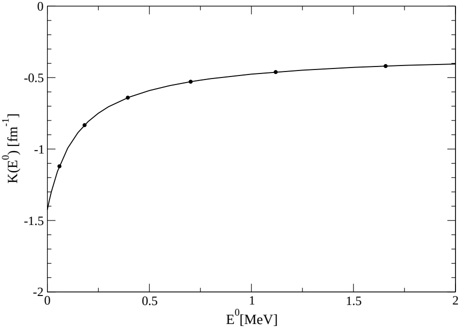
$$E_p = \frac{3}{4}(\hbar^2/m)k_0^2$$

$$a_{nd} \approx 0.7 \text{ fm}$$

$$E_p \approx 160 \text{ keV}$$

$$r_s \approx -127 \text{ fm}$$

C.R. Chen et al., PRC39, 1261 (1989)



Why there is a curvature in the effective range function?

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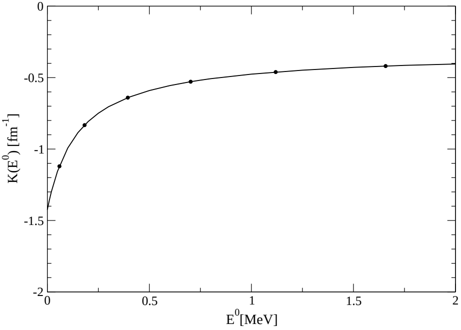


Why there is a curvature in the effective range function?

Universality in atom-dimer scattering

Efimov Theory: Zero-Range Theory for three bosons

$$E_3^n/(\hbar^2/ma^2) = \tan^2\xi$$

$$\kappa_* a = e^{\pi(n-n_*)/s_0} e^{-\Delta(\xi)/2s_0} / \cos\xi$$

- a is the two-body scattering length
- κ_* is the three-body parameter
- $\Delta(\xi)$ is an universal function

Efimov Theory: atom-dimer scattering length

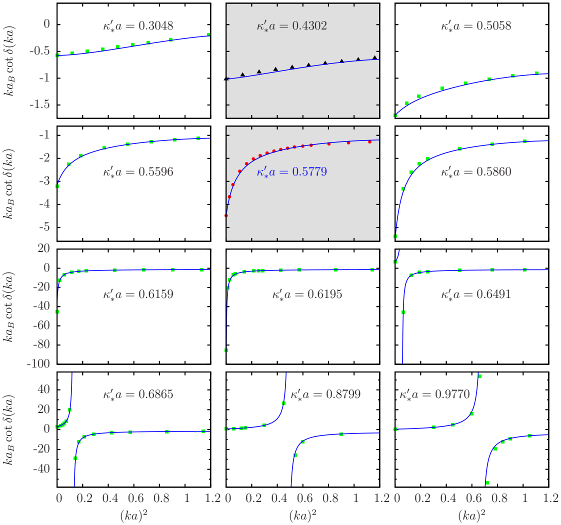
$a_{AD} = a(d_1 + d_2 \tan[s_0 \ln(\kappa_* a) + d_3])$ (Efimov 1979)
with d_1, d_2, d_3 universal constants (Braaten and Hammer, 2006)

Efimov Theory: atom dimer effective range

$k a \cot\delta_{AD} = c_1(ka) + c_2(ka)\cot[s_0 \ln(\kappa_* a) + \phi(ka)]$
with c_1, c_2, ϕ universal functions

Universal Effective Range Function

$$k\,a\cot\delta_{AD} = c_1(ka) + c_2(ka)\cot[s_0\ln(\kappa'_* a) + \phi(ka)]$$



Zero-Range vs. Finite-Range (three-body system)

zero-range	finite-range
$E_3/(\hbar^2/ma^2) = E_3/E_2 = \tan^2\xi$	$E_3/(\hbar^2/ma_B^2) = E_3/E_2 = \tan^2\xi$
$\kappa_* a = \frac{1}{\cos\xi} e^{-\Delta(\xi)/2s_0}$	$\kappa_* a_B = \frac{1}{\cos\xi} e^{-\tilde\Delta(\xi)/2s_0}$

