The Magnetic Structure of Light Nuclei from Lattice QCD

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The Emergence of Nuclei from QCD

Many-Body Methods
e.g. SM, GFMC, NCSM, DFT,...

JLab        RHIC        FRIB

Solve QCD
Lattice QCD
- a Discretized Spacetime

Lattice Spacing: \( a \ll 1/\Lambda \chi \)  
(Nearly Continuum)

Lattice Volume: \( m_\pi L \gg 2\pi \)  
(Nearly Infinite Volume)

Extrapolation to \( a = 0 \) and \( L = \infty \)

Systematically remove non-QCD parts of calculation

\[
\langle \hat{\theta} \rangle \sim \int DU_\mu \ \hat{\theta}[U_\mu] \ \det[\kappa[U_\mu]] \ e^{-SYM} \rightarrow \frac{1}{N} \sum_{\text{gluon cffgs}}^N \hat{\theta}[U_\mu]
\]
Energy Scales
Dynamical Degrees of Freedom

Nucleon
~ 1 GeV
size set by pion

Nucleus
~ 250 MeV

Lattice QCD
≳ 2 GeV

pionful EFT
ρ mass/2
~ 390 MeV

pionless EFT
~ 70 MeV
The results of a quenched Lattice QCD calculation of the $\pi$, $\Lambda$, and $H$-dibaryon correlation functions. The gauge-field configuration was generated with the DBW2 gauge action on a lattice with 16 sites in each spatial direction, 32 sites in the temporal direction and a lattice spacing of approximately 0.12 fermis. The masses of the light quarks were chosen to produce a pion mass of $m_\pi \sim 350$ MeV and a kaon mass of $m_K \sim 490$ MeV. The colors of the background show the (Gaussian-smeared) local action density, while the black contours are a topographical map of the given correlation function.
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Lattice QCD: Statistics of Correlation Functions

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Extensive study of s-shell nuclei and hypernuclei, and baryon-baryon interactions at SU(3) symmetric point
Deuteron appears to be unnatural but not finely-tuned ?? 
Generic feature of YM with $n_f=3$

$m_\pi \sim 800 \text{ MeV}$
Light Nuclei: Quark Mass Effects

Current production

Preliminary

Current production

Bd (MeV) vs mπ (MeV)

NPLQCD, anisotropic
Yamazaki et al.
NPLQCD, isotropic

3He: B (MeV) vs mπ (MeV)

NPLQCD
Yamazaki et al.
Magnetic Moments of Light Nuclei from Lattice Quantum Chromodynamics

Ab Initio Calculation of the np → dγ Radiative Capture Process
*arXiv:1505.02422*

The Magnetic Structure of Light Nuclei
*arXiv:1506.05518*
The Structure of Nuclei: Uniform, Time-independent Magnetic Fields

\[ U_1^{(Q)}(x) = \begin{cases} 
  1 & \text{for } x_1 \neq L - a \\
  \exp \left[ -i q \tilde{\alpha} \frac{6 \pi x_2}{L} \right] & \text{for } x_1 = L - a
\end{cases} \]

\[ U_2^{(Q)}(x) = \exp \left[ i q \tilde{\alpha} \frac{6 \pi a x_1}{L^2} \right] \]

\[ U_3^{(Q)}(x) = 1 \]

\[ U_4^{(Q)}(x) = 1 \]
Post-multiply QCD gauge-fields with U(1) gauge-field that produces a uniform, time-independent magnetic field - magnetic field NOT in gauge generation - i.e. no disconnected diagrams

At SU(3) point:
Magnetic moments are exact

Away from SU(3) point:
Isovector are exact
Isoscalar missing disconnected contributions
The Structure of Nuclei:
Background Magnetic Fields: $B^2$

At SU(3) point:
Isovector Magnetic Polarizabilities are exact
Isoscalar missing disconnected contributions

Away from SU(3) point:
All Magnetic Polarizabilities missing disconnected contributions
Landau levels present for charged particles contaminate the extraction of polarizabilities.
Constructing hadronic blocks to form nuclei is no longer optimal but remains doable at these masses

