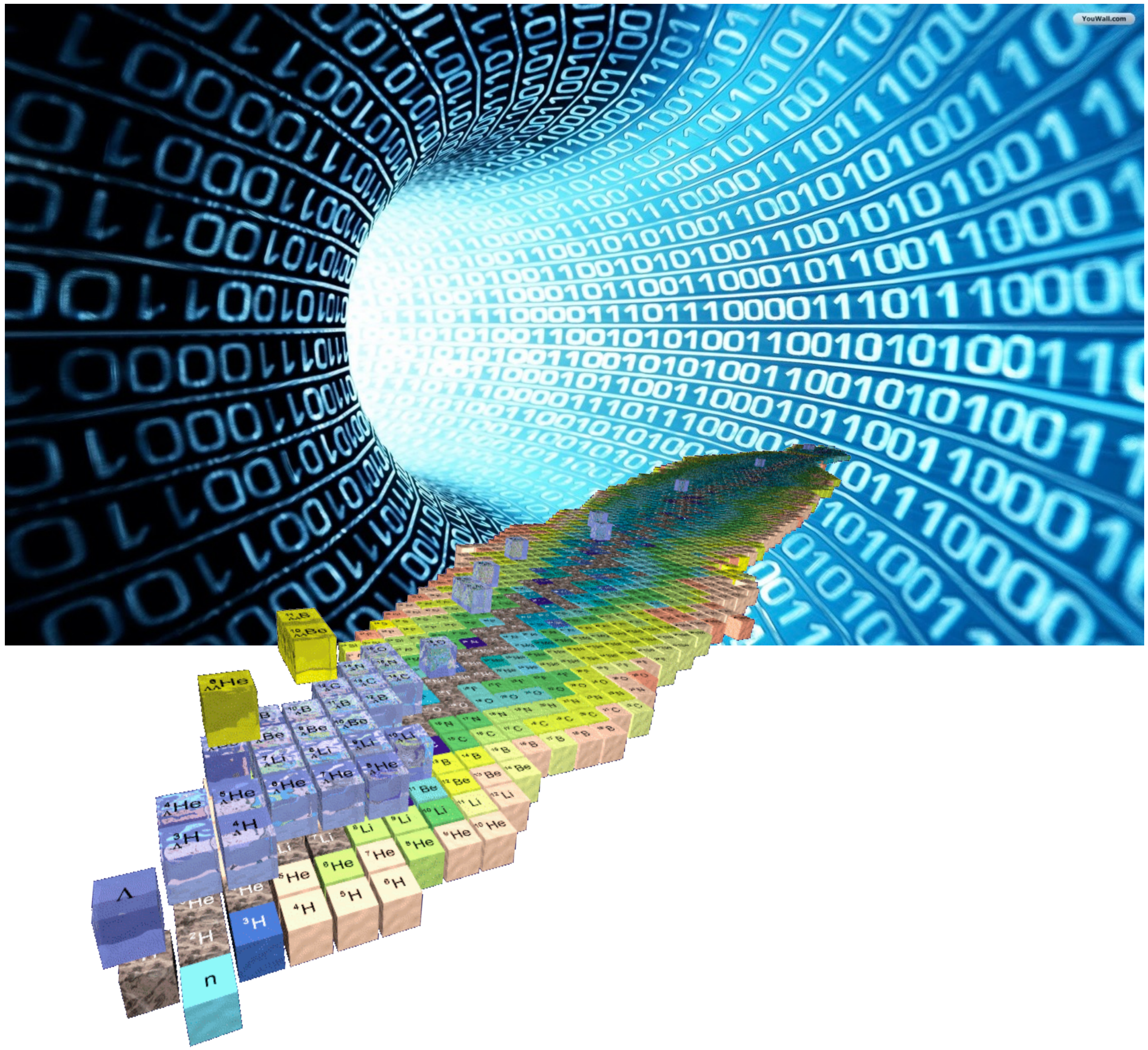
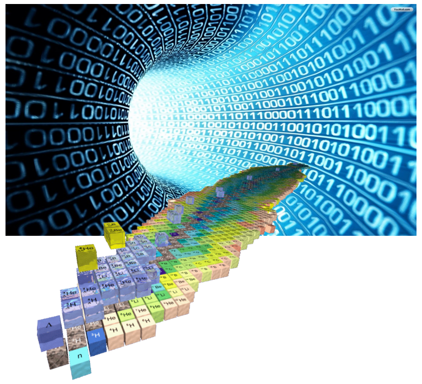


The Magnetic Structure of Light Nuclei from Lattice QCD

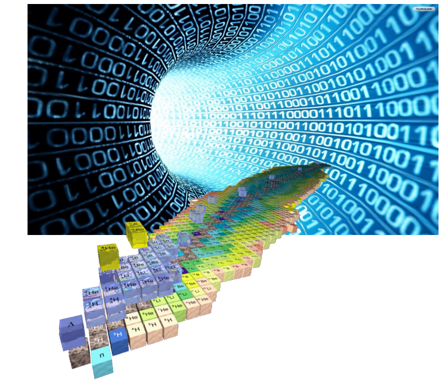
The 8th International Workshop on Chiral Dynamics 2015
Pisa, Italy, June 2015

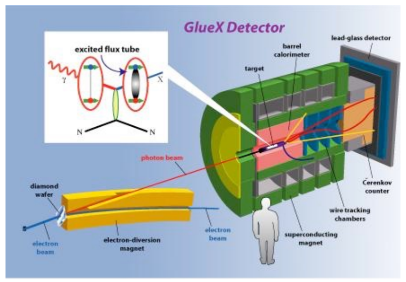
Martin J Savage

INSTITUTE for NUCLEAR THEORY

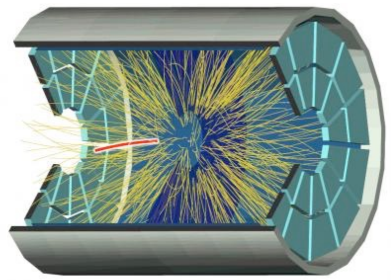


The Emergence of Nuclei from QCD

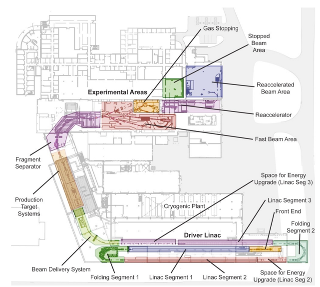




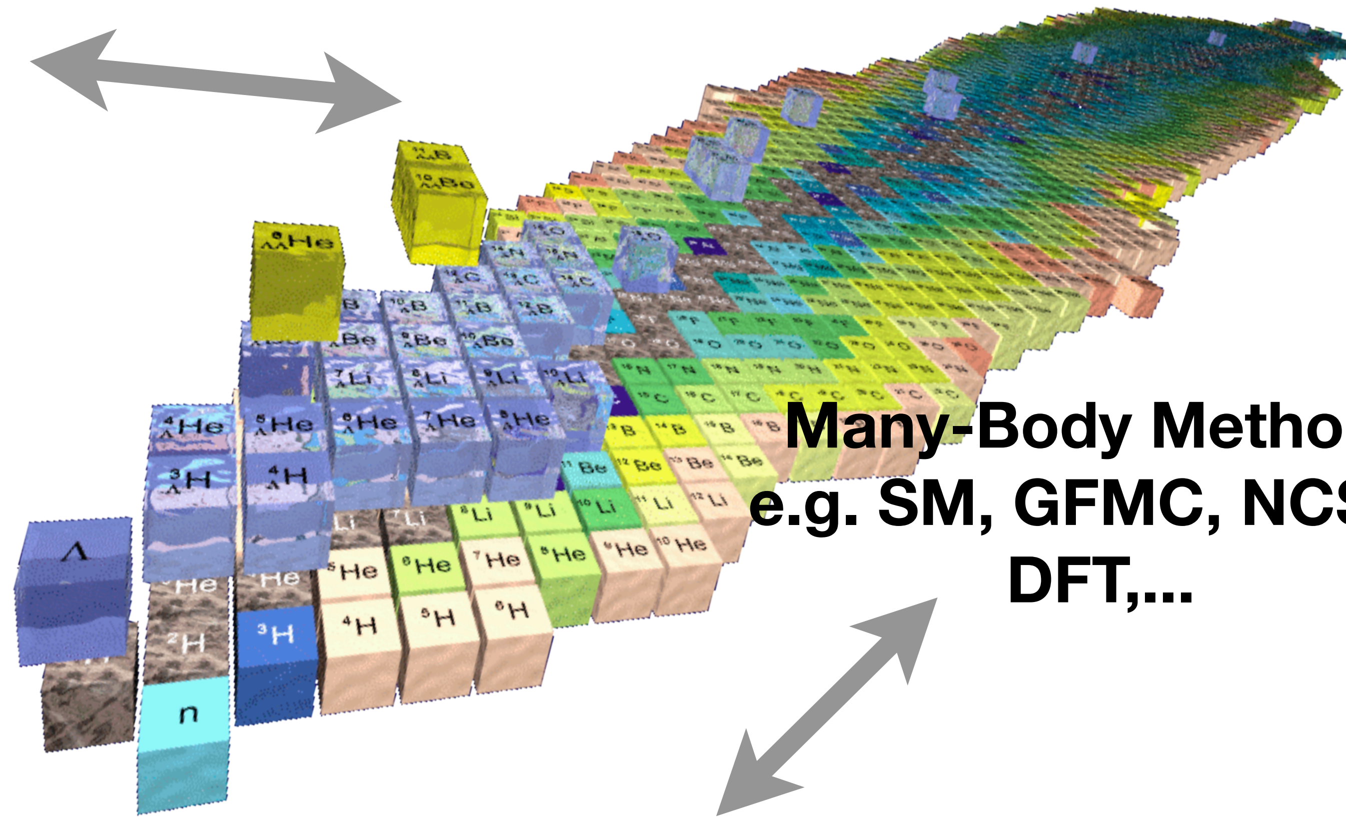
JLab



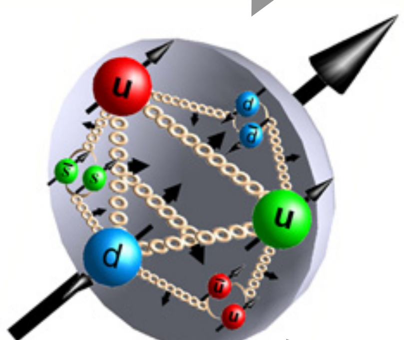
RHIC

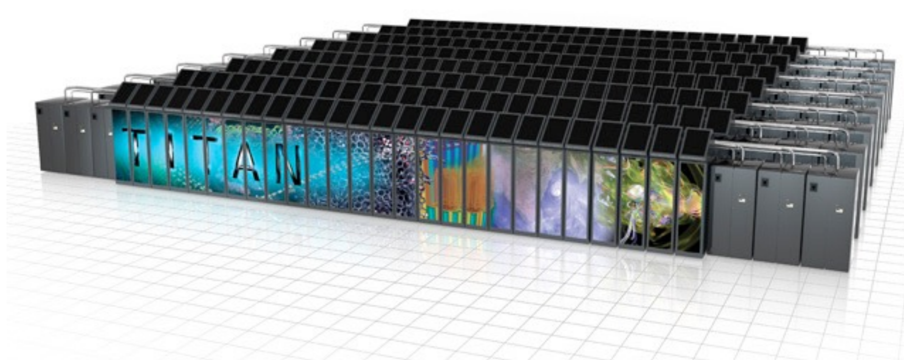


FRIB

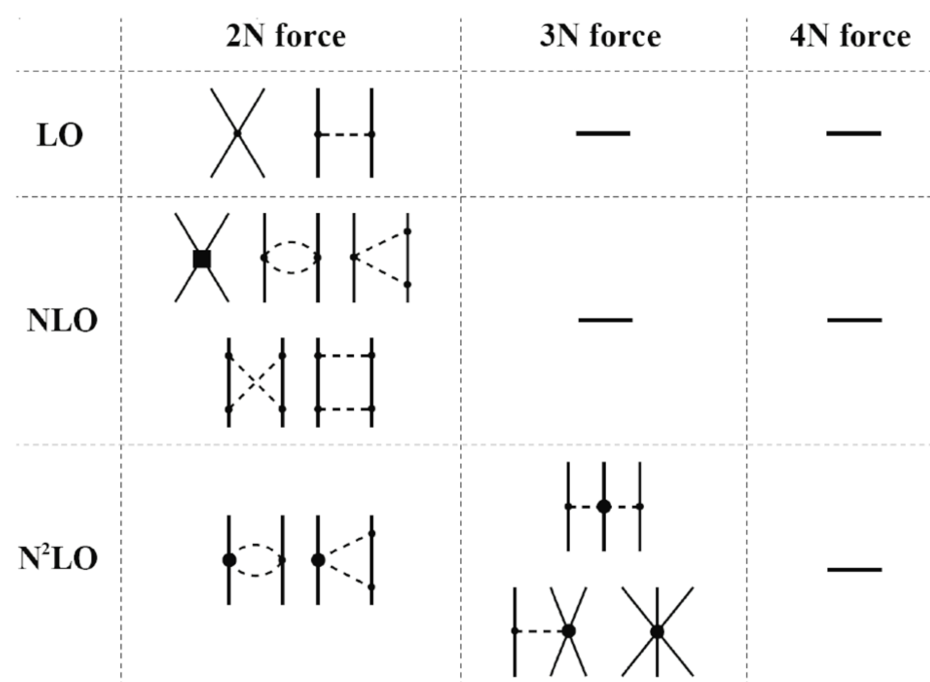


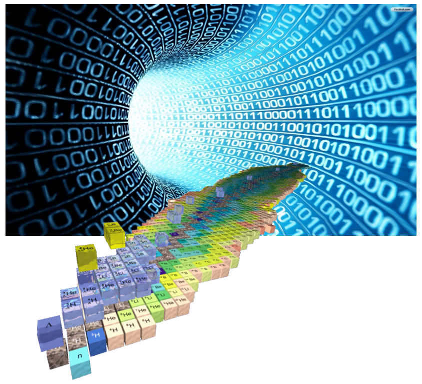
Many-Body Methods e.g. SM, GFMC, NCSM, DFT,...





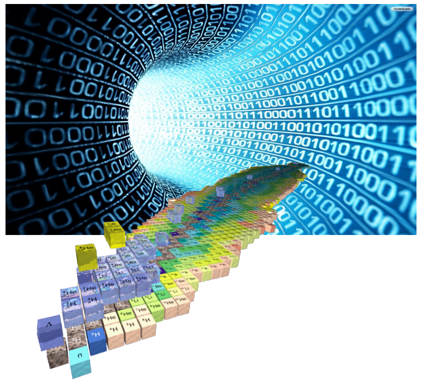
Solve QCD

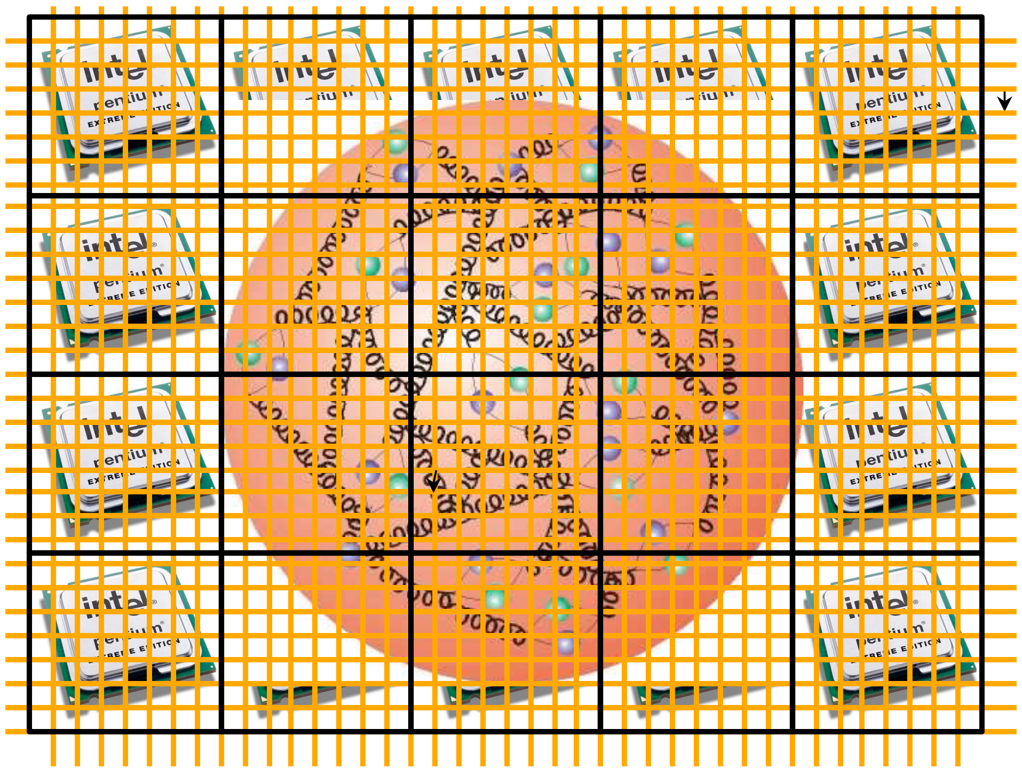




Lattice QCD

- a Discretized Spacetime





Lattice Spacing :

$a << 1/\Lambda\chi$

(Nearly Continuum)

Lattice Volume :

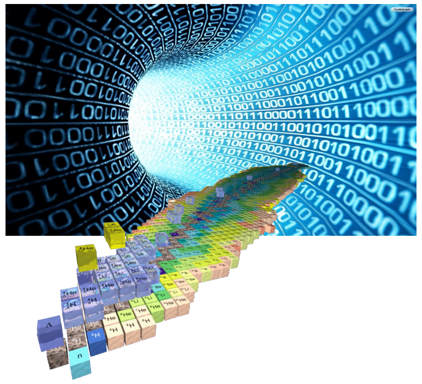
$m_\pi L >> 2\pi$

(Nearly Infinite Volume)

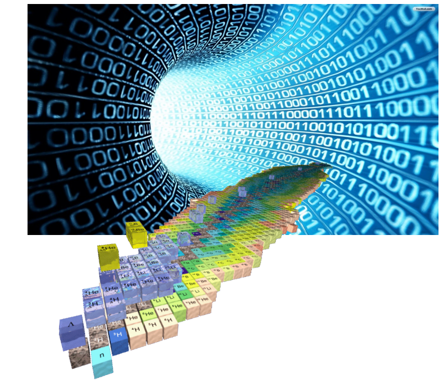
Extrapolation to $a = 0$ and $L = \infty$