M. G. and A. K., PRA 90, 012502 (2014)

$$\frac{1}{\cos\xi} e^{-\Delta(\xi)/2s_0} = \frac{1}{\cos\xi} e^{-\tilde\Delta(\xi)/2s_0} + \Gamma$$

or

$$2s_0\Gamma = \tan\tilde\phi - \tan\phi$$

with $\tan\phi, \tan\tilde\phi$ the derivatives of $\Delta(\xi), \tilde\Delta(\xi)$ at $\xi = -\pi/2$

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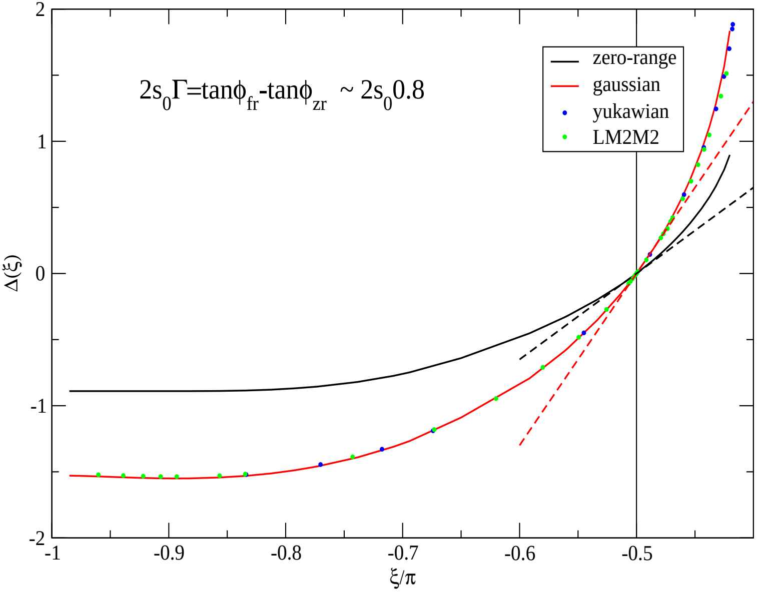


2s0Γ=tanφfr-tanφzr ~ 2s00.8
zero-range
gaussian
yukawian
LM2M2
Δ(ξ)
ξ/π
2
1
0
-1
-2
-1
-0.9
-0.8
-0.7
-0.6
-0.5

Zero-Range vs. Finite-Range (three-body system)

zero-range	finite-range
$K_3 a = K_3/K_2 = \tan\xi$	$K_3 a_B = K_3/K_2 = \tan\xi$
$\kappa_* a = \frac{1}{\cos\xi} e^{-\Delta(\xi)/2s_0}$	$\kappa_* a_B = \frac{1}{\cos\xi} e^{-\tilde\Delta(\xi)/2s_0}$

$$\frac{1}{\cos\xi} e^{-\Delta(\xi)/2s_0} = \frac{1}{\cos\xi} e^{-\tilde\Delta(\xi)/2s_0} + \Gamma$$

or

$$\kappa_*(a - a_B) = \Gamma$$

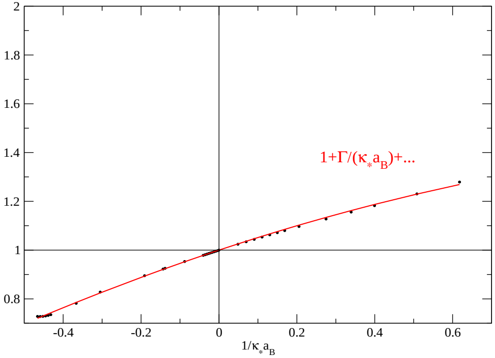
$$a - a_B = \frac{\Gamma}{\kappa_*} = r_* \approx constant \quad \text{(at equal values of } \xi\text{)}$$

Varying the depth of a potential around the unitary limit, the results can be reproduced by a two-parameter potential (as a gaussian) which produces an equivalent universal function, $\tilde\Delta(\xi)$, rotated with respect to the universal zero-range function $\Delta(\xi)$.