- neutron does not see Landau levels - momentum eigenstates
- proton sees level with $Q=1$ and $M=M_{\text{proton}}$
- triton sees level with $Q=1$ and $M=3 \times M_{\text{proton}}$
Magnetic Moments
Neutron Spin States

\( |eB| \sim 0.05 \text{ GeV}^2 \)

\( |eB| \sim 0.7 \text{ GeV}^2 \)

\( \sim 10^{20} \text{ Gauss} \)

\( 400 \text{ MeV} \)
Magnetic Moments
Neutron Spin States

- Lower state depends essentially linearly on $B$
- Polarizability results from upper level (essentially)
- Spin-dependences highly correlated

$|eB| \sim 0.7 \text{ GeV}^2$
$\sim 10^{20} \text{ Gauss}$

400 MeV
Magnetic Moments
Proton Spin States
Magnetic Moments

Magnetic moments of light nuclei from lattice quantum chromodynamics
Published in Phys.Rev.Lett. 113 (2014) 25, 252001
e-Print: arXiv:1409.3556 [hep-lat]

\[
\frac{e}{2M(m_\pi)}
\]

\[m_\pi \sim 800 \text{ MeV} \quad \text{Vs} \quad \text{Nature}\]
Magnetic Moments

Magnetic moments of light nuclei from lattice quantum chromodynamics
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\[ \frac{e}{2M(m_\pi)} \]

\[ m_\pi \sim 800 \text{ MeV} \quad \text{Vs} \quad \text{Nature} \]
Essentially ALL quark mass dependence of nucleon magnetic moments is due to the nucleon mass
Radiative Capture: 

\[ np \rightarrow d \gamma \]

\[ \mathcal{L} = \frac{e}{2M_N} N^\dagger \left[ \kappa_0 + \kappa_1 \tau^3 \right] \mathbf{\Sigma} \cdot \mathbf{B} N - \frac{e}{M_N} \left( \kappa_0 - \frac{\tilde{l}_2}{r_3} \right) i \epsilon_{ijk} t_i^\dagger t_j B_k \]

\[ + \frac{e}{M_N} \frac{l_1}{\sqrt{r_1 r_3}} \left[ t_j^\dagger s_3 B_j + \text{h.c.} \right] , \]
Radiative Capture:

\[ np \rightarrow d\gamma \]

Electroweak Matrix Elements in the Two-Nucleon Sector from Lattice QCD
William Detmold and MJS,

\[
\begin{align*}
\left[ p \cot \delta_1 - \frac{S_+ + S_-}{2\pi L} \right] \left[ p \cot \delta_3 - \frac{S_+ + S_-}{2\pi L} \right] &= \left[ \frac{|eB|l_1}{2} + \frac{S_+ - S_-}{2\pi L} \right]^2 \\
S_{\pm} &\equiv S \left( \frac{L^2}{4\pi^2} (p^2 \pm |eB|\kappa_1) \right)
\end{align*}
\]

\[
\Delta E_{3S_1,1S_0} = \pm Z_d^2 (\kappa_1 + \gamma_0 l_1) \frac{|eB|}{M} + \ldots = \mp (\kappa_1 + \bar{L}_1) \frac{|eB|}{M} + \ldots
\]
Radiative Capture:

\[ np \rightarrow d\gamma \]

Ab Initio Calculation of the \( np \rightarrow d\gamma \) Radiative Capture Process

NPLQCD, arXiv:1505.02422

Postdiction at the physical point (verification):

\[
\sigma^{lqcd} = 332.4 \pm 5.4 \text{ mb} \quad \nu = 2,200 \text{ m/s}
\]

\[
\sigma^{\text{exp}}(np \rightarrow d\gamma) = 334.2(0.5) \text{ mb}
\]
The Structure of Nuclei: Polarizabilities

The Magnetic Structure of Light Nuclei
NPLQCD, arXiv:1506.05518
The Structure of Nuclei:

Polarizabilities

Large isovector nucleon polarizability

Nuclear polarizabilities are similar to proton polarizability
Increasing B tends to dissociate dineutron
- if trend survives to physical point then neutron stars do not want to spontaneously generate B-fields

Possible Feshbach resonance at the physical point - system with infinite scattering length

Deuteron similar
Closing Remarks

Lattice QCD is revealing interesting magnetic properties of nucleons and light nuclei
END