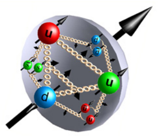
Systematically remove non-QCD parts of calculation

$$\langle \hat{\theta} \rangle \sim \int \mathcal{D}\mathcal{U}_\mu \, \hat{\theta}[\mathcal{U}_\mu] \, \det[\kappa[\mathcal{U}_\mu]] \, e^{-S_{YM}} \quad \rightarrow \frac{1}{N} \sum_{\text{gluon cfgs}}^{N} \hat{\theta}\,[\mathcal{U}_\mu]$$



Energy Scales
Dynamical Degrees of Freedom

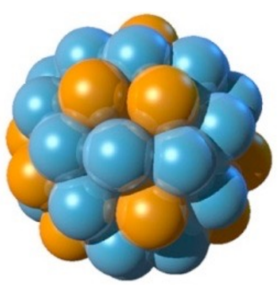




Nucleon

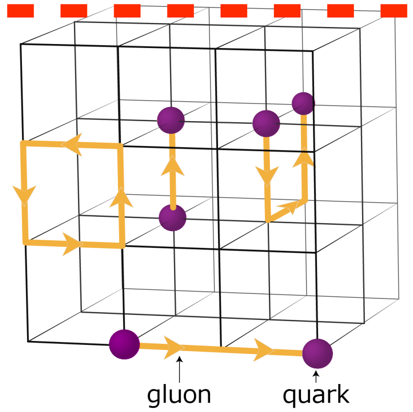
~ 1 GeV

size set by pion



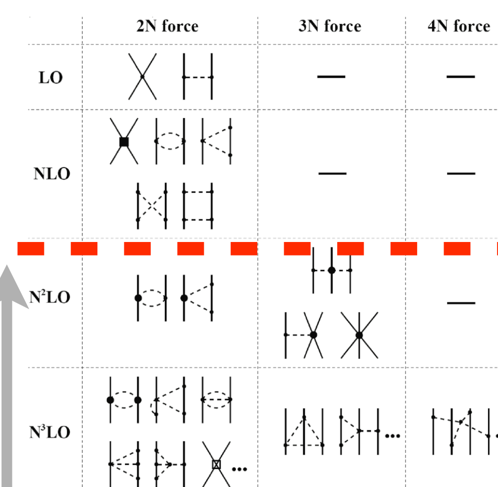
Nucleus

~ 250 MeV



Lattice QCD

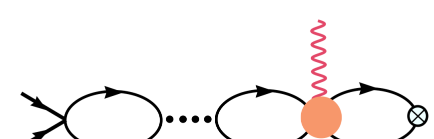
≳ 2 GeV



pionful EFT

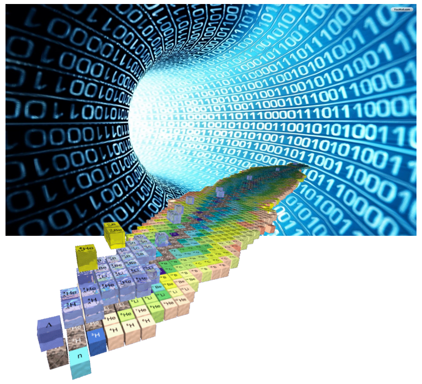
ρ mass/2

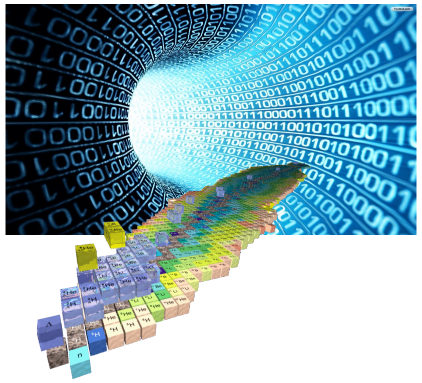
~ 390 MeV



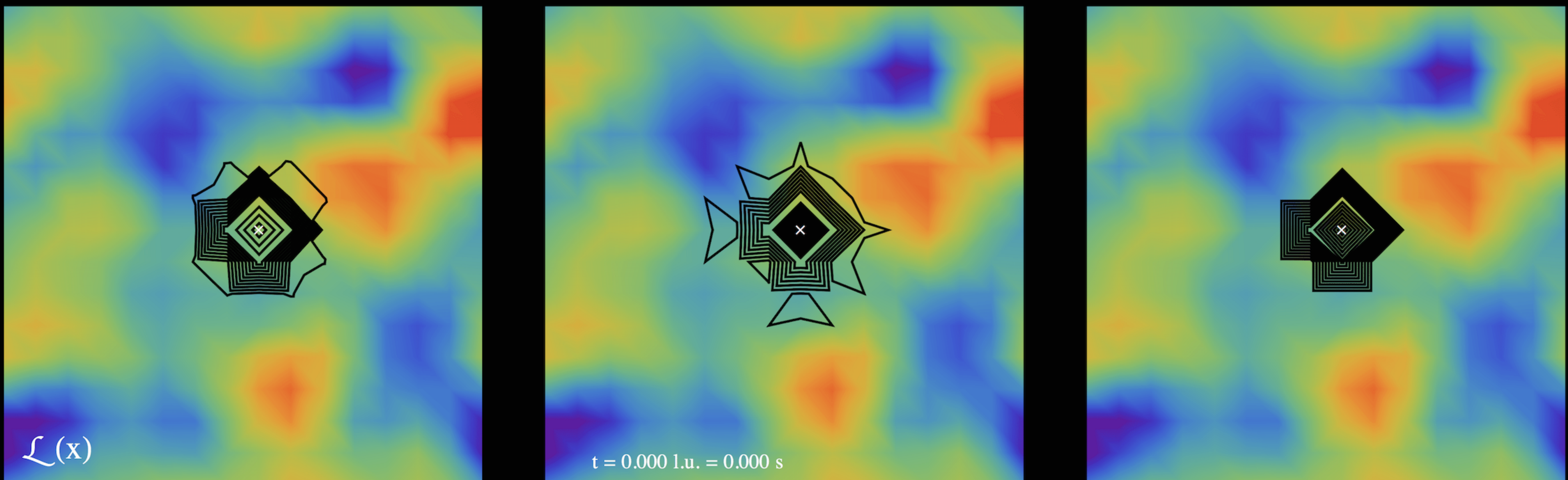
pionless EFT

~ 70 MeV





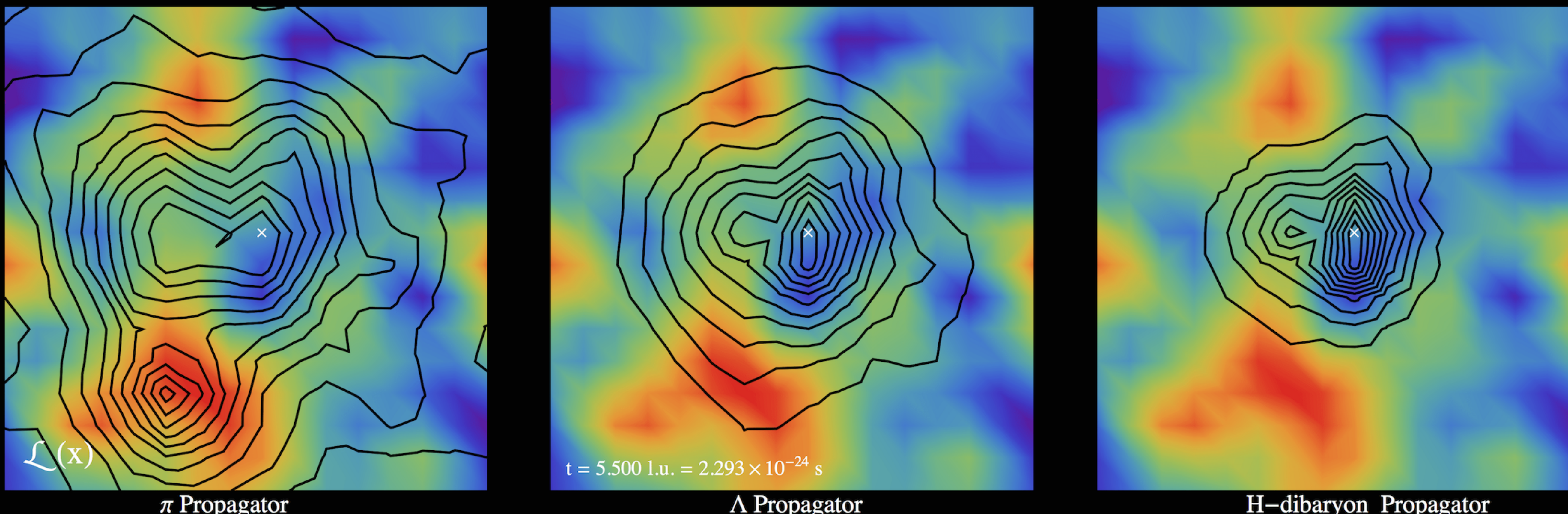
Lattice QCD:
Statistics of Correlation Functions



π Propagator | Λ Propagator | H−dibaryon Propagator

The results of a quenched Lattice QCD calculation of the π , Λ , and H−dibaryon correlation functions. The gauge−field configuration was generated with the DBW2 gauge action on a lattice with 16 sites in each spatial direction, 32 sites in the temporal direction and a lattice spacing of approximately 0.12 fermis. The masses of the light quarks were chosen to produce a pion mass of $m_\pi \sim$ 350 MeV and a kaon mass of $m_K \sim$ 490 MeV.
The colors of the background show the (Gaussian−smeared) local action density, while the black contours are a topographical map of the given correlation function.

Lattice QCD:
Statistics of Correlation Functions



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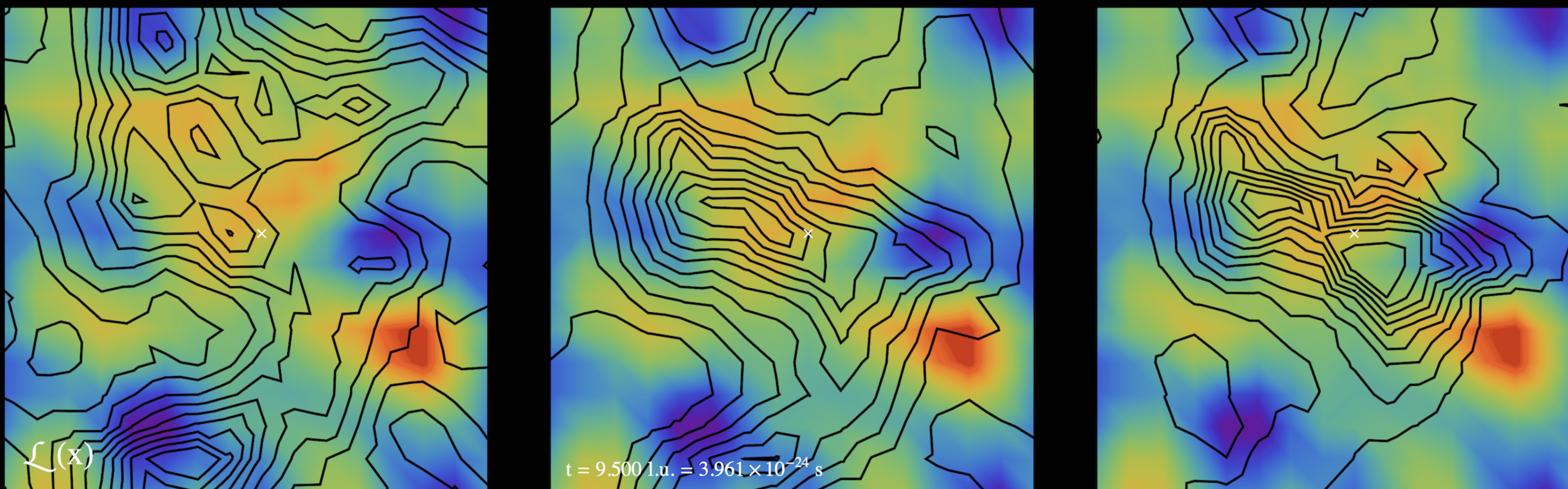
NPLQCD

Lattice QCD:
Statistics of Correlation Functions



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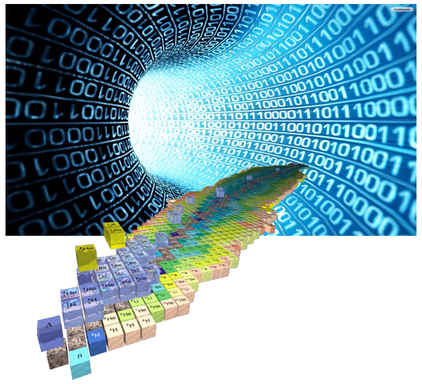
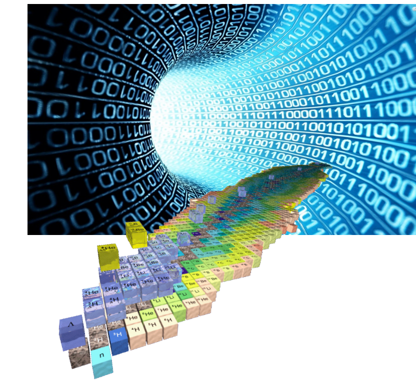
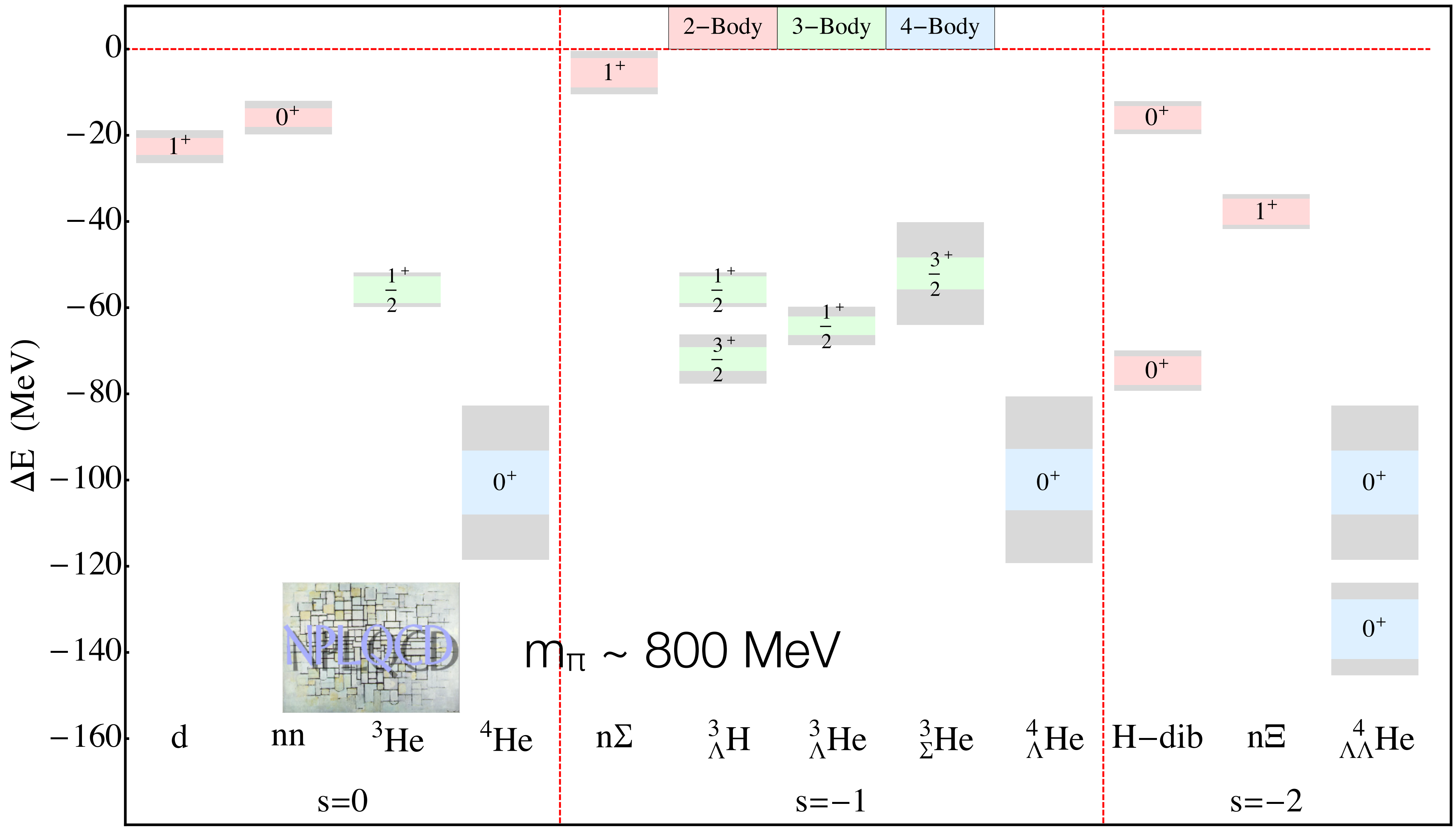