How constant is Γ?

$$\kappa_* a_B\left(1+\frac{\Gamma}{\kappa_* a_B}\right)=\frac{1}{\cos\xi}e^{-\Delta(\xi)/2s_0}=y(\xi)$$

$$1+\frac{r_*}{a_B}=y(\xi)/\kappa_* a_B$$



1/2-spin 1/2-isospin fermions close to the unitary limit

The 2N system in s-wave

This is a two-channel system with spin $S = 0$ and $S = 1$. For two nucleons the physical values are:

$E_d = -2.2245$ MeV, $a_B = 4.318$ fm

$a_1 = 5.424 \pm 0.003$ fm $\quad r_1^{eff} = 1.760 \pm 0.005$ fm

$a_0 = -23.740 \pm 0.020$ fm $\quad r_0^{eff} = 2.77 \pm 0.05$ fm

moving the system to the unitary limit

- The $S = 1$ channel:

  a gaussian $V_1 e^{-r^2/r_1^2}$ with V_0 and r_1 fixed to describe a_1 and a_B

  V_1 is varied: this path has the value $r_B = a_1 - a_B$ almost constant. For nuclear physics we have $r_B \approx 1.2$ fm

- The $S = 0$ channel:

  a gaussian $V_0 e^{-r^2/r_1^2}$ is used with V_0 fixed to describe a_1/a_0 constant

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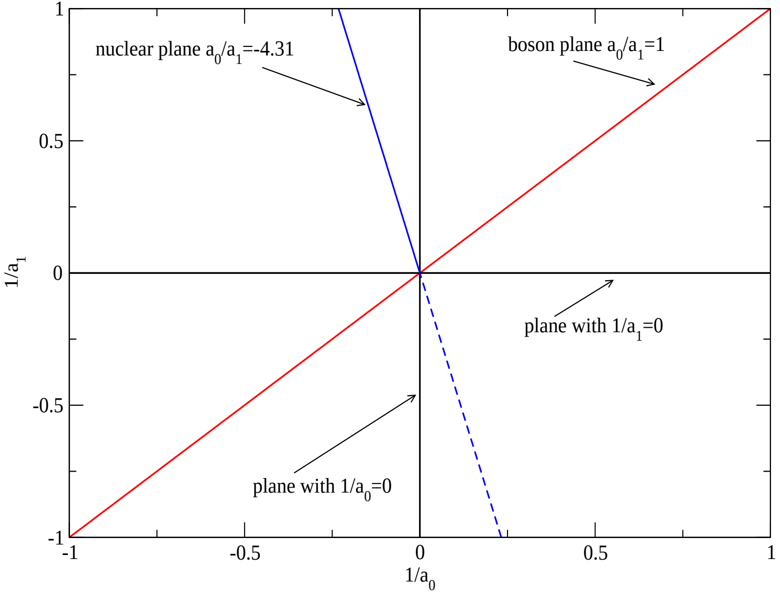


nuclear plane a_0/a_1=-4.31
boson plane a_0/a_1=1
plane with $1/a_1$=0
plane with $1/a_0$=0
$1/a_1$
$1/a_0$
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Three-body spectrum with spin-isospin symmetry

zero-range

$$K_3 a = K_3/K_2 = \tan\xi$$
$$\kappa_* a = \frac{1}{\cos\xi}\, e^{-\Delta(\xi)/2s_0}$$

finite-range

$$K_3 a_B = K_3/K_2 = \tan\xi$$
$$\kappa_* a_B = \frac{1}{\cos\xi}\, e^{-\tilde{\Delta}(\xi)/2s_0}$$

$$\frac{1}{\cos\xi} e^{-\Delta(\xi)/2s_0} = \frac{1}{\cos\xi} e^{-\tilde{\Delta}(\xi)/2s_0} - \Gamma$$

then the spectrum results

$$K_3 a = K_3/K_2 = \tan\xi$$
$$\kappa_* a_B + \Gamma = \frac{1}{\cos\xi}\, e^{-\Delta(\xi)/2s_0} = y(\xi)$$

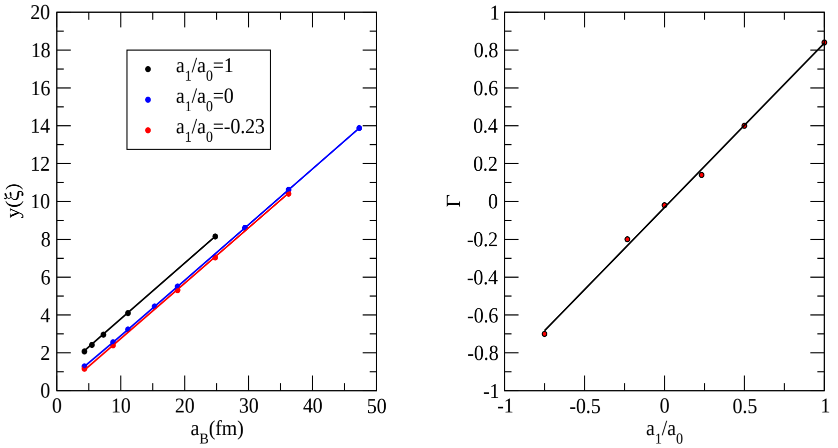
with

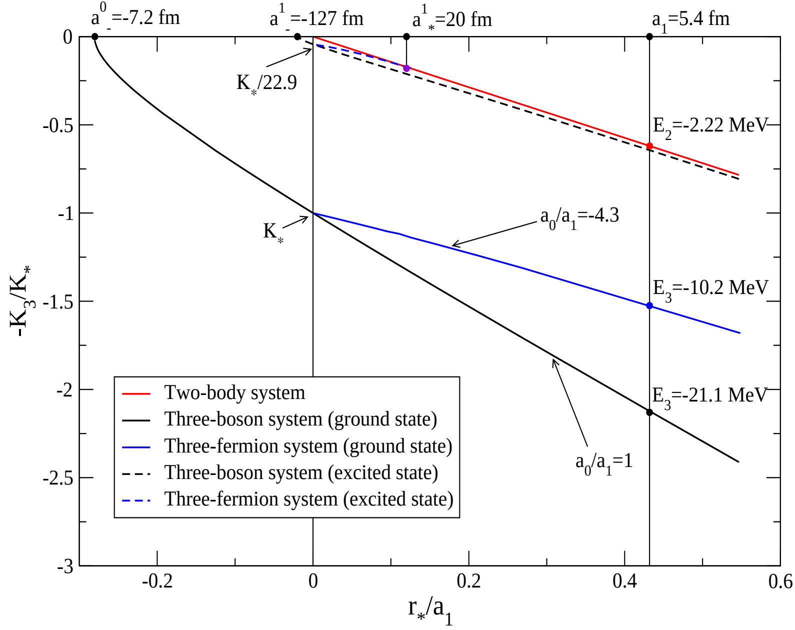
$$\Gamma = \Gamma(a_0/a_1)$$

determining Γ

for three bosons $\Gamma(1) \approx 0.8$

in the nuclear plane $\Gamma(a_0/a_1 = -4.3) \approx -0.2$


a_1/a_0=1
a_1/a_0=0
a_1/a_0=-0.23
y(ξ)
a_B(fm)
Γ
a_1/a_0



a^0_-=-7.2 fm
a^1_-=-127 fm
a^1_*=20 fm
a_1=5.4 fm
$K_*/22.9$
E_2=-2.22 MeV
a_0/a_1=-4.3
K_*
E_3=-10.2 MeV
E_3=-21.1 MeV
a_0/a_1=1
Two-body system
Three-boson system (ground state)
Three-fermion system (ground state)
Three-boson system (excited state)
Three-fermion system (excited state)
$-K_3/K_*$
r_*/a_1
0
-0.5
-1
-1.5
-2
-2.5
-3
-0.2
0
0.2
0.4
0.6