Nuclei from QCD

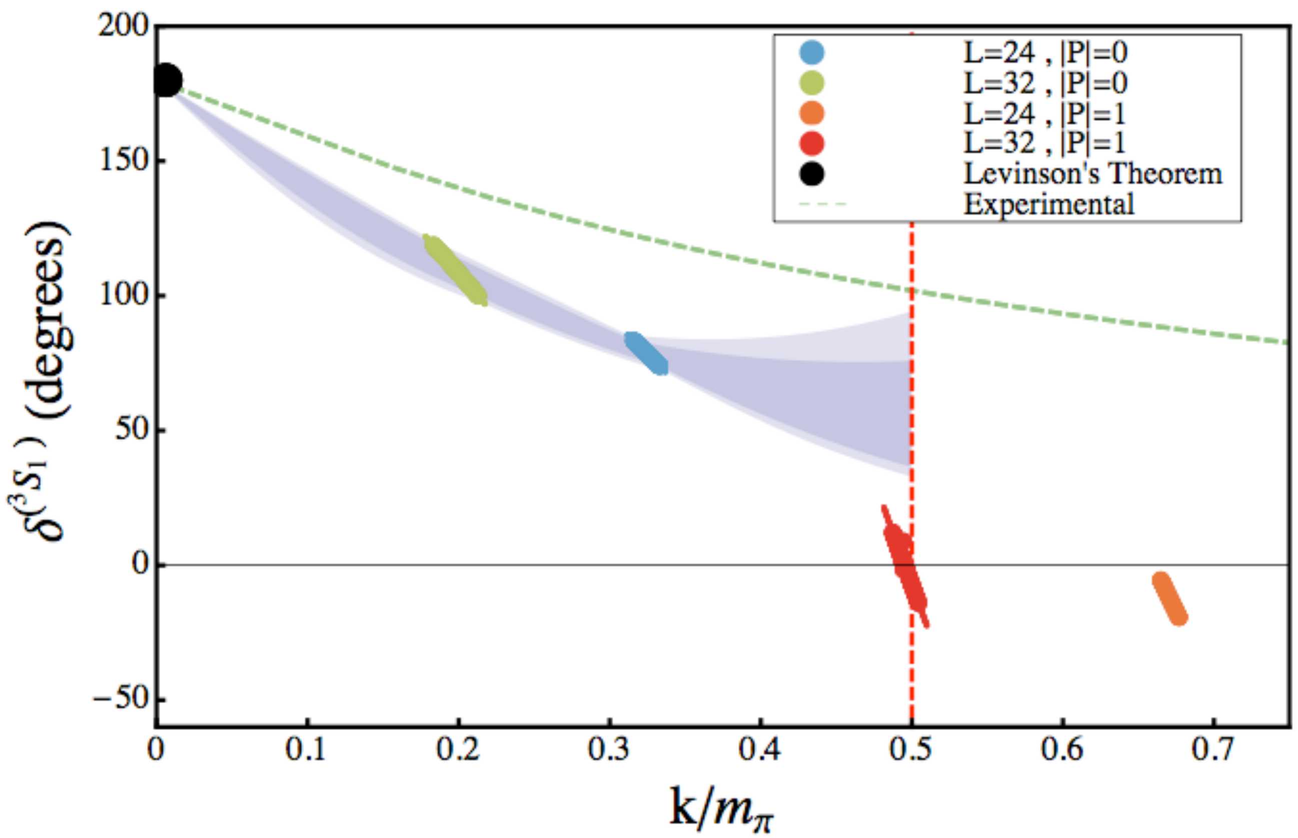


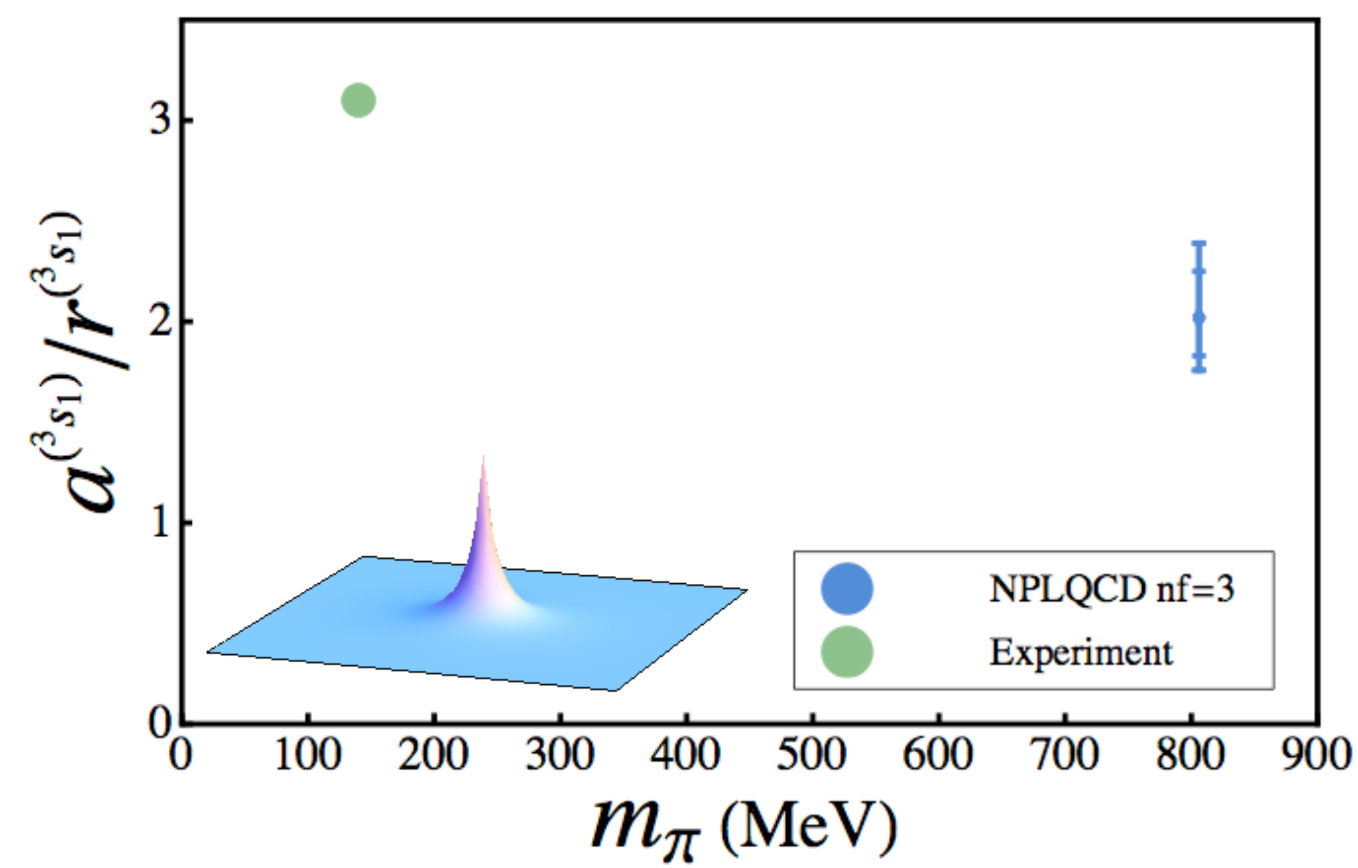
Beane *et al*, Phys.Rev. D87 (2013) 3, 034506, Phys.Rev. C88 (2013) 2, 024003



Extensive study of s-shell nuclei and hypernuclei, and baryon-baryon interactions at SU(3) symmetric point

NN Interactions

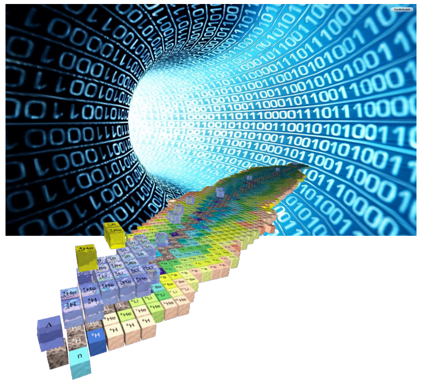


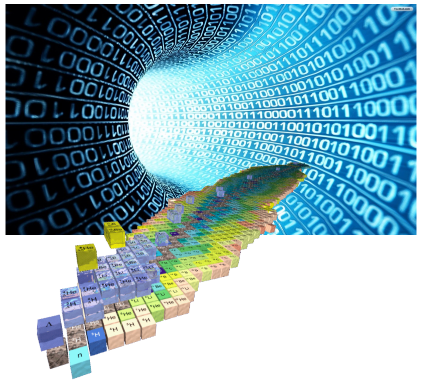


$m_\pi \sim 800$ MeV

Deuteron appears to be unnatural but not finely-tuned ??
Generic feature of YM with $n_f=3$

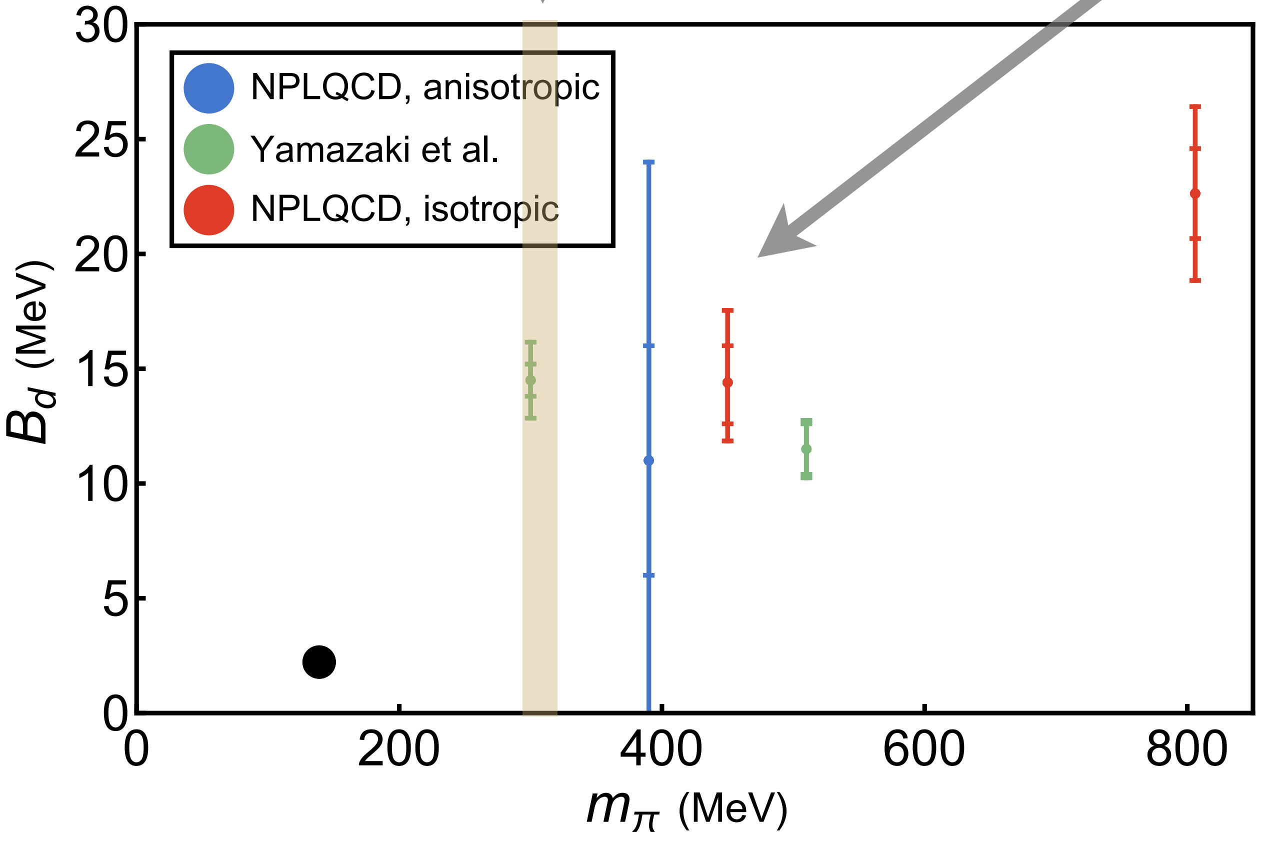
Light Nuclei : Quark Mass Effects

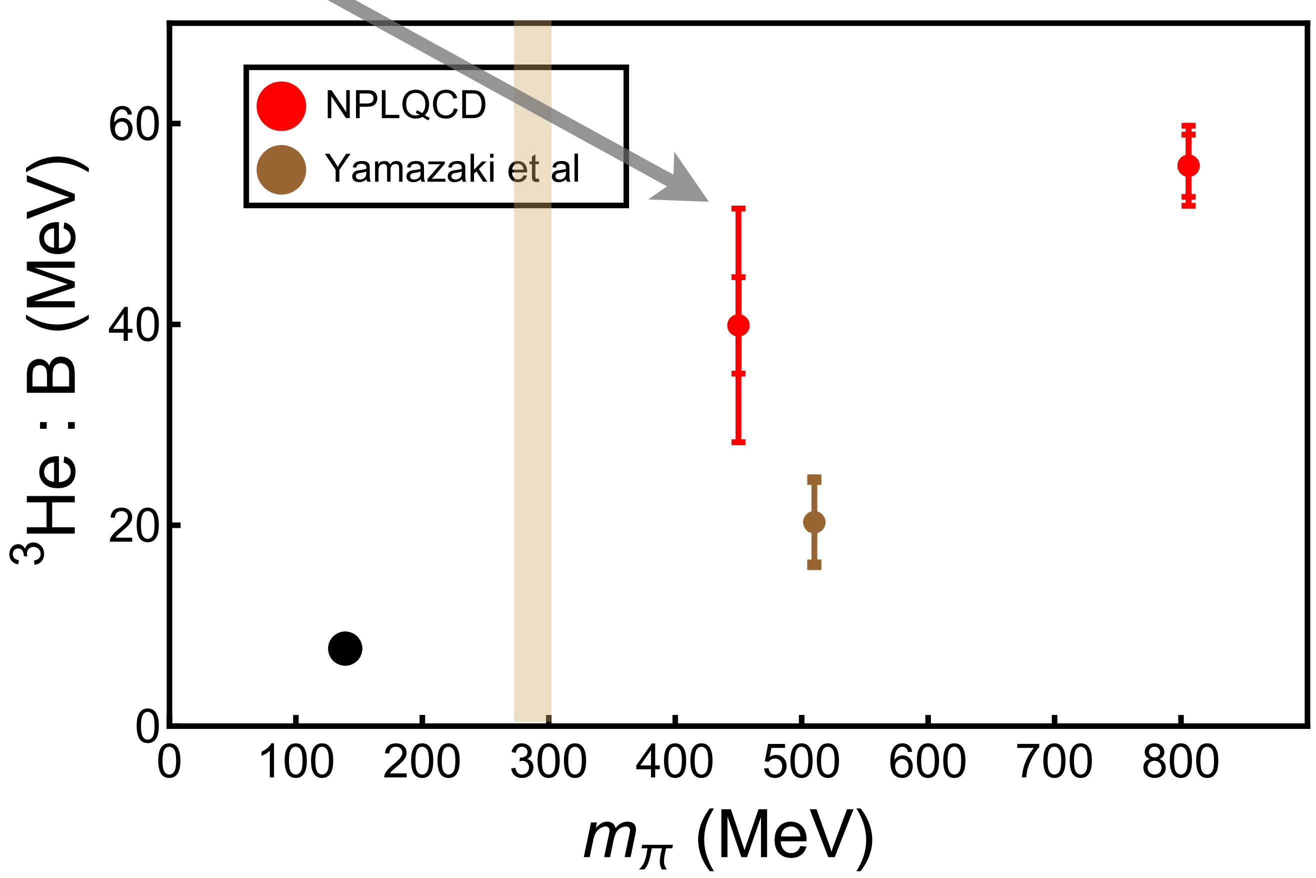




current production

preliminary





Light Nuclei in Magnetic Fields

Silas Beane
Emmanuel Chang
Saul Cohen
William Detmold
Huey-Wen Lin
Kostas Orginos
Assumpta Parreno
Martin Savage
Brian Tiburzi

Magnetic Moments of Light Nuclei from Lattice Quantum Chromodynamics

Phys. Rev. Lett. 113 (2014) 25, 252001 arXiv:1409.3556

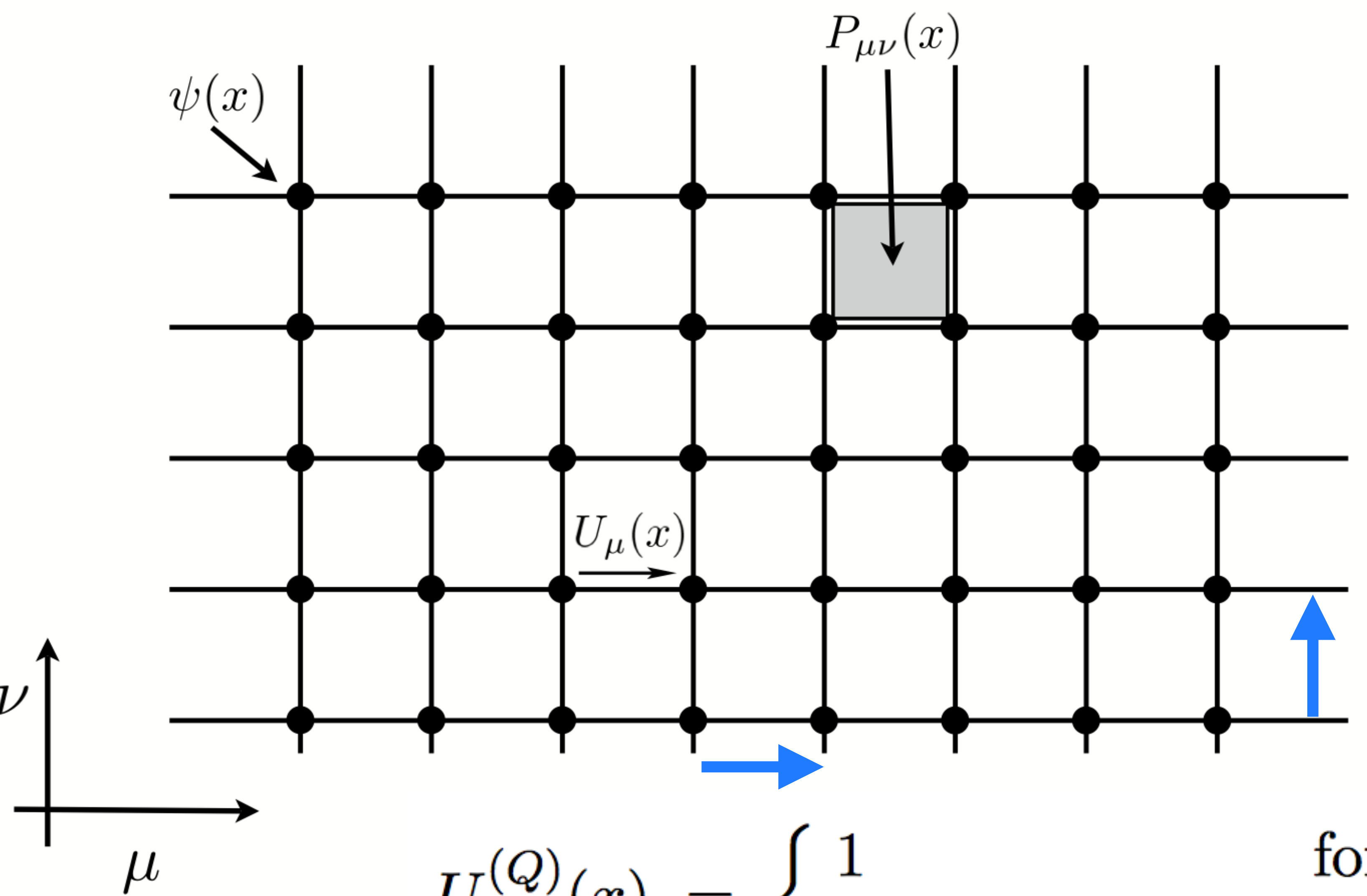
Ab Initio Calculation of the np → dγ Radiative Capture Process

arXiv:1505.02422

The Magnetic Structure of Light Nuclei

arXiv:1506.05518

The Structure of Nuclei : Uniform, Time-independent Magnetic Fields



$$U_2^{(Q)}(x) \; = \; \exp\left[i q_q \tilde{n} \frac{6\pi a\, x_1}{L^2} \right]$$

$$U_1^{(Q)}(x) \; = \; \begin{cases} 1 & \text{for } \; x_1 \neq L - a \\ \exp\left[-i q_q \tilde{n} \frac{6\pi x_2}{L} \right] & \text{for } \; x_1 = L - a \end{cases}$$

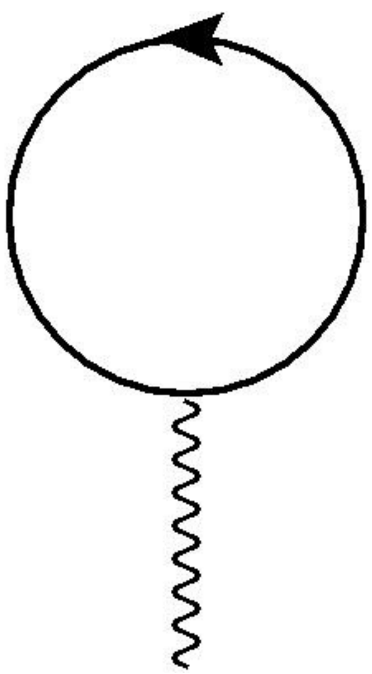
$$U_3^{(Q)}(x) \; = \; 1$$

$$U_4^{(Q)}(x) \; = \; 1$$

The Structure of Nuclei : Background Magnetic Fields : B[1]

Post-multiply QCD gauge-fields with U(1) gauge-field that produces a uniform, time-independent magnetic field

- magnetic field NOT in gauge generation - i.e. no disconnected diagrams



I=0

Tr[Q]=0

vanishes in SU(3) limit

At SU(3) point :

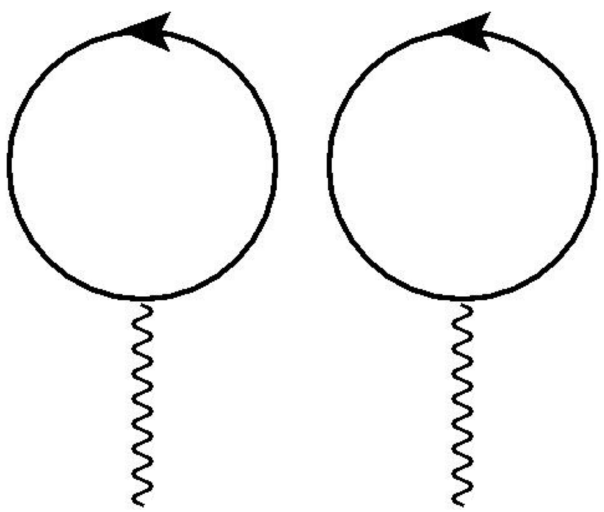
Magnetic moments are exact

Away from SU(3) point:

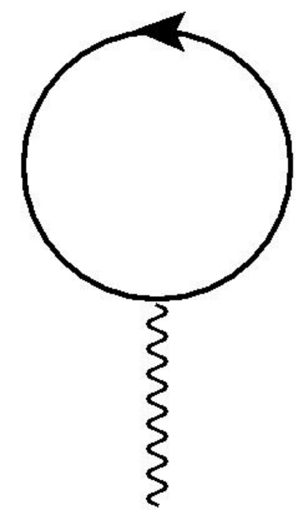
Isovector are exact

Isoscalar missing disconnected contributions

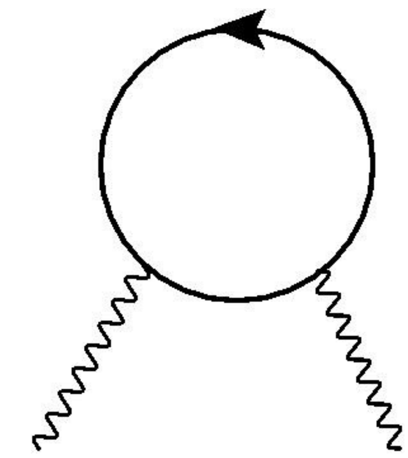
The Structure of Nuclei : Background Magnetic Fields : B^2



I=0
Tr[Q]=0



I=0
Tr[Q]=0



I=0
$Tr[Q^2] \neq 0$

At SU(3) point :

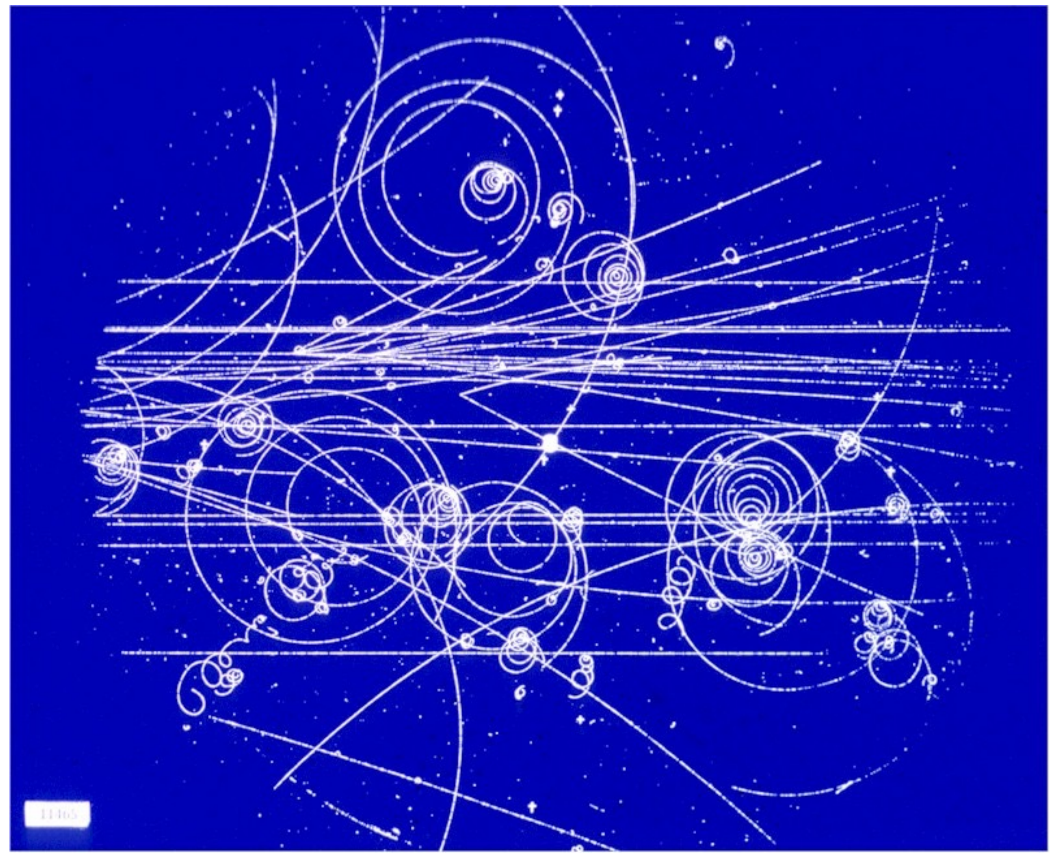
Isovector Magnetic Polarizabilities are exact

Isoscalar missing disconnected contributions

Away from SU(3) point:

All Magnetic Polarizabilities missing disconnected contributions

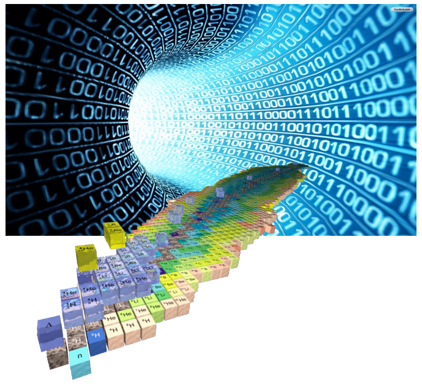
Magnetic Moments
Expectations and Landau Levels



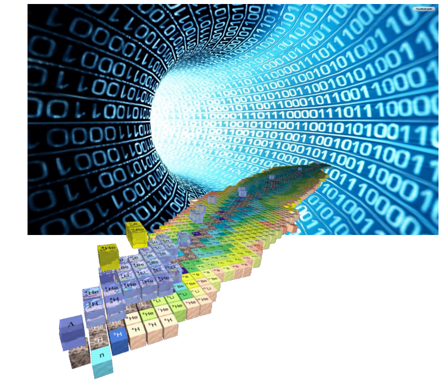
$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + P_{\|}^2 + (2n_L+1)|Q_h e\mathbf{B}|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j\rangle + \ldots$$

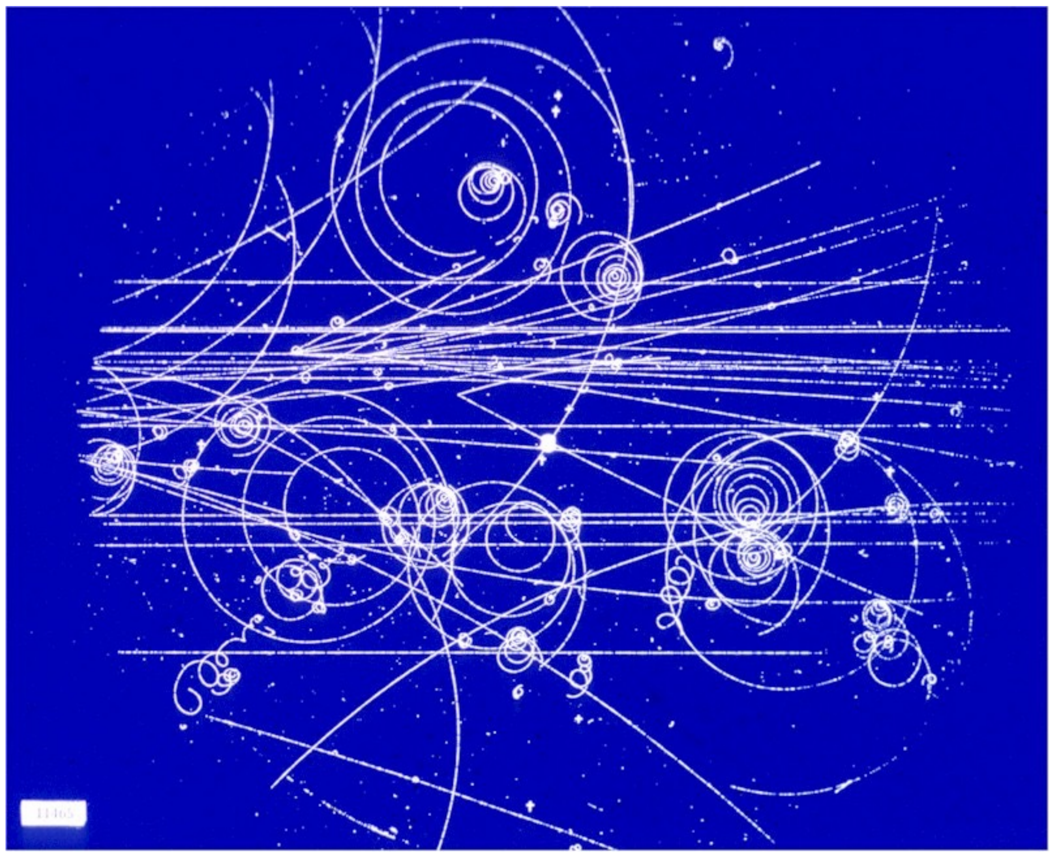
$$\hat{T}_{ij} \;=\; \frac{1}{2}\left[\, \hat{J}_i\hat{J}_j + \hat{J}_j\hat{J}_i - \frac{2}{3}\delta_{ij}\hat{J}^2 \,\right]$$

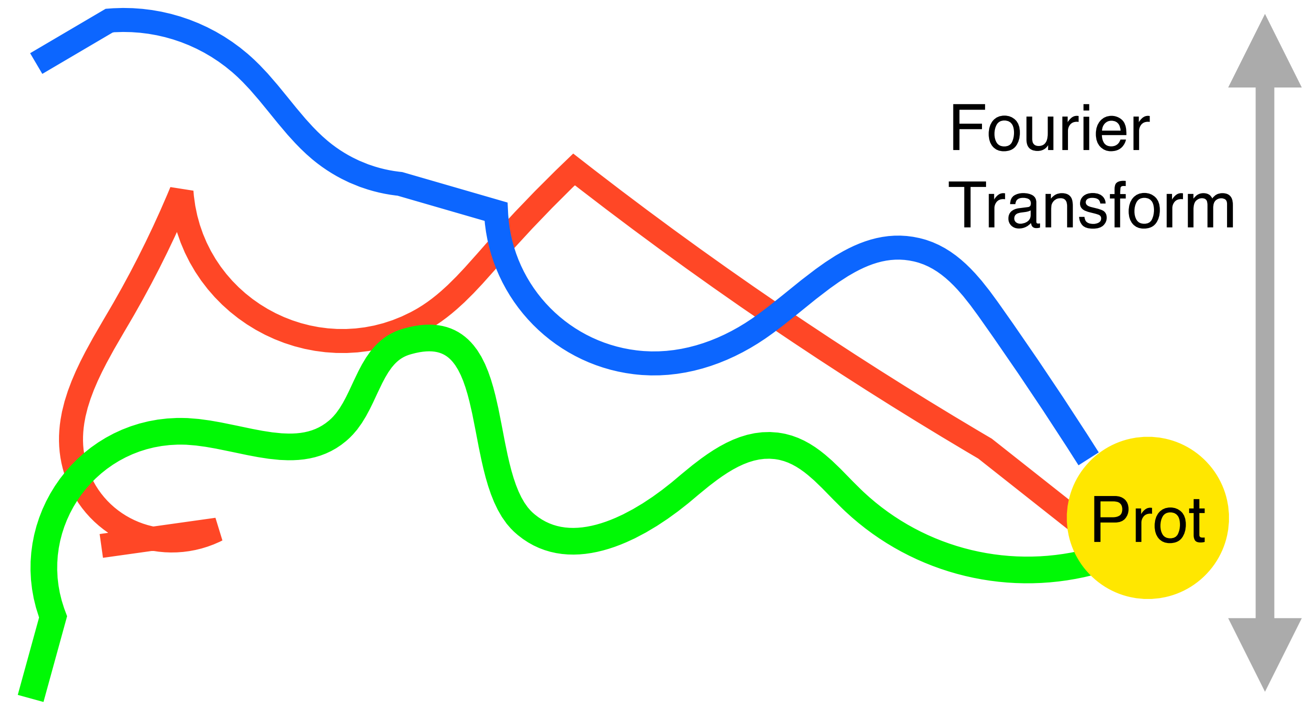
Landau levels present for charged particles contaminate the extraction of polarizabilities



Magnetic Moments
Expectations



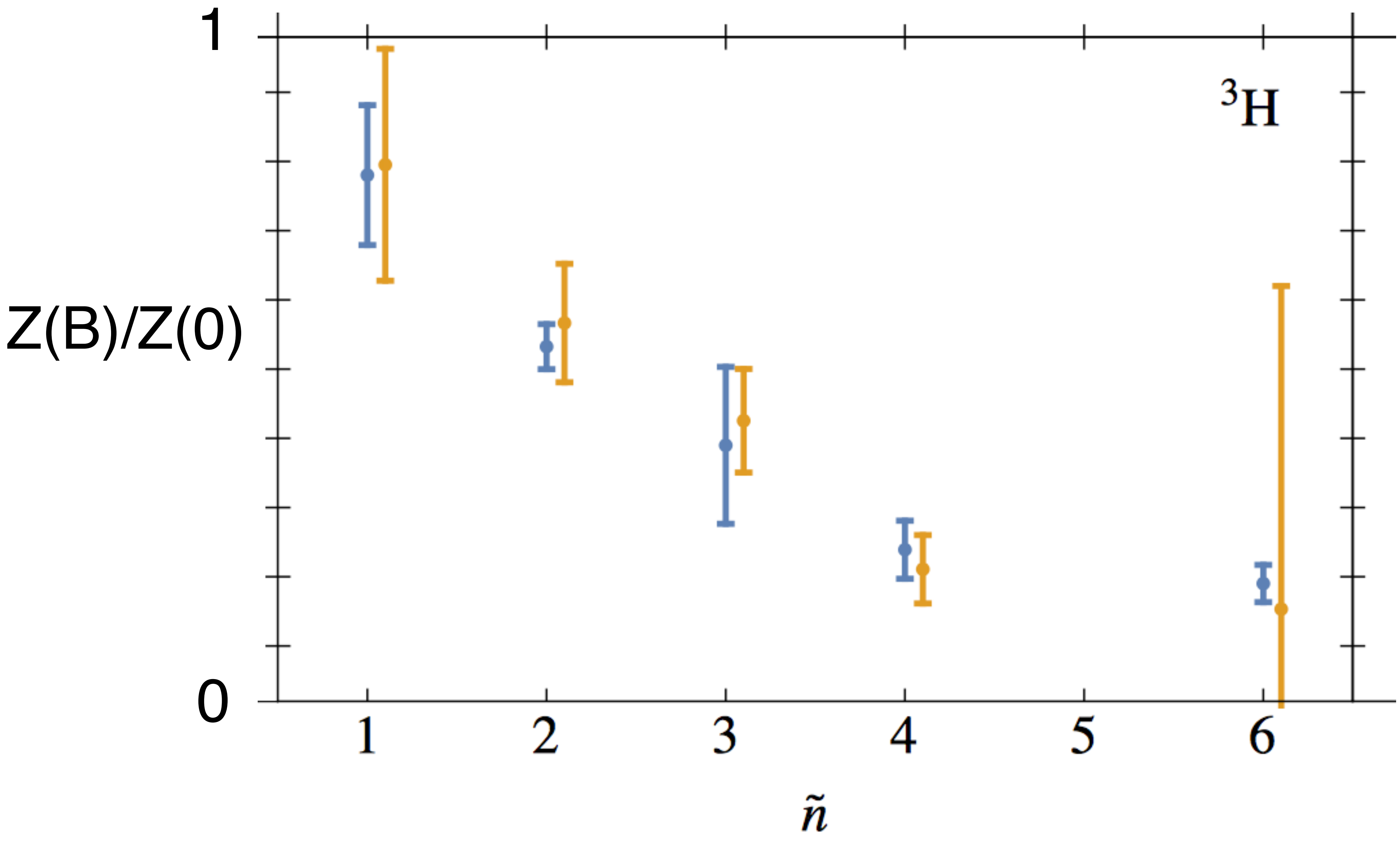


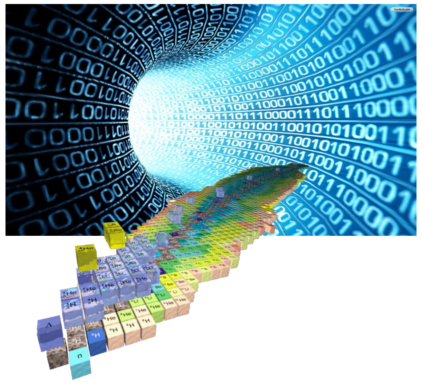


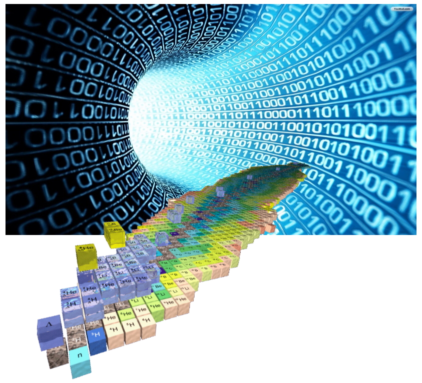
Constructing hadronic blocks to form nuclei is no longer optimal but remains doable at these masses