Comments on the two-channel plot

- Studying a three-boson system using finite-range potentials, the first excited state does not dispapear onto the two-body threshold
- In the two-channel system the excited state disappears on the two-body threshold as the ratio a_0/a_1 varies.
- The analysis of the nuclear plane produces a binding energy at the unitary limit of $E_u \approx 3.6$ MeV.
- However at the nuclear point the binding energy of $E_3 \approx 10.2$ MeV is far from the experimental value of 8.5 MeV
- A three-body force has to be included
- using a more realistic potential model and varying the depth, the unitary limit can be reached.
- The value obtained has been $E_u \approx 2.8$ MeV.

Working on the nuclear point

The 2N sector

Low Energy data:

$E_d = -2.2245$ MeV

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Constructing LO 2N potential

Two parameters corresponding to the $l = 0$ partial waves with $S = 0, 1$:

$V_0(r) = -V_0 e^{-r^2/r_0^2}$, $V_1(r) = -V_1 e^{-r^2/r_1^2}$

V_0[MeV]	r_0[fm]	a_0[fm]	r_0^{eff}[fm]	V_1[MeV]	r_1[fm]	a_1[fm]	r_1^{eff}[fm]
53.255	1.40	−23.741	2.094	79.600	1.40	5.309	1.622
42.028	1.57	−23.745	2.360	65.750	1.57	5.423	1.776
40.413	1.60	−23.745	2.407	63.712	1.60	5.447	1.802
37.900	1.65	−23.601	2.487	60.575	1.65	5.482	1.846
33.559	1.75	−23.745	2.644	55.036	1.75	5.548	1.930
30.932	1.82	−23.746	2.756				

Working on the nuclear point

The 3N sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	$^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	−12.40	−2.191	−2.175
42.028	1.57	65.750	1.57	−10.83	−2.199	−1.236
40.413	1.60	63.712	1.60	−10.59	−2.197	−1.097
37.900	1.65	60.575	1.65	−10.22	−2.199	−0.860
33.559	1.75	55.036	1.75	−9.584	−2.201	
30.932	1.82	65.750	1.57	−9.715		−0.285
Exp.				−8.482		0.645 ± 0.010

Introducing a Three-Body Force

We choose a simple (two-parameter) form:

$$W(\rho) = W_0 e^{-\rho^2/\rho_0^2}$$

with $\rho^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$

W_0 and ρ_0 fixed to describe $E(^3\mathrm{H})$ and $^2a_{nd}$

Working on the nuclear point

The 3N sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	$^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	-12.40	-2.191	-2.175
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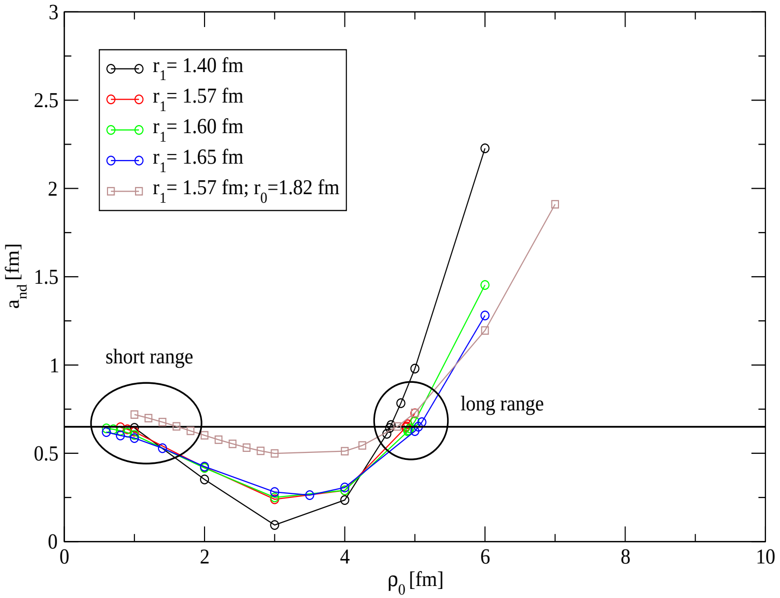
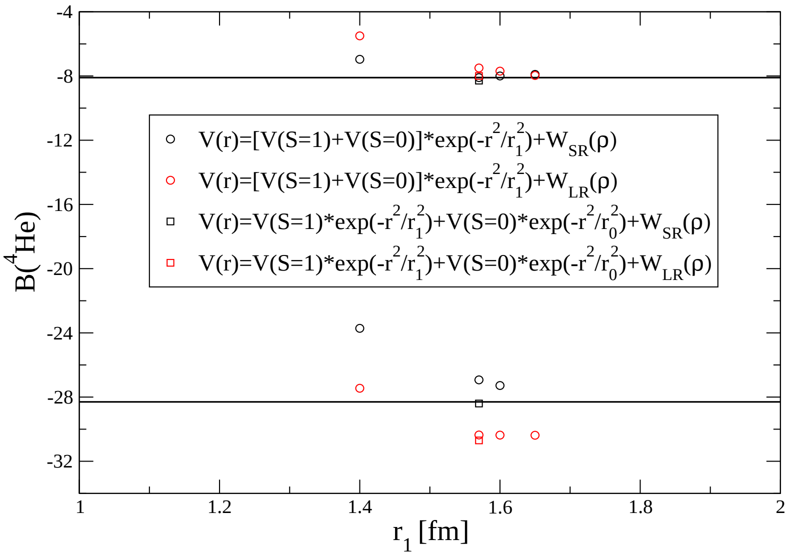
$$V(r)=[V(S=1)+V(S=0)]*\exp(-r^2/r_1^2)+W_0*\exp(-\rho^2/\rho_0^2)$$

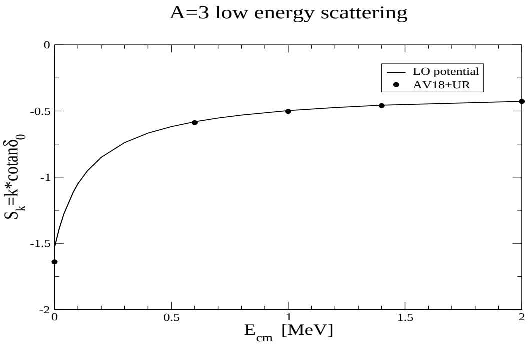


The N=4 ground and excited state



Summary of the LO potential

LO	E_d	$B(^3\mathrm{H})$	$B(^3\mathrm{He})$	$B(^3\mathrm{He}^*)$	$^2a_{nd}$
	-2.225	-8.480	-28.41	-8.29	0.652
Exp.	-2.225	-8.482	-28.296	-8.10	0.645



No bad for a 4-parameter 2N potential + 2-parameter 3N potential!
next step (in progress)→ ^{6}He and ^{6}Li ground states

Conclusions

- A path matching a physical point to the unitary limit has been analyzed
- Varying the depth of the potential the quantity $r_B = a - a_B$ remains almost constant
- Along this path different scale can be joined
- Finite-range effects have been analyzed
- Using this procedure a 1/2-spin 1/2-isospin fermion system has been studied
- A detailed study on the nuclear physics point has been performed with gaussian potentials
- Including a three-body force the doublet $n-d$ scattering length and the four-nucleon system have been studied
- Work in progress: extension to $A > 4$

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