- neutron does not see Landau levels - momentum eigenstates
- proton sees level with Q=1 and M=Mproton
- triton sees level with Q=1 and M=3 x Mproton

Magnetic Moments
Expectations - Source Overlaps

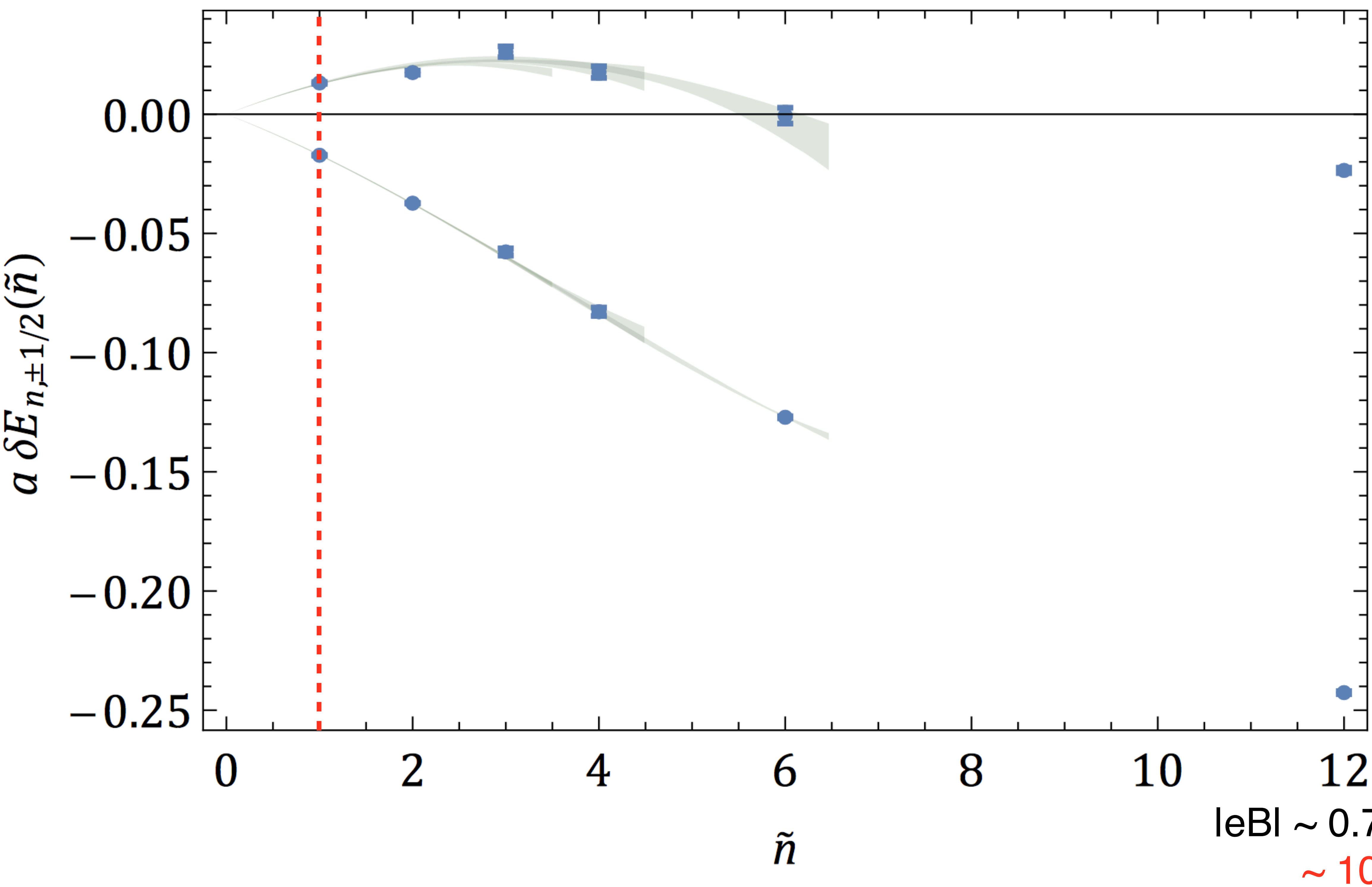






Magnetic Moments
Neutron Spin States

leBl ~ 0.05 GeV²

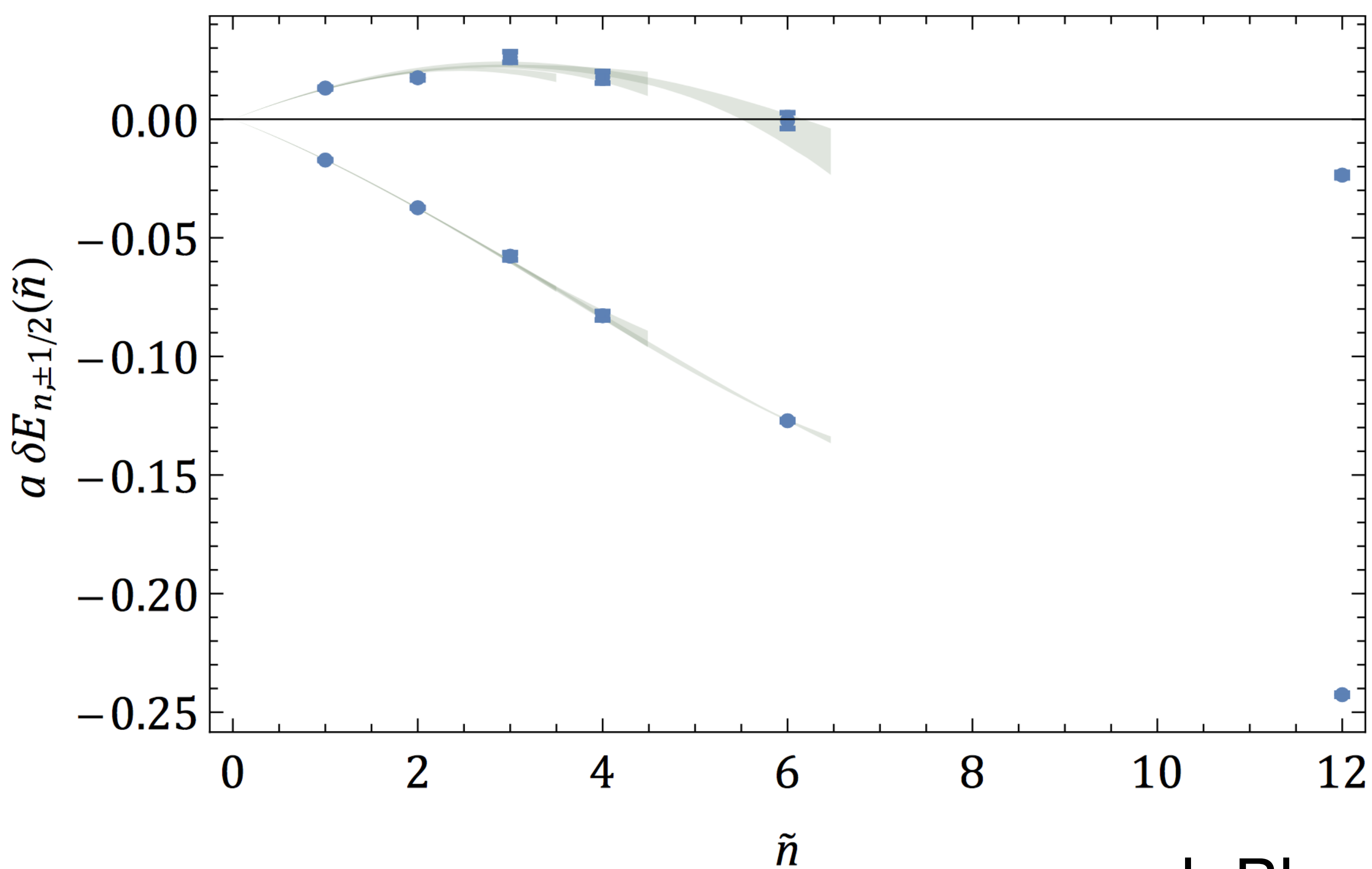


400 MeV

leBl ~ 0.7 GeV²

~ 10²⁰ Gauss

Magnetic Moments
Neutron Spin States

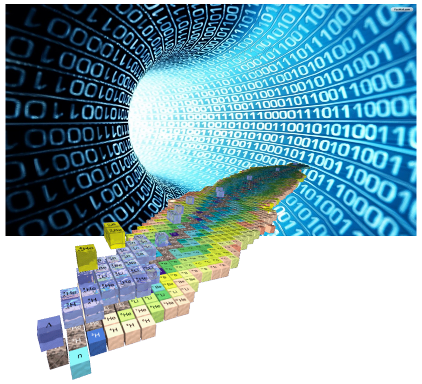


400 MeV

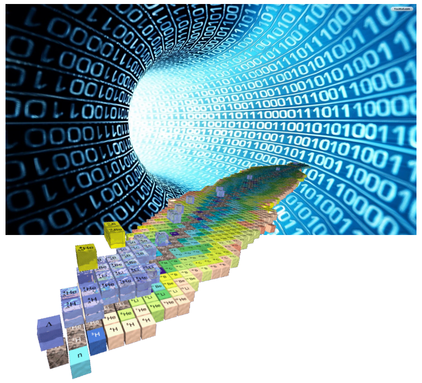
|eB| ~ 0.7 GeV^2

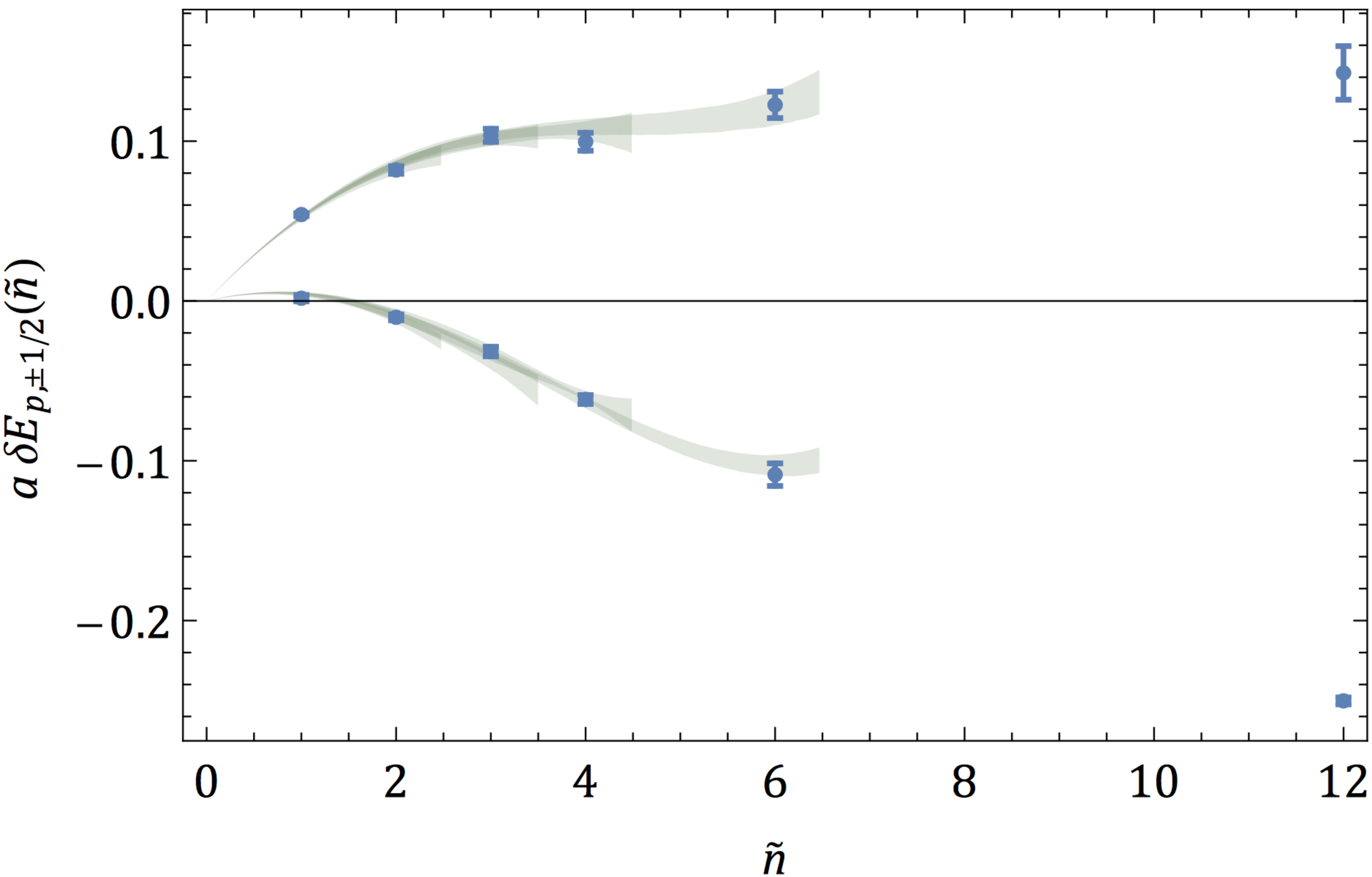
~ 10^{20} Gauss

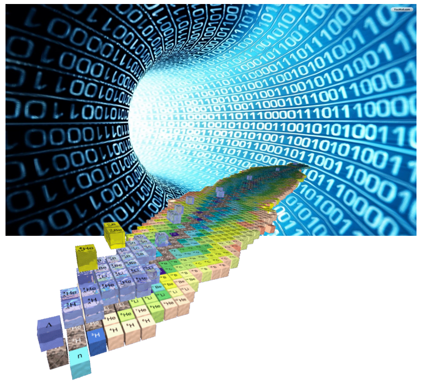
- Lower state depends essentially linearly on B
- Polarizability results from upper level (essentially)
- Spin-dependences highly correlated



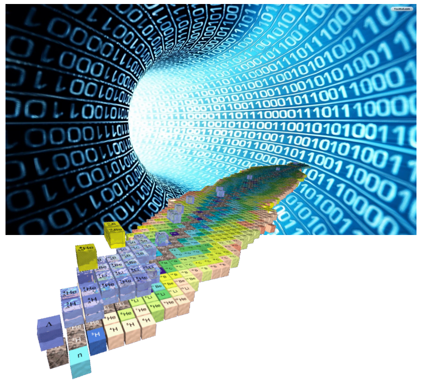
Magnetic Moments
Proton Spin States

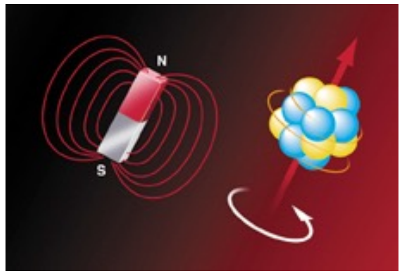




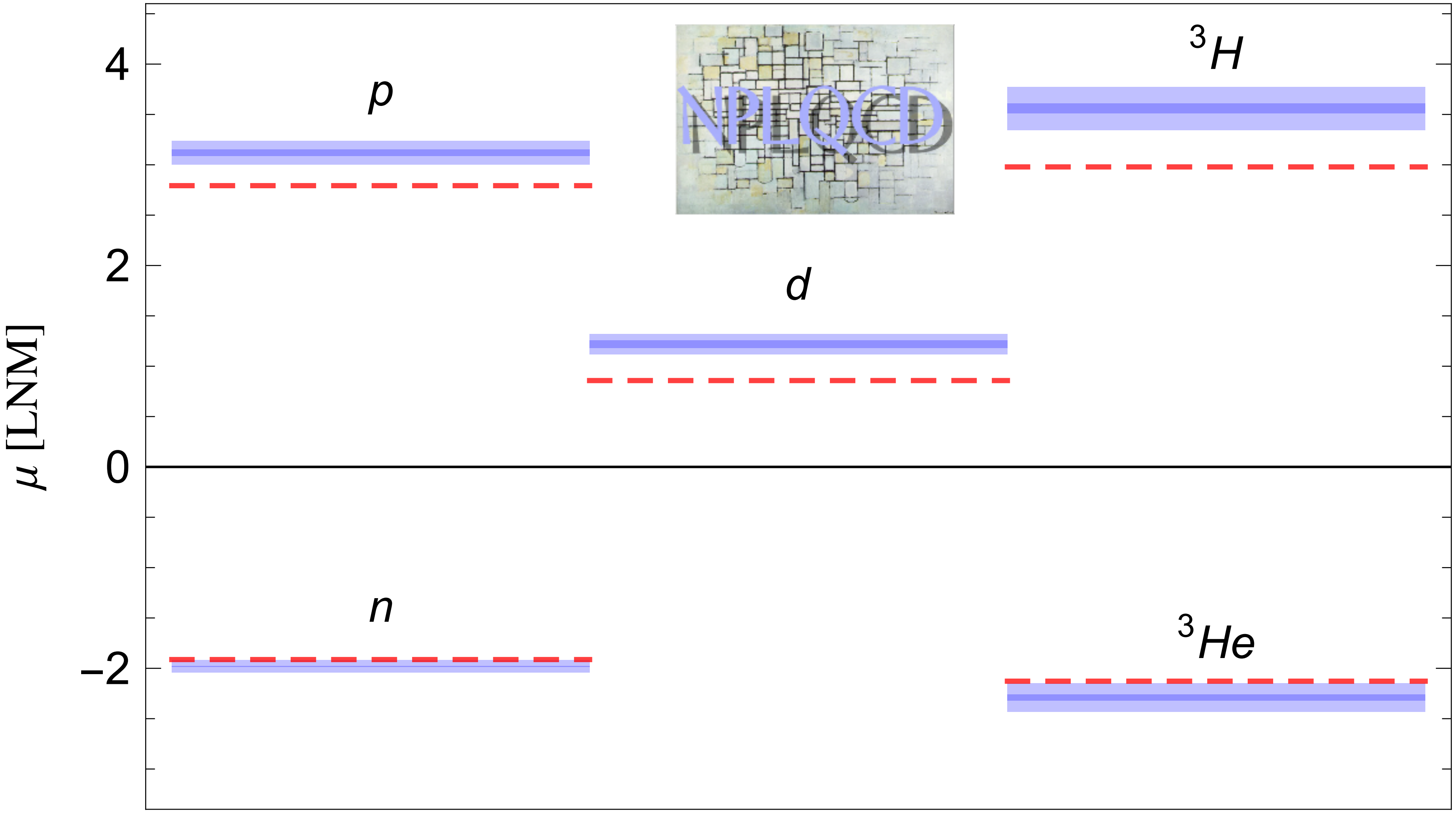


Magnetic Moments



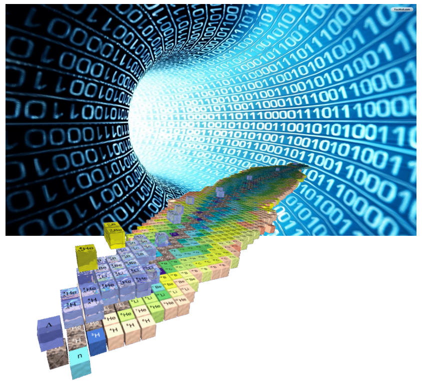


Magnetic moments of light nuclei from lattice quantum chromodynamics
S.R. Beane, E. Chang, S. Cohen, W. Detmold, H.W. Lin, K. Orginos, A. Parreno, M.J. Savage, B.C. Tiburzi
Published in **Phys.Rev.Lett. 113 (2014) 25, 252001**
e-Print: **arXiv:1409.3556 [hep-lat]**

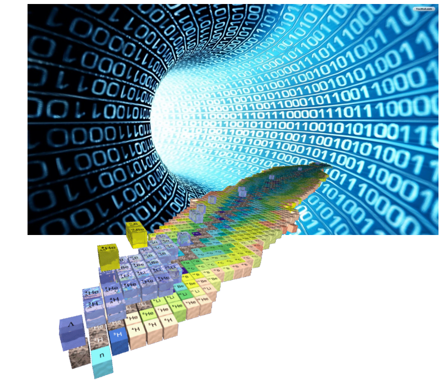


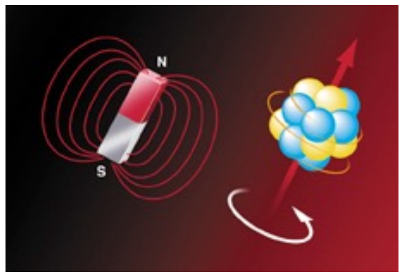
$$\frac{e}{2M(m_\pi)}$$

m_π ~ 800 MeV   Vs   Nature



Magnetic Moments

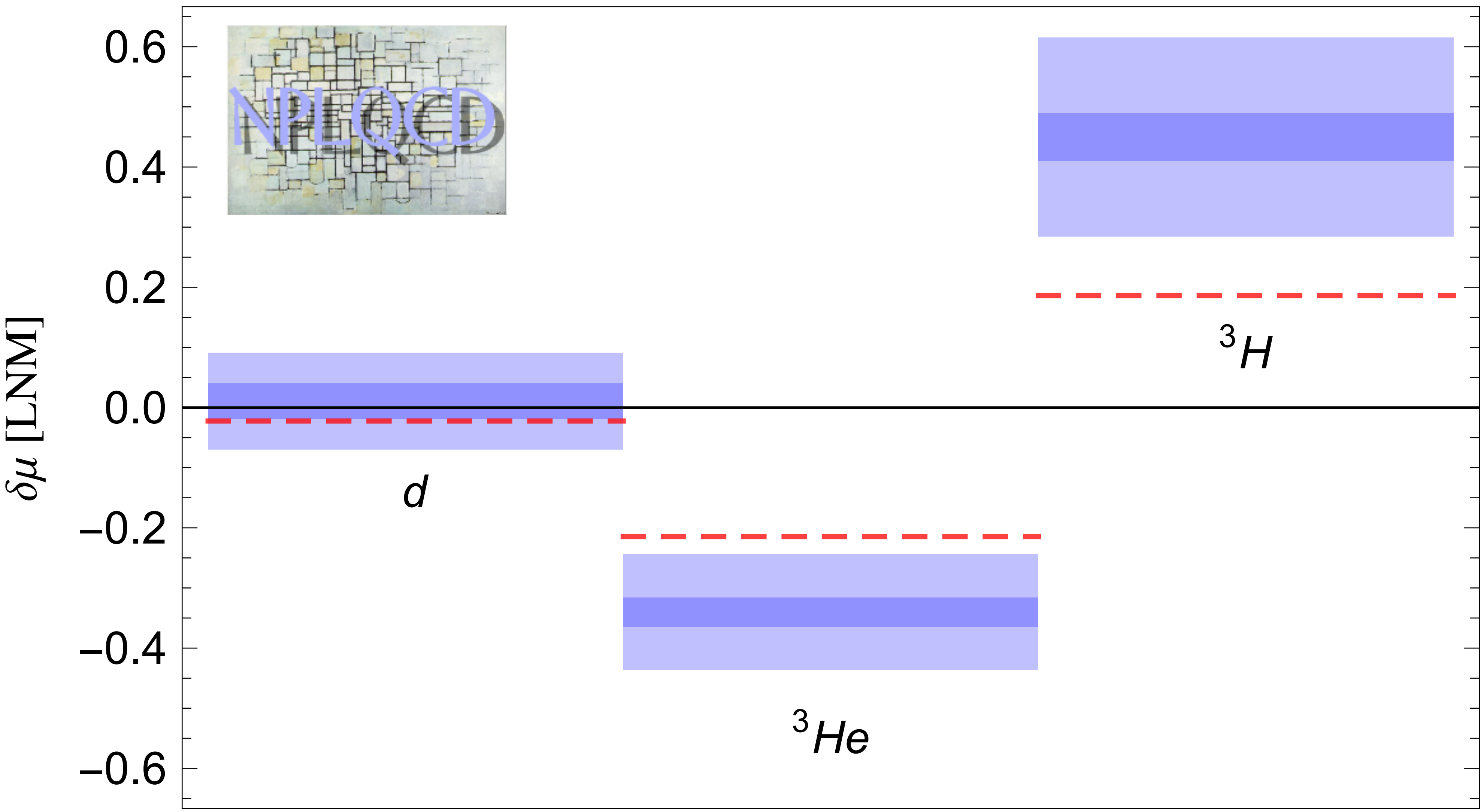




Magnetic moments of light nuclei from lattice quantum chromodynamics

S.R. Beane, E. Chang, S. Cohen, W. Detmold, H.W. Lin, K. Orginos, A. Parreno, M.J. Savage, B.C. Tiburzi

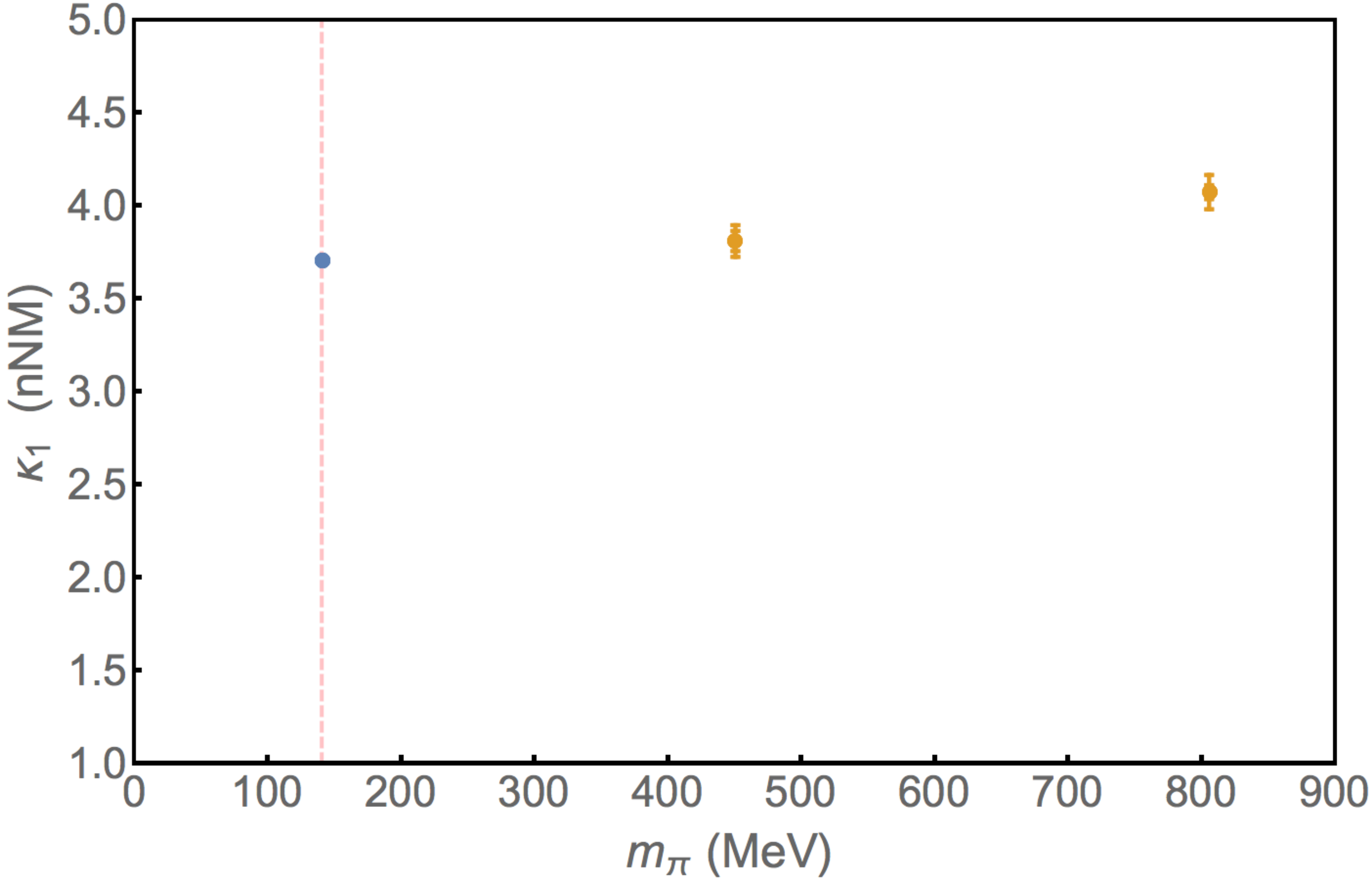
Published in **Phys.Rev.Lett. 113 (2014) 25, 252001**
e-Print: **arXiv:1409.3556 [hep-lat]**



$$\frac{e}{2M(m_\pi)}$$

m_π ~ 800 MeV Vs Nature

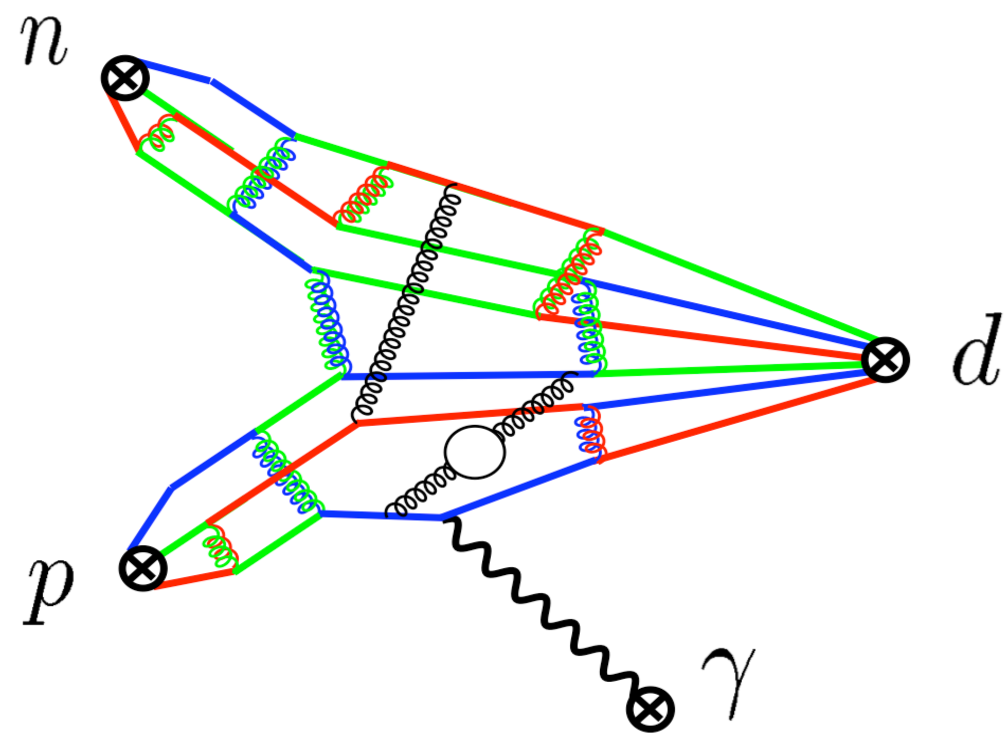
Magnetic Moments



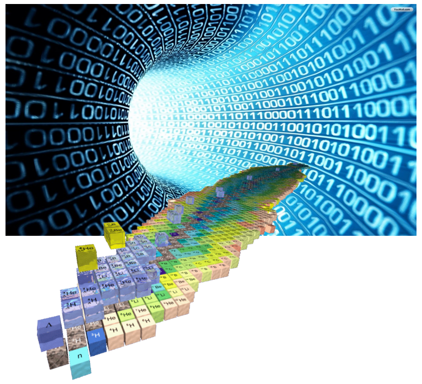
$$\frac{e}{2M(m_\pi)}$$

Essentially ALL quark mass dependence of nucleon magnetic moments is due to the nucleon mass

Radiative Capture : $np \to d\gamma$

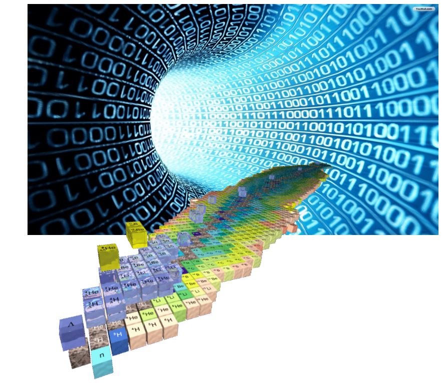


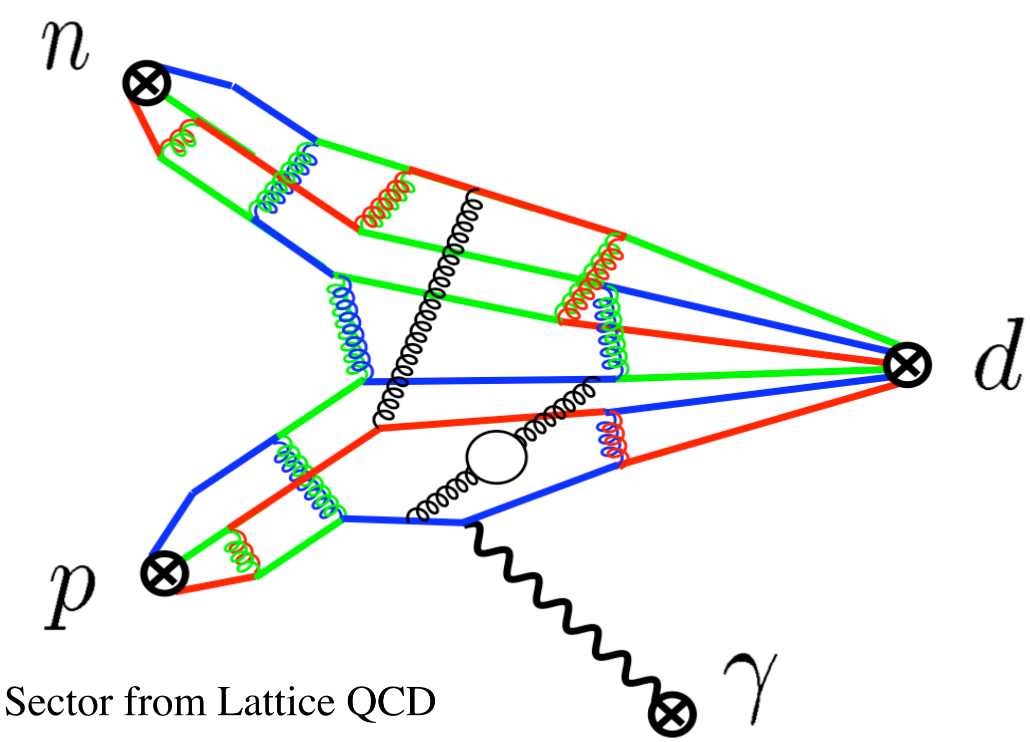
$$\mathcal{L} = \frac{e}{2M_N} N^\dagger \left[\kappa_0 + \kappa_1 \tau^3\right] \mathbf{\Sigma} \cdot \mathbf{B} N - \frac{e}{M_N}\left(\kappa_0 - \frac{\tilde{l}_2}{r_3}\right) i\epsilon_{ijk} t_i^\dagger t_j B_k + \frac{e}{M_N}\frac{l_1}{\sqrt{r_1 r_3}}\left[t_j^\dagger s_3 B_j + \text{h.c.}\right],$$

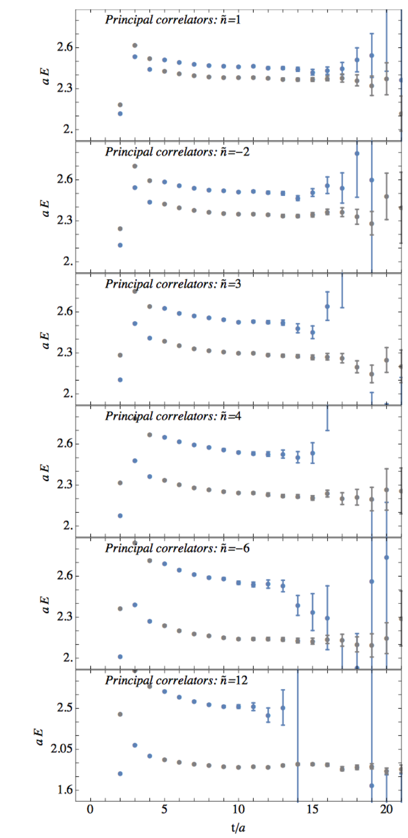


Radiative Capture :

$np \to d\gamma$





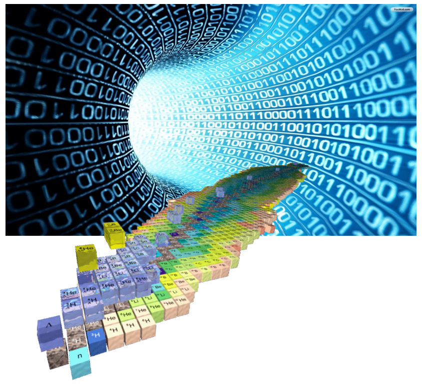


Electroweak Matrix Elements in the Two-Nucleon Sector from Lattice QCD
William Detmold and MJS,
Nucl. Phys. A 743, 170 (2004). hep-lat/0403005.

$$\left[p\cot\delta_1 - \frac{S_+ + S_-}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_+ + S_-}{2\pi L}\right] = \left[\frac{|e\mathbf{B}|l_1}{2} + \frac{S_+ - S_-}{2\pi L}\right]^2$$

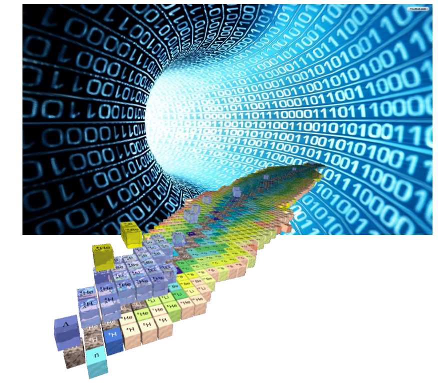
$$S_\pm \equiv S\left(\frac{L^2}{4\pi^2}(p^2 \pm |e\mathbf{B}|\kappa_1)\right)$$

$$\Delta E_{^3S_1,{}^1S_0} = \mp Z_d^2\,(\kappa_1 + \gamma_0 l_1)\,\frac{|e\mathbf{B}|}{M} + \ldots = \mp\left(\kappa_1 + \overline{L}_1\right)\frac{|e\mathbf{B}|}{M} + \ldots$$



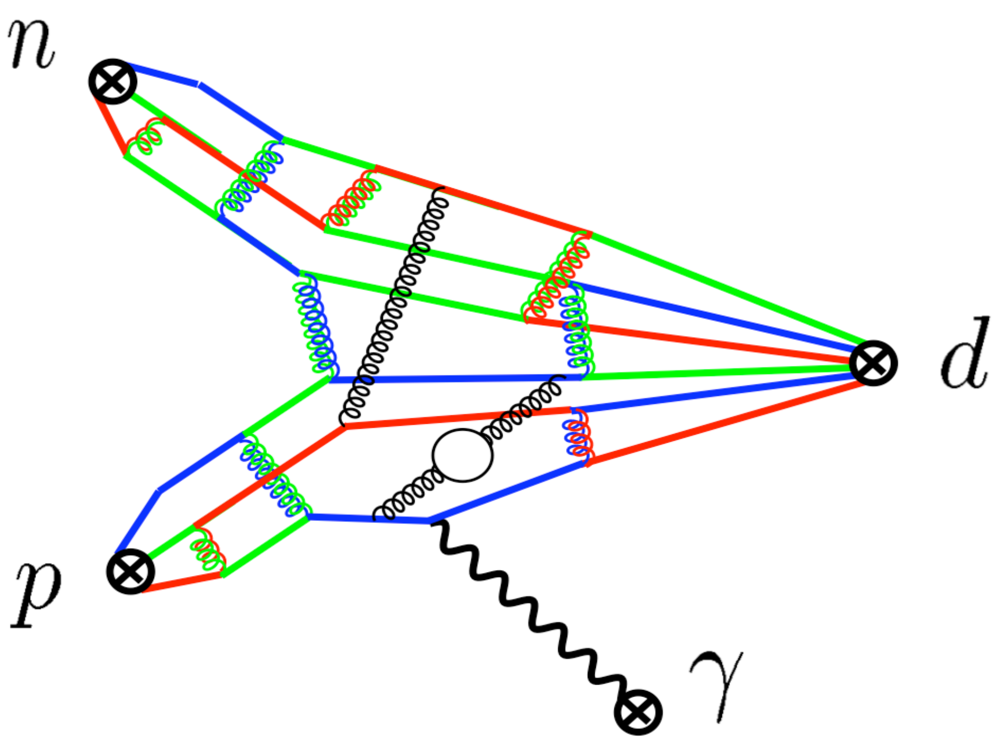
Radiative Capture :

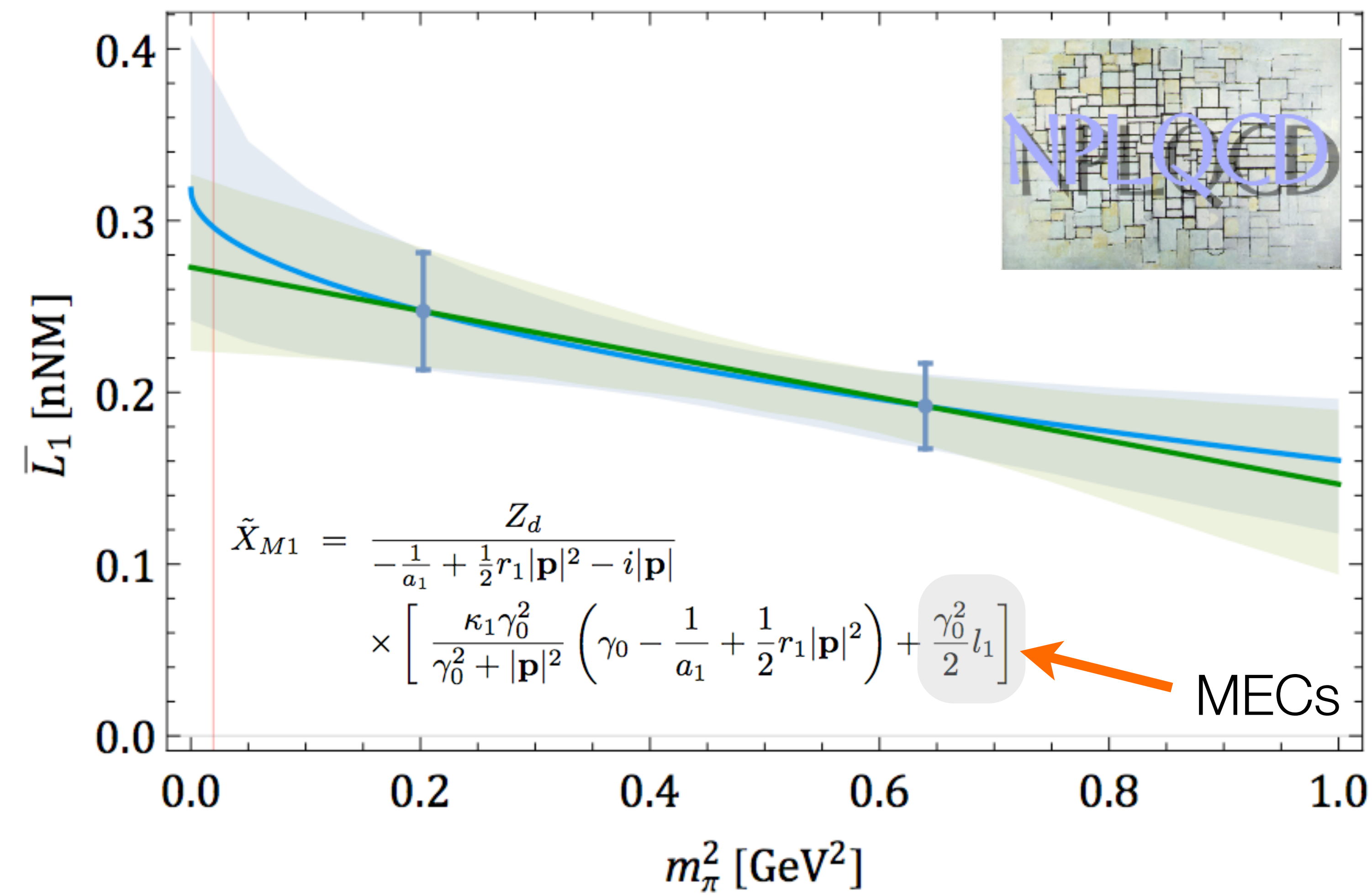
$$np \to d\gamma$$



Ab Initio Calculation of the np → dγ Radiative Capture Process

NPLQCD, arXiv:1505.02422

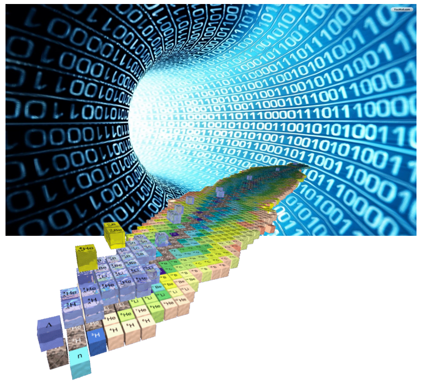




postdiction at the physical point (verification) :

$$\sigma^{\text{lqcd}} = 332.4(\,^{+5.4}_{-4.7}\,)\ \text{mb} \qquad v = 2,200\ \text{m/s}$$

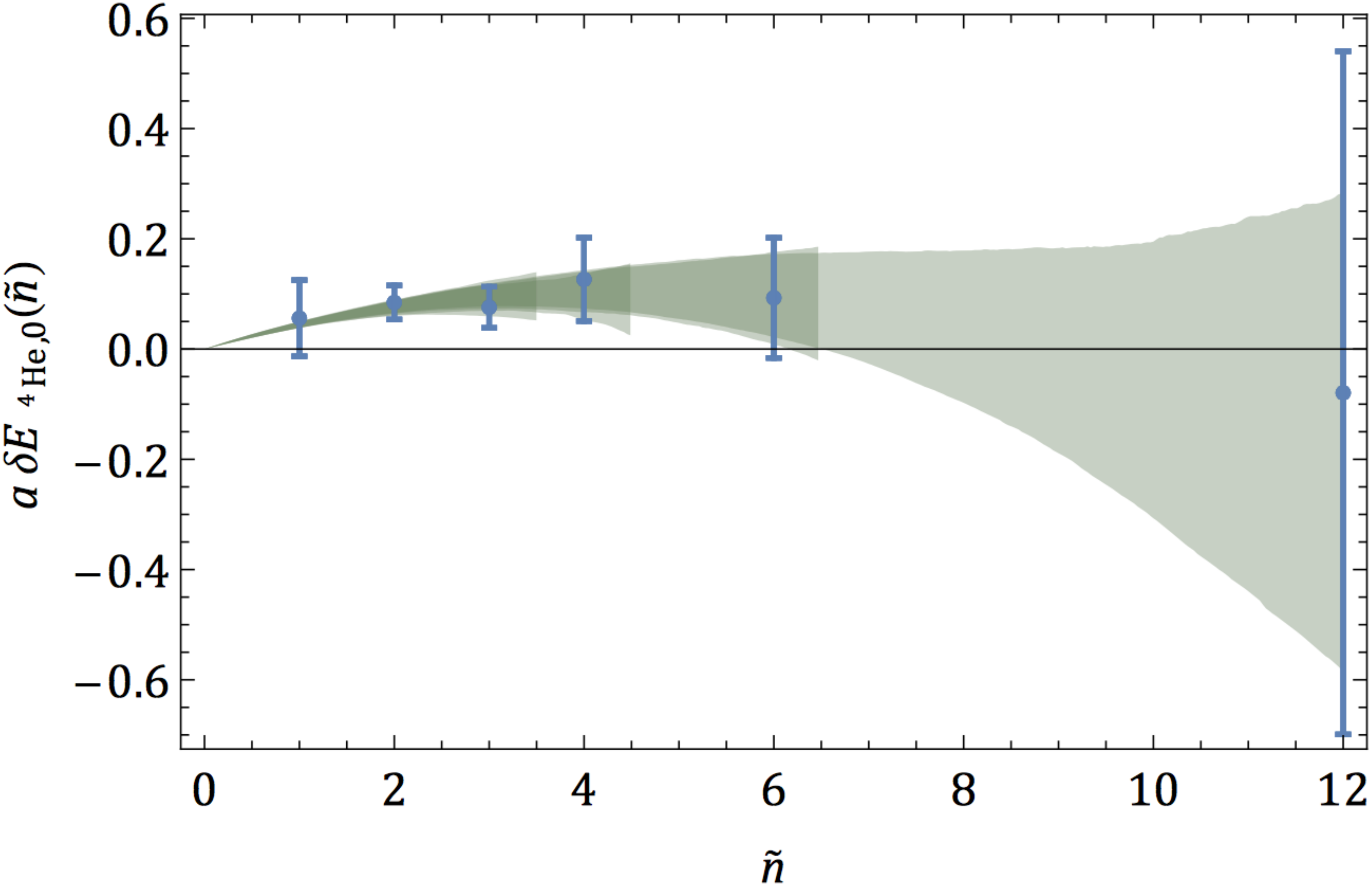
$$\sigma^{\text{expt}}(np \to d\gamma) = 334.2(0.5)\ \text{mb}$$

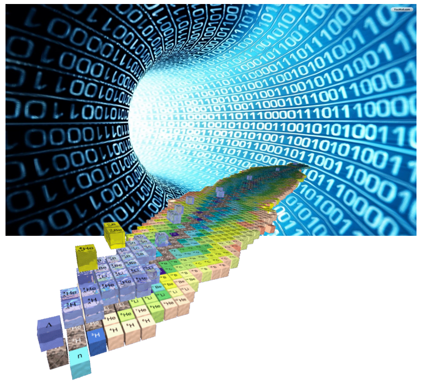


The Structure of Nuclei : Polarizabilities

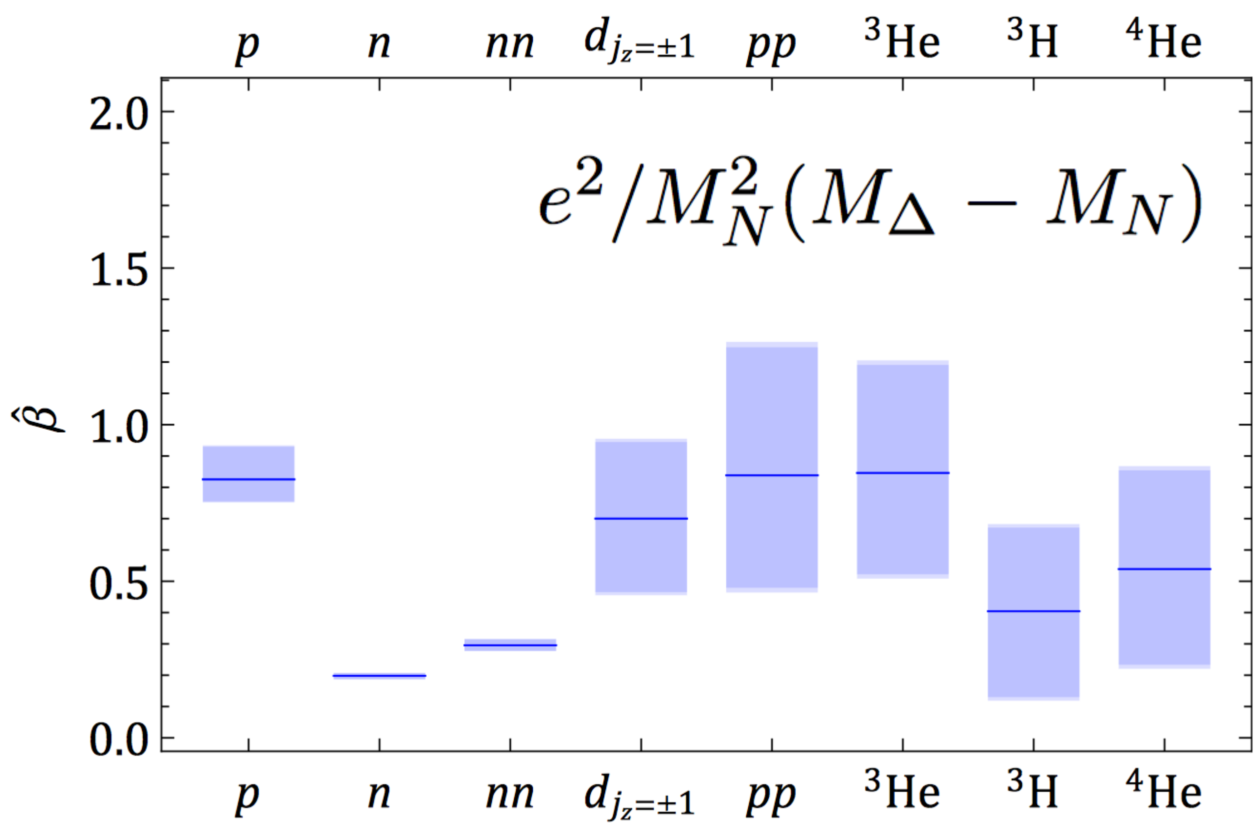
The Magnetic Structure of Light Nuclei

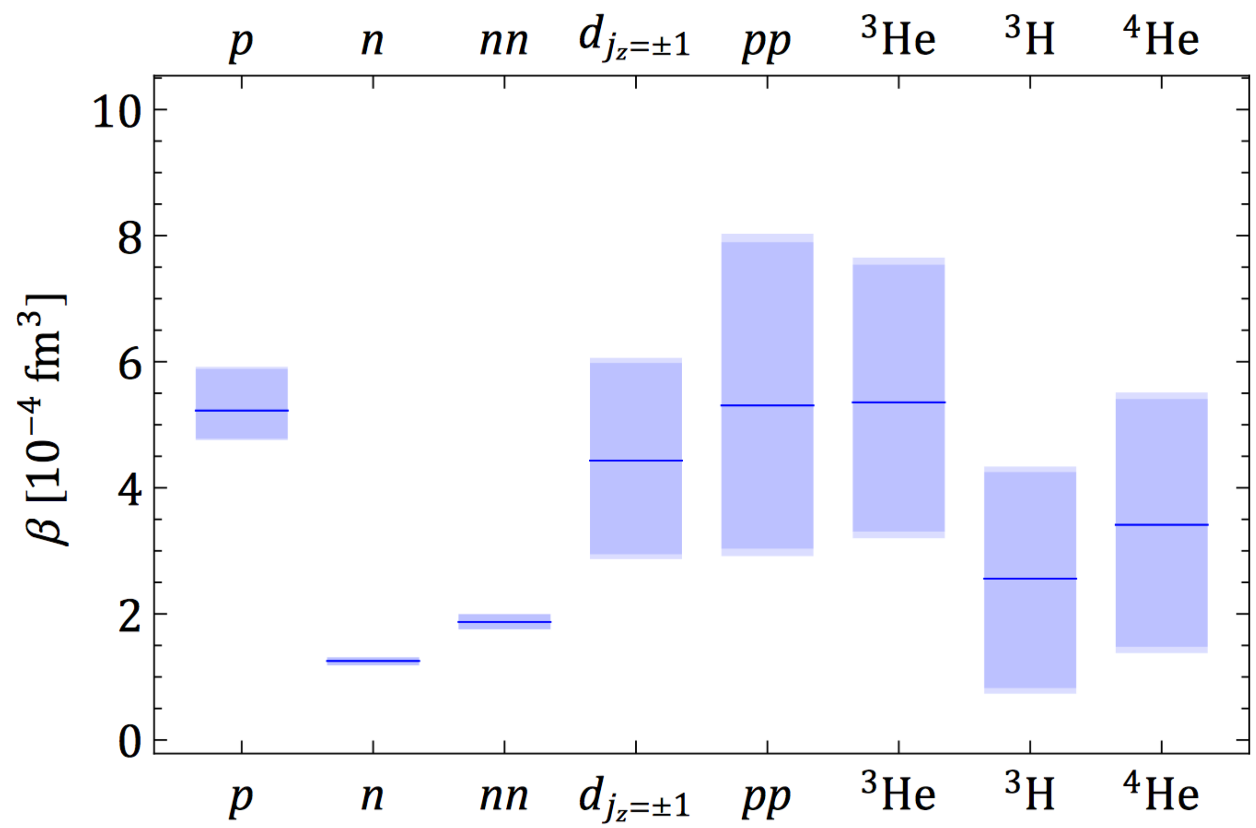
NPLQCD, arXiv:1506.05518





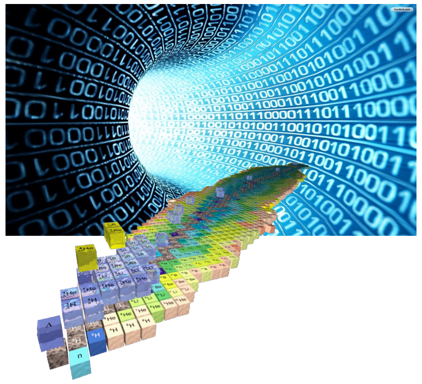
The Structure of Nuclei : Polarizabilities



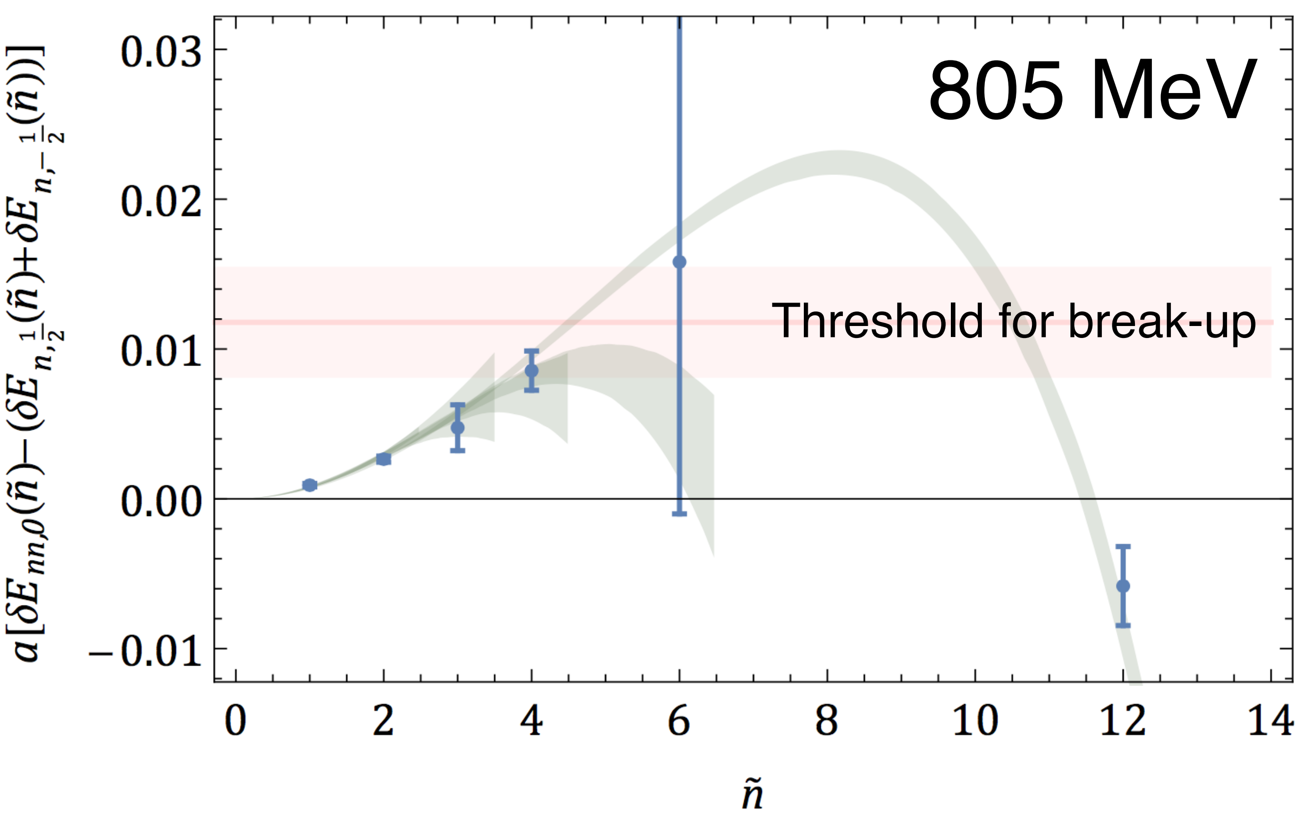


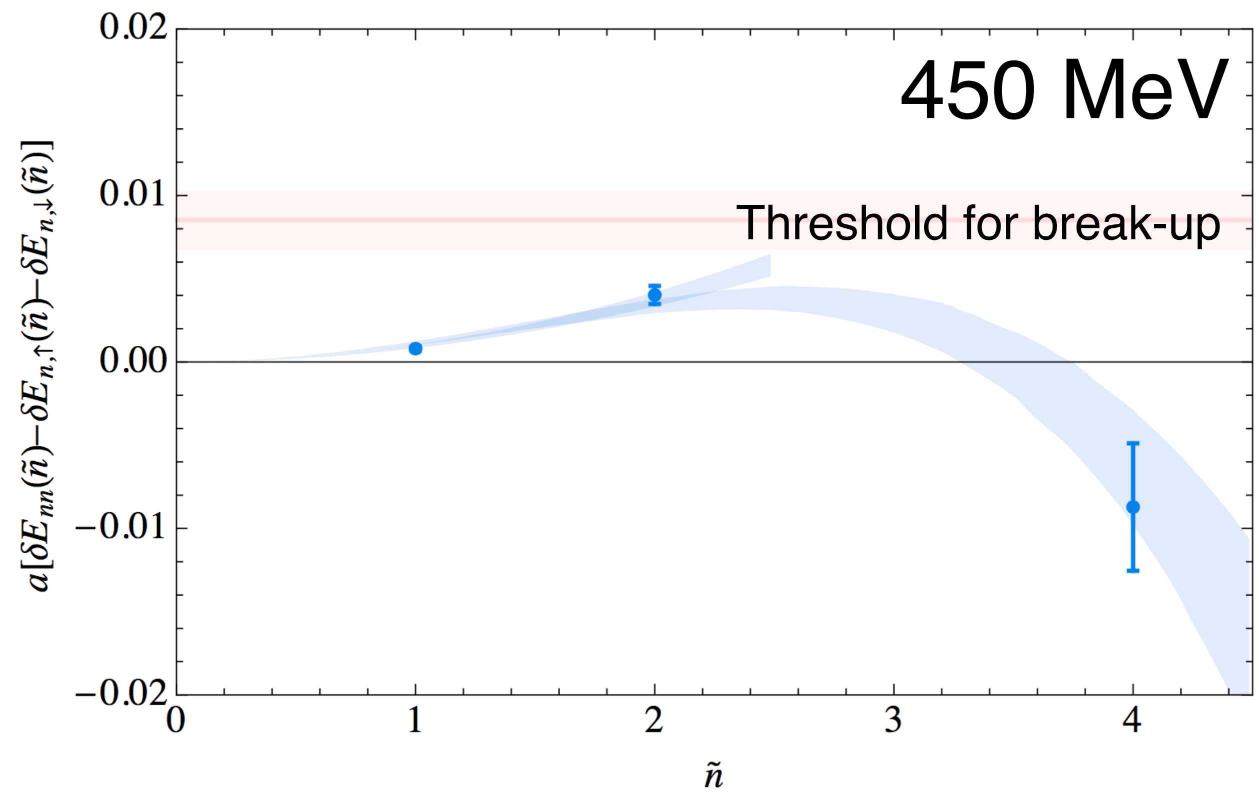
Large isovector nucleon polarizability

Nuclear polarizabilities are similar to proton polarizability



The Structure of Nuclei : Feshbach Resonances



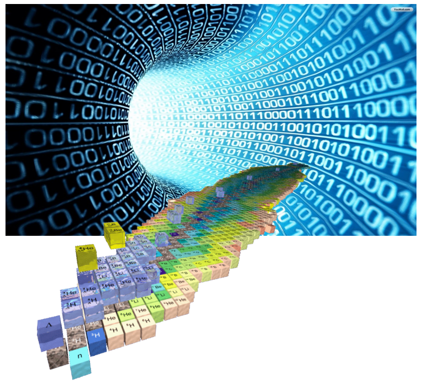


Increasing B tends to dissociate dineutron

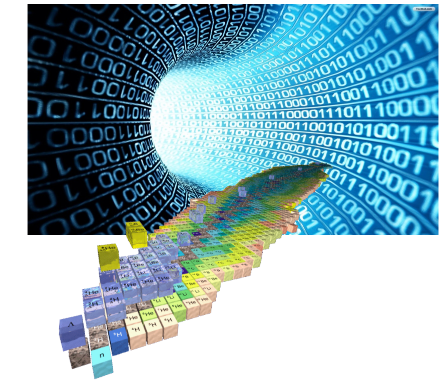
- if trend survives to physical point then neutron stars do not want to spontaneously generate B-fields

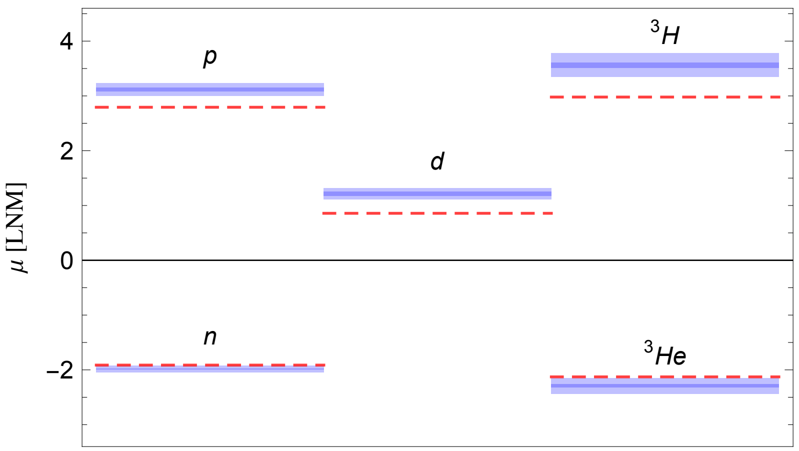
Possible Feshbach resonance at the physical point - system with infinite scattering length

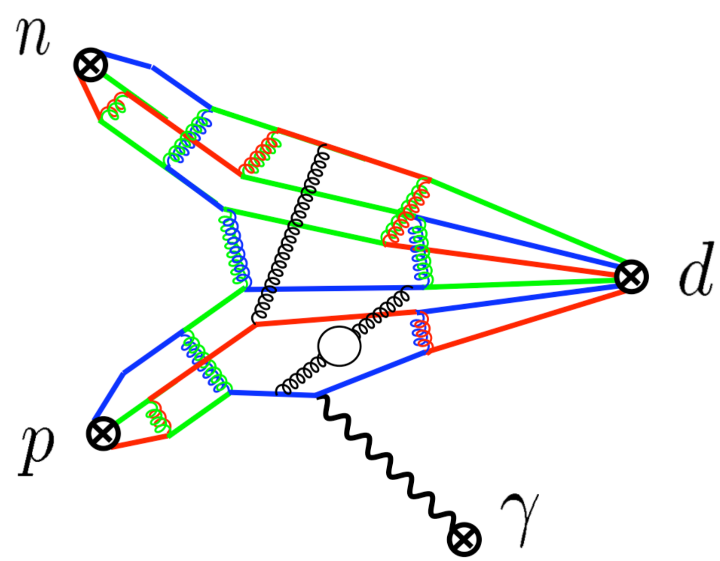
Deuteron similar

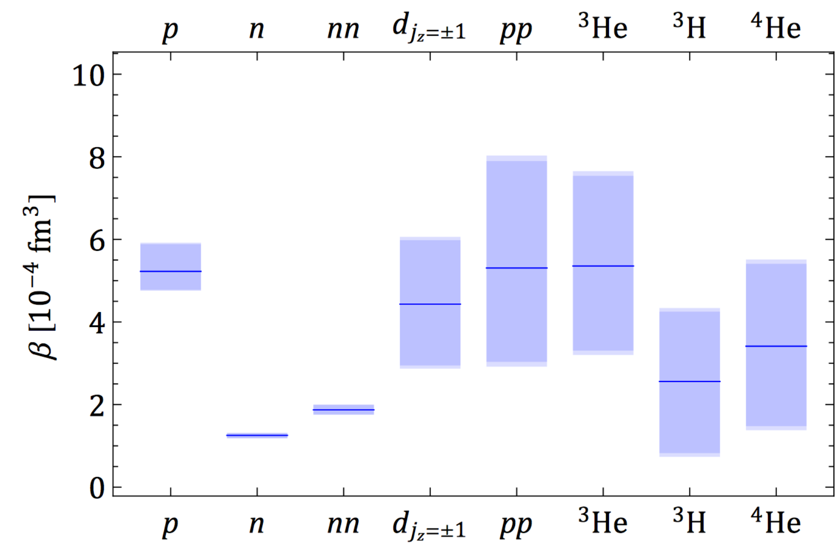


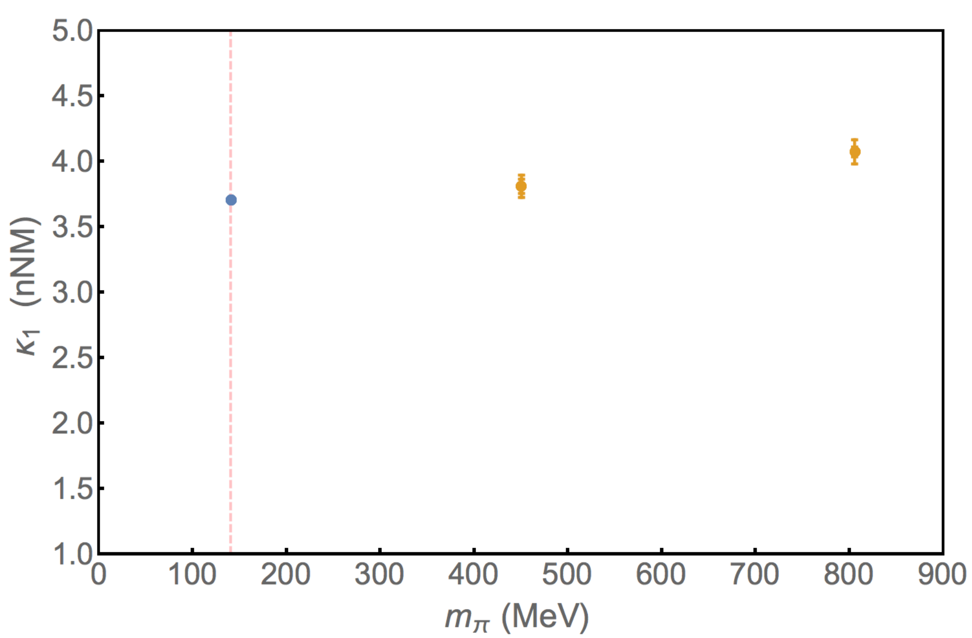
Closing Remarks











Lattice QCD is revealing interesting magnetic properties of nucleons and light nuclei

END